Example 1

A three-phase, wye-connected system is rated at 50MVA and 120kV. Express 40MVA of three-phase apparent power as a per unit value referred to:

a) The three-phase system as MVA as base, and

b) The per-phase system MVA as base.

Solution

a) For the three-phase system MVA as base,

$$S_{B3\phi} = 50MVA \qquad V_{B(line)} = 120kV$$

$$S_{pu\ 3\phi} = \frac{S_{actual}}{S_{B3\phi}} = \frac{40}{50} = 0.8pu$$

b) For the per phase base

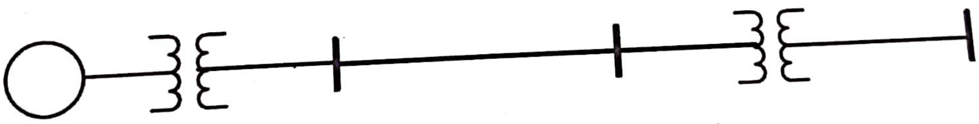
$$S_{B1\phi} = \frac{S_{B3\phi}}{3} = \frac{50M}{3} = 16.67MVA$$

$$V_{B(phase)} = \frac{V_{B(line)}}{\sqrt{3}} = \frac{120k}{\sqrt{3}} = 69.28kV$$

$$S_{pu\ 1\phi} = \frac{1}{3} \times \frac{40}{16.67} = 0.8pu$$

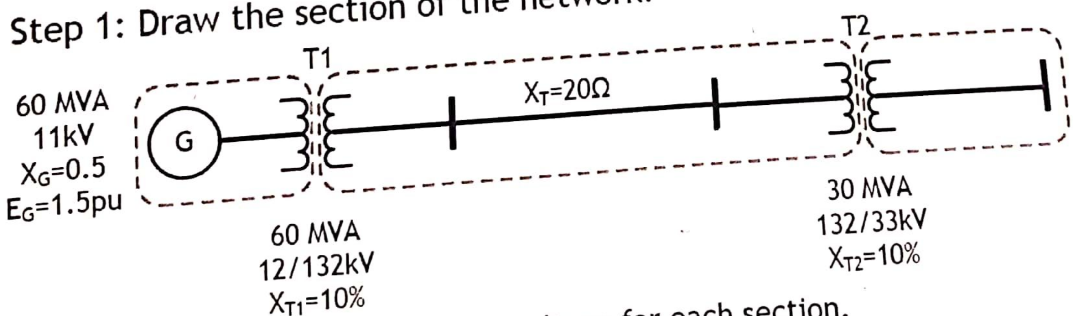
Example 2

Find the per unit value for X_{T1}, X_{T2} and X_T if the base values are 11kV and 60MVA.



Solution

Step 1: Draw the section of the network.



Step 2: Find the base value of the voltage for each section.

$$S_B = 60MVA$$

$$V_{B1} = 11kV$$

$$V_{B2} = \frac{V_2}{V_1} \times V_{B1} = \frac{132}{12} \times 11kV = 121kV$$

$$V_{B3} = \frac{V_3}{V_2} \times V_{B2} = \frac{33}{132} \times 121kV = 30.25kV$$

Step 3: Find the per unit values of each component

<u>Transformer T1 and T2</u>

Since both transformer voltage base are the same as their rated values, their p.u reactance on a 60MVA are:

$$Z_{pu}^{new} = Z_{pu}^{old} \times \left(\frac{V_B^{old}}{V_B^{new}}\right)^2 \times \left(\frac{S_B^{new}}{S_B^{old}}\right)$$

$$X_{T1}^{new} = 0.1 \times \left(\frac{12}{11}\right)^2 \times \left(\frac{60}{60}\right) = 0.12\,pu$$ :: Looking at the LV side of T1

$$X_{T2}^{new} = 0.1 \times \left(\frac{132}{121}\right)^2 \times \left(\frac{60}{30}\right) = 0.24\,pu$$ :: Looking at the HV side of T2

OR

$$X_{T1}^{new} = 0.1 \times \left(\frac{33}{30.25}\right)^2 \times \left(\frac{60}{30}\right) = 0.24\,pu$$ :: Looking at the LV side of T2

The per unit value for the transformer impedance is the same whether it is seen in the HV or LV side of the transformer- one of the advantages of per unit system..=)

<u>Line Impedance</u>

Since the impedance given in actual value, we have to find the base value for the impedance.

$$X_{T(base)} = \frac{(V_{B2})^2}{S_B{}^*} = \frac{(121k)^2}{60M} = 244\Omega$$

<u>Line Impedance</u>

Since the impedance given in actual value, we have to find the base value for the impedance.

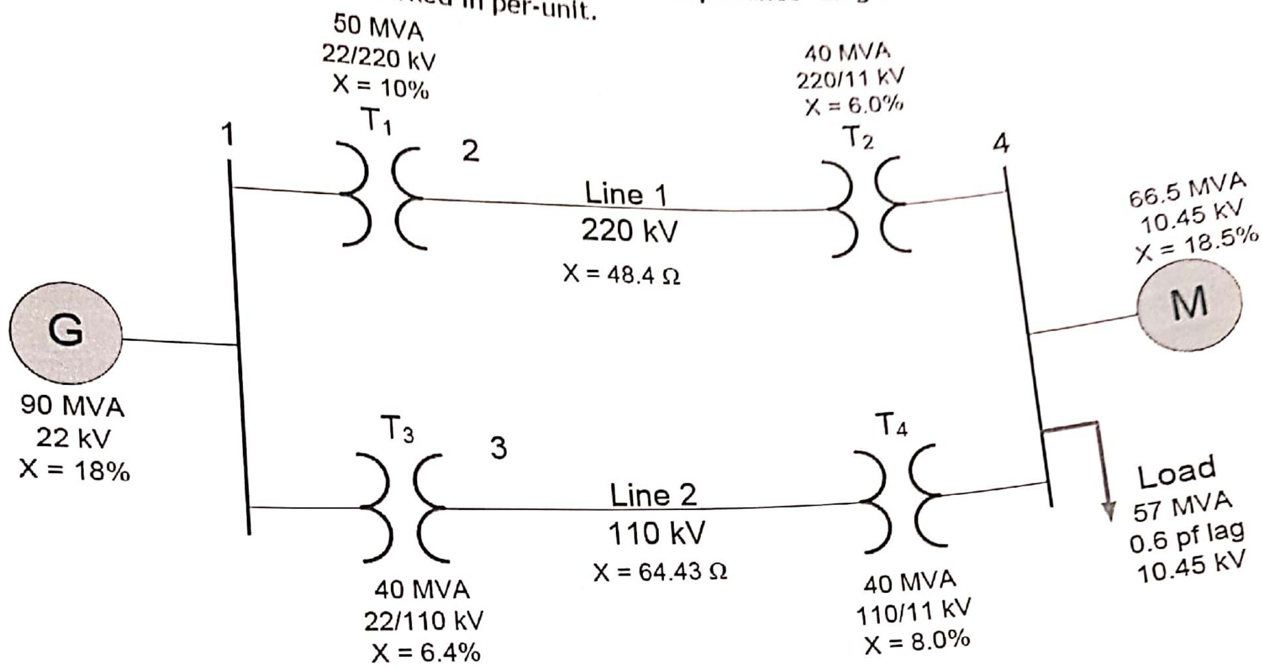
$$X_{T(base)} = \frac{(V_{B2})^2}{S_B{}^*} = \frac{(121k)^2}{60M} = 244\Omega$$

$$X_{T(pu)} = \frac{X_{T(actual)}}{X_{T(base)}} = \frac{20}{244} = 0.082\,pu$$

Note that the line impedance has only the resistive value.
Therefore the complex power conjugate value is the same since θ is equal to 0.

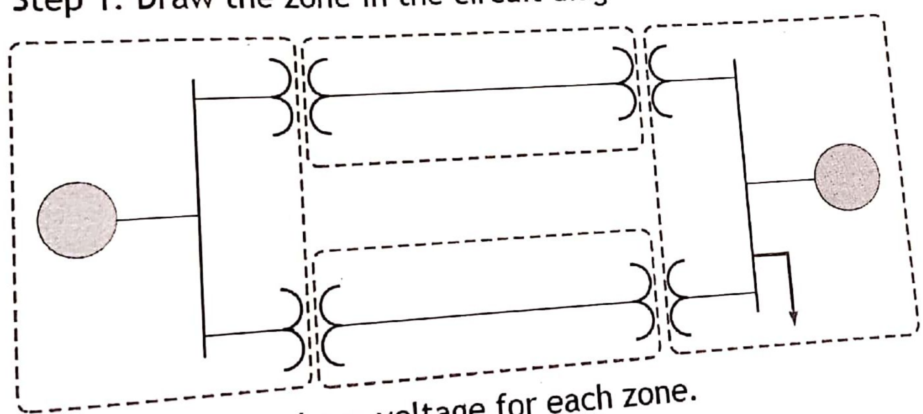
Example 3

The one-line diagram of three-phase power system is shown below. Select a common base of 100 MVA and 22 kV on the generator side. Draw an impedance diagram with all impedance including the load impedance marked in per-unit.



Solution

Step 1: Draw the zone in the circuit diagram



Step 2: Find the base voltage for each zone.

$$V_{b1} = 22kV$$

$$V_{b2} = \frac{V_2}{V_1} \times V_{b1} = \frac{220}{22} \times 22k = 220kV$$

$$V_{b3} = \frac{V_3}{V_1} \times V_{b1} = \frac{110}{22} \times 22k = 110kV$$

$$V_{b4} = \frac{V_4}{V_2} \times V_{b2} = \frac{11}{220} \times 220k = 11kV$$

Step 3: Find the per unit value

Generator and Transformer

Since generator & transformer voltage base are the same as their rated values, their p
reactance on a 100 MVA

$$Z_{pu}^{new} = Z_{pu}^{old}\left(\frac{V_B^{old}}{V_B^{new}}\right)^2\left(\frac{S_B^{new}}{S_B^{old}}\right)$$

$$Z_{pu}^{new} = Z_{pu}^{old}\left(\frac{S_B^{new}}{S_B^{old}}\right)$$

$$Z_G^{new} = 0.18\left(\frac{100}{90}\right) = 0.2pu$$

$$Z_{T1}^{new} = 0.10\left(\frac{100}{50}\right) = 0.2pu \qquad Z_{T3}^{new} = 0.064\left(\frac{100}{40}\right) = 0.16pu$$

$$Z_{T2}^{new} = 0.06\left(\frac{100}{40}\right) = 0.15pu \qquad Z_{T4}^{new} = 0.08\left(\frac{100}{40}\right) = 0.2pu$$

Motor

$$S_B^{new} = 100MVA \qquad S_B^{old} = 66.5MVA$$

$$V_B^{new} = 11kV \qquad V_B^{old} = 10.45kV$$

$$Z_{motor}^{old} = 0.18$$

$$Z_{motor}^{new} = Z_{motor}^{old} \times \left(\frac{V_B^{old}}{V_B^{new}}\right)^2 \times \frac{S_B^{new}}{S_B^{old}}$$

$$= 0.18 \times \left(\frac{10.45}{11}\right)^2 \times \left(\frac{100}{66.5}\right) = 0.25pu$$

Line Impedance

Line 1

$$V_{B2} = 220kV$$

$$S_B = 100MVA$$

$$Z_{Bline1} = \frac{(V_{B2})^2}{S_B} = \frac{(220k)^2}{100M} = 484\Omega$$

$$Z_{line1(pu)} = \frac{Z_{actual}}{Z_{Bline1}} = \frac{48.4}{484} = 0.1pu$$

Line 2

$$V_{B3} = 110kV$$

$$S_B = 100MVA$$

$$Z_{Bline2} = \frac{(V_{B3})^2}{S_B} = \frac{(110k)^2}{100M} = 121\Omega$$

$$Z_{line2(pu)} = \frac{Z_{actual}}{Z_{Bline2}} = \frac{65.43}{121} = 0.54pu$$

Load

$$S_{3\phi} = 57MVA \quad V_L = 10.45kV \quad pf = 0.6 lagging$$

$$\theta = \cos^{-1} 0.6 = 53.13°$$

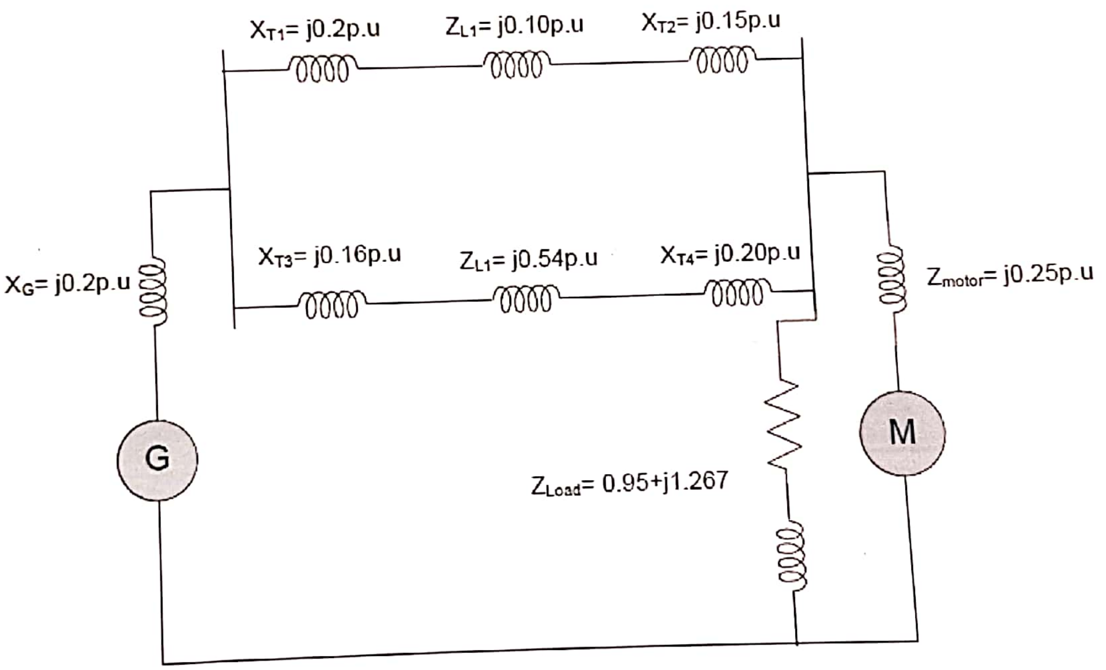
$$S_{3\phi load} = 57\angle 53.13° MVA$$

$$Z_{load(act)} = \frac{V_L^2}{S_{3\phi load}^*} = \frac{(10.45k)^2}{57M\angle 53.13°} = 1.1495 + j1.5327\Omega$$

$$Z_{load(base)} = \frac{(V_{B4})^2}{S_B} = \frac{(11k)^2}{100M} = 1.21\Omega$$

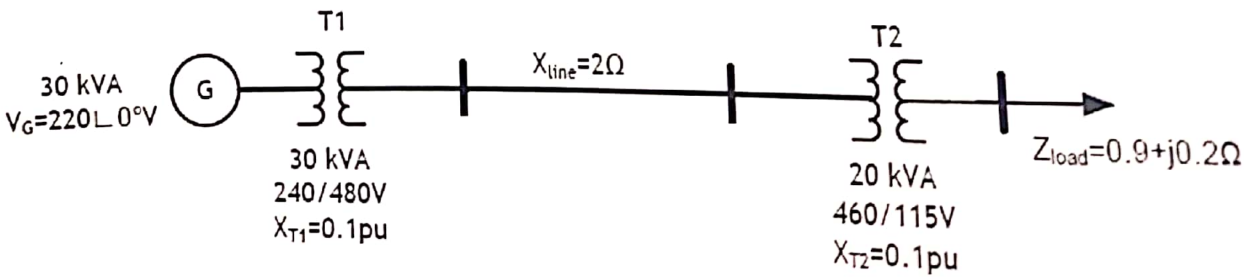
$$Z_{load(pu)} = \frac{Z_{load(act)}}{Z_{load(base)}} = \frac{1.1495 + j1.5327\Omega}{1.21\Omega} = 0.95 + j1.267\,pu$$

Step 4: Draw the per unit impedance diagram.



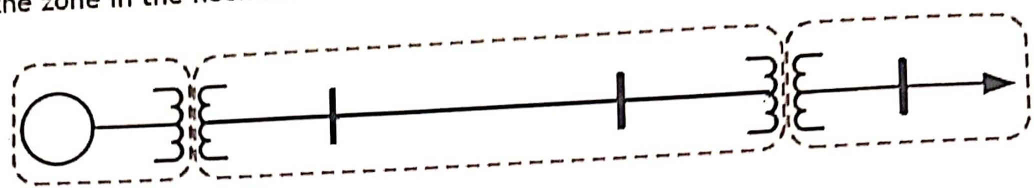
Example 4

Using base values of 30kVA and 240V in the generator side, draw the per unit circuit, and determine the per unit impedances and the per unit source voltage. Then calculate the load current both in per unit and in amperes. Transformer winding resistance and shunt admittance branches are neglected.

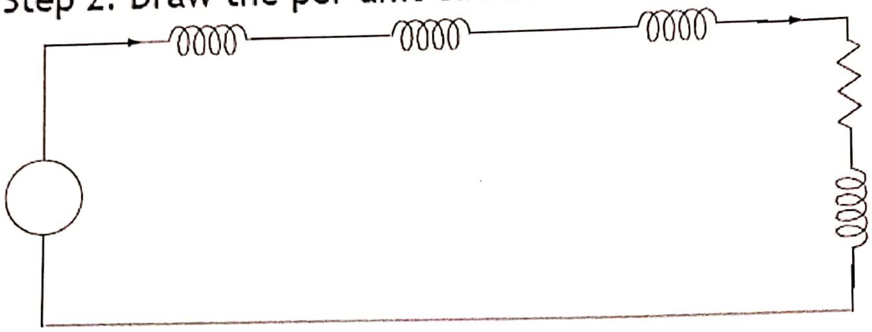


Solution

Step 1: Draw the zone in the network.



Step 2: Draw the per unit circuit



Step 3: Find the base values

S_B for the entire network is 30kVA. Find the base voltages and impedances of each zone.

$$Z_B = \frac{V_B^{\ 2}}{S_B}$$

$$V_{B1} = 240V \qquad Z_{B1} = \frac{240^2}{30k} = 1.92\Omega$$

$$V_{B2} = \frac{V_2}{V_1} \times V_{B1} = \frac{480}{240} \times 240 = 480V \qquad Z_{B2} = \frac{480^2}{30k} = 7.68\Omega$$

$$V_{B3} = \frac{115}{460} \times 480 = 120V \qquad Z_{B3} = \frac{120^2}{30k} = 0.48\Omega$$

Find base current in zone 3 [!:to calculate the load current later]

$$I_{B3} = \frac{S_B}{V_{B3}} = \frac{30k}{120} = 250A$$

Step 4: Find the per unit values

$$V_{s(pu)} = \frac{V_s}{V_{B1}} = \frac{220\angle 0^\circ}{240} = 0.9167\angle 0^\circ pu$$

$$X_{T1}^{new} = X_{T1}^{old} = 0.1pu$$

$$X_{line(pu)} = \frac{X_{line}}{Z_{B2}} = \frac{2}{7.68} = 0.2604pu$$

$$X_{T2}^{new} = X_{T2}^{old} \times \left(\frac{V_B^{old}}{V_B^{new}}\right)^2 \times \left(\frac{S_B^{new}}{S_B^{old}}\right)$$

$$= 0.1 \times \left(\frac{460}{480}\right)^2 \times \left(\frac{30}{20}\right) = 0.1378pu \quad :: \text{calculation using base value} \quad or$$

$$X_{T2}^{new} = X_{T2}^{old} \times \left(\frac{V_B^{old}}{V_B^{new}}\right)^2 \times \left(\frac{S_B^{new}}{S_B^{old}}\right)$$

$$= 0.1 \times \left(\frac{115}{120}\right)^2 \times \left(\frac{30}{20}\right) = 0.1378\,pu$$ :: calculation using voltage ratio

$$Z_{load(pu)} = \frac{Zload}{ZB3} = \frac{0.9 + j0.2}{0.48} = 1.875 + j0.4167\,pu$$

Step 5: Find the load current

$$I_{load(pu)} = I_{s(pu)} = \frac{V_{s(pu)}}{Z_{total(pu)}}$$

$$= \frac{V_{s(pu)}}{j(X_{T1} + X_{line} + X_{T2}) + Z_{load(pu)}}$$

$$= \frac{0.9167\angle 0^\circ}{j(0.1 + 0.2604 + 0.1378) + (1.875 + j0.4167)}$$

$$= \frac{0.9167\angle 0^\circ}{2.086\angle 26.01^\circ}$$

$$= 0.4395\angle -26.01^\circ\,pu$$

$$I_{load} = I_{load(pu)} \times I_{B3}$$

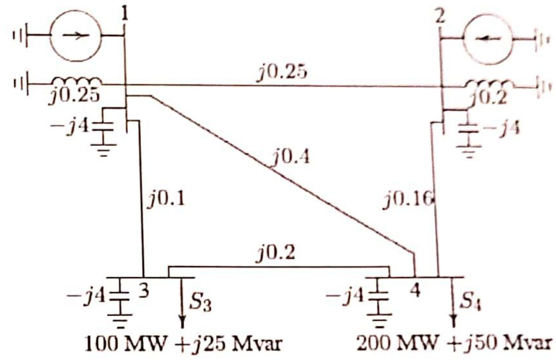
$$= 0.4395\angle -26.01^\circ \times 25$$

$$= 109.9\angle -26.01^\circ A$$

Problem # 1

A power system network is shown below. The generators at buses 1 and 2 are represented by their equivalent current sources with their reactances in per unit on a 100-MVA base. The lines are represented by ¼ model where series reactances and shunt reactances are also expressed in per unit on a 100 MVA base. The loads at buses 3 and 4 are expressed in MW and Mvar.

a. Assuming a voltage magnitude of 1.0 per unit at buses 3 and 4, convert the loads to per unit impedances. Convert network impedances to admittances and obtain the bus admittance matrix by inspection.
b. Use the function Y = ybus(zdata) to obtain the bus admittance matrix. The function argument zdata is a matrix containing the line bus numbers, resistance and reactance.



SOLUTION:

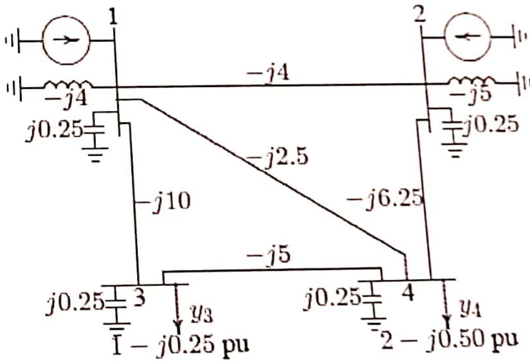
The load impedance in per unit is found from

$$Z = \frac{|V_{L-L}|^2}{S_L^*}\ \Omega \qquad Z_B = \frac{|V_B|^2}{S_B^*}\ \Omega \qquad Z = \frac{|V_{pu}|^2}{S_{pu}^*}\ \text{pu}$$

$$Z_3 = \frac{(1.0)^2}{1 - j0.25} = 0.9412 + j0.2353\ \text{pu}$$

$$Z_4 = \frac{(1.0)^2}{2 - j0.5} = 0.4706 + j0.11765\ \text{pu}$$

Converting all impedances to admittances results in the admittance diagram shown below



The self admittances are

$$Y_{11} = -j4 + j0.25 - j4 - j10 - j2.5 = -j20.25$$
$$Y_{22} = -j5 + j0.25 - j4 - j6.25 = -j15$$
$$Y_{33} = (1 - j0.25) + j0.25 - j10 - j5 = 1 - j15$$
$$Y_{44} = (2 - j0.5) + j0.25 - j2.5 - j6.25 - j5 = 2 - j14$$

Therefore, the bus admittance matrix is

$$Y_{bus} = \begin{bmatrix} -j20.25 & j4 & j10 & j2.5 \\ j4 & -j15 & 0 & j6.25 \\ j10 & 0 & 1 - j15 & j5 \\ j2.5 & j6.25 & j5 & 2 - j14 \end{bmatrix}$$

From the impedance diagram the following data is constructed for use with the function **Y=ybus(Z)**
The result is

```
z = [0   1   0        0.25
     0   1   0       -4.0
     0   2   0        0.2
     0   2   0       -4.0
     0   3   0       -4.0
     0   3   0.9412   0.2353
     0   4   0       -4.0
     0   4   0.4706   0.1176
     1   2   0        0.25
     1   3   0        0.10
     1   4   0        0.40
     2   4   0        0.16
     3   4   0        0.20];
Y=ybus(z)
```

```
Y =
   0 -20.25i     0 + 4.00i     0 +10.00i     0 + 2.50i
   0 + 4.00i     0 -15.00i     0             0 + 6.25i
   0 +10.00i     0             1 -15.00i     0 + 5.00i
   0 + 2.50i     0 + 6.25i     0 + 5.00i     2 -14.00i
```

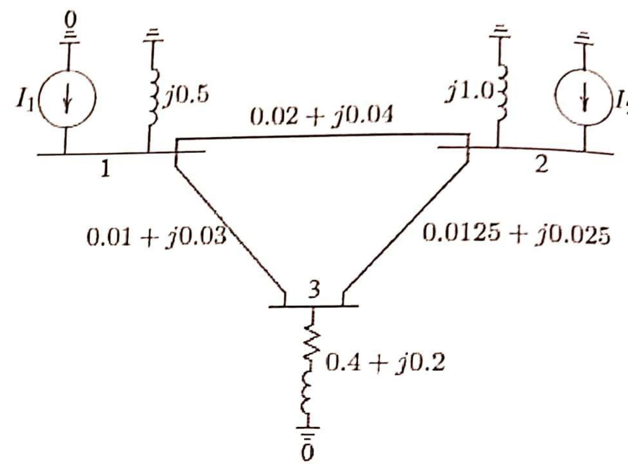
Problem # 2

A power system network is shown below. The values marked are impedances in per unit on a base of 100 MVA. The currents entering buses 1 and 2 are

$$I_1 = 1.38 - j2.72 \text{ pu} \qquad I_2 = 0.69 - j1.36 \text{ pu}$$

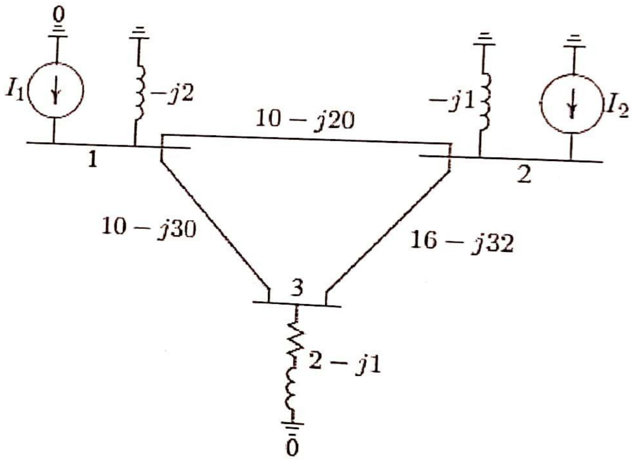
a. Determine the bus admittance matrix $\mathbf{Y}_{bus}$ by inspection.

b. Use the function **Y = ybus(zdata)** to obtain the bus admittance matrix. The function argument **zdata** is a matrix containing the line bus numbers, resistance and reactance. Write the necessary *MATLAB* commands to obtain the bus voltages.



SOLUTION:

Converting all impedances to admittances results in the admittance diagram shown below.



The self admittances are

$$Y_{11} = -j2 + (10 - j20) + (10 - j30) = 20 - j52$$
$$Y_{22} = -j1 + (10 - j20) + (16 - j32) = 26 - j53$$
$$Y_{33} = (2 - j1) + (10 - j30) + (16 - j32) = 28 - j63$$

Therefore, the bus admittance matrix is

$$Y_{bus} = \begin{bmatrix} 20 - j52 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j53 & -16 + j32 \\ -10 + j30 & -16 + j32 & 28 - j63 \end{bmatrix}$$

To obtain the bus admittance matrix using **Y = ybus(Z)**, and the bus voltages, we use the following commands

```
z = [0  1   0.0     0.5
     0  2   0.0     1.0
     0  3   0.4     0.2
     1  2   0.02    0.04
     1  3   0.01    0.03
     2  3   0.0125  0.025];
```

```
Y=ybus(z)
I=[1.38-j*2.72;  0.69-j*1.36;  0];
V=Y\I;
Vm=abs(V)
phase = 180/pi*angle(V)
```

The result is

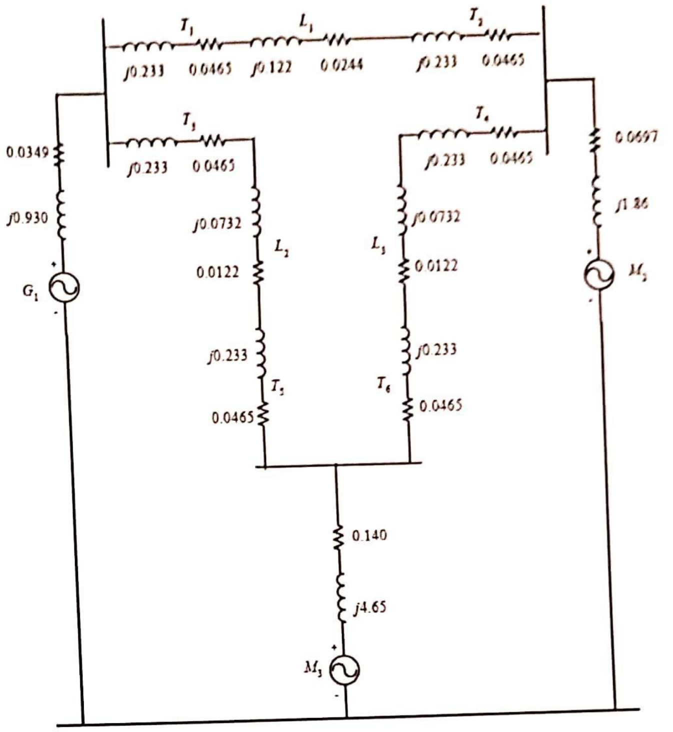
```
Y =
  20.0000-52.000i  -10.0000+20.000i  -10.0000+30.000i
 -10.0000+20.000i   26.0000-53.000i  -16.0000+32.000i
 -10.0000+30.000i  -16.0000+32.000i   28.0000-63.000i
Vm =                   phase =
   1.0293                 1.4596
   1.0217                 0.9905
   1.0001                -0.0150
```

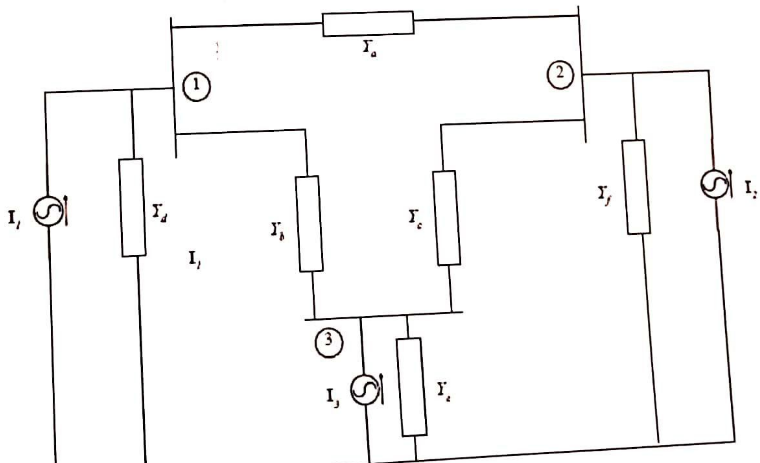
Problem # 3

Calculate the bus admittance matrix $\mathbf{Y}_{bus}$ and the bus impedance matrix $\mathbf{Z}_{bus}$ for the power system shown below.

SOLUTION:

The voltage sources can be converted to current sources, and the series impedances between each bus can be replaced by a single admittance, resulting in the system shown below. Note that we have labeled each bus with a number.





The admittances in this circuit are:

$$Y_a = \frac{1}{Z_{T1}+Z_{L1}+Z_{T2}} = \frac{1}{(0.0465+j0.233\text{ pu})+(0.0244+j0.122)+(0.0465+j0.233\text{ pu})}$$

$$Y_a = 0.3265 - j1.6355\text{ pu}$$

$$Y_b = \frac{1}{Z_{T3}+Z_{L2}+Z_{T5}} = \frac{1}{(0.0465+j0.233\text{ pu})+(0.0122+j0.0732)+(0.0465+j0.233\text{ pu})}$$

$$Y_b = 0.3486 - j1.7866\text{ pu}$$

$$Y_c = \frac{1}{Z_{T4}+Z_{L3}+Z_{T6}} = \frac{1}{(0.0465+j0.233\text{ pu})+(0.0122+j0.0732)+(0.0465+j0.233\text{ pu})}$$

$$= 0.3486 - j1.7866\text{ pu}$$

$$Y_d = \frac{1}{Z_{G1}} = \frac{1}{0.0349 + j0.930} = 0.3537 - j0.9425 \text{ pu}$$

$$Y_e = \frac{1}{Z_{M3}} = \frac{1}{0.140 + j4.65} = 0.0064 - j0.2149 \text{ pu}$$

$$Y_f = \frac{1}{Z_{M2}} = \frac{1}{0.0697 + j1.860} = 0.0201 - j0.5369 \text{ pu}$$

The bus admittance matrix $\mathbf{Y}_{bus}$ is:

$$\mathbf{Y}_{bus} = \begin{bmatrix} Y_a + Y_b + Y_d & -Y_a & -Y_b \\ -Y_a & Y_a + Y_c + Y_f & -Y_c \\ -Y_b & -Y_c & Y_b + Y_c + Y_e \end{bmatrix}$$

$$\mathbf{Y}_{bus} = \begin{bmatrix} 1.0288 - j4.3646 & -0.3265 + j1.6355 & -0.3486 + j1.786 \\ -0.3265 + j1.6355 & 0.6952 - j3.9590 & -0.3486 + j1.786 \\ -0.3486 + j1.7866 & -0.3486 + j1.7866 & 0.7036 - j3.7881 \end{bmatrix}$$

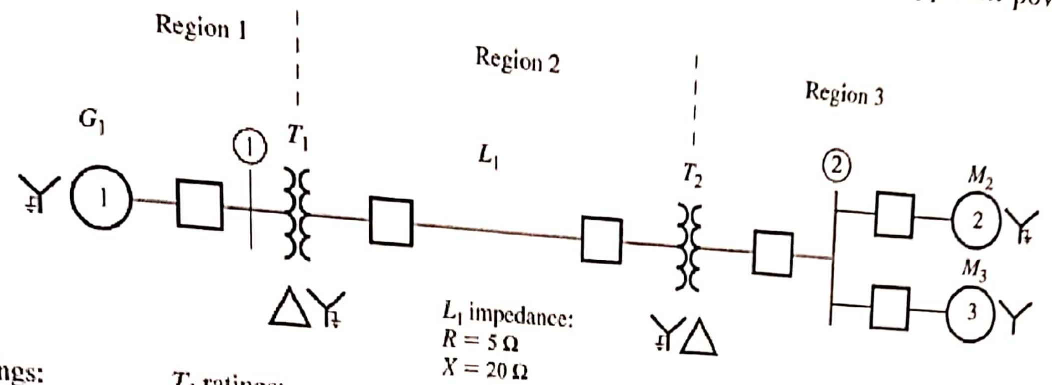
The bus impedance matrix $\mathbf{Z}_{bus}$ is:

$$\mathbf{Z}_{bus} = \mathbf{Y}_{bus}^{-1}$$

$$\mathbf{Z}_{bus} = \begin{bmatrix} 0.1532 + j0.6048 & 0.1068 + j0.4875 & 0.1181 + j0.5172 \\ 0.1068 + j0.4875 & 0.1251 + j0.7046 & 0.1045 + j0.5642 \\ 0.1181 + j0.5172 & 0.1045 + j0.5642 & 0.1479 + j0.7670 \end{bmatrix}$$

Problem # 4

The power system below shows a one-line diagram of a simple power system. Assume that the generator G_1 is supplying a constant 13.8 kV at Bus 1, that load M_2 is consuming 20 MVA at 0.85 PF lagging, and that load M_3 is consuming 10 MVA at 0.90 PF leading. Calculate the bus admittance matrix $\mathbf{Y}_{bus}$ for this system. (*Note:* assume that the system base apparent power is 30 MVA.)



G_1 ratings:
30 MVA
13.8 kV
$R = 0.1$ pu
$X_S = 1.0$ pu

T_1 ratings:
35 MVA
13.2/115 kV
$R = 0.01$ pu
$X = 0.10$ pu

T_2 ratings:
30 MVA
120/12.5 kV
$R = 0.01$ pu
$X = 0.08$ pu

M_2 ratings:
20 MVA
12.5 kV
$R = 0.1$ pu
$X_S = 1.1$ pu

M_3 ratings:
10 MVA
12.5 kV
$R = 0.1$ pu
$X_S = 1.1$ pu

SOLUTION:

The system base apparent power is $S_{base} = 30$ MVA, and the system base voltages in each region are:

$$V_{base,1} = 13.8 \text{ kV}$$

$$V_{base,2} = \left(\frac{115 \text{ kV}}{13.2 \text{ kV}}\right) V_{base,1} = \left(\frac{115 \text{ kV}}{13.2 \text{ kV}}\right)(13.8 \text{ kV}) = 120 \text{ kV}$$

$$V_{base,3} = \left(\frac{12.5 \text{ kV}}{120 \text{ kV}}\right) V_{base,2} = \left(\frac{12.5 \text{ kV}}{120 \text{ kV}}\right)(120 \text{ kV}) = 12.5 \text{ kV}$$

The base impedance of Region 2 is:

$$Z_{base,2} = \frac{\left(V_{LL,\,base,2}\right)^2}{S_{3\phi,\,base}} = \frac{(120,000 \text{ V})^2}{30,000,000 \text{ VA}} = 480 \ \Omega$$

The per unit resistance and reactance of G_1 are already on the proper base:

$$Z_{G1} = 0.1 + j1.0 \text{ pu}$$

The per unit resistance and reactance of T_1 are:

$$\text{per-unit } Z_{new} = \text{per-unit } Z_{given}\left(\frac{V_{given}}{V_{new}}\right)^2\left(\frac{S_{new}}{S_{given}}\right)$$

$$Z_{T1} = (0.01 + j0.10)\left(\frac{13.2 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{30,000 \text{ kVA}}{35,000 \text{ kVA}}\right) = 0.00784 + j0.0784 \text{ pu}$$

The per unit resistance and reactance of the transmission line are:

$$Z_{line} = \frac{Z}{Z_{base}} = \frac{5 + j20 \ \Omega}{480 \ \Omega} = 0.0104 + j0.0417 \text{ pu}$$

The per unit resistance and reactance of T_2 are already on the right base:

$$Z_{T2} = 0.01 + j0.08 \text{ pu}$$

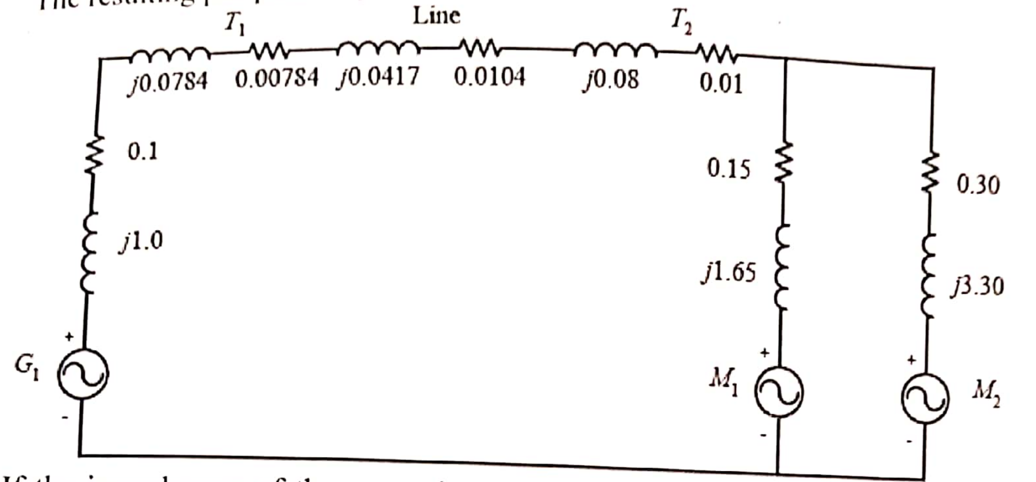
The per unit resistance and reactance of M_1 are:

$$Z_{M1} = (0.1 + j1.1)\left(\frac{12.5 \text{ kV}}{12.5 \text{ kV}}\right)^2\left(\frac{30,000 \text{ kVA}}{20,000 \text{ kVA}}\right) = 0.15 + j1.65 \text{ pu}$$

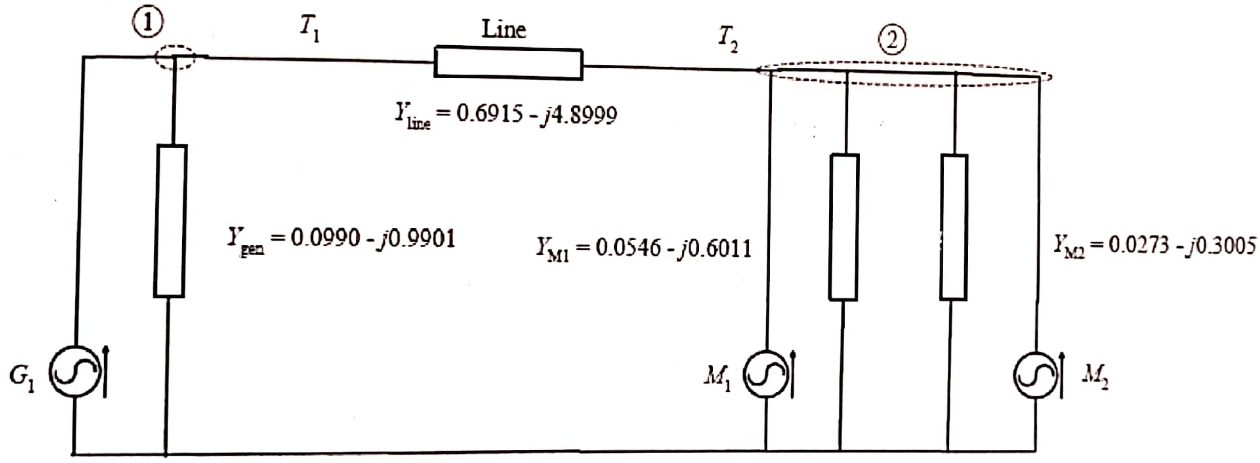
The per unit resistance and reactance of M_1 are:

$$Z_{M2} = (0.1 + j1.1)\left(\frac{12.5\text{ kV}}{12.5\text{ kV}}\right)^2\left(\frac{30{,}000\text{ kVA}}{10{,}000\text{ kVA}}\right) = 0.30 + j3.30\text{ pu}$$

The resulting per-phase equivalent circuit is:



If the impedances of the transmission line and the transformers are combined, all voltage sources are converted to their Norton equivalents, and all impedances are converted to admittances, the resulting circuit is:



For power flow studies, we ignore the self admittances of each node. There are only two busses in this circuit, so the bus admittance matrix will appear as follows:

$$\mathbf{Y}_{bus} = \begin{bmatrix} 0.6915 - j4.8999 & -0.6915 + j4.8999 \\ -0.6915 + j4.8999 & 0.6915 - j4.8999 \end{bmatrix}$$

The loads on this power system are:

$$P_{M2} = (20\text{ MVA})(0.85) = 17\text{ MW} = 0.567\text{ pu}$$

$$Q_{M2} = (20\text{ MVA})\sin\left[\cos^{-1}(0.85)\right] = 10.5\text{ MVAR} = 0.350\text{ pu}$$

$$P_{M3} = (10\text{ MVA})(0.9) = 9\text{ MW} = 0.300\text{ pu}$$

$$Q_{M3} = (10\text{ MVA})\sin\left[-\cos^{-1}(0.9)\right] = -4.36\text{ MVAR} = -0.145\text{ pu}$$

Problem # 1

A 69-kV, three-phase short transmission line is 16 km long. The line has a per phase series impedance of $0.125 + j0.4375\ \Omega$ per km. Determine:

a. the sending end voltage,
b. the voltage regulation,
c. the sending end power, and
d. the transmission efficiency when the line delivers
   i. 70 MVA, 0.8 lagging power factor at 64 kV.
   ii. 120 MW, unity power factor at 64 kV.

Solution:

The line impedance is $Z = (0:125 + j0:4375)(16) = 2 + j7\ \Omega$
The receiving end voltage per phase is

$$V_R = \frac{64\angle 0^\circ}{\sqrt{3}} = 36.9504\angle 0^\circ \text{ kV}$$

(a) The complex power at the receiving end is

The current per phase is given by

$$S_{R(3\phi)} = 70\angle\cos^{-1} 0.8 = 70\angle 36.87^\circ = 56 + j42 \text{ MVA}$$

$$I_R = \frac{S^*_{R(3\phi)}}{3V^*_R} = \frac{70000\angle -36.87^\circ}{3 \times 36.9504\angle 0^\circ} = 631.477\angle -36.87^\circ \text{ A}$$

The sending end voltage is

$$V_S = V_R + ZI_R = 36.9504\angle 0^\circ + (2 + j7)(631.477\angle -36.87^\circ)(10^{-3})$$
$$= 40.708\angle 3.9137^\circ \text{ kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 70.508 \text{ kV}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I^*_S = 3 \times 40.708\angle 3.9137 \times 631.477\angle 36.87^\circ \times 10^{-3}$$
$$= 58.393 \text{ MW} + j50.374 \text{ Mvar}$$
$$= 77.1185\angle 40.7837^\circ \text{ MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{70.508 - 64}{64} \times 100 = 10.169\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{56}{58.393} \times 100 = 95.90\%$$

(b) The complex power at the receiving end is

$$S_{R(3\phi)} = 120\angle 0^\circ = 120 + j0 \text{ MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3V^*_R} = \frac{120000\angle 0^\circ}{3 \times 36.9504\angle 0^\circ} = 1082.53\angle 0^\circ \text{ A}$$

The sending end voltage is

$$V_S = V_R + ZI_R = 36.9504\angle 0^\circ + (2 + j7)(1082.53\angle 0^\circ)(10^{-3})$$
$$= 39.8427\angle 10.9639^\circ \text{ kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 69.0096 \ \text{kV}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I_S^* = 3 \times 39.8427\angle 10.9639 \times 1082.53\angle 0^\circ \times 10^{-3}$$
$$= 127.031 \ \text{MW} + j24.609 \ \text{Mvar}$$
$$= 129.393\angle 10.9639^\circ \ \text{MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{69.0096 - 64}{64} \times 100 = 7.8275\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{120}{127.031} \times 100 = 94.465\%$$

Problem # 2

A 230-kV, three-phase transmission line has a per phase series impedance of z = 0:05+j0:45 Ω per Km and a per phase shunt admittance of $y = j3{:}4\times10^{-6}$ siemens per km. The line is 80 km long. Using the nominal π model, determine

a. the transmission line ABCD constants.
b. the sending end voltage and current,
c. the voltage regulation,
d. the sending end power and
e. the transmission efficiency when the line delivers
   i. 200 MVA, 0.8 lagging power factor at 220 kV.
   ii. 306 MW, unity power factor at 220 kV.

Solution:

The line impedance and shunt admittance are

$$Z = (0.05 + j0.45)(80) = 4 + j36\ \Omega$$

$$Y = (j3.4 \times 10^{-6})(80) = j0.272 \times 10^{-3} \text{ siemens}$$

The ABCD constants of the nominal π model are

$$A = (1 + \frac{ZY}{2}) = (1 + \frac{(4 + j36)(j0.272 \times 10^{-3})}{2}) = 0.9951 + j0.000544$$

$$B = Z = 4 + j36$$

$$C = Y(1 + \frac{ZY}{4}) = j0.0002713$$

The receiving end voltage per phase is

$$V_R = \frac{220\angle 0^\circ}{\sqrt{3}} = 127\angle 0^\circ \text{ kV}$$

(a) The complex power at the receiving end is

$$S_{R(3\phi)} = 200\angle \cos^{-1} 0.8 = 200\angle 36.87^\circ = 160 + j120 \text{ MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3V^*_R} = \frac{200000\angle -36.87^\circ}{3 \times 127\angle 0^\circ} = 524.864\angle -36.87^\circ \text{ A}$$

The sending end voltage is

$$V_S = AV_R + BI_R = 0.9951 + j0.000544)(127\angle 0^\circ) + (4 + j36)$$
$$(524.864 \times 10^{-3}\angle -36.87^\circ) = 140.1051\angle 5.704^\circ \text{ kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}|V_S| = 242.67 \text{ kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.0002713)(127000\angle 0^\circ) + (0.9951 + j0.000544)$$
$$(524.864\angle -36.87^\circ) = 502.38\angle -33.69^\circ \text{ A}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I^*_S = 3 \times 140.1051\angle 5.704 \times 502.38\angle 33.69^\circ \times 10^{-3}$$
$$= 163.179 \text{ MW} + j134.018 \text{ Mvar}$$
$$= 211.16\angle 39.396^\circ \text{ MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{242.67}{0.9951} - 220}{220} \times 100 = 10.847\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{160}{163.179} \times 100 = 98.052\%$$

(b) The complex power at the receiving end is

$$S_{R(3\phi)} = 306\angle 0^\circ = 306 + j0 \text{ MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3V^*_R} = \frac{306000\angle 0^\circ}{3 \times 127\angle 0^\circ} = 803.402\angle 0^\circ \text{ A}$$

The sending end voltage is

$$V_S = AV_R + BI_R = 0.9951 + j0.000544)(127\angle 0^\circ) + (4 + j36)$$
$$(803.402 \times 10^{-3}\angle 0^\circ) = 132.807\angle 12.6^\circ \text{ kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 230.029 \text{ kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.0002713)(127000\angle 0^\circ) + (0.9951 + j0.000544)$$
$$(803.402\angle 0^\circ) = 799.862\angle 2.5^\circ \text{ A}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I^*_S = 3 \times 132.807\angle 12.6 \times 799.862\angle -2.5^\circ \times 10^{-3}$$
$$= 313.742 \text{ MW} + j55.9 \text{ Mvar}$$
$$= 318.68\angle 10.1^\circ \text{ MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{230.029}{0.9951} - 220}{220} \times 100 = 5.073\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{306}{313.742} \times 100 = 97.53\%$$

Problem # 3

The ABCD constants of a three-phase, 345-kV transmission line are, determine

$$A = D = 0.98182 + j0.0012447$$
$$B = 4.035 + j58.947$$
$$C = j0.00061137$$

The line delivers 400 MVA at 0.8 lagging power factor at 345 kV. Determine

a. the sending end quantities.
b. the voltage regulation, and
c. the transmission efficiency.

Solution:

The receiving end voltage per phase is

$$V_R = \frac{345\angle 0^\circ}{\sqrt{3}} = 199.186\angle 0^\circ \text{ kV}$$

(a) The complex power at the receiving end is

$$S_{R(3\phi)} = 400\angle \cos^{-1} 0.8 = 400\angle 36.87^\circ = 320 + j240 \text{ MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3V^*_R} = \frac{400000\angle -36.87^\circ}{3 \times 199.186\angle 0^\circ} = 669.392\angle -36.87^\circ \text{ A}$$

The sending end voltage is

$$V_S = AV_R + BI_R = 0.98182 + j0.0012447)(199.186\angle 0^\circ) + (4.035 + j58.947)$$
$$(668.392 \times 10^{-3}\angle -36.87^\circ) = 223.449\angle 7.766^\circ \text{ kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 387.025 \text{ kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.00061137)(199.186\angle 0^\circ) + (0.98182 + j0.0012447)$$
$$(669.392\angle -36.87^\circ) = 592.291\angle -27.3256^\circ \text{ A}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I^*_S = 3 \times 223.449\angle 7.766 \times 592.291\angle 27.3256^\circ \times 10^{-3}$$
$$= 324.872 \text{ MW} + j228.253 \text{ Mvar}$$
$$= 397.041\angle 35.0916^\circ \text{ MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{387.025}{0.98182} - 345}{345} \times 100 = 14.2589\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{320}{324.872} \times 100 = 98.500\%$$

Problem # 4

The ABCD constants of a lossless three-phase, 500-kV transmission line are

$$A = D = 0.86 + j0$$
$$B = 0 + j130.2$$
$$C = j0.002$$

a. Obtain the sending end quantities and the voltage regulation when line delivers 1000 MVA at 0.8 lagging power factor at 500 kV.

To improve the line performance, series capacitors are installed at both ends in each phase of the transmission line. As a result of this, the compensated ABCD constants become.

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2}jX_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2}jX_c \\ 0 & 1 \end{bmatrix}$$

where X_c is the total reactance of the series capacitor. If $X_c = 100\ \Omega$

b. Determine the compensated ABCD constants.

c. Determine the sending end quantities and the voltage regulation when line delivers 1000 MVA at 0.8 lagging power factor at 500 kV.

Solution:

(a) The receiving end voltage per phase is

$$V_R = \frac{500\angle 0°}{\sqrt{3}} = 288.675\angle 0° \ \ \text{kV}$$

(a) The complex power at the receiving end is.

$$S_{R(3\phi)} = 1000\angle\cos^{-1}0.8 = 1000\angle 36.87° = 800 + j600 \ \ \text{MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{1000000\angle-36.87°}{3 \times 288.675\angle 0°} = 1154.7\angle-36.87° \ \ \text{A}$$

The sending end voltage is

$$V_S = AV_R + BI_R = (0.86)(288.675\angle 0°) + (j130.2)$$
$$(1154.7 \times 10^{-3}\angle{-36.87°}) = 359.2\angle 19.5626° \ \ \text{kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 622.153 \ \ \text{kV}$$

The sending end current is

$$I_S = CV_R + DI_R = (j0.002)(288675\angle 0°) + (0.86)$$
$$(1154.7\angle{-36.87°}) = 794.647\angle{-1.3322°} \ \ \text{A}$$

The sending end power is

$$S_{S(3\phi)} = 3V_SI^*_S = 3 \times 359.2\angle 19.5626 \times 794.647\angle 1.3322° \times 10^{-3}$$
$$= 800 \text{ MW} + j228.253 \text{ Mvar}$$
$$= 831.925\angle 15.924° \ \ \text{MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{622.153}{0.86} - 500}{500} \times 100 = 44.687\%$$

(b) The compensated $ABCD$ constants are

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2}j100 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.86 & j139.2 \\ j0.002 & 0.86 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2}j100 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.96 & j39.2 \\ j0.002 & 0.96 \end{bmatrix}$$

(c) Repeating the analysis for the new $ABCD$ constants result in
The sending end voltage is

$$V_S = AV_R + BI_R = (0.96)(288.675\angle 0°) + (j39.2)$$
$$(1154.7 \times 10^{-3}\angle -36.87°) = 306.434\angle 6.7865° \text{ kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 530.759 \text{ kV}$$

$$I_S = CV_R + DI_R = (j0.002)(288675\angle 0°) + (0.96)$$
$$(1154.7\angle -36.87°) = 891.142\angle -5.6515° \text{ A}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I_S^* = 3 \times 306.434\angle 6.7865 \times 891.142\angle 5.6515° \times 10^{-3}$$
$$= 800 \text{ MW} + j176.448 \text{ Mvar}$$
$$= 819.227\angle 12.438° \text{ MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{\frac{530.759}{0.96} - 500}{500} \times 100 = 10.5748\%$$

Problem # 5

A three-phase 420-kV, 60-HZ transmission line is 463 km long and may be assumed lossless. The line is energized with 420 kV at the sending end. When the load at the receiving end is removed, the voltage at the receiving end is 700 kV, and the per phase sending end current is $646.6\angle 90°$ A.

a. Find the phase constant β in radians per Km and the surge impedance Z_c in Ω.
b. Ideal reactors are to be installed at the receiving end to keep $|V_S| = |V_R| = 420$ kV when load is removed. Determine the reactance per phase and the required three-phase MVAR.

Solution:

(a) The sending end and receiving end voltages per phase are

$$V_S = \frac{420}{\sqrt{3}} = 242.487 \text{ kV}$$

$$V_{Rnl} = \frac{700}{\sqrt{3}} = 404.145 \text{ kV}$$

With load removed $I_R = 0$,

OR

$$242.487 = (\cos \beta\ell)(404.145)$$

$$\beta\ell = 53.13° = 0.927295 \text{ Radian}$$

$$j646.6 = j\frac{1}{Z_c}(\sin 53.13°)(404.145)10^3$$

OR

$$Z_c = 500\ \Omega$$

(b) For $V_S = V_R$, the required inductor reactance is given by

$$X_{Lsh} = \frac{\sin(53.13°)}{1 - \cos(53.13°)}(500) = 1000\ \Omega$$

The three-phase shunt reactor rating is

$$Q_{3\phi} = \frac{(KV_{Lrated})^2}{X_{Lsh}} = \frac{(420)^2}{1000} = 176.4 \text{ Mvar}$$

Problem # 6

A three-phase power of 3600 MW is to be transmitted via four identical 60-Hz transmission lines for a distance of 300Km. From a preliminary line design, the line phase constant and surge impedance are given by $\beta = 9{:}46\times10^{-4}$ radian/Km and $Z_c = 343\Omega$, respectively. Based on the practical line loadability criteria determine the suitable nominal voltage level in kV for each transmission line. Assume V_S=1.0 pu, V_R=0.9 pu, and the power angle $\delta = 36{:}9°$.

Solution:

$$\beta\ell = (9.46 \times 10^{-4})(300)(\frac{180}{\pi}) = 16.26^\circ$$

The real power per transmission circuit is

$$P = \frac{3600}{4} = 900 \text{ MW}$$

From the practical line loadability given by(5.97), we have

$$900 = \frac{(1.0)(0.9)(SIL)}{\sin(16.26^\circ)} \sin(36.87^\circ)$$

Thus

$$SIL = 466.66 \text{ MW}$$

$$KV_L = \sqrt{(Z_c)(SIL)} = \sqrt{(343)(466.66)} = 400 \text{ kV}$$

and also $$V_{pu} = Z_{pu}\, I_{pu} \qquad ...(5.8)$$

The power consumed by the load at its rated voltage can also be expressed by per-unit impedance. The three-phase complex load power can be given as:

$$S_{load(3\phi)} = 3\, V_{phase} I_L^* \qquad ...(5.9)$$

Here $S_{load(3\phi)}$ = three-phase complex load power

V_{phase} = phase voltage

I_L^* = complex conjugate of per-phase load current I_L.

The phase load current can be given as:

$$I_L = \frac{V_{phase}}{Z_L} \qquad ...(5.10)$$

where Z_L is load impedance per phase.

Substituting I_L from eqn. (5.10) in eqn. (5.9), we get,

$$S_{load(3\phi)} = 3.\, V_{phase}\left(\frac{V_{phase}}{Z_L}\right)^*$$

$$\therefore \quad S_{load(3\phi)} = \frac{3|V_{phase}|^2}{Z_L^*}$$

$$\therefore \quad Z_L = \frac{3|V_{phase}|^2}{S_{load(3\phi)}^*} \qquad ...(5.11)$$

Also, load impedance in per-unit can be given as

$$Z_{pu} = \frac{Z_L}{Z_B} \qquad ...(5.12)$$

Substituting Z_L from eqn. (5.11) and Z_B from eqn. (5.6) into eqn. (5.12), we obtain

$$Z_{pu} = \frac{3|V_{phase}|^2}{S_{load\,(3\phi)}^*} \cdot \frac{(MVA)_B}{(KV)_B^2} \qquad ...(5.13)$$

Now $$|V_{L-L}| = \sqrt{3}\,|V_{phase}| \qquad ...(5.14)$$

$$\therefore \quad 3|V_{phase}|^2 = |V_{LL}|^2$$

Using eqns. (5.13) and (5.14), we get

$$Z_{pu} = \frac{|V_{L-L}|^2}{(KV)_B^2} \cdot \frac{(MVA)_B}{S_{load(3\phi)}^*} \qquad ...(5.15)$$

$$Z_{pu} = \frac{|V_{pu}|^2}{S_{load\,(pu)}^*}$$