PAGE 5th. Feb.
DATE

① Fetch
② Multiplication
③ Addition.

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}_{2\times2} \begin{bmatrix} 4 & 6 \\ 2 & 2 \end{bmatrix}_{2\times2} = \begin{bmatrix} 1.4+2.2 & 1.6+2.2 \\ 3.4+3.2 & 2.6+3.2 \end{bmatrix}_{2\times2}$$

# of total operations
↳ # of multiplication = 8
↳ # of Addition = 4
↳ # of fetch = 4

$\Sigma = 16$

*

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 1 & 2 \end{bmatrix}_{3\times3} \begin{bmatrix} 2 & 3 & 1 \\ 1 & 5 & 3 \\ 4 & 2 & 1 \end{bmatrix}_{3\times3} = \begin{bmatrix} 1.2+2.1+3.4 & 1.3+2.5+3.2 & 1.1+2.3+3.1 \\ 3.2+2.1+1.4 & 3.3+2.5+1.2 & 3.1+2.3+1.1 \\ 4.2+1.1+2.4 & 4.3+1.5+2.2 & 4.1+1.3+2.1 \end{bmatrix}_{3\times3}$$

# of operations
↳ # of multiplications = 27
↳ # of Addition = 18
↳ # of fetch = 9

$\Sigma = 54$

Total # of operations $= 2(n)^3$
for $(n\times n)(n\times n)$ matrices.

* Microprocess at 1GHZ

  $\hookrightarrow \tau = \frac{1}{1G} = 1ns = 1G$ cycle/S.

- DRAM latency = 100 ns (without cache)
- MP has 2 Multiply - Add Units.
- MP excutes 4 inst./cycle

a) MP rating = 1GHZ $\Rightarrow$ 1G * 4 = 4G Flops.

b) Block Size = 1 word

Mp waiting time = 100 ns = $\frac{100\ ns}{1\ ns}$ = 100 cycle

c) dot product

$$[1 \quad 2 \quad 3] \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} = 1 \cdot 2 + 2 \cdot 4 + 3 \cdot 5.$$

Limit peak speed = $100 \frac{ns}{inst}$

$= \frac{1}{100\ ns} = 0.01 * 10^{9}$ Inst/S

$= 10 * 10^{6}$ FlopS

= 10M FlopS.

d) Using cache: Size 32 KB & latency 1 ns

| [A] | x | [B] | = | [C] |
|---|---|---|---|---|
| 32x32 | | 32x32 | | 32x32 |
| $2^5 \times 2^5$ | | $2^5 \times 2^5$ | | $2^5 \times 2^5$ |
| $2^{10} = \boxed{1K}$ | | $2^{10} = \boxed{1K}$ | | |

- To multiply (A) & (B)
  - ↳ we need to fetch 2K words into cache

e) Time to fetch $= 2K \times 100ns$
$= 2 \times 10^3 \times 100 \times 10^{-9}$
$= 200\ \mu s$

f) To multiply $[A]_{n \times n}$ & $[B]_{n \times n}$
↳ we need $2n^3$ operations.
$\rightarrow n = 32 \rightarrow 2(32)^3 = 64\ K$ operations.

g) $4 \frac{\text{Inst}}{\text{cycle}} \Rightarrow$ Time to mutiply (A) & (B)

$$= \frac{64\ K}{4 \frac{ops}{cycle}} = \frac{64\ K}{4\left(\frac{ops}{\frac{1}{1ns}}\right)} = 16\ \mu s$$

h) Total time $= 200\ \mu s + 16\ \mu s = 216\ \mu s$

i) Peak Processing rate $= \frac{64\ K}{216\ \mu s} = 303$ MFLOPS.

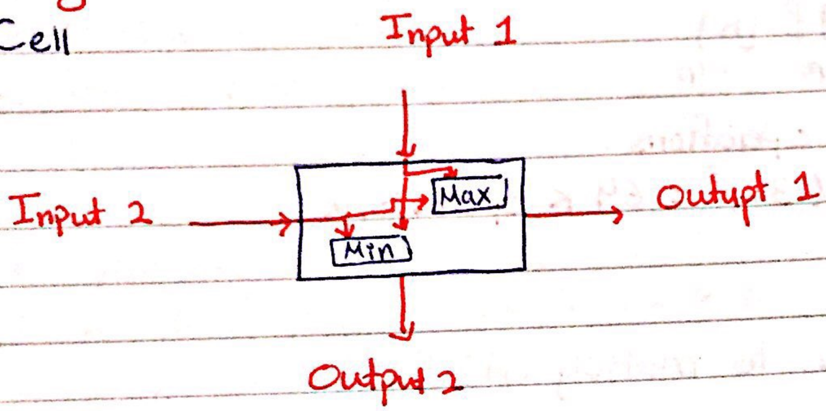
j) New block size = 4 words.
100 ns ⟶ 1 word
now 100 ns ⟶ 4 word
200 ns ⟶ 8 word
8 word ⟶ 8 FLOPS ⟹ 4 Mult-Add in 200 ns
⟹ 200 cycle.

# * Systolic Architectures :

- Enhancing performance (Speedup)
- Less power consumption.
- Less complex
- Ease of manufactorability.
- Less expensive.

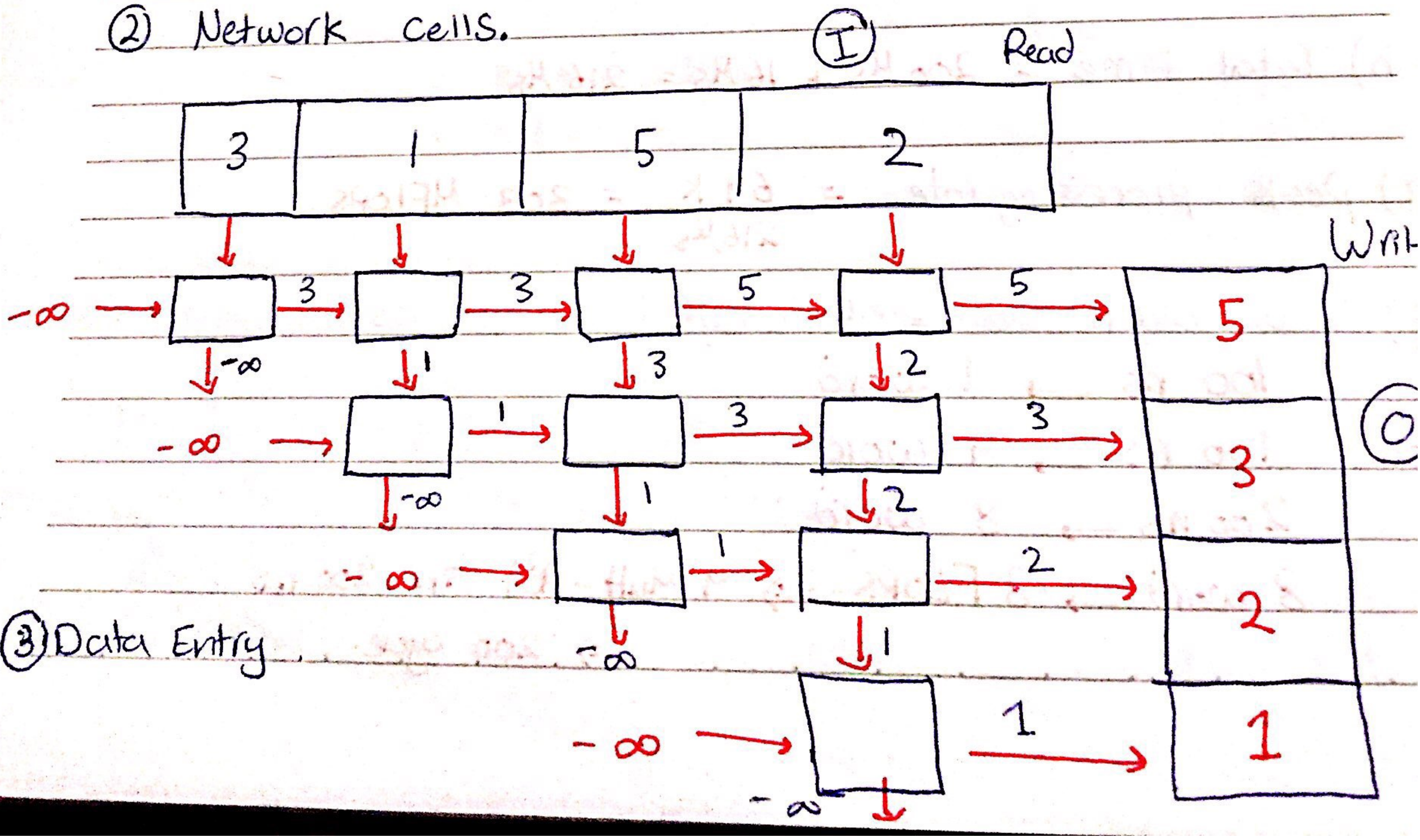
## * Sorting

① Cell




Sorting Cell.

② Network cells.




③ Data Entry

14th. feb.

*Band_Matrix Vector Multiplication

$$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 \\ 0 & a_{42} & a_{43} & a_{44} & a_{45} \\ 0 & 0 & - & - & - \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}$$

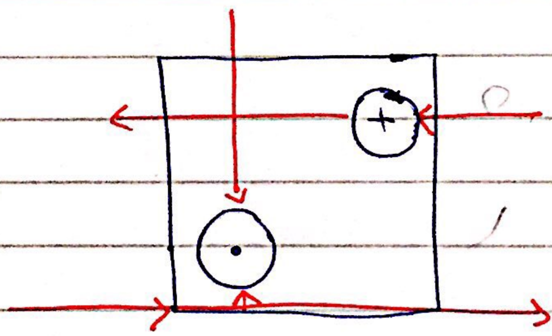
$y_1 = a_{11}x_1 + a_{12}x_2$

$y_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3$

$y_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4$

$y_4 = a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5$

(1) PE (CELL) :



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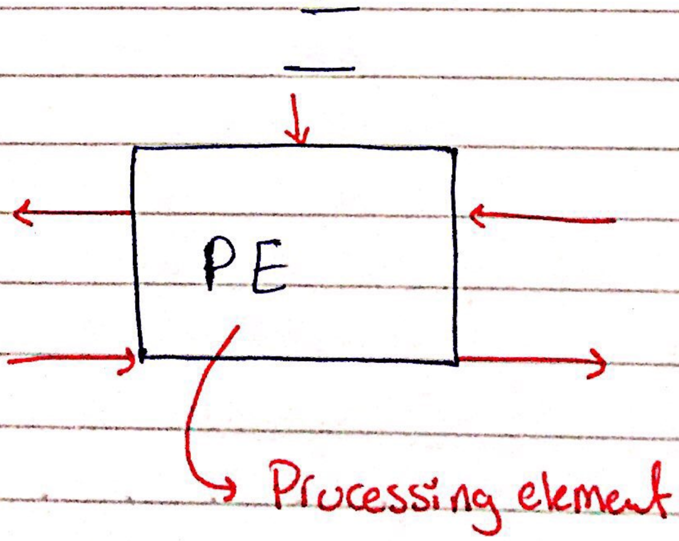
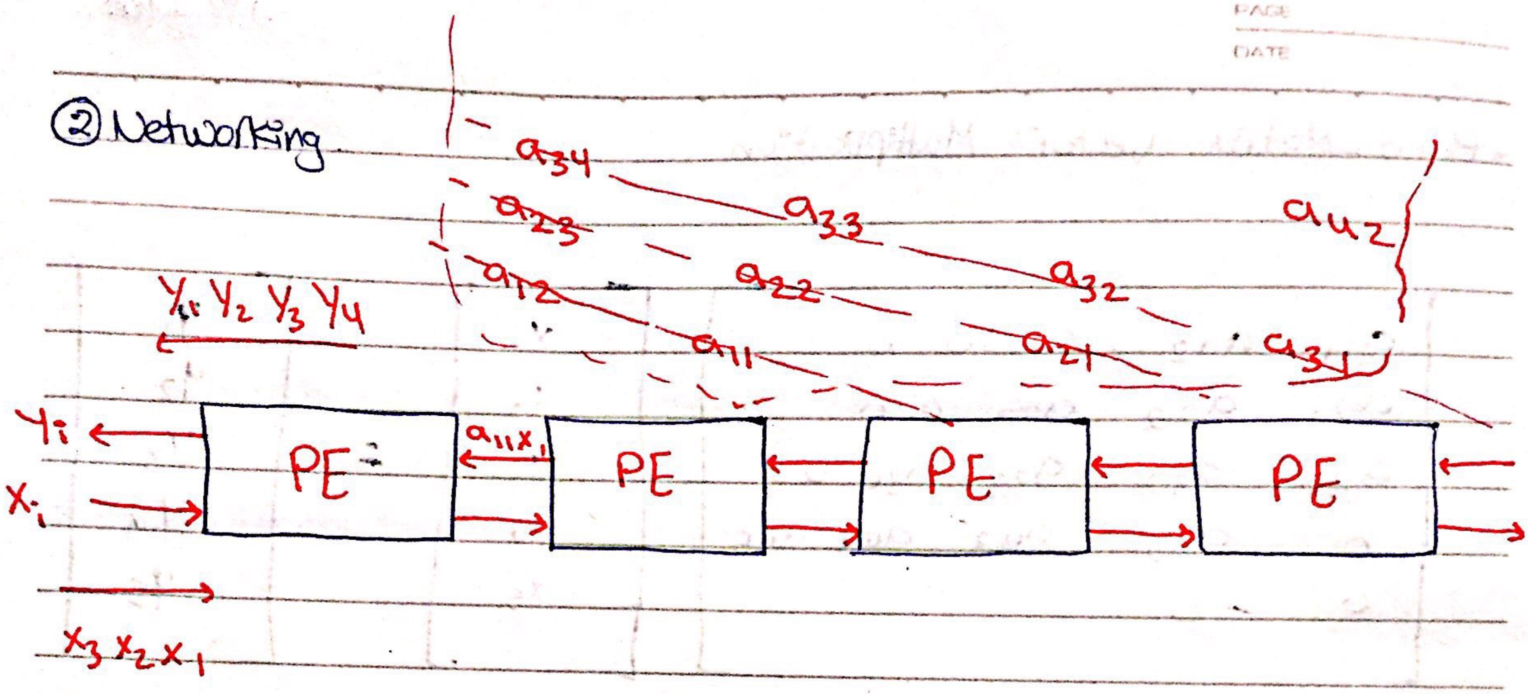


② Networking

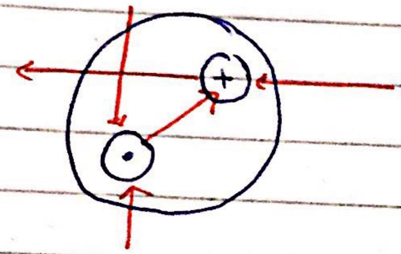


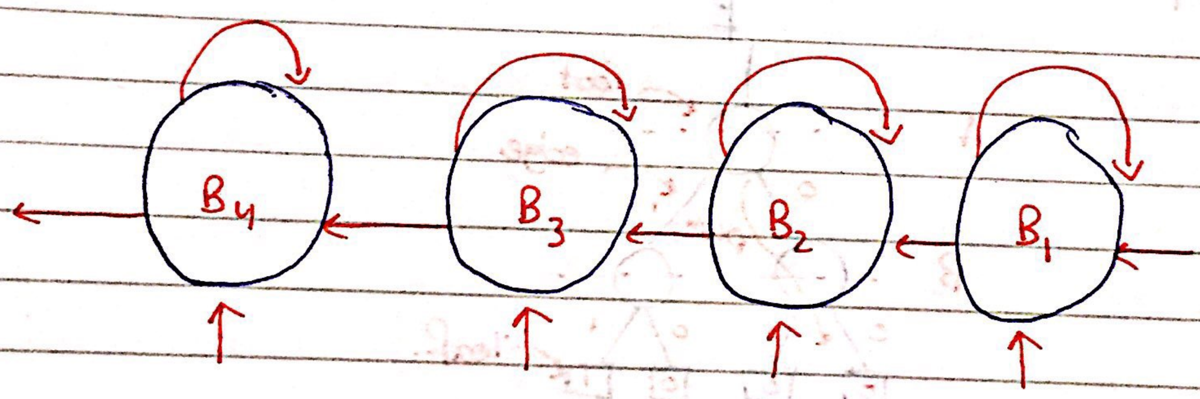

③ Data Entry

PAGE
DATE 24th. Feb.

## * inner product systolic array:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{33} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} A_{11}B_1 + A_{12}B_2 + A_{13}B_3 + A_{14}B_4 \text{ - -} \\ \text{- - - -} \\ \text{- - - -} \\ \vdots \end{bmatrix}$$




| | | | |
|---|---|---|---|
| 0 | 0 | 0 | $A_{11}$ |
| 0 | 0 | $A_{12}$ | $A_{21}$ |
| 0 | $A_{13}$ | $A_{22}$ | $A_{31}$ |
| $A_{14}$ | $A_{23}$ | $A_{32}$ | $A_{41}$ |
| $A_{24}$ | $A_{33}$ | $A_{42}$ | 0 |
| $A_{34}$ | $A_{43}$ | 0 | 0 |
| $A_{44}$ | 0 | 0 | 0 |

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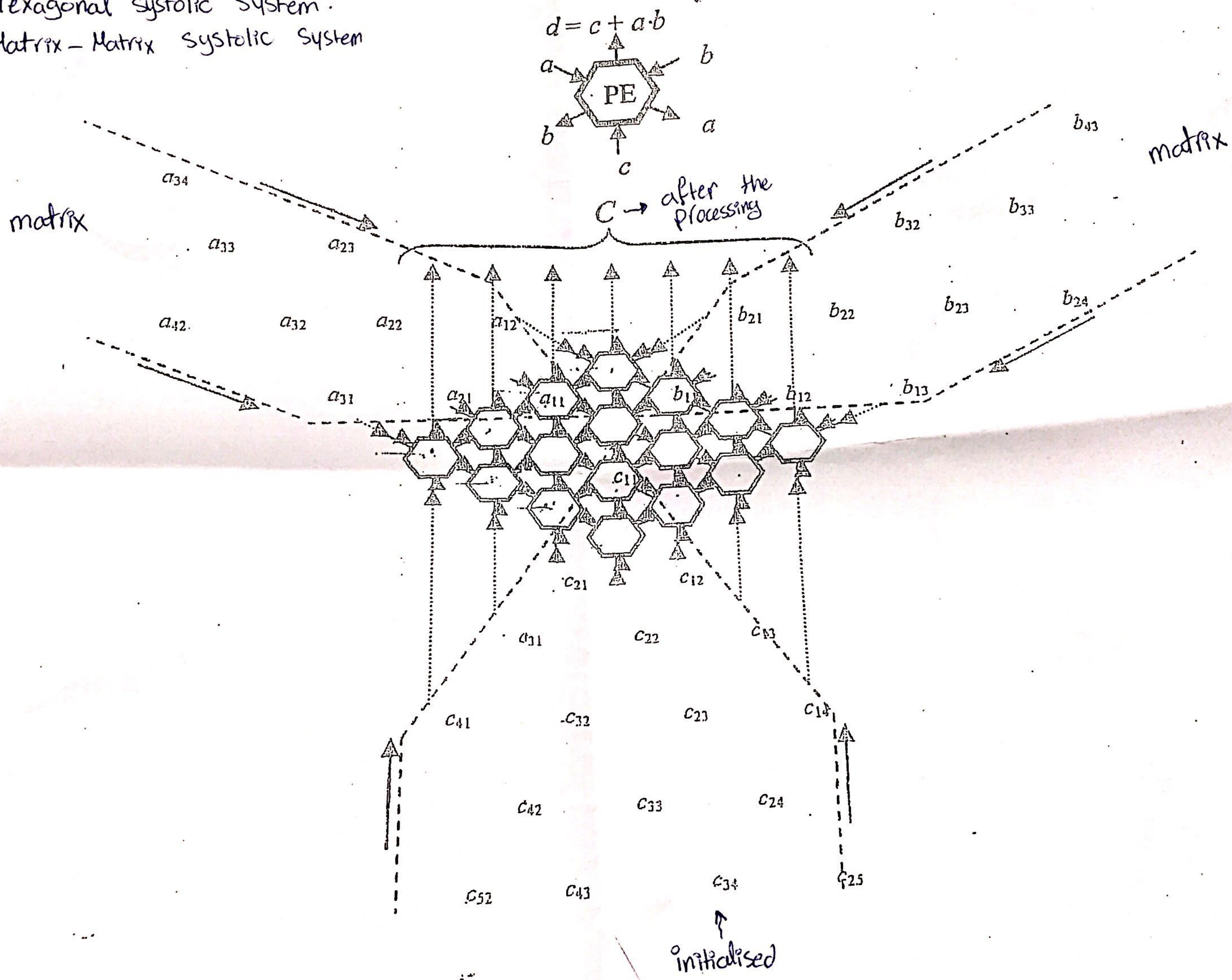
~~Kung~~-Hexagonal systolic system.

Matrix – Matrix systolic system




29/8/2012

10th. Mar

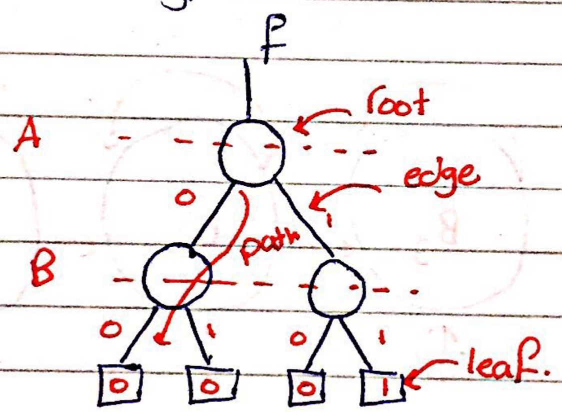
*Binary Diagrams: Trees and Binary Diagrams

* $f = A$ AND $B$ → Boolean equation
$= A \cdot B$
$= A \cap B$

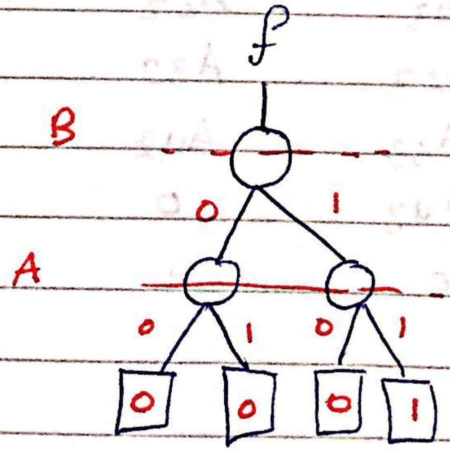
- Truth Table

| A | B | f |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

- CAD Representation
↳ Computer Aided Design.



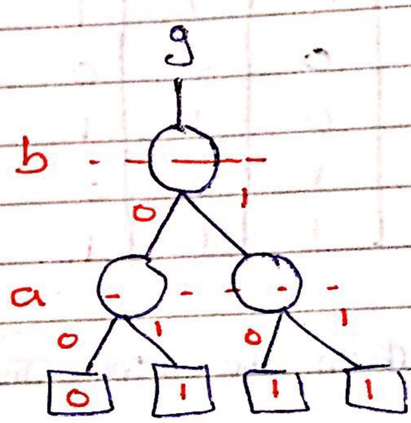

Binary Tree.



* $g = A \text{ or } B = A + B = A \cup B$.

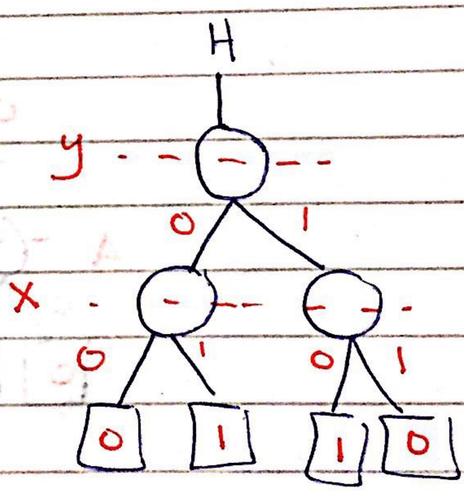
$A$, $B$ → Control or input variables.

| a | b | g |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |




* $H = X \oplus Y$

| X | Y | H |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

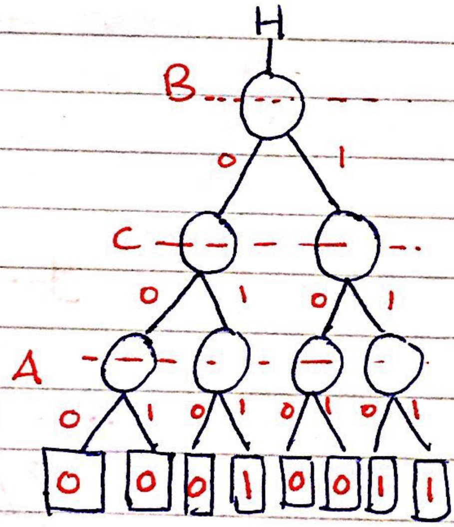


17th. Mar

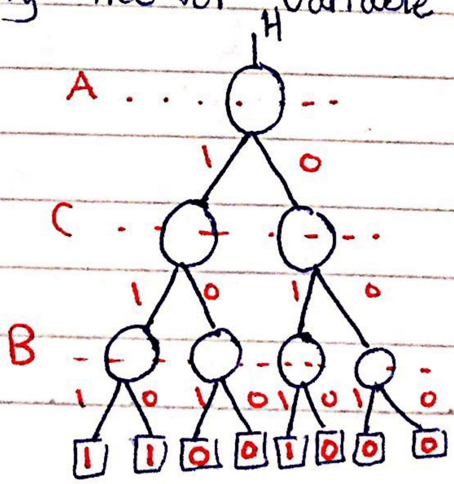
* $H = \bar{A}BC + CA$

| A | B | C | H |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

* perform the Binary Tree for variable order (B, C, A).




* perform the Binary Tree for variable order (A, C, B)

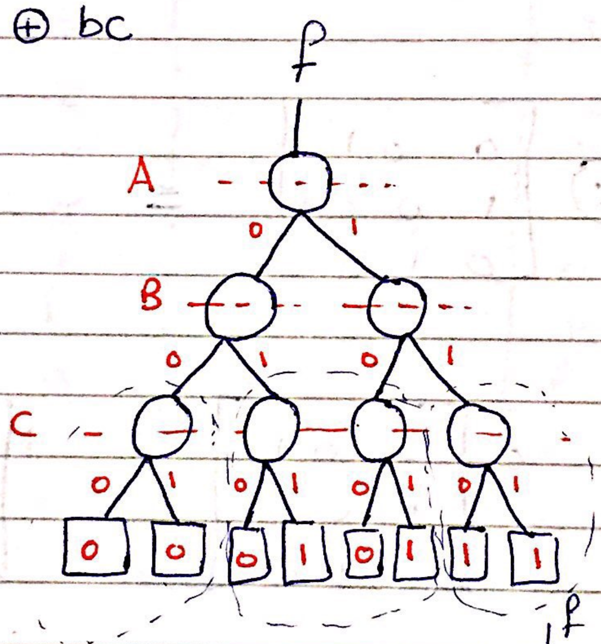


* Binary Diagram (BD)
Binary Desicion Diagram (BDD)

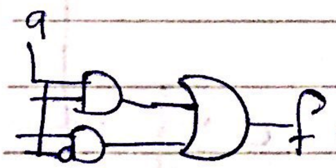
① Join all isomorphic nodes.
② Remove all redundant nodes.

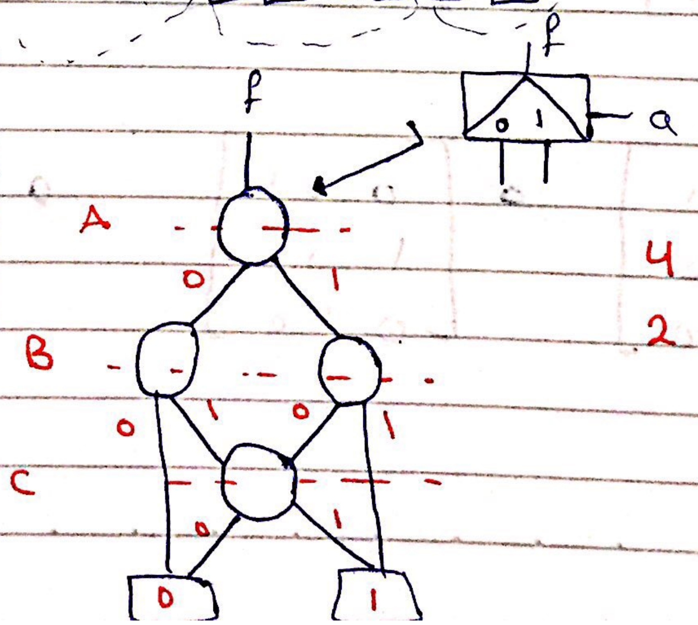
- Advantages:
  ① Reduce Size
  ② Reduce power consumption.
  ③ Reduce delay
  ④ Reduce cost.

* $f = ab \oplus ac \oplus bc$




7 nodes
8 leafs.



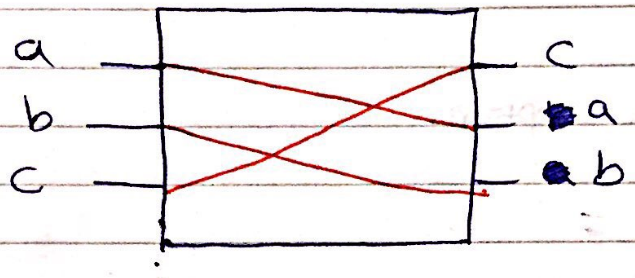




Multiplixer.

4 nodes
2 leafs.

* Butterfly Diagrams.

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} c \\ a \\ b \end{bmatrix}$$




Swapping

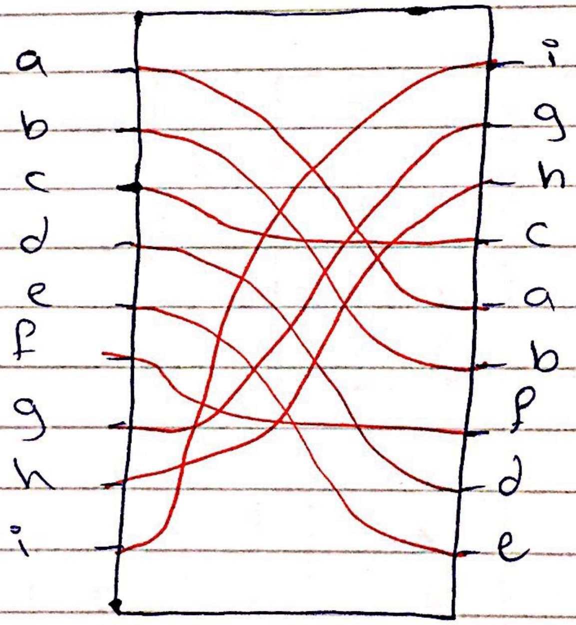
*

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} & 0\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} & 1\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\ 1\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} & 0\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} & 0\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{bmatrix}$$

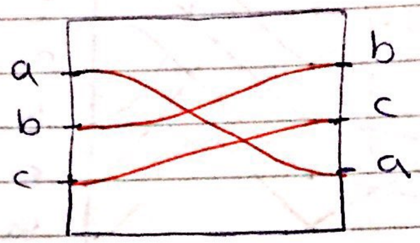
$9 \times 9$

$$\begin{bmatrix} 000 & 000 & 001 \\ 000 & 000 & 100 \\ 000 & 000 & 010 \\ 001 & 000 & 000 \\ 100 & 000 & 000 \\ 010 & 000 & 000 \\ 000 & 001 & 000 \\ 000 & 100 & 000 \\ 000 & 010 & 000 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{bmatrix} = \begin{bmatrix} i \\ g \\ h \\ c \\ a \\ b \\ f \\ d \\ e \end{bmatrix}$$



26th. Mar

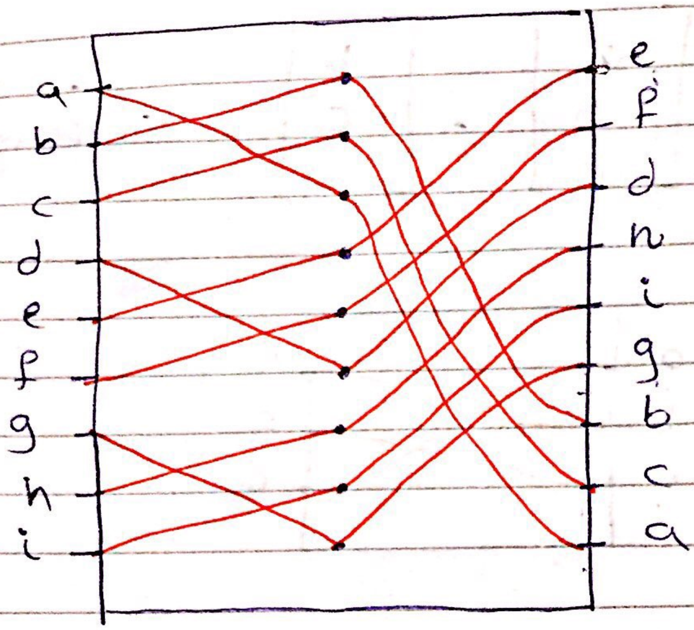
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} b \\ c \\ a \end{bmatrix}$$ Swapping.




* Kronikal product.

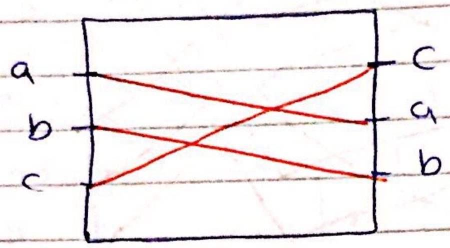
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} & 1\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} & 0\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \\ 0\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} & 0\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} & 1\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 000 & 010 & 000 \\ 000 & 001 & 000 \\ 000 & 100 & 000 \\ 000 & 000 & 010 \\ 000 & 000 & 001 \\ 000 & 000 & 100 \\ 010 & 000 & 000 \\ 001 & 000 & 000 \\ 100 & 000 & 000 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{bmatrix} = \begin{bmatrix} e \\ f \\ d \\ h \\ i \\ g \\ b \\ c \\ a \end{bmatrix}$$



a
b
c
d
e
f
g
h
i
e
f
d
h
i
g
b
c
a

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} c \\ a \\ b \end{bmatrix}$$




kernel.

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0&0&0 & 0&0&0 & 0&0&1 \\ 0&0&0 & 0&0&0 & 1&0&0 \\ 0&0&0 & 0&0&0 & 0&1&0 \\ 0&0&1 & 0&0&0 & 0&0&0 \\ 1&0&0 & 0&0&0 & 0&0&0 \\ 0&1&0 & 0&0&0 & 0&0&0 \\ 0&0&0 & 0&0&1 & 0&0&0 \\ 0&0&0 & 1&0&0 & 0&0&0 \\ 0&0&0 & 0&1&0 & 0&0&0 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{bmatrix} = \begin{bmatrix} i \\ g \\ h \\ c \\ a \\ b \\ f \\ d \\ e \end{bmatrix}$$

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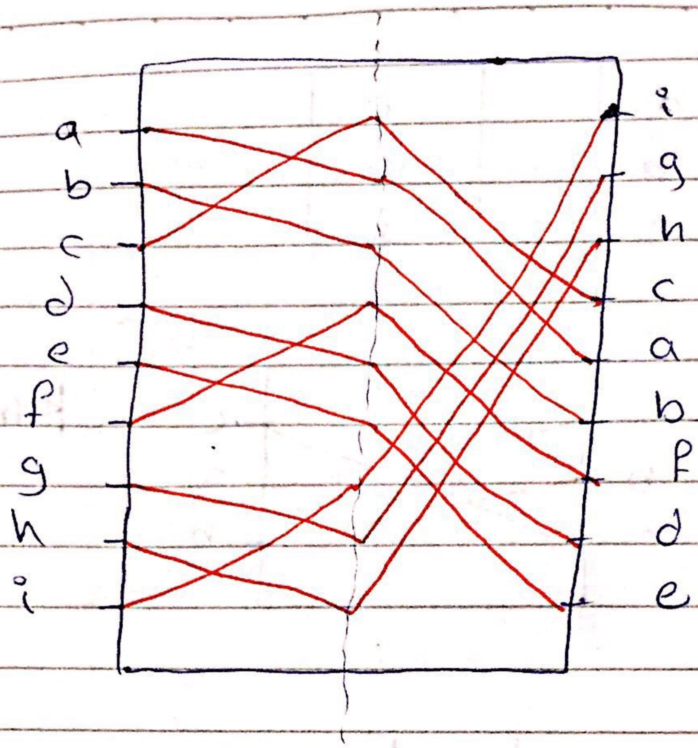
a
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e

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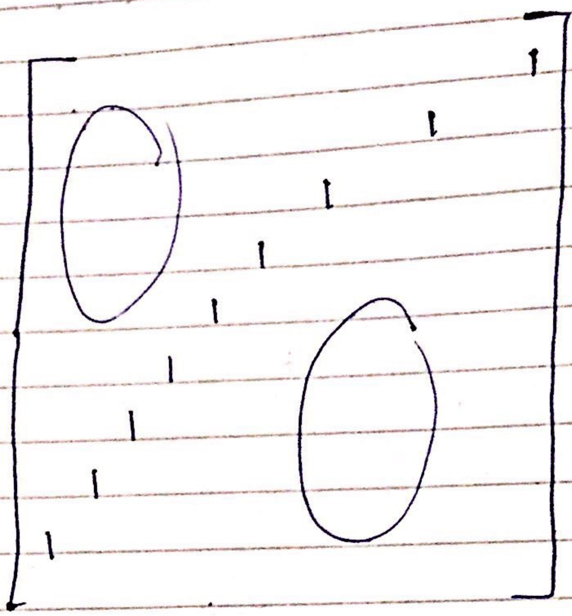
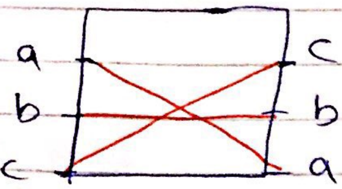
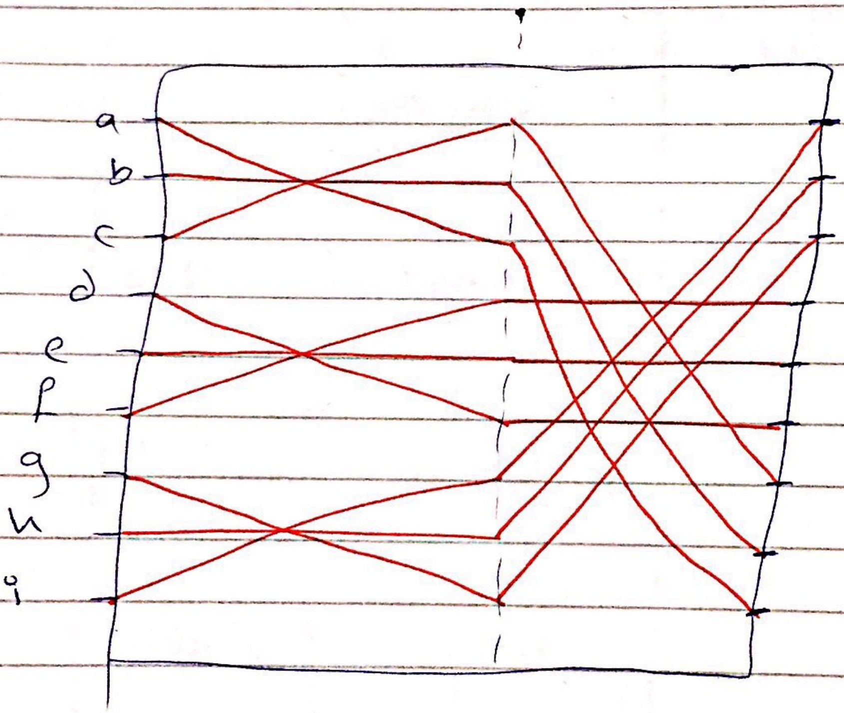
$$* \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow$$



7th. Apr

* FFT
Fast Forrier Transform.

* C FT (Continuos)
↓
DFT (~~[illegible]~~). → FFT
Discrete

* FFT reduces DFT complexity from
$O(N^2)$ to $O(N \log_2 N)$

| * operation | DFT | FFT |
|---|---|---|
| - Complex Multiplication | $N^2$ | $\frac{N}{2}(\log_2 N - 1)$ |
| - Complex Addition | $N(N-1)$ | $N \log_2 (N)$ |
| - Real Multiplication | $4N^2$ | $2N(\log_2 (N) - 1)$ |
| - Real Addition | $N(4N-2)$ | $2N \log_2 (N)$ |

N: # of points.

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9th. Apr

## * Weight Factor

$$W_N^{s} = e^{-2\pi i\left(\frac{s}{N}\right)}$$

$i = \sqrt{-1}$ , $N$: # of points

$$\boxed{s=0} \Rightarrow W_N^{0} = e^{-2\pi i\left(\frac{0}{N}\right)} = \boxed{1}$$

$$\boxed{s=\frac{N}{2}} \Rightarrow W_N^{N/2} = e^{-2\pi i\left(\frac{N/2}{N}\right)} = e^{-\pi i}$$

$$= \cos(-\pi) + \cancel{i\sin(-\pi)}^{\,0}$$

$$= \boxed{-1}$$

$$\boxed{S = s+\frac{N}{2}} \Rightarrow W_N^{s+\frac{N}{2}} = e^{-2\pi i\left(\frac{s+\frac{N}{2}}{N}\right)} = e^{-2\pi i\left(\frac{s}{N}\right)} \cdot \cancel{e^{-2\pi i\left(\frac{N/2}{N}\right)}}^{\,-1}$$

$$= -e^{-2\pi i\left(\frac{s}{N}\right)}$$

$$= \boxed{-W_N^{s}}$$

$$\boxed{s=nk} \Rightarrow W_N^{nk} = \boxed{e^{-2\pi i\left(\frac{nk}{N}\right)}}$$

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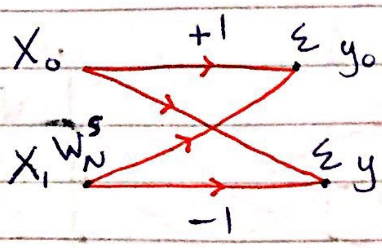
DATE 11th. Apr

## *2-point FFT (Butterfly diagram)

$(x_0, x_1) \longrightarrow (y_0, y_1)$




$$y_0 = X_0 + X_1 e^{-2\pi i\left(\frac{s}{N}\right)}$$

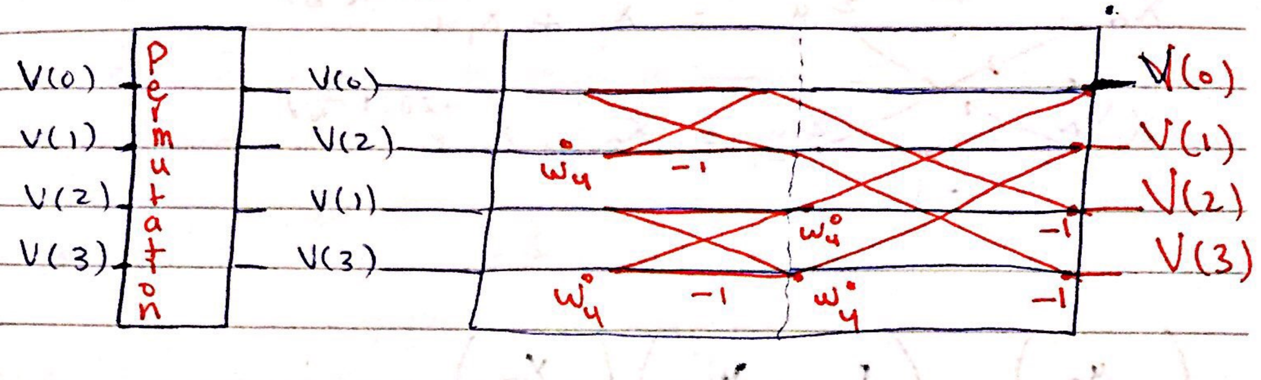
$$y_1 = X_0 - X_1 e^{-2\pi i\left(\frac{s}{N}\right)}$$

$$\begin{pmatrix} y_0 \\ y_1 \end{pmatrix} = \begin{pmatrix} 1 & W_N^s \\ 1 & -W_N^s \end{pmatrix} \begin{pmatrix} X_0 \\ X_1 \end{pmatrix}$$

14th. Apr

## * 4-point FFT:

$\{v(0),\ v(1),\ v(2),\ v(3)\} \longrightarrow \{V(0),\ V(1),\ V(2),\ V(3)\}$



*8-point FFT:

x(0), x(1), ——, x(7) ⟶ (X(0), X(1), ——, X(7))