logic :- propositional

proposition : statement that is true (T) or False (F)

Ex :

* Today is Monday F (proposition)
* 1+1 = 2 T (proposition)
* what time is it ? (not proposition) تحتمل أكثر من جواب
* Eat your lunch (not proposition)

* Proposition
  - atomic (no connectors)
  - compound (connectors)

* connectors :

1. Negation (not) $\neg$ (N)

Ex : * I don't speak Japanese $\neg X \Rightarrow$ (not X)

$\underbrace{\text{speak Japanese}}_{X}$

| * X | $\neg X$ |
|---|---|
| T $\Rightarrow$ | F |
| F $\Rightarrow$ | T |

2. Conjunction (and) $\wedge$ , * $\cup \Rightarrow$ and أيضاً التقاطع وتكافئ ال

Ex : * I like tea and coffee

$\underbrace{\text{I like tea}}_{P}$ and $\underbrace{\text{I like coffe}}_{Q}$ $\Rightarrow$

| P | Q | $P \wedge Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

* Tom and sami are brothers

هذه and ليست connector

3. Disjunction (or) $\vee$ (inclusive) , $\oplus$ (exclusive)

Ex : * Today is hot or sunny

$\underbrace{\text{Today is hot}}_{X}$ or $\underbrace{\text{Today is sunny}}_{Y}$ $\Rightarrow X \vee Y \Rightarrow$ لأنه احتمال الاثنين مع بعض يكونو

* Today is sunday or Monday

$\underbrace{\text{Today is sunday}}_{X}$ or $\underbrace{\text{Today is Monday}}_{Y}$ $\Rightarrow X \oplus Y$ لأنه الاثنين ما يكونوا مع بعض .

$\Rightarrow$

| X | Y | X ∨ Y | X ⊕ Y |
|---|---|---|---|
| T | T | T | F |
| T | F | T | T |
| F | T | T | T |
| F | F | F | F |

* and / or $\Rightarrow$ " إشارات تبديلية "

4. Implication ( if ---- then ) $\rightarrow$

Ex : * if $\underbrace{\text{you study}}_{P}$ then $\underbrace{\text{you pass}}_{Q}$

$P \longrightarrow Q$ ( وهي ليست تبديلية )

$\Rightarrow$

| P | Q | P → Q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

$\Rightarrow$ F هاي الوحيدة اللي بتعطي

* $P \longrightarrow Q$ ( جمل تدل على (Implication) )

1. if P then Q
2. if P , Q
3. P implies Q
4. Q if P
5. P is sufficient for Q
6. P only if Q
7. Q is necessary for P
8. Q whenever P

$P \rightarrow Q$

* inverse : $\neg P \rightarrow \neg Q$

* converse : $Q \rightarrow P$

* contrapositive : $\neg Q \rightarrow \neg P$

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5. Biconditional $\leftrightarrow$ (if and only if) (iff)

(تبديلية) (sufficient and necessary)

Ex: $\underbrace{\text{An integer is even}}_{P}$ iff $\underbrace{\text{it's divisable by 2}}_{Q}$

$P \leftrightarrow Q$

$\Rightarrow$

| P | Q | $P \leftrightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

* ال Biconditional هي معكوس ال $\oplus$

* Translate : English $\rightarrow$ logic (propositions)

1. $\boxed{\text{you'll get an A}}_{P}$ iff $\boxed{\text{you do every exercise}}_{Q}$ or $\boxed{\text{you get over 35 in final}}_{R}$.

$P \leftrightarrow (Q \vee R) \Rightarrow$ ولو تشليتهم عادي حسب الأولوية

* precedence : (الأولوية)
* $\neg$
* $\wedge$
* $\vee$
* $\rightarrow$
* $\leftrightarrow$

2. you'll get an A if you do every exercise or you get over 35 in final

$(Q \vee R) \rightarrow P$

3. you'll get an A only if you do every exercise or you get over 35 in final

$P \rightarrow (Q \vee R)$

4. $\boxed{\text{You get an A}}_{P}$ but $\boxed{\text{you}}$ don't $\boxed{\text{do every exercise}}$ Q

and نفي

$P \wedge \neg Q$

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5. $\boxed{\text{Getting over 35}}$ and $\boxed{\text{doing every exercise}}$ is sufficient $\boxed{\text{for getting A}}$

P = Getting over 35, Q = doing every exercise, R = for getting A

$(P \land Q) \rightarrow R$

* H.W :

1. You get over 35 (P) but you don't do every exercise (Q) nevertheless you get an A (R).

and لعن نفس

$$P \land \neg Q \land R$$

2. You can't take a medicine ($\neg P$) unless you are sick (Q).

$$\neg Q \rightarrow \neg P$$

3. You can't get at A (P) unless you get over 35 (Q) and do every exercise (R)

$$\neg (Q \land R) \rightarrow \neg P \quad / \quad \neg Q \lor \neg R \rightarrow \neg P$$

* Tautology : compound proposition that is always true

Ex : $P \lor T$ , $P \rightarrow T$ , $F \rightarrow Q$ , $P \lor \neg P$

* Contradiction : compound proposition that is always false

Ex : $P \land F$ , $P \land \neg P$ , $P \leftrightarrow \neg P$ , $P \oplus P$

* contingency : not tautology nor contradiction

Ex : $P \land Q$, $P \lor Q$ , $P \rightarrow Q$, ..... ( كل الجداول السابقة )

* proof :- Equivalence :
  - ① By truth table
  - ② Equivalence rules

① Truth table :

Ex : Show if $P \rightarrow Q$ and its converse are equivalent :

Sol. $P \rightarrow Q \overset{?}{\equiv} Q \rightarrow P$ $\qquad Q \rightarrow P$

Ex: show that $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

left ← | | | | | Right
| P | Q | $P \vee Q$ | $\neg(P \vee Q)$ | $\neg P$ | $\neg Q$ | $\neg P \wedge \neg Q$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | T | F | F | T | F |
| F | T | T | F | T | F | F |
| F | F | F | T | T | T | T |

$\therefore \neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

② Using rules

* Equivelence rules :-

① $P \wedge T \Leftrightarrow P$ ⑤ $P \vee P \Leftrightarrow P$

② $P \vee F \Leftrightarrow P$ ⑥ $P \wedge P \Leftrightarrow P$

③ $P \wedge F \Leftrightarrow F$ ⑦ $\neg(\neg P) \Leftrightarrow P$

④ $P \vee T \Leftrightarrow T$ ⑧ $P \vee Q \Leftrightarrow Q \vee P$

⑨ $P \wedge Q \Leftrightarrow Q \wedge P$

⑩ $P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$

⑪ $P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$

⑫ $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$

⑬ $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$

⑭ $P \rightarrow Q \Leftrightarrow \neg P \vee Q$ ⑮ $P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$

⑯ $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ ⑰ $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$ $\equiv \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

Ex] show that $\underbrace{\neg(P \vee (\neg P \wedge Q))}_{L} \Leftrightarrow \underbrace{\neg P \wedge \neg Q}_{R}$

* نبدأ بالأطول :

$\neg(P \vee (\neg P \wedge Q)) \Leftrightarrow$

$\neg P \wedge \neg(\neg P \wedge Q) \Leftrightarrow \neg P \wedge (P \vee \neg Q)$

$\Leftrightarrow (\neg P \wedge P) \vee (\neg P \wedge \neg Q)$

$\Leftrightarrow F \vee (\neg P \wedge \neg Q) \Leftrightarrow \neg P \wedge \neg Q$

Ex : Show that : $(P \wedge Q) \rightarrow (P \vee Q)$ is tautology

$(P \wedge Q) \rightarrow (P \vee Q) \Leftrightarrow T$ ( Tautology )

$\therefore (P \wedge Q) \rightarrow (P \vee Q) \Leftrightarrow \neg (P \wedge Q) \vee (P \vee Q)$

$\Leftrightarrow (\neg P \vee \neg Q) \vee (P \vee Q)$

$\Leftrightarrow (\neg P \vee P) \vee (\neg Q \vee Q) \Leftrightarrow T \vee T \Leftrightarrow T$

Ex : Show that : $(P \leftrightarrow Q) \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

$P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$

$(\neg P \vee Q) \wedge (\neg Q \vee P)$

$(\neg P \wedge \neg Q) \vee (\neg P \wedge P) \vee (Q \wedge \neg Q) \vee (Q \wedge P)$

$(\neg P \wedge \neg Q) \vee F \vee F \vee (Q \wedge P)$

$(\neg P \wedge \neg Q) \vee (Q \wedge P)$

Ex : Show that : $[(P \rightarrow Q) \wedge (Q \rightarrow R) \rightarrow (P \rightarrow R)] \Leftrightarrow T$

Ex : Show that : $P \wedge (P \vee Q) \Leftrightarrow P$

$(P \wedge P) \vee (P \wedge Q) \Leftrightarrow P \vee (P \wedge Q) \Leftrightarrow P$

* Mathematical reasoning :-

Prove that conclusion logically follows from premises → Truth table / → Rules (inference)

EX: if you get full mark in final, you well get an A.  (P = you get full mark in final, Q = you well get an A)

you get a full mark in final.

⇒ You'll get an A

1. $P \rightarrow Q$ ] → premises (always true) $(P_1)$ $(P_2)$
2. $P$
3. $\therefore Q$ ] → conclusion (c)

⇒ Truth table :

| P ($P_2$) | Q (c) | $P \rightarrow Q$ ($P_1$) |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

* ما بنطلع عليهم كلهم باخذ بس اللي ال premises كله True

أو بكمل الجدول هيك

| $P_1 \wedge P_2$ | $P_1 \wedge P_2 \rightarrow C$ → (always true) |
|---|---|
| T | T |
| F | T |
| F | T |
| F | T |

$\therefore P_1 \wedge P_2 \Rightarrow C$

$((P \rightarrow Q) \wedge P) \Rightarrow Q$

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Ex: if it's hot then we will swim.
we will swim
=> it's hot

1. $P \to Q$
2. $Q$

$\therefore P$

| C | $P_2$ | $P_1$ | | |
|---|---|---|---|---|
| $P$ | $Q$ | $P \to Q$ | $P_1 \land P_2$ | $P_1 \land P_2 \to C$ |
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | T | F |
| F | F | T | F | T |

F لما ماخذناه conclusion لأنه F

$\therefore$ Not tautology, conclusion is not correct (fallicy)

$\therefore ((P \to Q) \land Q) \Rightarrow P$ X

Ex: if n is odd then 2n is even.
n is not odd therfor 2n is not even.

1. $P \to Q$
2. $\neg P$

$\therefore \neg Q$

وبالتالي

| $P$ | $Q$ | $P \to Q$ ($P_1$) | $\neg P$ ($P_2$) | $\neg Q$ (C) |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | F | F | T |
| F | T | T | T | F |
| F | F | T | T | T |

بعد ما أخذنا ← الخط اللي فيه

$P_1 / P_2 \Leftarrow$ true يشوف C لازم تكون true فيهم

$\therefore$ fallicy

Ex: If you send me email, I will finish the program.
If you don't send me email, I will sleep
If I go sleep, then I wake up fresh therfore If I don't finish the program then I will wake up fresh

تدل على الـ C

1. $E \to P$
2. $\neg E \to S$
3. $S \to W$

$\therefore \neg P \to W$

| $E$ | $P$ | $S$ | $W$ | $\neg E$ | $\neg P$ | $P_1$: $E \to P$ | $P_2$: $\neg E \to S$ | $P_3$: $S \to W$ | $C$: $\neg P \to W$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | F | F | T | T | T | T |
| T | T | T | F | F | F | T | T | F | T |
| T | T | F | T | F | F | T | T | T | T |
| T | T | F | F | F | F | T | T | T | T |
| T | F | T | T | F | T | F | T | T | T |
| T | F | T | F | F | T | F | T | F | F |
| T | F | F | T | F | T | F | T | T | T |
| T | F | F | F | F | T | F | T | T | F |
| F | T | T | T | T | F | T | T | T | T |
| F | T | T | F | T | F | T | T | F | T |
| F | T | F | T | T | F | T | F | T | T |
| F | T | F | F | T | F | T | F | T | T |
| F | F | T | T | T | T | T | T | T | T |
| F | F | T | F | T | T | T | T | F | F |
| F | F | F | T | T | T | T | F | T | T |
| F | F | F | F | T | T | T | F | T | F |

∴ It's tautology ⇒ the conclusion is correct

$$P_1 \wedge P_2 \wedge P_3 \Longrightarrow C$$

* Reasoning :

⇒ Inference rules ;

1. $\dfrac{P}{P \vee Q}$ $\quad P \Rightarrow P \vee Q$

2. $\dfrac{\begin{array}{l} P \to \text{true} \\ Q \to \text{true} \end{array}}{P \wedge Q}$ $\quad (P) \wedge (Q) \Longrightarrow P \wedge Q$

3. $\dfrac{P \wedge Q}{P}$ $\quad P \wedge Q \Rightarrow P$

$\dfrac{P \wedge Q}{Q}$ $\quad P \wedge Q \Rightarrow Q$

4. $\dfrac{\begin{matrix} P \to Q \\ P \end{matrix}}{Q}$ $\quad P \to Q \wedge P \Rightarrow Q$

5. $\dfrac{\begin{matrix} P \to Q \\ \neg Q \end{matrix}}{\neg P}$ $\quad P \to Q \wedge \neg Q \Rightarrow \neg P$

6. $\dfrac{\begin{matrix} P \to Q \\ Q \to R \end{matrix}}{P \to R}$ $\quad (P \to Q) \wedge (Q \to R) \Rightarrow P \to R$ (is tautology)

7. $\dfrac{\begin{matrix} P \vee Q \\ \neg P \end{matrix}}{Q}$ $\quad (P \vee Q) \wedge \neg P \Rightarrow Q$

EX 8 If you send me email ($E$), I will finish the program ($P$). If you don't send me email ($\neg E$), I will sleep ($S$). If I go sleep ($S$), then I wake up fresh ($W$) therefore If I don't finish the program then I will wake up fresh.

تطبيق

Sol. $E \to P$

$\neg E \to S$

$\underline{S \to W}$ $\quad \to$ 6 قانون

$\neg P \to W$

① $\dfrac{\begin{matrix} \neg E \to S \\ S \to W \end{matrix}}{\neg E \to W}$

② $\neg E \to W \Leftrightarrow \neg W \to E$

$\Downarrow$

③ $\dfrac{\begin{matrix} \neg W \to E \\ E \to P \end{matrix}}{\neg W \to P}$ $\quad$ 6 قانون

④ $\neg W \to P \Leftrightarrow \neg P \to W$

Ex 8: 1. $\neg P \wedge Q$

2. $R \rightarrow P$

3. $\neg R \rightarrow S$

4. $\underline{S \rightarrow A}$

$A$

*ممكن تستخدمي المعطى أكثر من مرة*

* بس لا يفضل

① $\dfrac{\neg R \rightarrow S \qquad S \rightarrow A}{\neg R \rightarrow A}$ $\Rightarrow$ ② $\neg R \rightarrow A \Leftrightarrow \neg A \rightarrow R$

③ $\dfrac{\neg A \rightarrow R \qquad R \rightarrow P}{\neg A \rightarrow P}$

④ $\dfrac{\neg P \wedge Q}{\neg P}$

⑤ $\dfrac{\neg A \rightarrow P \qquad \neg P}{\neg\neg A \Leftrightarrow A}$

Ex 8: 1. $P \rightarrow Q \vee R$

2. $P \wedge S$

3. $\underline{\neg Q}$

$R \vee S$

① $\dfrac{P \wedge S}{P}$ ② $\dfrac{P \rightarrow Q \vee R \qquad P}{Q \vee R}$ ③ $\dfrac{Q \vee R \qquad \neg Q}{R}$

④ $\dfrac{R}{R \vee S}$

Ex 8: 1. $P \vee Q \rightarrow R$

2. $S \vee \neg Q$

3. $Z$

4. $P \wedge R \rightarrow \neg S$

5. $\underline{\neg P \rightarrow \neg Z}$

$\neg Q$

① $\neg P \to \neg Z$

$\dfrac{Z}{\boxed{P}}$

② $S \lor \neg Q \iff Q \to S$

③ $Q \to S \iff \neg S \to \neg Q$

④ $\neg S \to \neg Q$

$\dfrac{P \land R \to \neg S}{\boxed{P \land R \to \neg Q}}$

⑤ $\dfrac{P}{P \lor Q}$

⑥ $P \lor Q \to R$

$\dfrac{P \lor Q}{\boxed{R}}$

⑦ $P$

$\dfrac{R}{P \land R}$

⑧ $P \land R \to \neg Q$

$\dfrac{P \land R}{\neg Q}$

---

EX: Every student has a book

$\dfrac{\text{Ali is a student}}{\text{Ali has a book}}$

$P$

$\dfrac{Q}{R}$ ⇒ هونا مابنقع نستخدم خطوات قبل لأنه بطلعلنا ناتج جديد. نستخدم ( predicates )

* predicates :-

* $\underbrace{\text{Ali}}_{\text{Subject}}$ $\underbrace{\text{has a book}}_{\text{Predicate}}$.

book ( Ali ) arity = 1 ( 1 subject )

* $\underbrace{\text{Ali}}_{\text{Subject 1}}$ and $\underbrace{\text{sami}}_{\text{Subject 2}}$ are $\underbrace{\text{friends}}_{\text{predicate}}$

Friend (Ali, sami) arity = 2 ( 2 subjects )

* Ali has a book and sami has a book.

book (Ali) $\wedge$ book (sami)

* $1 + 1 = 2$

Sum (1, 1, 2) arity = 3 (3 subjects)

* عادةً بالسؤال يطلب منك تحكي على proposition أو على predicate

* كل جملة proposition هي predicate بس العكس لا.

* 2 is negative

$n(2) \Rightarrow F \rightarrow$ ( predicate and proposition ) * arity = 1

* x is negative

$n(x) \Rightarrow$ No truth value $\Rightarrow$ No proposition * arity = 1

* Every x is negative

معناها for all $\leftarrow \forall x \Rightarrow n(x) \Rightarrow F \Rightarrow$ It's proposition * arity = 1

* Some x is negative : $\exists x \; n(x) \; T \Rightarrow$ It's Proposition * arity = 1

* Quantifiers :-

① Universal :

$\forall \Rightarrow$ " ويجي بعده متغير دائماً "

$\forall$ ( Every , All, Any , Each, ---- ) $\Rightarrow$ دلالاته

② Existental :

$\exists$ ( some, few, at least, there is , --- )

Ex 8 * $\underbrace{\text{square of}}_{P}$ $\underbrace{3}_{S_1}$ is $\underbrace{9}_{S_2}$

S (3, 9)

* square of 3 is 9 and square of 9 is 81

S (3, 9) $\wedge$ S (9, 81) $\Rightarrow$ لازم نفس الترتيب.

* square of x is y

S (x, y)

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* 100 is square of some number

$\exists x\ S(x, 100)$ => It's proposition (true قيمة)

* 9 is square of every number

$\forall x\ S(x, 9)$ => it's proposition (false قيمة)

* Every number is square of 2

$\forall x\ S(2, x)$ => It's proposition (false قيمة)

* There is number which is square of 2

$\exists x\ S(2, x)$ => It's proposition (true قيمة)

* 2 has a square

$\exists x\ S(2, x)$ => It's proposition (true قيمة)

* predicats :-

EX] Assume

$V(x, y)$ : x visited y

$T(x)$ : x is teacher

$S(x)$ : x is student

Domain : people

1. Ali visited Sami

$V(Ali, Sami)$

2. Ali visited someone.

$\exists x\ V(Ali, x)$

↳ Someone شان

3. Ali visited every one

$\forall x\ V(Ali, x)$

↳ every one شان

4. Someone visited Ali

$\exists x\ V(x, Ali)$

↳ Someone شان

5. everyone visited Ali

$\forall x\ V(x, Ali)$

↳ everyone شان

6. No one visited Ali

$\neg \exists x\ V(x, Ali)$

«لا تنسونا من صالح دعائكم»

- Atomic
- Compound

7. Everyone didn't visit Ali

$\forall x \; \neg V(x, Ali)$ => وهي نفسها ٦ بس مكان الـ "¬" يتنقل

8. Not every one visited Ali ≡ Someone didn't visit Ali

$\neg \forall x \cdot V(x, Ali) \equiv \exists x \; \neg V(x, Ali)$

9. Someone visited Someone

$V(x, y)$

10. Everyone visited everybody

$\forall x \forall y \; V(x, y) \equiv \forall y \forall x \; V(x, y)$

11. Someone visited everyone → "هون الشخص نفسه هو اللي زار الكل"

$\exists x \forall y \; V(x, y)$ → « هنا مابصير أعكس »

12. Everyone was visited by Someone → "هونا مش نفس الشخص اللي زار الكل"

$\forall y \exists x \; V(x, y)$

13. Some teachers visited Ali

$\exists x \; [\, t(x) \land V(x, Ali) \,]$

( قاعدة بحطوها عشان أعرف إنه الـ Teachers هم "x" وبينها إشارة الـ

and (∧) )

14. Some student visited Ali.

$\exists x \; [\, S(x) \land U(x, Ali) \,]$

15. All teachers visited Ali.

$\forall x \; [\, t(x) \rightarrow V(x, Ali) \,]$ → ( هاي القاعدة بيستخدموها لما يكون All مش

Someone وعشان أعرف إنه x <= Teachers )

16. All Students visited Ali..

$\forall x \; [\, S(x) \rightarrow V(x, Ali) \,]$

17. Every teacher visited or has been visited by Ali

$\forall x \; [\, t(x) \rightarrow (V(x, Ali) \lor V(Ali, x)) \,]$

← أو بدون أقواس على كلٍ الأولوية لـ or

18. Every teacher visited or has been visited by some student

Some or a

$\forall x [t(x) \rightarrow \exists y \ (s(y) \wedge v(x,y) \ \vee \ v(y,x))]$

19. There is a teacher who has never been visited by any student.

$\exists x [t(x) \wedge \forall(y) ( s(y) \rightarrow \neg V(x,y) ]$

$\exists x [t(x) \wedge \neg \exists(y) (s(y) \wedge V(x,y) ]$

20. Some students have visited every teacher

$\exists x [ s(x) \wedge \forall(y) ( t(y) \rightarrow V(x,y))]$

21. All students visited Ali and some teachers too

$\forall x [ S(x) \rightarrow \exists y ( t(y) \wedge V(x, Ali) \wedge V(x,y))]$

22. Ali visited everyone but no one visited him

$\forall x [V(Ali, x) \wedge \neg \exists x \ V(x, Ali)]$

- Predicates :-

Ex : Find truth value :-

1. $\forall x \, P(x)$ $\Rightarrow$ True

$P(x) : x+1 > x$ Domain : $\mathbb{R}$

2. $\exists x \ P(x)$

$P(x) = x+1 > x$ $\Rightarrow$ True

Domain : $\mathbb{R}$

3. $\forall(x) \ Q(x)$

$Q(x) : x < 2$ $\Rightarrow$ False $\Leftrightarrow$ $\exists(x) \ p(x)$ مافيرأحدد

Domain : $\mathbb{R}$

4. $\exists(x)\ p(x)$ $\Rightarrow$ True

$p(x): x > 3$

Domain: $\mathbb{R}$

5. $\exists x\ Q(x)$

$Q(x): x = x+1$ $\Rightarrow$ False. $\Leftrightarrow$ $\forall(x)\ Q(x) \Rightarrow$ False يرهنه

Domain: $\mathbb{R}$

6. $\forall x\ \forall y\ p(x,y)$

$p(x,y): x+y = y+x$ $\Rightarrow$ True

Domain: $\mathbb{R}$

7. $\exists y\ \forall x\ Q(x,y)$ $\Rightarrow$ False

$Q(x,y): x+y = 0$

Domain: $\mathbb{R}$

8. $\exists y\ \forall x\ Q(x,y)$

$Q(x,y): y * x = 0$ $\Rightarrow$ True

Domain: $\mathbb{R}$

9. $\forall x\ \exists y\ Q(x,y)$

$Q(x,y): x+y = 0$ $\Rightarrow$ True

Domain: $\mathbb{R}$

10. $\forall x\ \exists y\ Q(x,y)$

$Q(x,y): x+y = 0$ $\Rightarrow$ True

Domain: $\mathbb{R}$

11. $\exists x \; \forall y \; \forall z \; Q(x,y,z)$

$Q(x,y,z): \; x = y + z$ $\Rightarrow$ False

Domain: $\mathbb{R}$

لأنه معناها كل الأعداد حاصل جمعها x دائماً.

12. $\forall y \; \forall z \; \exists x \; Q(x,y,z)$

$Q(x,y,z): \; x = y + z$ $\Rightarrow$ True

Domain: $\mathbb{R}$

معناها أي عددين إذا جمعتهم إلهم قيمة تساوي x

13. $\forall x \; P(x)$

$P(x): \; x^2 < 10$

Domain: positive integers not exceeding 4 $\Rightarrow$ False

↳ $\{1, 2, 3, 4\}$

* وإذا كانت $\exists x \; P(x)$

بنفس الـ domain و $P(x): x^2 < 10$

$\Leftarrow$ True

- Reasoning :-

Inference rule :-

① $\dfrac{\forall x \; P(x)}{P(c)}$

↳ arbitary (c is element)

متغير عشوائي

$\dfrac{P(c) \rightarrow \text{متغير عشوائي}}{\forall x \; P(x)}$

② $\dfrac{\exists x \; P(x)}{P(c)}$ → (particular element) متغير محدد

$\dfrac{P(c) \rightarrow \text{متغير محدد}}{\exists x \; P(x)}$

Ex: Every student has a book

Ali is a student

---

Ali has a book

1. $\forall x \; [\, S(x) \longrightarrow b(x)$ $\Rightarrow$

2. $S(\text{Ali})$

---

$b(\text{Ali})$

* $\dfrac{\forall x \,(S(x) \rightarrow b(x))}{S(\text{Ali}) \rightarrow b(\text{Ali})}$

* $\overset{P}{S(\text{Ali})} \longrightarrow \overset{Q}{b(\text{Ali})}$

وعلى القوانين السابقة ← $P$ $S(\text{Ali})$

---

$b(\text{Ali})$

Ex: Everyone who wins is rich
Mary wins
Someone is rich

و كشان الفيا بعوض متغير عندي

$* \ \forall x [w(x) \rightarrow r(x)]$

$w(Mary) \rightarrow r(Mary)$

1. $\forall x [w(x) \rightarrow r(x)]$
2. $w(Mary)$

$\exists x \ r(x)$

$\Rightarrow$

$* \ w(Mary) \rightarrow r(Mary)$

$w(Mary)$

$r(Mary)$

$* \ \dfrac{r(Mary)}{\exists x \ r(x)}$ « لأنه Mary اسم محدد »

Ex: Every IT student likes programming
Everyone is IT student
Everyone likes programming

1. $\forall x (ITs(x) \rightarrow p(x))$
2. $\forall x \ ITs(x)$

$\forall x \ p(x)$

$\Rightarrow$

1. $\forall x [ITs(x) \rightarrow P(x)]$

$ITs(e) \rightarrow P(e)$

(كشوائي منكشي) arbitary element

2. $\forall x \ ITs(x)$

$IT \ s(e)$

3. $ITs(e) \rightarrow P(e)$

$IT \ s(e)$

$P(e)$

arbitary

4. $P(e)$

$\forall x \ p(x)$

$\Leftarrow$

Ex : Every IT students like programming
Some are IT students
Some like programming

1. $\forall x [ ITs(x) \rightarrow p(x) ]$
2. $\exists x \; ITs(x)$

$\exists x \; p(x)$

* (لازم أبدأ بهذه الخطوة)

$\Rightarrow$

$\exists x \; ITs(x)$

$ITs(e) \rightarrow$ particular

* $\forall x [ ITs(x) \rightarrow p(x) ]$

$ITs(e) \rightarrow p(e)$

* $p(e)$ $\rightarrow$ particular

$\exists x \; p(x)$

$\Leftarrow$

* $ITs(e)$

$ITs(e) \rightarrow p(e)$

$p(e)$

Ex : $c(Ali)$

$j(Ali)$

$\forall x [ j(x) \rightarrow h(x) ]$

$\exists x [ c(x) \wedge h(x) ]$

1. $\forall x [ j(x) \rightarrow h(x) ]$

$j(Ali) \rightarrow h(Ali)$

2. $j(Ali) \rightarrow h(Ali)$

$j(Ali)$

$h(Ali)$

3. $h(Ali)$

$c(Ali)$

$h(Ali) \wedge c(Ali)$

4. $h(Ali) \wedge c(Ali)$

$\exists x [ c(x) \wedge h(x) ]$

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Ex : $\forall x\,[J(x) \to f(x)]$

$\exists x\,[J(x) \wedge \neg s(x)]$

---

$\exists x\,[f(x) \wedge \neg s(x)]$

① $\exists x\,[J(x) \wedge \neg S(x)]$

$J(c) \wedge \neg S(c)$ محددة

⑧ $J(c) \wedge \neg S(c)$

$J(c)$

② $\forall x\,[J(x) \to f(x)]$

$J(c) \to f(c)$

④ $J(c) \to f(c)$

③ من ← $J(c)$

$f(c)$

⑤ $J(c) \wedge \neg S(c)$

$\neg S(c)$ $\Rightarrow$ ⑥ $f(c)$ → ④ من

⑤ من ← $\neg S(c)$

محددة $f(c) \wedge \neg S(c)$

⑦ $\exists x\,[f(x) \wedge \neg s(x)]$ $\Leftarrow$ $f(c) \wedge \neg S(c)$

* Theorem Proof :-

1. Direct $(P \to Q)$

Assume P is true ( "True" ⇐ P بفترض )

Prove Q

Ex : if $\underbrace{n \text{ is odd}}_{P}$ then $\underbrace{n^2 \text{ is odd}}_{Q}$

Assume n is odd : $n = 2k+1$

$n^2 = (2k+1)^2 \Rightarrow n^2 = 4k^2 + 4k + 1$

$\therefore 2(2k^2 + 2k) + 1 = 2(L) + 1$ odd

"L عدد معين"

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Ex: if n is odd then 3n+2 is odd

Assume n is odd $\Rightarrow n = 2K+1$

$3n+2 = 3(2K+1)+2 = 6K+3+2 = 6K+5$

$= 6K+4+1 = 2(3K+2)+1 = 2L+1$ odd

* نستخدم بعد الافتراض الطرفين للإثبات ..

Ex: if $3n+2$ is $\underbrace{\text{not odd}}_{\text{even}}$ then n is $\underbrace{\text{not odd}}_{\text{even}}$

Assume $3n+2$ is even

$3n+2 = 2K$ (صورة العدد الزوجي) $\Rightarrow n = \frac{2K-2}{3}$

لـ هنا المثال صعب اشتغل في direct إذا بروح

للـ indirect

2. Indirect $(P \rightarrow Q)$

1. Find contrapositive
2. Prove directly

Ex: if $n^2$ is odd then n is odd

① contrapositive

if n is even then $n^2$ is even

$n = 2K \Rightarrow n^2 = 4K^2 \Rightarrow n^2 = 2(2K^2) = 2L$ (even)

Ex: if $3n+2$ is odd then n is odd $\rightarrow$ indirect أسهل

* Indirect $\Rightarrow$ contrapositive: If n is even then $3n+2$ is even

$n = 2K \Rightarrow 3n+2 = 3(2K)+2 = 6K+2 = 2(3K+1)$

$= 2L$ (even)

EX: prove that $\sqrt{2}$ is irrational

نستخدمها على الطريقة الثالثة لأنه بس P عندي ماقي Q

عدد لايكتب على شكل $\frac{a}{b}$

3. Contradiction

Ex: prove that $\sqrt{2}$ is irrational

Assume $\sqrt{2}$ is rational → أخذت عكس اللي عندي

$\Rightarrow \sqrt{2} = \frac{a}{b} \Rightarrow a = \sqrt{2}\, b$ و rational تعني $\frac{a}{b}$

$\Rightarrow a^2 = 2b^2$

$\Rightarrow a^2$ is even $\Rightarrow a$ is even → "بالنظرية السابقة"

$a = 2K$

$(2K)^2 = 2b^2 \Rightarrow b^2 = 2K^2 \Rightarrow b^2$ is even → $b$ is even

$\therefore \sqrt{2} = \frac{a \to \text{even}}{b \to \text{even}}$ * مستحيل ألاقي كسر بسطه ومقامه even وموجود بأبسط صورة

$\therefore \sqrt{2} = \frac{a}{b}$ not simplest form

$\therefore \sqrt{2}$ is rational X

$\therefore \sqrt{2}$ is irrational

Ex: by contradiction, prove that:

if $\underbrace{3n+2 \text{ is odd}}_{P\ T}$ then $\underbrace{n \text{ is odd}}_{Q\ F}$

* Assume if $3n+2$ is odd then $n$ is even
* Indirect ⇒ if $n$ is odd then $3n+2$ is even

$n = 2K+1 \Rightarrow 3n+2 = 3(2K+1)+2 = 6K+3+2 = 6K+4+1$

$= 2(3K+2)+1 = 2L+1$ (odd) → اللي أنا فرضته غلط إذاً الجواب اللي بالسؤال صح.

H.W

Ex: ① The sum of 2 odd integers is even.

② If $m, n$ are 2 squares, then $mn$ is square

③ If $n$ is odd then $n^2$ is odd (contradiction)

① $n = (2K+1) + (2L+1) \Rightarrow 2K + 2L + 2$

$\Rightarrow 2(K + L + 1) \Rightarrow 2C$ (even)

② $m = K^2 \qquad n = R^2$

$\Rightarrow mn \Rightarrow K^2 * R^2 \Rightarrow (K * R)^2 \Rightarrow (C)^2$

③ $n = 2K+1$

$n^2 = (2K+1)^2$

$n^2 = 4K^2 + 4K + 1 \Rightarrow 2(2K^2 + 2) + 1$

$= 2L + 1 \Rightarrow$ odd

* contradiction → طريقة الحل

Assume if $n$ is odd then $n^2$ is even

$n = 2k+1 \quad / \quad n^2 = (2k+1)^2 = 4k^2 + 4k + 1$

$\Rightarrow 2(2k^2 + 2) + 1 = 2L + 1 \Rightarrow$ not even

لو فيه غلط إذاً الجواب اللي بالسؤال هو الصح

# Ch 2:

## * Sets :-

Sets : group of elements

Represent :-

1. Setbuilder

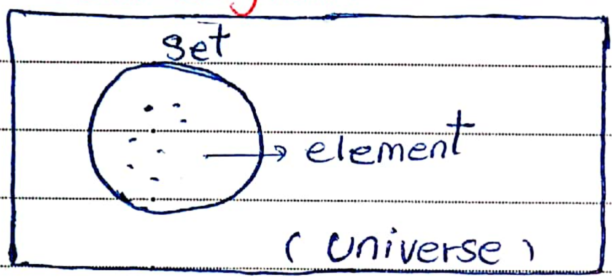
* $S = \{x \mid x$ is positive odd integer between 0 and 10 $\}$

such that

2. Listing

* $S = \{1, 3, 5, 7, 9\}$
* $U = \{10, 20, 30, 40, 50, ---\}$
* $V = \{---, -4, -2, 0, 2, 4, 6, ----\}$

3. Venn diagram

* 


## * Notes :

* $R \Rightarrow$ real numbers ( كل الأرقام )
* $Z \Rightarrow$ integers ( الأعداد الصحيحة )
* $Z^+ \Rightarrow$ Positive integers ( الأعداد الموجبة وتبدأ من 1 )
* $N \Rightarrow$ natural numbers ( الأعداد الموجبة وتبدأ من الصفر )

EX : $S = \{\emptyset, 1, \{1\}, \{1, 2\}\} \rightarrow$ 4 elements

$\{\ \}$ لـ نفسها

belongs ( الانتماء )

element $\in$ set

subset ( الاحتواء )

set $\underline{A} \subset$ set $\underline{B}$

$\forall x\, (x \in A \rightarrow x \in B)$

$\in$ $\subseteq$ * في المثال السابق:

يعني "set"

* $\emptyset \in S$ * $\{\emptyset\} \subseteq S$ * $\{\emptyset, 1\} \subseteq S$
* $1 \in S$ * $\{1\} \subseteq S$ * $\{\{1\}, \{1,2\}\} \subseteq S$
* $\{1\} \in S$ * $\{\{1\}\} \subseteq S$ * $\{\{1\}, 2\} \not\subseteq S$
* $\{1,2\} \in S$ * $\{\{1,2\}\} \subseteq S$ * $\{\emptyset, 1, \{1\}\} \subseteq S$
* $2 \notin S$ * $\emptyset \subseteq S$ * $\{\emptyset, \{1\}, \{1,2\}\} \subseteq S$
* $\{\{\emptyset\}, \{1\}, \{2\}\} \not\subseteq S$
* $\{\emptyset, 1, \{1\}, \{1,2\}\} \subseteq S$

* $|S|$ : cardinality : no of elements $\in S$

Ex : * $|S| \rightarrow$ في المثال السابق $= 4$

* $|\emptyset| \rightarrow = 0$
* $|\{\emptyset\}| \rightarrow = 1$
* $|\{\{\}\}| \rightarrow = 1$
* $|\{\{1,2,3\}\}| \rightarrow = 1$
* $|\{\{1,2\}, \{3\}\}| \rightarrow = 2$
* $|\{\{\{1,2\}, \{3\}\}\}| \rightarrow = 1$
* $|\{1,1\}| \rightarrow = 1$
* $|\{\{1\}, \{1\}\}| \rightarrow = 1$
* $|\{1, \{1\}\}| \rightarrow = 2$

* Notes :- $\subseteq$

1. $\emptyset \subseteq S$ any $S$
2. $A \subseteq B \cap B \subseteq A$ , A,B are sets $\Rightarrow A = B$
3. $\subseteq$ subset * $\{1,2,3\} \subseteq \{1,2,3\}$ / $\{1,2\} \subseteq \{1,2,3\}$
4. $\subset$ proper subset * $\{1,2\} \subset \{1,2,3\}$
* $\{1,2,3\} \not\subset \{1,2,3\}$

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* set :

1. $\in$ $\subset / \subseteq$

2. $|\ |$

3. Powerset : Set of all subsets : $P(A)$ or $\wp(A)$

Ex : * $A = \{1, 2\}$

$P(A) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

* $A = \{1, 2, 3\}$ → $\{1,2,3\}$ من دون $7 =$ proper subset عدد ال

$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$|A| = n$

* $|P(A)| = 2^n$

* $A = \emptyset$

$P(A) = \{\emptyset\}$

* $A = \{\emptyset\}$

$P(A) = \{\emptyset, \{\emptyset\}\}$

4. $X$ : Cartesian product

$\underbrace{A \times B}_{\text{sets}} = \{(x, y) \mid x \in A \wedge y \in B\}$

Ex : $A = \{1, 2, 3\}$

$B = \{a, b\}$

$C = \emptyset$ / $D = \{5\}$

* $A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

* $B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)$

* $B \times C = \emptyset$

$2 \times 0 = 0$

$0 \Leftarrow \emptyset$ ?

$* A \times B \times D = \{(1,a,5), (1,b,5), (2,a,5), (2,b,5), (3,a,5), (3,b,5)\}$

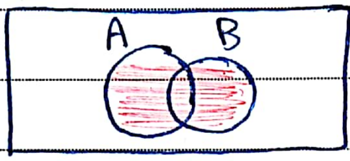
$1 = \Leftarrow \{\emptyset\}$ *

$0 = \Leftarrow \emptyset$ *

* operation :-

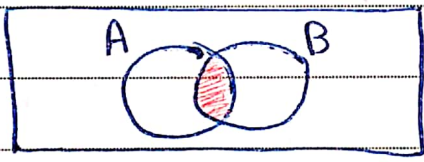
1. $\cup$ Union ( اتحاد )

$\underbrace{A \cup B}_{\text{( Sets )}} = \{ x \mid x \in A \vee x \in B \}$




2. $\cap$ Intersection ( تقاطع )

$\underbrace{A \cap B}_{\text{Sets}} = \{ x \mid x \in A \wedge x \in B \}$

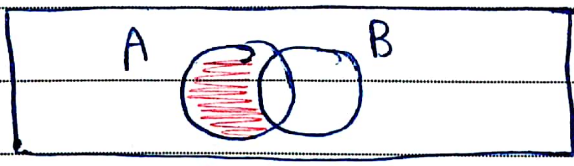



* Note : $|A \cup B| = |A| + |B| - |A \cap B|$

3. Difference $-$

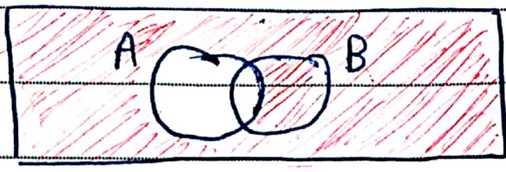
$\underbrace{A - B}_{\text{Sets}} = \{ x \mid x \in A \wedge x \notin B \ (\neg(x \in B)) \}$

→ ليست تبديلية *




4. complement $-$

set $\leftarrow \overline{A} = \{ x \mid x \notin A \}$



EX: $U = \{1, 2, 3, ---, 10\}$

$A = \{1, 2, 5, 8\}$

$B = \{2, 3, 6\}$

$C = \emptyset$

$D = \{3, 4\}$

* $A \cup B \Rightarrow \{1, 2, 5, 8, 3, 6\}$
* $A \cup C \Rightarrow \{1, 2, 5, 8\} = A$
* $A \cup U \Rightarrow U$
* $A \cap B \Rightarrow \{2\}$
* $A \cap C \Rightarrow \emptyset$
* $A \cap U \Rightarrow A$
* $A \cap D \Rightarrow \emptyset$ (A, D) are disjoint (تقاطعهم = 0)
* $A - B \Rightarrow \{1, 5, 8\}$
* $B - A \Rightarrow \{3, 6\}$
* $A - C \Rightarrow \{1, 2, 5, 8\} = A$
* $C - A \Rightarrow \emptyset$
* $A - U \Rightarrow \emptyset$
* $U - A \Rightarrow \{3, 4, 6, 7, 9, 10\}$
* $\overline{A} \Rightarrow \{3, 4, 6, 7, 9, 10\}$
* $\overline{B} \Rightarrow \{1, 4, 5, 7, 8, 9, 10\}$
* $\overline{C} \Rightarrow U$
* $\overline{U} \Rightarrow \emptyset$
* $\overline{(\overline{A})} \Rightarrow A$

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* sets :-

Prove 2 sets are equal
- → membership table
- → venn diagram
- → set identities (rules)

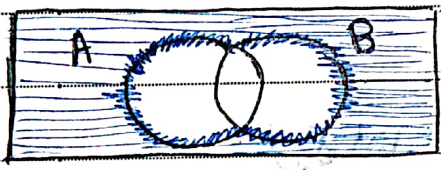
Ex: $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

① membership table :

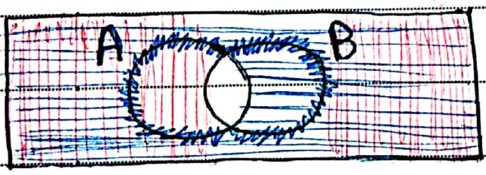
| | A | B | $A \cup B$ | $\overline{A \cup B}$ ① | $\bar{A}$ | $\bar{B}$ | $\bar{A} \cap \bar{B}$ ② |
|---|---|---|---|---|---|---|---|
| $x \in A \leftarrow$ | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| $x \notin A \leftarrow$ | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| | | | like (or) | like (not) | | | like (and) |

① = ②

② venn diagram :

① =



→ جميع المنطقة ماعدا
تقاطع A مع B

② =



① = ②

③ set identities : (A, B, C : sets)

① $A \cup A = A$
② $A \cap A = A$
③ $A \cup \emptyset = A$
④ $A \cap U = A$
↓
"Universe"

⑤ $A \cap \emptyset = \emptyset$
⑥ $A \cup U = U$
⑦ $\overline{(\bar{A})} = A$
⑧ $A \cup B = B \cup A$

9. $A \cap B = B \cap A$
10. $A \cup (B \cup C) = (A \cup B) \cup C$
11. $A \cap (B \cap C) = (A \cap B) \cap C$
12. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
13. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
14. $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$
15. $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

Ex: $A \cup (A \cap B) = A$

1. $(A \cap U) \cup (A \cap B)$ → نرجع لقبل التوزيع
2. $A \cap (U \cup B)$ شكلها كان ←
3. $A \cap U$
4. $A$

Ex: $A \cap (A \cup B) = A$

1. $(A \cap A) \cup (A \cap B)$
2. $A \cup (A \cap B)$
3. $(A \cap U) \cup (A \cap B)$ → نرجع لشكلها قبل التوزيع
4. $A \cap (U \cup B)$
5. $A \cap U$
6. $A$

## * Functions :-

$f: \underbrace{A \to B}_{\text{sets}}$ such that every element $\in A$ is related to only one element in set B

$f: A \to B$ coDomain

Domain

$f(a) = b$

$a \in A$

$b \in B$ — $\{b's\}$ : range

image

Preimage

Ex: $f: \{1, 2, 3\} \to \{a, b, c, d\}$

$f(1) = a$

$f(2) = c$

$f(3) = d$

Domain $\{1, 2, 3\}$

codomain $\{a, b, c, d\}$

Range $\{a, c, d\}$

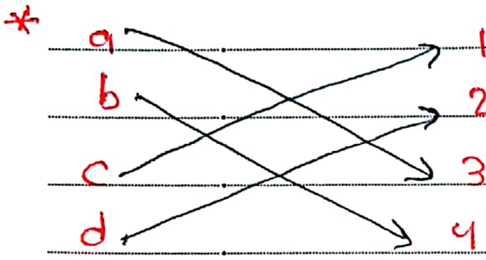
## * Types of functions :-

1. one to one (injective) : different elements in domain, have different images.

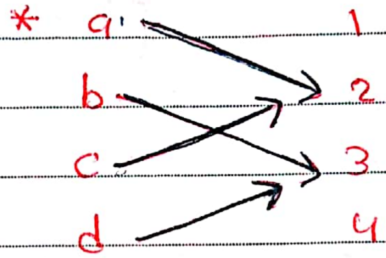
2. onto (surjective) : Every element in codomain is image

③ Bijective : one to one and onto

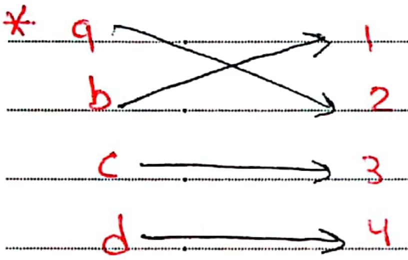
Ex: ① f : {a, b, c, d} → {1, 2, 3, 4}



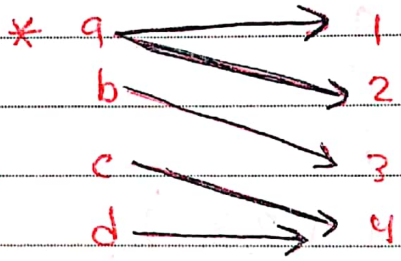

one to one / onto / Bijective




not one to one / not onto




one to one / onto / Bijective




Not function

(لأنه ال element لازم يكون الها صورة واحدة فقط

② $f : R \to R$

$f(x) = x + 1$ ⇒ نجرب كذا رقم $1.5 \to 2.5$

∴ one to one / onto $1 \to 2$

* strictly increasing and decreasing are one to one ⇐ $0 \to 1$

$-1 \to 0$

③ $f : Z \to Z$

$f(x) = x^2$

∴ not one to one / not onto

④ $f : Z^+ \to Z$

$f(x) = x^2$

∴ one to one / not onto

↲ بخلي onto انا غيرت ال codomain بالك {1, 4, 9, 25, 16, ...}

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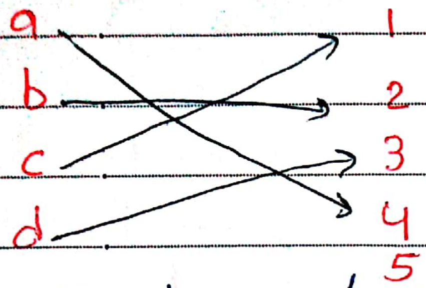
⑤ $f: \{a, b, c, d\} \to \{1, 2, 3, 4, 5\}$




one to one / not onto

$*$ Floor and ceiling

$\lfloor x \rfloor$      $\lceil x \rceil$

$\therefore f: \mathbb{R} \to \mathbb{Z}$

nearest integer $\leq x$      nearest integer $\geq x$

$\lfloor 3.5 \rfloor = 3$      $\lceil 3.5 \rceil = 4$

$\lfloor 3.1 \rfloor = 3$      $\lceil 3.1 \rceil = 4$

$\lfloor 3.9 \rfloor = 3$      $\lceil 3.9 \rceil = 4$

$\lfloor 3 \rfloor = 3$      $\lceil 3 \rceil = 3$

$\lfloor -3.5 \rfloor = -4$      $\lceil -3.5 \rceil = -3$

$*$ $-\lfloor x \rfloor = \lceil -x \rceil$

$*$ $\lfloor x+m \rfloor = \lfloor x \rfloor + m$

$m \in \mathbb{Z}$

$\therefore$ Floor and ceiling : Not one to one / Not bijective / onto

لـ إذا كان Codomain " Z " أما إذا كان " R " يصير not onto

# * Functions operations :-

① Inverse :-

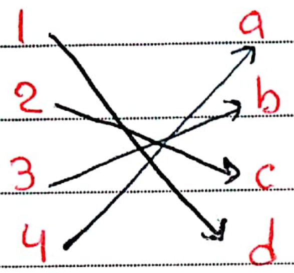
$f : A \to B$ $\qquad$ $f^{-1} : B \to A$

$f(a) = b$ $\qquad$ $f^{-1}(b) = a$

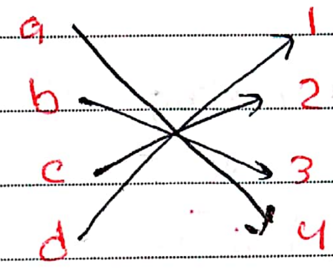
* Note : function to be invertible must be bijective.

(*) $f$ $\qquad$ $f^{-1}$

Ex :




$\Rightarrow$




(*) $f : \mathbb{Z} \to \mathbb{Z}$ $\qquad$ $f^{-1} : \mathbb{Z} \to \mathbb{Z}$

$f(x) = x + 1 \Rightarrow f^{-1}(x) = x - 1$

$y = x + 1$

$x = y - 1$

(*)

$f : \mathbb{R} \to \mathbb{R}$ $\Rightarrow$ Not invertible

$f(x) = x^2$

(*) $f : \mathbb{Z}^+ \to \{1, 4, 9, 16, 25, \dots\}$

$f(x) = x^2$

$f^{-1} : \{1, 4, 9, 16, 25, \dots\} \to \mathbb{Z}^+$

$f^{-1}(x) = \sqrt{x}$

② Composition :-

$\circ$ after

$f: A \to \boxed{B}$
$g: \boxed{B} \to C$

$\Rightarrow * \; g \circ f(x) = g(f(x))$

$* \; f \circ g(x) =$ Not defined

Ex : ⊛ $f: \mathbb{Z} \to \mathbb{Z}$ $\quad g: \mathbb{Z} \to \mathbb{Z}$

$f(x) = 2x + 3 \quad g(x) = 3x + 2$

a:

$(f \circ g)(x) = f(g(x)) = f(3x+2) = 2(3x+2) + 3$

$= 6x + 4 + 3 = 6x + 7$

b: $(g \circ f)(x) = g(f(x)) = g(2x+3) = 3(2x+3) + 2$

$= 6x + 9 + 2 = 6x + 11$

$* \; f \circ g(x) \neq g \circ f(x)$

* Not onto لأنهم Not invertible $\Leftarrow g(x), f(x)$ في المثال السابق *

$f^{-1}(x) = \frac{x-3}{2} \Leftarrow f: R \to R$ في حال كان المثال السابق *

$* \; (f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{x-3}{2}\right) = 2\left(\frac{x-3}{2}\right) + 3 = x$

$* \; (f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(2x+3) = \frac{(2x+3)-3}{2} = x$

# * Sequences and Summations : ($\Sigma$)

• Sequence : function $A \rightarrow B$ $\{0, 1, 2, 3, \ldots\}$ / $\{1, 2, 3, \ldots\}$ Positions

Ex : * $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

Set * $1, 2, 3, 4, 5$ ⇒ positions

Ex : $a_n = 5^n$

Terms ← 0 1 2

$1, 5, 25, 125, \ldots$

Ex : $5, 11, 17, 23, 29, \ldots$ (+6 +6 +6 +6)

Term to term
$a_0 = 5$ ⇒ first element
$a_n = a_{n-1} + 6$

n'th term
$a_n = 5 + 6n$ $n \geq 0$

Sequences طرق حل ال * ⇐
* Arithmetic sequence

Ex : $1, 2, 4, 8, 16, 32, 64, 128, 256, \ldots$ (×2 ×2 ×2 ×2 ×2 ×2 ×2 ×2)

* geometric sequences.

Term to Term :
$a_0 = 1$
$a_n = 2 * a_{n-1}$

nth term :
$a_n = 2^n, \quad n \geq 0$

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Ex : 1, 7, 25, 79, 241, ---

→ Term to Term :

$a_0 = 1$

$a_n = a_n \times 3 + 4$

→ n th term :

$a_n = 3^n - 2 \quad , \quad n \geq 1$

Ex : 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, ------

n appears n times , $(n \geq 0)$ or $(n \geq 1)$

لـ "تتكرر"

$$* \sum_{i=L}^{U} a_i = a_L + a_{L+1} + a_{L+2} + --- a_U$$

U → upper bound

$a_i$ → index of som

L → lower bound

Ex : $\sum_{i=3}^{6} \frac{i^2}{2} = \frac{3^2}{2} + \frac{4^2}{2} + \frac{5^2}{2} + \frac{6^2}{2}$

* Rules :-

① $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Ex : $\sum_{i=1}^{10} i = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$

or

$\frac{(10)(10+1)}{2} = 55$

(2) $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

(3) $$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

(4) $$\sum_{i=0}^{n} ar^i = a * \frac{r^{n+1}-1}{r-1}$$

Ex: $$\sum_{i=2}^{10} i = \sum_{i=1}^{10} i - 1 = 55-1 = 54$$

بلشت من 2 مش 1

* Given $\sum_{i=L}^{U} a_i = n \Rightarrow \sum_{i=L+1}^{U} a_i = n - a_L$

Ex: $$\sum_{i=1}^{11} i = \sum_{i=1}^{10} i + 11 = 55+11 = 66$$

(or) $$\frac{11(11+1)}{2} = 66$$

* $$\sum_{i=L}^{U+1} a_i = n + a_{U+1}$$

Ex: $$\sum_{i=0}^{10} i = 55+0$$

* $$\sum_{i=L-1}^{U} a_i = n + a_{L-1}$$

Ex: $$\sum_{i=1}^{9} i = 55-10 = 45$$

* $$\sum_{i=L}^{U-1} a_i = n - a_U$$

Ex : $\sum_{i=50}^{100} i$ using roles :

$$\sum_{i=1}^{100} i - \sum_{i=1}^{i=49} i = 3150$$

$$\sum_{j=1}^{3} i + \sum_{j=1}^{3} j = 3i + 6$$

* Ex : Find $\sum_{i=1}^{2} \sum_{j=1}^{3} (i+j)$

$$= \sum_{i=1}^{2} (i+1)+(i+2)+(i+3) = \sum_{i=1}^{2} (3i+6) = (3+6)+(6+6)$$

$$= 9 + 12 = 21$$

* $\sum_{i=1}^{3} 6 = 6(3-1+1) = 6+6+6 = 18$

H.w * $\sum_{j=1}^{20} \sum_{i=1}^{30} (1+j) =$ باستخدام القوانين

$$\downarrow \rightarrow \sum_{i=1}^{30} (1+j)$$

$$30(1+j) = 30+30j$$

$$\sum_{j=1}^{20} 30+30j$$

$$k = 1+j$$

$$= 30 \sum_{j=1}^{20} (1+j)$$

$$= 30 \sum_{k=2}^{20} k = \qquad = 30\left(\sum_{k=1}^{21} k - \sum_{k=1}^{1} k\right)$$

$$3(1+j) \qquad = 30(210-1)$$

$$K = 1+j \qquad 30 \times 209 = 6270$$

Ex 8 $\sum_{i=0}^{2} \sum_{j=0}^{3} (2i+3j)$

$$\sum_{i=0}^{2} (2i) + (2i+3) + (2i+9) + (2i+6)$$

$$\sum_{i=0}^{2} 8i+18 = 18+18+8+16+18 = 78$$

* $\sum_{i=1}^{3} \sum_{j=0}^{2} i$

$$= \sum_{i=1}^{3} \sum_{j=0}^{2} i = \sum_{i=1}^{3} 3i = 3(1)+3(2)+3(3) = 18$$

* $\sum_{i=0}^{2} \sum_{j=1}^{3} ij$

$$\sum_{i=0}^{2} i + 2i + 3i = \sum_{i=0}^{2} 6i = 6(0)+6(1)+6(2) = 18$$

* $\sum_{i=1}^{10} 3 = 3(10-1+1) = 30$

* $\sum_{j=0}^{4} (-2)^j = (-2)^0 + (-2)^1 + (-2)^2 + (-2)^3 + (-2)^4$

$$= 1 - 2 + 4 - 8 + 16 = 11$$

* $\sum_{k=0}^{9} \lfloor \sqrt{k} \rfloor = \lfloor \sqrt{0} \rfloor + \lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor +$

$\lfloor \sqrt{3} \rfloor + \lfloor \sqrt{4} \rfloor + \lfloor \sqrt{5} \rfloor + \lfloor \sqrt{6} \rfloor + \lfloor \sqrt{7} \rfloor +$
$\lfloor \sqrt{8} \rfloor + \lfloor \sqrt{9} \rfloor$

$$= 0 + 1 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + 3$$

$$= 16$$

* $\sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3$

$$\sum_{i=0}^{2} 0 + i^2 + 8i^2 + 27i^2$$

$$= \sum_{i=0}^{2} 36 i^2$$

$= (36 * 0^2) + (36 * 1^2) + (36 * 2^2) = 36 + 144 = 180$

* $\sum_{i=1}^{3} \sum_{j=0}^{2} j = \sum_{i=1}^{3} 3 = 3(3-1+1) = 9$

* $\sum_{j=3}^{3} j = 0$

Find rule :

$\boxed{1}$ $\overset{1}{3}, \overset{2}{5}, \overset{3}{8}, \overset{4}{12}, \overset{5}{17}, \overset{6}{23}, 30, 38, ---$

$a_n = a_{n-1} + n$

$a_1 = 3$

$\boxed{2}$ $2, 16, 54, 128, 250, 432$

$2 \times n^3$

$\boxed{3}$ $2, 3, 7, 25, 121$

$a_1 = 2$

$a_n = n! + 1$

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## * Mathrix :- ( مصفوفة )

$$M = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$m \times n$

rows ↲ ↳ coloms

Ex : $M = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 0 & -1 \end{bmatrix}$

$a_{22} = 0$

$a_{13} = 5$

$-1 = a_{23}$

عمود ↳ له صف

## Arithmetic op :-

### ① Summation (لازم تكون نفس الحجم)

$$C_{m\times n} = A_{m\times n} + B_{m\times n}$$

$$C_{rj} = a_{rj} + b_{rj}$$

Ex :

$A = \begin{bmatrix} 1 & 2 & 4 \\ 4 & 3 & 0 \end{bmatrix}_{2\times 3}$ $D = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}_{2\times 2}$

$B = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 0 & -1 \end{bmatrix}_{2\times 3}$ $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}_{3\times 2}$

$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

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* $A + B$

$$\begin{bmatrix} 3 & 3 & 5 \\ 10 & 3 & -1 \end{bmatrix}_{2\times3}$$

* $C + D$

$$\begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}_{2\times2}$$

* $B + B$

$$\begin{bmatrix} 4 & 2 & 2 \\ 12 & 0 & -2 \end{bmatrix}_{2\times3}$$

«من المثال السابق»

② Multiplication

$$C_{m\times n} = A_{m\times k} \times B_{k\times n}$$

$$C_{ij} = a_{i2} \times b_{2j} + a_{i2} \times b_{2j} + \cdots a_{ik} \times b_{kj}$$

* $A \times E$

$$\begin{bmatrix} \boxed{1 \quad 2 \quad 4} \\ 4 \quad 3 \quad 0 \end{bmatrix}_{2\times3} \times \begin{bmatrix} 1 & 6 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}_{3\times2}$$

$$= \begin{bmatrix} 9 & 6 \\ 4 & 3 \end{bmatrix}_{2\times2}$$

«من المثال السابق»

* $E \times A$

$$\begin{bmatrix} \boxed{1 \quad 6} \\ 6 \quad 1 \\ 2 \quad 1 \end{bmatrix}_{3\times2} \times \begin{bmatrix} 1 & 2 & 4 \\ 4 & 3 & 6 \end{bmatrix}_{2\times3}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 4 & 3 & 0 \\ 6 & 7 & 8 \end{bmatrix}_{3\times3}$$

(الصف الأول في العامود الأول والصف الأول
في العمود الثاني وهكذا)

* $C \times D = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}_{2\times 2}$

$$\overset{C}{\begin{bmatrix} \boxed{1 \quad 0} \\ 0 \quad 1 \end{bmatrix}}_{2\times 2} \times \overset{D}{\begin{bmatrix} \boxed{\begin{matrix}1\\2\end{matrix}} & \begin{matrix}1\\1\end{matrix} \end{bmatrix}}_{2\times 2}$$

* $D \times C = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}_{2\times 2}$

$$\overset{D}{\begin{bmatrix} \boxed{1 \quad 1} \\ 2 \quad 1 \end{bmatrix}}_{2\times 2} \times \overset{C}{\begin{bmatrix} \boxed{\begin{matrix}1\\0\end{matrix}} & \begin{matrix}0\\1\end{matrix} \end{bmatrix}}_{2\times 2}$$

"في المثال السابق"

* $D \times F = [\quad]_{2\times 2}$

* $F \times D = [\quad]_{2\times 2}$

③ Power

$A^0_{n\times n} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Identity

$A^1_{n\times n} = A$

$A^2_{n\times n} = A \times A$

$A^n_{n\times n} = A \times A \times A \times \text{——} A$ n times

* $D^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

* $D^2 = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$

* $D^1 = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

* $D^3 = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ 10 & 7 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ 10 & 7 \end{bmatrix}$

## 4 Transpose (العمود بيصير صف والصف عمود)

$A_{m \times n}$ $\qquad a_{ij}$ in $A$

$A^t_{n \times m}$ $\qquad a_{ji}$ in $A^t$

* $A^t = \begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 4 & 0 \end{bmatrix}_{3 \times 2}$

* $D^t = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}_{2 \times 2}$

* Note: if $A = A^t \Rightarrow A$ is symmetric

Ex: $X = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 7 \end{bmatrix}_{3 \times 3}$

$X^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 4 \\ 3 & 4 & 7 \end{bmatrix}$

* Matrics :-

Boolean operations :- Zero one matrices

① Meat

$C_{m\times n} = \underset{m\times n}{A} \text{ meet } B_{m\times n}$

* نفس الحجم

$C_{ij} = a_{ij} \wedge b_{ij}$

② Join

$C_{m\times n} = A_{m\times n} \text{ Join } B_{m\times n}$

* نفس الحجم

$C_{ij} = a_{ij} \vee b_{ij}$

Ex :

$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}_{2\times 3}$

$B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{2\times 3}$

$C = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}_{2\times 2}$

$D = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}_{2\times 2}$

* $A \text{ join } B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{2\times 3}$

* $C \text{ meet } D = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}_{2\times n}$

## ③ Boolean product ⊙

$C_{m\times n} = A_{m\times k} \odot B_{k\times n}$

$c_{ij} = (a_{i1} \wedge b_{1j}) \vee (a_{i2} \wedge b_{2j}) \vee \text{-----} (a_{ik} \wedge b_{kj})$

* من المثال السابق:

* $C \odot D = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}_{2\times 2}$

* $D \odot C = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}_{2\times 2}$

→ ليست تبديلية

* $C \odot A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}_{2\times 3}$

## ④ Boolean power

$A^{[0]} = I^{A_{n\times n}}$ / $A^{[2]} = A \odot A$

$A^{[1]} = A$ / $A^{[n]} = \underbrace{A \odot A \odot A - A}_{\text{"n times"}}$

Ex : * من المثال السابق *

* $A + B = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

* $C^2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

* $C \times D = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

* $C^{[2]} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

* نفسها بس استبدلنا الـ 2 بـ 1 *

* $C^1 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

* $C^{[0]} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Identities ↵

* Identities :

$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$I_{4\times4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

* Integers and division
  - ↳ without mod
  - ↳ with mod

Def : " | divides

$a \mid b \Rightarrow \frac{b}{a} = k \in \mathbb{Z}$

a: factor of → عامل

b: multiple of

Ex : 3|6 ✓ / 6|3 ✗ / 3|125 ✗ / 1|7 ✓ / 8|8 ✓

Thm :-

1. if $a \mid b$ then $a \mid bn$, $n \in \mathbb{Z}$

proof : Directly

Assume $a \mid b \Rightarrow \frac{b}{a} = K \quad K \in \mathbb{Z} \Rightarrow b = aK$

$a \mid bn \Rightarrow \frac{bn}{a} = \frac{aKn}{a} \in \mathbb{Z} \Rightarrow a \mid bn$

Ex : $3 \mid 6$ / $3 \mid 12$ / $3 \mid 18$ / $3 \mid 600$

2. if $a \mid b$ and $a \mid c$ then $a \mid (b+c)$

3. if $a \mid b$ and $b \mid c$ then $a \mid c$

* Primes : p ( عدد أولي )

$p \geq 2$ is prime iff it's only factors are 1 and p

Ex : 2, 3, 5, 7, 11, 13, 17, ....

Mersenne primes $2^n - 1$

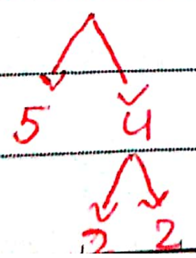
* $3 = 2^2 - 1$
* $7 = 2^3 - 1$
* $15 = 2^4 - 1$ X لأن 15 ليس prime

Thm :-

Any composite (not prime) number can be written as Product of primes ( uniquely )

$20 = 2.2.5 = 2^2.5$ / $20 = 2.10$, $10 \rightarrow 2.5$

أحللها لعواملها الأولية



Thm :-

if $n$ has no factors $\leq \sqrt{n} \Rightarrow n$ is prime

Ex : 101 $\Rightarrow \sqrt{101} \approx 10$

وبناخذ من 2 لحد 10 وإذا ماكان العوامل = أولي

$2 | 101$ x / $3 | 101$ x / $5 | 101$ x / $7 | 101$ x

خلصنا الأعداد الأولية اللي قبل 10 :

101 is prime number

قاسم مشترك أكبر

* GCD

Greatest common Divisor

مضاعف مشترك أصغر

* LCM

Least common multipliers

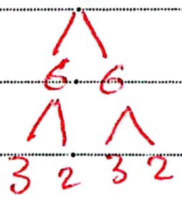
Ex : GCD (36, 24) = 12

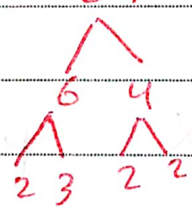
36 : 1, 2, 3, 4, 6, 9, 12, 18, 36

24 : 1, 2, 3, 4, 6, 8, 12, 24

* $GCD(x, y) = P_1^{min} \cdot P_2^{min} \cdot P_3^{min} --- P_n^{min}$

$36 = 2^2 \cdot 3^2$ / $24 = 2^3 \cdot 3$





(طريقة الشجرة)

$GCD(36, 24) = 2^2 \cdot 3 = 12 \Leftarrow$ (طريقة الأس)

LCM (36, 12) = 72

36 : 36, 72, 144, ---

24 : 24, 48, 72, 96, ....

* $LCM(x, y) = P_1^{max} \cdot P_2^{max} \cdot P_3^{max}$ ------

$LCM(36, 24) = 2^3 \cdot 3^2 = 72$

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Thm :- $GCD(x,y) \cdot LCM(x,y) = x \cdot y$

* Integers and division :-

Ex : $GCD(8,15) = 1$

$$8 \to 2^3 \qquad 15 \to 3 \cdot 5$$

8, 15 are relativly prime

* $GCD(8,15) = 1$
$GCD(8,7) = 1$
$GCD(7,15) = 1$

* 7, 8, 15 are pairwise relatively Prime

Thm :-

$a \in Z$
$d \in Z^+$
$a = dq + r$

a: divident مقسوم
d: divsor مقسوم عليه
q: quotient حاصل القسمة
r: remainder الباقي (div) م

$0 \leq r < d$

Ex :

* $\frac{23}{5}$ = $23 = 5 \cdot 4 + 3$ → quotient = 4, remainder (mod) م = 3

* $\frac{77}{7}$ = $77 = 11 \cdot 7 + 0$ → quotient = 11, remainder = 0

* $\frac{-12}{8}$ = $-12 = 8 \cdot -2 + 4$ → quotient = -2, remainder = 4

* $\frac{-33}{7}$ = $-33 = 7 \cdot -5 + 2$ → quotient = -5, remainder = 2

Def : $a \equiv b \pmod m$ , $(a,b) \in Z$ / $m \in Z^+$

$\equiv$ is congruent (تطابق)

iff $a \bmod m = b \bmod m$

Ex : * $7 \equiv 17 \pmod 5$

$7 \bmod 5 = 2$

$17 \bmod 5 = 2$

* $7 \equiv 27 \pmod 5$

* $17 \equiv 27 \pmod 5$

Thm : if $a \equiv b \pmod m$ $a = b + mk$ $k \in \mathbb{Z}$

$m \mid (a-b)$

Ex : * $2 \equiv \boxed{8} \pmod 6$

$\boxed{14}$

$\boxed{-4}$

* $3 \equiv 9 \pmod{\boxed{6}}$

$\boxed{3}$

$\boxed{2}$

Thm : if $a \equiv b \pmod m$

$c \equiv d \pmod m$

then : $(a+c) \equiv (b+d) \pmod m$

$(a.c) \equiv (b.d) \pmod m$

Ex : * $2 \equiv 5 \pmod 3$

* $7 \equiv 10 \pmod 3$

* $(2+7) \equiv (5+10) \pmod 3$

$9 \equiv 15 \pmod 3$

* $(2.7) \equiv (5.10) \pmod 3$

$14 \equiv 50 \pmod 3$

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## * Proof by induction :-

EX : $2^0 \quad 2^1 \quad 2^2 \quad 2^3 \quad 2^4$

$1 + 2 + 4 + 8 + 16 + \cdots 2^n = 2^{n+1} - 1$ OR

$$\sum_{i=0}^{n} 2^i$$

1- Base : $2^0 = 2^1 - 1$

$1 = 1$ / صحيحة يكمل

غير صحيحة يوقف

2- Hypothesis :

Assume $2^0 + 2^1 + \cdots 2^n = 2^{n+1} - 1$

3- step :

Prove $\underbrace{2^0 + 2^1 + \cdots 2^n}_{2^{n+1}-1} + 2^{n+1} = 2^{n+2} - 1$

$2^{n+1} - 1 + 2^{n+1} = 2^{n+2} - 1 \Rightarrow 2^{n+1} - 1 + 2^{n+1} = 2^1 \cdot 2^{n+1} - 1 = 2^{n+2} - 1$

4- conclusion :

$2^0 + 2^1 + \cdots 2^n = 2^{n+1} - 1$

$\forall n$

Steps :

1. Base : Prove for first element.
2. Hypothesis : Assume that thm is correct for $n$
3. Step : Prove for $n+1$ using step 2
4. Conclusion : thm is correct $\forall n$

* $P(0)$ / $\dfrac{P(n)}{P(n+1)}$

$\dfrac{P(0) \rightarrow P(1)}{P(1)}$

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Ex: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$1 + 2 + 3 + 4 + \cdots n$

Sol: 1. Base: $\sum_{i=1}^{1} i = \frac{1(2)}{2} \Rightarrow 1 = 1$ correct

2. Hypothesis: Assume $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3. step: Prove $\sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2}$

$\sum_{i=1}^{n+1} i = \sum_{i=1}^{n} i + (n+1)$

$\downarrow$

$\frac{n(n+1)}{2} + (n+1) \Rightarrow \frac{n(n+1)+2(n+1)}{2} = \frac{(n+1)(n+2)}{2}$

4. Conclusion: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad \forall n$

* $\sum_{i=1}^{n+1} i^2 = \sum_{i=1}^{n} i + (n+1)^2 \Rightarrow$ « لو كانت تربيع »

Ex: prove: $3 \mid (n^3 - n)$ or $(n^3 - n)$ is divisable by 3, $n \geq 0$

"قابل للقسمة بدون باقي"

Sol: 1. Base: $3 \mid (0-0) = 3 \mid 0$ أخذنا ال 0

2. Hypotheses: Assume $3 \mid (n^3 - n)$ → فكيناه

3. Step: Prove $3 \mid ((n+1)^3 - (n+1))$

$= n^3 + 3n^2 + 3n + \cancel{1} - n - \cancel{1} = \underbrace{(n^3 - n)}_{3 \mid (n^3 - n)} + \underbrace{(3n^2 + 3n)}_{3 \mid (3n^2 + 3n)}$

$= \cancel{3}(n^2 + n)$

if $a \mid b$ and $a \mid c$ then $a \mid (b+c)$ ⟸

$\Rightarrow 3 \mid (n^3 - n + 3n^2 + 3n) = 3 \mid ((n+1)^3 - (n+1))$ = devise 3

4. Conclusion: $3 \mid (n^3 - n) \quad \forall n$

الأكمال في الأمام .....

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$1 = 0!$ *

Ex: Prove: $2^n < n!$ , $n \geq 4$

Sol: 1. Base: $2^4 < 4!$ $\Rightarrow$ "correct"

$16 < 24$

2. Hypothesis:

$2^n < n!$

3. Step: prove $2^{n+1} < (n+1)!$

بس بيلش من Hypothesis الحل

$2^n < n! \Rightarrow 2 \cdot 2^n < n! \cdot 2 \Rightarrow 2^{n+1} < 2n! < (n+1)\,n!$

من السؤال $\geq 5$

$\Rightarrow 2^{n+1} < 2n! < (n+1)!$

$\Rightarrow 2^{n+1} < (n+1)!$

4. conclusion: $2^n < n!$ $\forall n$

Ex: Prove: $n < 2^n$ , $n \geq 1$ — H.W

1. Base: $1 < 2^1$ $\Rightarrow$ correct
2. Hypothesis: Assume $n < 2^n$
3. step: Prove $(n+1) < 2^{(n+1)}$

$n+1 < 2n < 2^1 \cdot 2^n$

$n+1 < 2n < 2^{n+1} \Rightarrow n+1 < 2^{n+1}$

4. conclusion:

$n < 2^n$ $\forall n$

Induction:-

Ex: $\sum_{i=1}^{n} (2i-1) = n^2$ or $1+3+5+7+9 \cdots (2n-1) = n^2$

$5^2$

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1. Base:
$$\sum_{i=1}^{1}(2i-1) = 1^2$$
$$1 = 1$$

2. Hypothesis:
Assume $\sum_{i=1}^{n}(2i-1) = n^2$

3. Step:
Prove $\sum_{i=1}^{n+1}(2i-1) = (n+1)^2$

$$\sum_{i=1}^{n+1}(2i-1) = \sum_{i=1}^{n}(2i-1) + (2(n+1)-1)$$

$$= n^2 + 2n + 1 = (n+1)(n+1) = (n+1)^2$$

4. Conclusion: $\sum_{i=1}^{n}(2i-1) = n^2 \quad \forall n$

H.W:
1. $4n < 2^n \quad n \geq 5$
2. $1\times2 + 2\times3 + 3\times4 + \cdots n(n+1) = n(n+1)(n+2) \quad n > 1$
3. $5 \mid (8^n - 3^n)^3 \quad n \geq 1$

*Relations : → Binary (set of pairs)
→ n - any

① Representation

1. Set builder      2. Listing      3. Table

4. Matrix      5. Graph

Ex : Domain $S = \{1, 2, 3\}$

$R : S \to S$

* $R = \{ (x, y) \mid x = y \}$

* $R = \{ (1,1), (2,2), (3,3) \}$

*

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | ▨ | | |
| 2 | | ▨ | |
| 3 | | | ▨ |

$(1,1) \in R$      $(1,3) \notin R$

* $\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & & & \\ 2 & & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \\ 3 & & & \end{array}$ 3x3

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} 1 \quad 2 \quad 3 \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \quad 3\times3$$

ب ,اذا ما أعطاكي هون الترتيب تلقائي = 3 2 1

OR

$$\begin{array}{c} \\ 3 \\ 1 \\ 2 \end{array}\begin{array}{c} 2 \quad 1 \quad 3 \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array} \to (3,3)$$

(2,2)      (2,2)

* 1 ⟶ 1

2 ⟶ 2

3 ⟶ 3

OR


* properties

- Reflexivity (انعكاس) → reflexive $\forall x\, (x \in S) \to (x,x) \in R$
  - reflexive عكس ← irreflexive $(x, x) \notin R$
- Symmetricity (تماثل) → symmetric $\forall x \forall y\, ((x,y) \in R \to (y,x) \in R)$
  - antisymmetric $(x,y) \in R \to (y,x) \notin R$
- Transitivity → transitive if $((x,y) \in R \wedge (y,z) \in R) \to (x,z) \in R$

Ex : Domain $S = \{1, 2, 3\}$

--(2,2) , (1,1) في Symmetric ماوخلنا ب

$R_1 = \{(1,1), (2,2), (2,3)\}$ → في (2,3) ما في (3,2)

Not Reflexive, Not Irreflexive, Not symmetric, antisymmetric, transitive

$R_2 = \{(2,3), (3,2)\}$

Not Reflexive, irreflexive, symmetric, Not antisymmetric, not transitive

$R_3 = \{(1,2), (2,1), (1,1), (3,1)\}$ → لأنه (3,1) ما الها (1,3) لازم الكل موجود

Not Reflexive, Not irreflexive, Not symmetric, Not antisymmetric, not transitive → مدام عندي خالف إذاً

$R_4 = I_R = \{(1,1), (2,2), (3,3)\}$

Reflexive, Not irreflexive, symmetric, untisymmetric, Transitive

$R_5 = \emptyset$

Not Reflexive, irreflexive, symmetric, untisymmetric, transitive

$$M_{R_6} = \begin{array}{c} \begin{array}{ccc} & (1,2) & (1,3) \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \\ \begin{array}{ccc} & (3,2) & \end{array} \end{array}$$

إذا كان في (1,1), (2,2), (3,3) وكان في زيادة عادي → بتنفع Reflexive

هون لازم (3,3)

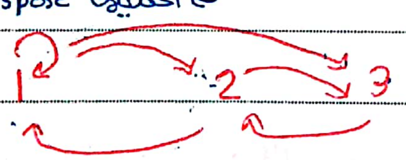
Not Reflexive ← تكون 1 بس ما في :

, Not irreflexive, Not symmetric, Not untisymmetric, Not transitive

له اكملها transpose إذا ماعرفني

له بالتجربة

$R_7$



Not Reflexive, Not irreflexive, Not symmetric, Not untisymmetric, Not transitive

# Relations :-

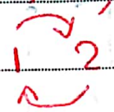
## Operations :

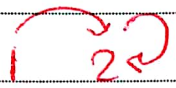
1. Union 'U'
2. Intersection '∩'
3. Difference −
4. Complement ¯
5. Inverse ~ or −1
6. Composition ∘ (after)

$\{ (1,2),$

Universal $R = \{(1,1), (1,2), (2,1), (2,2)\}$

Ex : $S = \{1, 2\}$ $R_1 = \{(1,1), (1,2)\}$

$R_2 = \{(2,1)\}$ $R_3 =$ 

$R_4 =$ 

M universe $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $M_{R_1} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ $M_{R_2} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

* $R_1 \cup R_2 = \{(1,1), (1,2), (2,1)\}$

* $M_{R_1}$ Join $M_{R_2}$ ($\vee$) $= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

* $R_1 \cap R_2 = \emptyset$

* $M_{R_1}$ Meet $M_{R_2}$ ($\wedge$) $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

* $R_1 - R_2 = \{(1,1), (1,2)\} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ → في ال diff ⇐ بطرح من ال1 بس ما بطرح من ال0 خليه زي ماهو

* $R_2 - R_1 = \{(2,1)\}$

* $\overline{R_1} = \{(2,1), (2,2)\}$

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* $R_1^{-1}$ or $R_1^{N}$ = $\{(1,1), (2,1)\}$

* $R_1^{-1} = M_{R_1}^{t} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

* $R_1 \circ R_2 = \{(2,1), (2,2)\}$

$R_1 = \{(1,1), (1,2)\}$

$R_2 = \{(2,1)\}$

اللي بنتهي بـ $R_2$ بلش فيه بـ $R_1$

=

* $R_2 \circ R_1 = \{(1,1)\}$

* $M_{R_2} \odot M_{R_1} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

* $M_{R_1} \odot M_{R_2} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ =

* $R_1 \circ R_1 = \{(1,1), (1,2)\}$

$R_1 \circ R_1 \subseteq R_1 \Rightarrow R$ is transitive

$|S| = n$

$R \circ R$

$R \circ R \circ R$

$R \circ R \circ R \circ \ldots$ N times

$\subseteq R \Rightarrow R$ is transitive

→ نفس القاعدة

* $M_{R_1}^{[2]} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

* $R_2 \circ R_2 = \{\ \} \Rightarrow$ transitive

## closures :-

- reflexive: $R^{r} = R \cup R_{I}$ (closures; relation)
- symmetric: $R^{s} = R \cup R^{-1}$
- transitive: $R^{t}$ or $R^{*} = R \cup R \circ R \cup \underbrace{R \circ R \circ R}_{n \text{ times } |s|}$

«من المثال السابق»

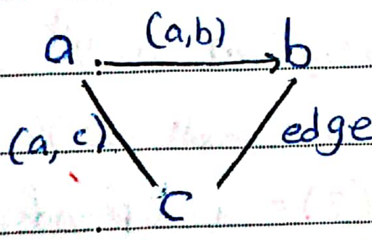
$* \; R_1^{r} = \{(1,1),(1,2)\} \cup \underbrace{\{(1,1),(2,2)\}}_{\text{Identity}}$

$= \{(1,1),(1,2),(2,2)\}$

$* \; R_1^{s} = \{(1,1),(1,2)\} \cup \underbrace{\{(1,1),(2,1)\}}_{\text{inverse}}$

$= \{(1,1),(1,2),(2,1)\}$

## Graph :-



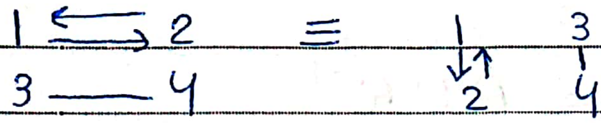

vertex node (vertices) رؤوس

edge ( حد )

set of vertices — set of edges

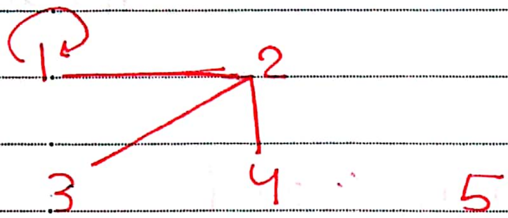
$G = (\{a,b,c\}, (\overrightarrow{(a,b)}, \{a,c\}, \{b,c\}))$

$G = (\{1,2,3,4\}, \{(1,2),(2,1),\{3,4\}\})$




Terminology

1. undirected graph (no direction)




- Adjacent $\{x,y\}$ : x is adjacent with y (المجاور)
  Ex: 1 is adjacent with 2
- Incident $\{x,y\}$ : is incident with x and y (الواقع)
  Ex: $\{3,2\}$ is incident with 3 and 2 (الحادث)
- loop: $\{x,x\}$ : is a loop
  Ex: $\{1,1\}$ or $\{1\}$
- path: Sequence of edges مسار
  Ex: 1, 2, 3 of length 2
- cycle: Path that begins and ends at the same node
  Ex: 3, 2, 3 of length 2 (مسار بيبدأ وبينتهي بنفس المكان) نقطة التقاء
- degree: No. of adjacent nodes (عدد المجاورين)
  Ex: deg (1) = 3 , deg(2) = 3 , deg(3) = 1 (pendant)
  deg (4) = 1 , deg(5) = 0 (isolated)

$\sum deg(v) = 8$ ( even لازم يطلع )

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Thm:

Handshaking thm.

$$\sum_{v \in V} \deg(v) = 2|E|$$

↳ set of edges

Ex: $|V| = 5$ (→ vertices)

$\deg(v) = 6$

$|E| = ?$

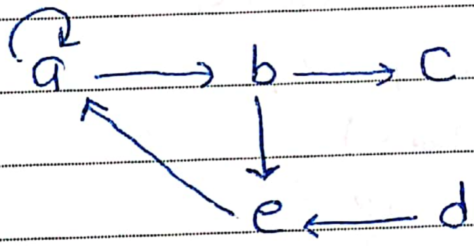
Sol. $\sum \deg(v) = 5 * 6 = 30$

$\sum \deg(v) = 2|E|$

$30 = 2|E| \Rightarrow |E| = 15$

Graph: terminology

Directed:



هنا نهتم للاتجاه.

* adjacent: * b is adjacent to e / to c

* b is adjacent from a

| out | in |
|---|---|
| * $\deg^+(a) = 2$ | $\deg^-(a) = 2$ |
| $\deg^+(b) = 2$ | $\deg^-(b) = 1$ |
| $\deg^+(c) = 0$ | $\deg^-(c) = 1$ |
| $\deg^+(d) = 1$ | $\deg^-(d) = 0$ |
| $\deg^+(e) = 1$ | $\deg^-(e) = 2$ |
| $\sum_{v \in V} \deg^+(v) = 6$ | $\sum_{v \in V} \deg^-(v) = 6$ |

$\therefore |E| = 6$

ممكن تيجي ومطلوب زي

(path, cycle, Incident, loop) no direction الباقي نفس ال *

Thm:

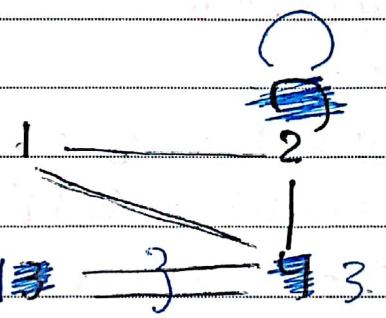
$$\sum_{v \in V} deg^{+}(v) = \sum_{v \in V} deg^{-}(v) = |E|$$

* Representation :-

Matrix : Adjacency

$$M = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 0 & 2 \\ 4 & 0 & 0 & 2 & 0 \end{array}_{4\times 4}$$

$\Leftarrow$



↳ multiple ⇒ 2 بحطها

ل القطر بيبنلي إذا في loop

ل ال symmetric بيبنلي إنو Undirect

deg(3) = Σ elements of raw or colums

deg (2) = 4 ( لأنه ⇒ loop = 2)

* |V| = 4 (size of matrix بيعطيني)

* $|E| = 6 = \frac{\sum deg(v)}{2} = \frac{12}{2}$

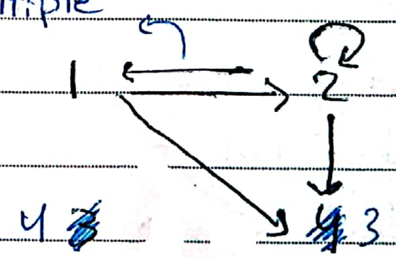
---

Initial ↲    1 2 3 4 ↝ : edge from    not multiple

$$M = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{array}_{4\times 4}$$

1 to 2

$\Leftarrow$



Not symmetric ⇒ directed * → raw

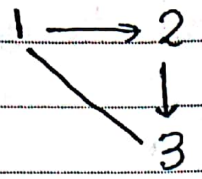
Multiple لازم يكونو نفس الاتجاه ونفس النقطتين

|V| = 4    $deg^{+}(2) = 3$

|E| = 5    $deg^{-}(2) = 2$

colum

## * Graph :-

⇒ 


$G = (\{1,2,3\}, \{(1,2), (2,3), \{1,3\}\})$

$$M = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 1 \\ 2 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 \end{array}$$

## * Simple graph

→ no direction
→ no loops
→ no parallel edges

Types :

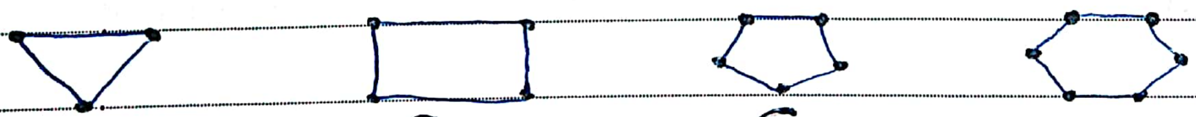
① Complete (K) :

each node must adjacent with others (تبلش من 2)

X $K_2$ $K_3$ $K_4$ $K_5$

② cycle (C) :

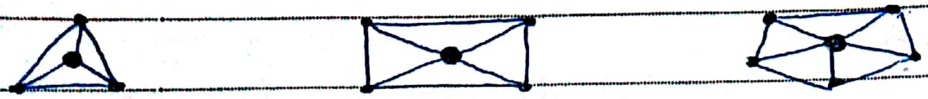
each node is adjacent with 2 around (تبلش من 3)



$C_3$ $C_4$ $C_5$ $C_6$

③ wheel (W):

cycle + 1 node inside adjacent to cycle nodes

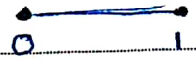


$W_3$ $W_4$ $W_5$

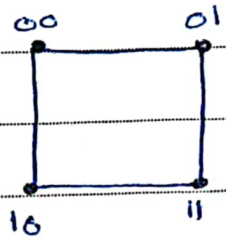
④ n-cubes (Q):

nodes in binary adjacent nodes differ in 1 bit

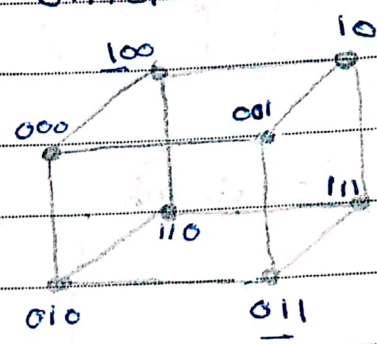
$Q_1$



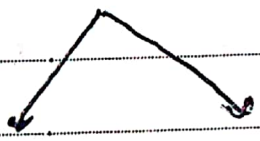

$Q_2$




$Q_3$



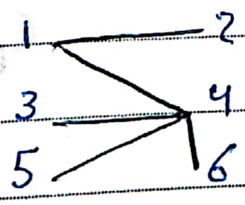

* Tree :- Graph



undirected
- no direction
- no loops
- no siple cycles
- connected

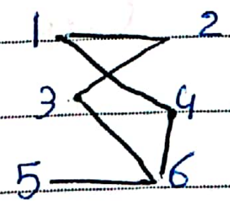
EX:

*



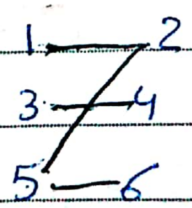

Tree

*




Simple cycle / X Tree

*

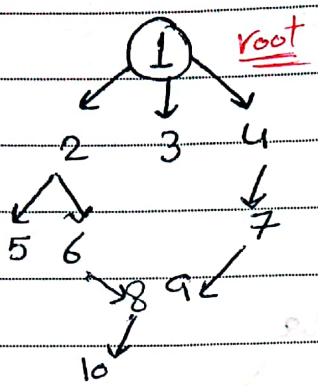



Not connected / X tree

# Tree

## * Directed :

There only one path from a root to any node.




لازم عدد الأسهم الداخلة لكل واحد تكون واحد ماعدا ال root ولا واحد.

## Terminology :-

* مثال *

root : 1

Parent : parent of 9 is 7

child : child of 1 are 2,3,4

Siblings : Siblings ( 5,6 ) (اخوة)

ancestor : ancestor of 7 ( 4, 1 )

يعني الرقم واطلعي لتوصلي لل root

descendent : descendent of 2 :

( 5, 6, 8, 10 ) الأحفاد

اللي تحتهم فش اشي

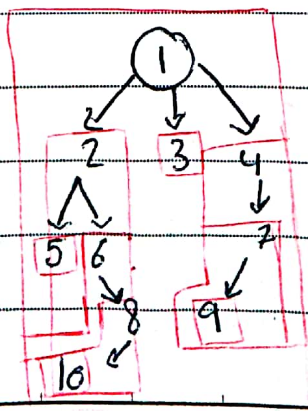
leaf (leaves) : ( 5, 3, 9, 10 )

internal : ( 1, 2, 4, 6, 7, 8 )

اللي تحتهم في اشي

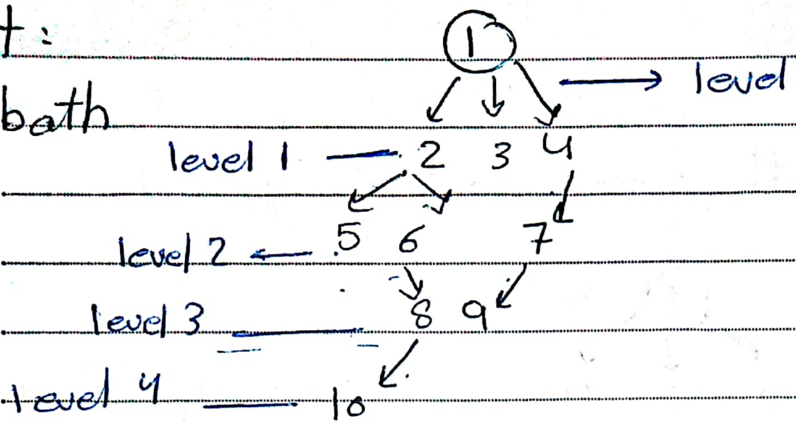
مجموعهم بيعطيني عدد الأحداث

Subtree :



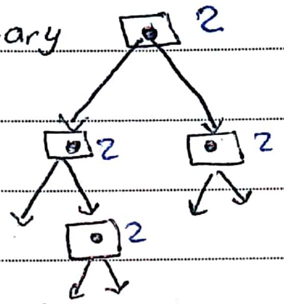
مجموعهم بساوي عدد الأحداث

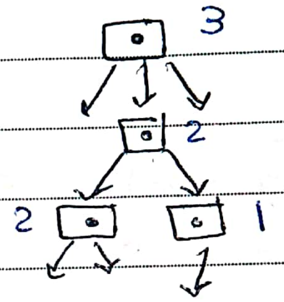
Height :
max bath




* Tree types

→ Size
- → Full-m-ary
- → m-ary



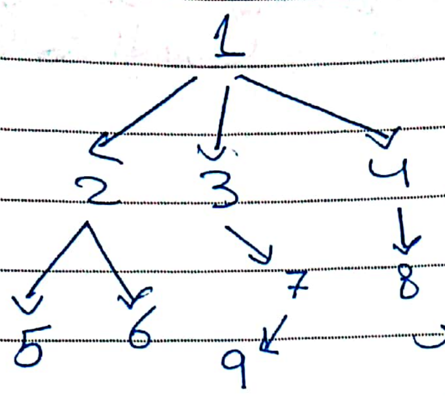




لــ نفس الحجم كل node
وهذه تسمى Full m-ary
هون Full 2-ary
Full binary
( Balance )

لـ هون عشان
نفس عدد الـ Child
إذاً M-ary
هون 3-ary
( Not Balanced )

→ balance : Difference between leaves levels لـ
لـ الفرق بين الـ leaves ما يكون أكبر من 1
- → Balanced
- → Not Balanced

# Tree Traversal :



في ال pree ال root
أول وفي ال post ال root آخر

* pre order : root, left subtree, right subtree :

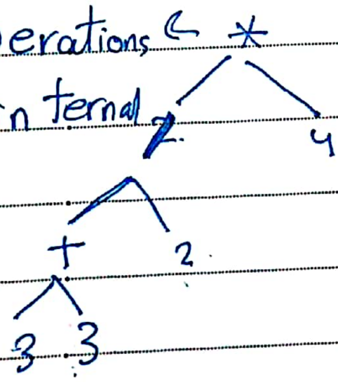
( 1, 2, 5, 6, 3, 7, 9, 4, 8 )

* in order : left sub, root, right sub :
( 5, 2, 6, 1, 3, 9, 7, 8, 4 )

* post order : left sub, right sub, root :
( 5, 6, 2, 9, 7, 3, 8, 4, 1 )

* expression tree :
(3+3) / 2 * 4

operations
internal



leaves operands

* in order : in fix : ( 3+3 ) / 2 * 4 = 12
* post order : post fix : 3 3 + 2 / 4 * العملية آخر شي
6 2 / 4 * بيلش من اليسار
3 4 *

Preorder : prefix

العملية تكون في البداية
بلش من اليمين

* / + 3 3 2 4

* / 6 2 4

* 3 4

12