سنفور مهندس

# Discrete

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## Logic :

propostion: Statement that is True (T) or False (F)

Ex. Today is sunday T ↘

1+1=3 F ↘

what is your name ? X

Do your H.w X

Propositions → Atomic (No connectors)

→ Compound (Connectors: not, and, or, if, iff)

## Connectors:

1) Negation (not) ¬ (~)

Ex: I (don't) like big sections → ¬P

| P | ¬P |
|---|---|
| T | F |
| F | T |

2) Conjunction: (and) ∧

Ex: I teach Discrete and C++

$\underbrace{\text{I teach Discreete}}$ (and) $\underbrace{\text{I teach C++}}$ → X ∧ Y

Sami and Rami are brothers → P

| X | Y | X ∧ Y |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

3) Disjunction: (or) V ⊕

inclusive exclusive

⊕

$\underbrace{\text{I Like tea}}_{P}$ (or) $\underbrace{\text{coffee}}_{Q}$ → P V Q

$\underbrace{\text{Today is sunday}}_{P}$ or $\underbrace{\text{monday}}_{Q}$ → P⊕Q

| P | Q | P V Q | P⊕Q |
|---|---|---|---|
| T | T | T | F |
| T | F | T | T |
| F | T | T | T |
| F | F | F | F |

## 4) Implication (if ∴ then) $(\longrightarrow)$

Ex: $\boxed{\text{if}}\ \overbrace{\text{you do every exercise}}^{P},\ \boxed{\text{then}}\ \overbrace{\text{you'll get full mark.}}^{Q}$

$$P \rightarrow Q \neq Q \rightarrow P$$

| | P | Q | $P \rightarrow Q$ |
|---|---|---|---|
| | T | T | T |
| | T | F | F |
| (كذب) | F | T | T |
| | F | F | T |

$P \longrightarrow Q$

1) if P then Q
2) if P, Q
3) P implies Q
4) P is sufficient for Q (كفاية)
5) Q if P
6) $\boxed{\text{P only if Q}}$ $Q \not\rightarrow P$
7) Q whenever P
8) Q is necessary for P (Sufficientness)

$$P \rightarrow Q$$

* Inverse: $\neg P \rightarrow \neg Q$

* Converse: $Q \rightarrow P$

* Contrapositive: $\neg Q \rightarrow \neg P$

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⑤ Biconditional $\longleftrightarrow$

if and only if (iff)

Sufficient and necessary

Ex. $\underbrace{\text{An integer is even}}_{P}$ $\boxed{\text{iff}}$ $\underbrace{\text{it's divisable by 2}}_{Q}$.

$P \longleftrightarrow Q$

عكس ال ④

| P | Q | $P \longleftrightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

Precedence
$\neg$
$\wedge$
$\vee$
$\rightarrow$
$\longleftrightarrow$

Translate: English $\longrightarrow$ logic (propositions).

1) $\underbrace{\text{You'll get an A}}_{P}$ $\overset{}{\text{(iff)}}$ $\underbrace{\text{you do every exercise}}_{Q}$ (or) $\underbrace{\text{you get over 35 in final.}}_{R}$

if
only if

$P \longleftrightarrow (Q \vee R)$

$(Q \vee R) \longrightarrow P$

$P \longrightarrow (Q \vee R)$

الاقواس مش ضرورية
لانه الاولوية اعلى

2) $\overset{P}{\boxed{\text{You get an A}}}$ $\overset{\text{And نفس}}{\text{(but)}}$ $\boxed{\text{you}}$ (don't) $\overset{Q}{\boxed{\text{do every exercise.}}}$

$P \wedge \neg Q$

If then زي كانا

3) $\overset{P}{\boxed{\text{Getting over 35}}}$ (and) $\overset{Q}{\boxed{\text{doing every exercise}}}$ $\boxed{\text{is Sufficient for}}$ $\overset{R}{\boxed{\text{getting A}}}$

$(P \wedge Q) \longrightarrow R$

مش
الاقواس غير ضرورية

H.W: 1) $\overset{P}{\boxed{\text{you get over 35}}}$ (but) $\boxed{\text{you}}$ (don't) $\overset{Q}{\boxed{\text{do every exercise}}}$

نفس And
nevertheless $\boxed{\text{you get an A}}$ R

$P \wedge \neg Q \wedge R$

P Q

2) you can't take a medicine unless you are sick

$\neg Q \to \neg P$

Q ⟶ R

P Q

3) you can't get an A unless you get over 35 and do every exercise.

~~$\neg Q \wedge \neg R \to \neg P$~~

$\neg Q \vee \neg R \to \neg P$ . or $\neg(Q \wedge R) \to \neg P$

Proposition

* Tautology: Compound proposition that is always True

$P \vee T$ , $P \to T$ , $F \to Q$ , $P \vee \neg P$

* Contradiction: Compound proposition that is always False

$P \wedge F$ , $P \wedge \neg P$ , $P \leftrightarrow \neg P$ , $P \oplus P$

* Contingency: not tautology not Contradiction

$P \wedge Q$ , $P \vee Q$ , ....

Proof: Equivalence
- By truth table
- Equivalence rules

① Truth table:

$\boxed{Q \to P}$

Ex. Show if $\boxed{P \to Q}$ and its converse are equivalent

$P \to Q \overset{?}{=} Q \to P$

| P | Q | P→Q | Q→P | L↔R |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

Not tautology

$P \to Q \neq Q \to P$

$P \to Q \not\Leftrightarrow Q \to P$

Ex: Show that $\underbrace{\neg(P\vee(\neg P\wedge Q))}_{L} \Longleftrightarrow \underbrace{\neg P\wedge Q}_{R}$

| P | Q | ¬P | ¬P∧Q | P∨(¬P∧Q) | ¬(P∨(¬P∧Q)) |
|---|---|---|---|---|---|
| T | T | F | F | T | F |
| T | F | F | F | T | F |
| F | T | T | T | T | F |
| F | F | T | F | F | T |

$\neg(P\vee(\neg P\wedge Q)) \not\Longleftrightarrow \neg P\wedge Q$

Ex. Show that $\neg(P\vee Q) \Longleftrightarrow \neg P\wedge\neg Q$

| P | Q | P∨Q | ¬(P∨Q) | ¬P | ¬Q | ¬P∧¬Q |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | T | F | F | T | F |
| F | T | T | F | T | F | F |
| F | F | F | T | T | T | T |

$\neg(P\vee Q) \Longleftrightarrow \neg P\wedge\neg Q$

Logical Equivalence:
- → truth table
- → using rules

Equivalence rules:-

1) $P\wedge T \Longleftrightarrow P$ تعتمد ع P
2) $P\vee F \Longleftrightarrow P$
3) $P\wedge F \Longleftrightarrow F$
4) $P\vee T \Longleftrightarrow T$
5) $P\vee P \Longleftrightarrow P$
6) $P\wedge P \Longleftrightarrow P$
7) $\neg(\neg P) \Longleftrightarrow P$
8) $P\vee Q \Longleftrightarrow Q\vee P$
9) $P\wedge Q \Longleftrightarrow Q\wedge P$
10) $P\vee(Q\vee R) \Longleftrightarrow (P\vee Q)\vee R$ نفسها
11) $P\wedge(Q\wedge R) \Longleftrightarrow (P\wedge Q)\wedge R$
12) $P\wedge(Q\vee R) \Longleftrightarrow (P\wedge Q)\vee(P\wedge R)$
13) $P\vee(Q\wedge R) \Longleftrightarrow (P\vee Q)\wedge(P\vee R)$
14) $P\rightarrow Q \Longleftrightarrow \neg P\vee Q$ عشان نتخلص منها
15) $P\leftrightarrow Q \Longleftrightarrow (P\rightarrow Q)\wedge(Q\rightarrow P)$ عشان نتخلص منها

$\Longleftrightarrow (P\wedge Q)\vee(\neg P\wedge\neg Q)$

16) $\neg(P\wedge Q) \Longleftrightarrow \neg P\vee\neg Q$
17) $\neg(P\vee Q) \Longleftrightarrow \neg P\wedge\neg Q$

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Ex. show that $\underbrace{\neg(P \vee (\neg P \wedge Q))}_{L} \Longleftrightarrow \underbrace{\neg P \wedge \neg Q}_{R}$

$\neg(P \vee (\neg P \wedge Q))$

$\Longleftrightarrow \neg P \wedge \neg(\neg P \wedge Q)$

$\Longleftrightarrow \neg P \wedge (P \vee \neg Q)$

$\Longleftrightarrow (\neg P \wedge P) \vee (\neg P \wedge \neg Q)$

$\Longleftrightarrow F \vee (\neg P \wedge \neg Q)$

$\Longleftrightarrow \neg P \wedge \neg Q$

Ex. show that $(P \wedge Q) \rightarrow (P \vee Q)$ is tautology

$(P \wedge Q) \rightarrow (P \vee Q) \Longleftrightarrow T$

$(P \wedge Q) \rightarrow (P \vee Q)$

$\Longleftrightarrow \neg(P \wedge Q) \vee (P \vee Q)$

$\Longleftrightarrow (\neg P \vee \neg Q) \vee (P \vee Q)$

$\Longleftrightarrow (\neg P \vee P) \vee (\neg Q \vee Q)$

$\Longleftrightarrow T \vee T$

$\Longleftrightarrow T$

H.W

Ex. 1) $(P \leftrightarrow Q) \Longleftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

2) $[(P \leftrightarrow Q) \wedge (Q \rightarrow R) \rightarrow (P \rightarrow R)] \Longleftrightarrow T$

3) $P \wedge (P \vee Q) \Longleftrightarrow P$

## Mathematical reasoning:

prove that conclusion logically follows from premises.
- ↳ truth table
- ↳ Rules (inference)

Ex. If $\overbrace{\text{you get full mark in final}}^{P}$, $\overbrace{\text{you'll get an A}}^{Q}$

you get a full mark in final

you'll get an A

1. $P \rightarrow Q$ ] premises
2. $P$

$Q$ ] Conclusion

| | $\boxed{P_2}$ P | $\boxed{C}$ Q | $\boxed{P_1}$ $P \rightarrow Q$ | $P_1 \wedge P_2$ | $P_1 \wedge P_2 \rightarrow C$ |
|---|---|---|---|---|---|
| | T | T | T | T | T |
| α | T | F | F | F | T |
| α | F | T | T | F | T |
| α | F | F | T | F | T |

اللي فيه جميع ال Premises true

$P_1 \wedge P_2 \Rightarrow C$

~~((P∧~~ $((P \rightarrow Q) \wedge P) \rightarrow Q$

Ex. If $\overbrace{\text{it's hot}}^{P}$ then $\overbrace{\text{we will swim}}^{Q}$.

we will swim

it's hot

1. $P \rightarrow Q$
2. $Q$

$P$

| | $\boxed{C}$ P | $\boxed{P_2}$ Q | $\boxed{P_1}$ $P \rightarrow Q$ | $P_1 \wedge P_2$ | $P_1 \wedge P_2 \rightarrow C$ |
|---|---|---|---|---|---|
| | T | T | T | T | T |
| α | T | F | F | F | T |
| | F | T | T | T | F |
| α | F | F | T | F | T |

$((P \rightarrow Q) \wedge Q) \rightarrow P$ X

Not tautology

Conclusion is not correct

Fallicy

Ex IF $\overbrace{\text{N is odd}}^{P}$ then $\overbrace{\text{2n is even}}^{Q}$

N is not odd

2n is not ~~odd~~ even

1. $P \to Q$

2. $\neg P$

$\neg Q$

| | | $P_1$ | $P_2$ | C | |
|---|---|---|---|---|---|
| P | Q | $P \to Q$ | $\neg P$ | $\neg Q$ | |
| T | T | T | F | F | α |
| T | F | F | F | T | α |
| F | T | T | T | F | Fallicy |
| F | F | T | T | T | |

Ex. If $\overbrace{\text{you send me}}^{E}$ Email $\overbrace{\text{I will Finish the program}}^{P}$ $P_1$

If you dont send me Email $\overbrace{\text{I will go sleep}}^{S}$ $P_2$

If I ~~will~~ go sleep, then $\overbrace{\text{I will wake ~~g~~ up fresh}}^{W}$

If I dont finish the program therefore I will wake up Fresh

1. $E \to P$

2. $\neg E \to S$ 16 احتمال

3. $S \to W$

$\neg P \to W$

|  |  |  |  | $P_1$ |  | $P_2$ | $P_3$ |  | C |
|---|---|---|---|---|---|---|---|---|---|
| E | P | S | W | $E \to P$ | $\neg E$ | $\neg E \to S$ | $S \to W$ | $\neg P$ | $\neg P \to W$ |
| T | T | T | T | T | F | T | T | F | T |
| T | T | T | F | T | F | T | F | F | T |
| T | T | F | T | T | F | T | T | F | T |
| T | T | F | F | T | F | T | T | F | T |
| T | F | T | T | F | F | T | T | T | T |
| T | F | T | F | F | F | T | F | T | F |
| T | F | F | T | F | F | T | T | T | T |
| T | F | F | F | F | F | T | T | T | F |
| F | T | T | T | T | T | T | T | F | T |
| F | T | T | F | T | T | T | F | F | T |
| F | T | F | T | T | T | F | T | F | T |
| F | T | F | F | T | T | F | T | F | T |
| F | F | T | T | T | T | T | T | T | T |
| F | F | T | F | T | T | T | F | T | F |
| F | F | F | T | T | T | F | T | T | T |
| F | F | F | F | T | T | F | T | T | F |

$(P_1 \wedge P_2 \wedge P_3) \longrightarrow C$

Reasoning

- Truth table
- Inference rules :-

1) $$\frac{P}{P \vee Q} \qquad P \Rightarrow P \vee Q$$

2) $$\begin{array}{c} P \\ Q \\ \hline P \wedge Q \end{array} \qquad (P) \wedge (Q) \Longrightarrow P \wedge Q$$

3) $$\frac{P \wedge Q}{P} \qquad P \wedge Q \Rightarrow P$$

$$\frac{P \wedge Q}{Q} \qquad P \wedge Q \Rightarrow Q$$

4) $$\begin{array}{c} P \rightarrow Q \\ P \\ \hline Q \end{array} \qquad P \rightarrow Q \wedge P \Longrightarrow Q$$

5) $$\begin{array}{c} P \rightarrow Q \\ \neg Q \\ \hline \neg P \end{array} \qquad P \rightarrow Q \wedge \neg Q \Rightarrow \neg P$$

6) $$\begin{array}{c} P \rightarrow Q \\ Q \rightarrow R \\ \hline P \rightarrow R \end{array} \qquad (P \rightarrow Q) \wedge (Q \rightarrow R) \Longrightarrow P \rightarrow R$$

7) $$\begin{array}{c} P \vee Q \\ \neg P \\ \hline Q \end{array} \qquad (P \vee Q) \wedge \neg P \Longrightarrow Q$$

(الجدول)

Ex. $$\begin{array}{cl} E \rightarrow P & (P_1) \\ \neg E \rightarrow S & ((P_2) \\ S \rightarrow W & (P_3) \\ \hline \neg P \rightarrow W & \end{array}$$

① $$\begin{array}{c} \neg E \rightarrow S \\ S \rightarrow W \\ \hline \neg E \rightarrow W \end{array}$$

③ $$\begin{array}{c} \neg W \rightarrow E \\ E \rightarrow P \\ \hline \neg W \rightarrow P \end{array}$$

② $\neg E \rightarrow W \overset{=}{\Longleftrightarrow} \neg W \rightarrow E$

$\neg W \rightarrow P \overset{=}{\Longleftrightarrow} \neg P \rightarrow W$

$$\begin{array}{ll} \neg P \wedge Q & (P_1) \\ R \to P & (\cancel{P_2}) \\ \neg R \to S & (\cancel{P_3}) \\ S \to A & (\cancel{P_4}) \\ \hline A & \end{array}$$

① $$\begin{array}{l} \neg R \to S \\ S \to A \\ \hline \neg R \to A \end{array}$$

② $\neg R \to A \Leftrightarrow \neg A \to R$

③ $$\begin{array}{l} \neg A \to R \\ R \to P \\ \hline \neg A \to P \end{array}$$

④ $$\begin{array}{l} \neg P \wedge Q \\ \hline \neg P \end{array}$$

⑤ $$\begin{array}{l} \neg A \to P \\ \neg P \\ \hline \neg\neg A \Leftrightarrow \boxed{A} \end{array}$$

Ex. * $$\begin{array}{ll} P \to Q \vee R & (\cancel{P_1}) \\ P \wedge S & (\cancel{P_2}) \\ \neg Q & (\cancel{P_3}) \\ \hline R \vee S & \end{array}$$

① $$\begin{array}{l} P \wedge S \\ \hline P \end{array}$$

② $$\begin{array}{l} P \to Q \vee R \\ P \\ \hline Q \vee R \end{array}$$

③ $$\begin{array}{l} Q \vee R \\ \neg Q \\ \hline R \end{array}$$

④ $$\begin{array}{l} R \\ \hline R \vee S \end{array}$$

$P \vee Q \to R$ ($P_1$)

$S \vee \neg Q$ ($P_2$)

$Z$ ($P_3$)

$P \wedge R \to \neg S$ ($P_4$)

$$\begin{array}{l} \neg P \to \neg Z \quad (P_5) \\ \hline \neg Q \end{array}$$

① $$\begin{array}{c} \neg P \to \neg Z \\ Z \\ \hline P \end{array}$$

② $S \vee \neg Q \iff Q \to S$

③ $Q \to S \iff \neg S \to \neg Q$

④ $$\begin{array}{c} \neg S \to \neg Q \\ P \wedge R \to \neg S \\ \hline P \wedge R \to \neg Q \end{array}$$

⑤ $$\begin{array}{c} P \\ \hline P \vee Q \end{array}$$

⑥ $$\begin{array}{c} P \vee Q \to R \\ P \vee Q \\ \hline R \end{array}$$

⑦ $$\begin{array}{c} P \\ R \\ \hline P \wedge R \end{array}$$

⑧ $$\begin{array}{c} P \wedge R \to \neg Q \\ P \wedge R \\ \hline \neg Q \end{array}$$

Ex: Every Student has a book.

$\underline{\text{Ali is a student}}$

Ali has a book

$$\begin{array}{c} P \\ Q \\ \hline R \end{array}$$

Predicates:-

$\underbrace{\text{Ali}}_{\text{Subject}}$ $\underbrace{\text{has a book}}_{\text{Predicate}}$

book(Ali) arity = 1

$\underbrace{\text{Ali}}_{\text{Subject 1}}$ and $\underbrace{\text{Sami}}_{\text{Subject 2}}$ aref $\underbrace{\text{Friends}}_{\text{predicate}}$

F(Ali, Sami) arity = 2

Ali has a book and Sami has a book.

book(Ali) ∧ book(Sami)

1+1 = 2

S (1,1,2) arity = 3

2 is negative

n(2) F $\overset{\text{proposition}}{✓}$

X is negative

n(X) No truth value $\overset{✗}{\text{Proposition}}$

every $\overset{\text{number}}{X}$ is negative

∀x n(X) F ✓ proposition

Some X are negative

∃x n(x) T ✓ proposition

Quantifiers:

1) Universal

$\forall$ ( ~~Ev~~ Every , All , Any , Each ---

2) Existential

$\exists$ ( Some , Few , at least , there is

- Square of $\underbrace{3}_{S_1}$ is $\underbrace{9}_{S_2}$

اذا كان في Square وحدة الترتيب يهمهم

اما اذا تكررت يهم الترتيب

S(3,9) T ✓Proposition

- Square of 3 is 9 and Square of 9 is 81

$S(3,9) \cap S(9,81)$

- Square of x is y

$S(x,y)$

- 100 is Square of $\underbrace{\text{Some} \overset{x}{\ } \text{number}}$

$\exists x \; S(x,100)$

- ~~Every~~ 9 is square of $\underbrace{\text{exery} \overset{x}{\ } \text{number}}$ .

$\forall x \; S(x,9)$

- $\underbrace{\text{Every} \overset{x}{\ } \text{number}}$ is square of 2

$\forall x \; S(2,x)$

- $\underbrace{\text{There} \overset{x}{\text{is}} \text{number}}$ which is Square of $\underset{\smile}{2}$ ( 2 has a square )  نفسها

$\exists x \; S(2,x)$

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Predicates :-

Ex: Assume

$V(x, y)$ : x visited y

$t(x)$ : x is teacher

$S(x)$ : x is student

Domain: People

1) Ali visited Sami

$V(Ali, Sami)$

2) Ali visited Someone

$\exists x \; V(Ali, x)$

3) Ali visited everyone

$\forall x \; V(Ali, x)$

4) Someone visited Ali

$\exists x \; V(x, Ali)$

5) Everyone visited Ali

$\forall x \; V(x, Ali)$

6) No one visited Ali. ≡ Everyone didn't visit Ali

$\neg \exists x \; V(x, Ali) \equiv \forall x \; \neg V(x, Ali)$

7) Not everyone visited Ali ≡ Someone didn't visit Ali.

$\neg \forall x \; V(x, Ali) \equiv \exists x \; \neg V(x, Ali)$

8) Someone visited Someone.

$\exists x \exists y \; V(x, y) \equiv \exists y \exists x \; V(x, y)$

9) everyone visited everybody

$\forall x \forall y \; V(x, y) \equiv \forall y \forall x \; V(x, y)$

10) Someone visited everyone.

Same ~~Sing~~ $\underline{\underline{\exists x}} \; \forall y \; V(x, y)$

11) Everyone was visited by Someone

$\forall y \exists x \; V(x, y)$

12) Some teachers visited Ali

$\exists x \ V(X, Ali)$ X غلط

$\exists x [t(x) \land V(X, Ali)]$ Rule

Some Students visited Ali

$\exists x [S(x) \land V(X, Ali)]$ لازم الاقتران

13) All teachers visited Ali

$\forall x [t(x) \rightarrow V(X, Ali)]$ Rule②

14) Every teacher visited or has been visited by Ali

$\forall x [t(x) \longrightarrow V(X, Ali) \ \lor \ V(Ali, X)]$ لو ما حطينا الأقواس عادي

نفس الاشي (41)

15) Every teacher visited or has been visited by Some students

~~$\forall x \ t(x) \longrightarrow V(X, Ali)$~~

$\forall x [t(x) \longrightarrow \exists y (S(y) \land V(x, y)) \ \lor \ V(y, x))]$

(H.w) - There is a teacher who has never been visited by Any Student.

- Some students have visited every teacher.
- All students visited Ali and some teachers too.
- Ali visited everyone but no one visited him.

① $\exists x [t(x) \land \neg \exists y (S(y) \land V(y, x))]$

~~① $\exists x [t(x) \land \neg$~~

② $\exists x [S(x) \land \forall y (t(y) \rightarrow V(y, x))]$

③ $\forall x [S(x) \longrightarrow \exists (y) \ t(y) \land V(X, Ali) \land V(X, y)]$

④ $\forall x [V(Ali, x) \land \neg \exists x \ V(x, Ali)]$

## Predicates :

Ex:- Find truth value:-

1) $\forall x\, P(x)$ True $\Longrightarrow$ $\exists x\, P(x)$ T

$P(x): x+1 > x$

Domain R

2) $\forall x\, Q(x)$

$Q(x): x < 2$ F

Domain R

3) $\exists x\, P(x)$ T

$P(x): x > 3$

domain R

4) $\exists x\, Q(x)$ F $\Longrightarrow$ $\forall x\, Q(x)$ F

$Q(x): x = x+1$

domain R

5) $\forall x \forall y\; P(x,y)$ T

$P(x,y): x+y = y+2$

domain R

y وحدة لكل

6) $\exists y \forall x\; Q(x,y)$ F

$Q(x,y): x+y=0$

$* \longrightarrow$ T

Domain R

لكل x في y

7) $\forall x \exists y\; Q(x,y)$

$Q(x,y) = x+y=0$ T

$* \longrightarrow$ T

Domain R

8) $\exists x \forall y \forall z \; Q(x,y,z)$ F

$Q(x,y,z) = x = y + z$

Domain R

$\forall y \forall z \exists x \; Q(x,y,z)$ T

9) $\forall x \; P(X)$ F

$P(x) : x^2 < 10$

Domain : positive integers not exceeding 4

$\{1, 2, 3, 4\}$

## Reasoning:-

### Inference rules:

1) $\dfrac{\forall x\, P(x)}{P(C)}$ → عشوائي arbitrary element

$\dfrac{P(c)}{\forall x\, P(x)}$

2) $\dfrac{\exists x\, P(x)}{P(C)}$ → Particular element

$\dfrac{P(c)}{\exists x\, P(x)}$

إذا طلب الـ Domain نأخذ الشكل Domain

Ex. Domain People

Every student has a book

Ali is a student

Ali has a book

1) $\forall x\,[S(x) \rightarrow b(x)]$

2) $\dfrac{S(Ali)}{b(Ali)}$

$$\dfrac{\forall x\,(S(x) \rightarrow b(x))}{S(Ali) \rightarrow b(Ali)}$$

$$\dfrac{\begin{array}{l} P\ \boxed{S(Ali)} \rightarrow \boxed{b(Ali)}\ Q \\ P\ \boxed{S(Ali)} \end{array}}{b(Ali)}$$

اول ما اشوف everyone ---- Predic..

Ex.

Everyone who wins is rich

Mary wins

Someone is rich

1) $\forall x\,[w(x) \rightarrow r(x)]$

2) $\dfrac{W(Mary)}{\exists x\ r(x)}$

$$\dfrac{\forall x\,[w(x) \rightarrow r(x)]}{w(Mary) \rightarrow r(Mary)}$$

$$\dfrac{\begin{array}{l} w(Mary) \rightarrow r(Mary) \\ w(Mary) \end{array}}{r(Mary)}$$

$$\dfrac{r(Mary)}{\exists x\ r(x)}$$

Ex. Every IT student likes programming

Everyone is IT student

Everyone likes programming

اذا التنتين ∀ نثبت باي وحدة

العنوان لانه نفس x

1) $\forall x [ITS(x) \longrightarrow P(x)]$

$\forall x \ ITS(x)$

$\forall x \ P(x)$

$\forall x \ [ITS(x) \rightarrow P(x)]$

عشوائي $ITS(e) \rightarrow P(e)$ ↳ arbitrary

$\forall x \ ITS(x)$

$ITS(e)$

$\overset{p}{ITS(e)} \rightarrow \overset{q}{P(e)}$

$\overset{p}{ITS(e)}$

$P(e)$

$P(e)$ → arbitrary

$\forall x P(x)$

Ex. Every IT students like programming

لما يكون في ∃ و ∀ منبلش ب ∃

Some are IT students

Some like programming

$\exists x \ ITS(x)$

$ITS(e)$ ↳ Particular

1) $\forall x [ITS \rightarrow P(x)]$

2) $\exists x \ ITS(x)$

$\exists x \ P(x)$

$\forall x [ITS(x) \rightarrow P(x)]$

$ITS(e) \rightarrow P(e)$

$ITS(e)$

$ITS(e) \rightarrow P(e)$

$P(e)$

$P(e)$ → Particular

$\exists x P(X)$

H.W

1)
$$\begin{array}{l} C(Ali) \\ J(Ali) \\ \forall x[J(x) \rightarrow h(x)] \\ \hline \exists x[C(x) \wedge h(x)] \end{array}$$

2)
$$\begin{array}{l} \forall x[J(x) \rightarrow f(x)] \\ \exists x[J(x) \wedge \neg S(x)] \\ \hline \exists x[F(x) \wedge \neg S(x)] \end{array}$$

مستحيل يطلع ∃

Theorem proof :-

1) Direct $(P \rightarrow Q)$

Assume P is true

prove Q

Ex: if $\underbrace{n \text{ is odd}}_{P}$ then $\underbrace{n^2 \text{ is odd}}_{Q}$

Assume n is odd

$n = 2K+1$

$n^2 = (2k+1)^2$

$= 4k^2 + 4K + 1$

$= 2(2k^2+2k) + 1$

$= 2L + 1$ odd

Ex: If n is odd then 3n+2 is odd

Assume n is odd

$n = 2K+1$

$3n+2$

$= 3(2K+1) + 2$

$= 3\ 6K + 3 + 2$

$= 6K + 5$

عشان أخد
عامل مشترك

$= 6K + 4 + 1$

$= 2(3K+2) + 1$

$= 2n + 1$ odd

Contrape-

Ex: if 3n+2 is $\underset{\text{even}}{\underline{\text{not odd}}}$ then n is $\underset{\text{even}}{\underline{\text{not odd}}}$

Assume 3n+2 even

بعكس السؤال نحل
indirect

$3n+2 = 2K$

$n = \frac{2K-2}{3}$

2) Indirect $(P \to Q)$

1) Find Contrapositive

2) Prove directly

Ex: if $n^2$ is odd then $n$ is odd

① Contrapositive

if $n$ is even then $n^2$ is even

$n = 2k$

$n^2 = 4k^2$

$= 2(2k^2)$

$= 2L$ even

Ex: if $3n+2$ is odd then $n$ is odd.

Indirect

if $n$ is even then $3n+2$ is even

$n = 2k$

$3n+2 = 3(2k)+2$

$= 6k+2$

$= 2(3k+1)$

$2L$ even

3) Contradiction

Ex: Prove that $\sqrt{2}$ is irrational

Assume $\sqrt{2}$ is rational

$\sqrt{2} = \frac{a}{b} \Rightarrow a = \sqrt{2}\,b$

$a = 2k$

$a^2 = 2b^2 \longrightarrow a^2$ is even $\Rightarrow \boxed{a \text{ is even}}$

~~a~~ $(2k)^2 = 2b^2 \to b^2 = 2k^2 \to b^2$ even $\to \boxed{b\text{even}}$

$\sqrt{2} = \frac{a}{b}$ ~~most~~ not Simplest form

↳ $\sqrt{2}$ is rational

↳ $\sqrt{2}$ is irrational

Ex: by contradiction

if $\underbrace{3n+2 \text{ is odd}}_{P \quad T}$ then $\underbrace{n \text{ is odd}}_{Q \quad F}$

Assume if $3n+2$ is odd then $n$ is even

Indirect  if $n$ is odd then $3n+2$ is even

$n = 2k+1$

$3n+2 = 3(2k+1)+2$

$6k+3+2$

$6k+4+1$

$= 2(3k+2)+1$

$= 2L+1$ odd        الجملة ✓ Assume X

H.W

Ex: - the sum of 2 odd integers is even        ( direct , in )

- if $m, n$ are 2 squares then $mn$ is square
- if $n$ is odd then $n^2$ is odd        (Contra)

Sets :- group of elements

Represent :-

1) Set builder

$S = \{ x \mid x$ is positive odd integer between 0 and 10 $\}$

such that

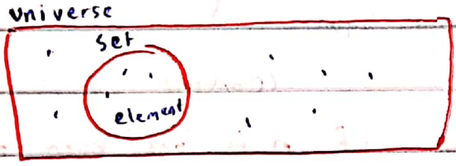
2) listing

$S = \{1, 3, 5, 7, 9\}$

$u = \{10, 20, 30, 40, 50, ----\}$

$v = \{----, -4, -2, 0, 2, 4, 6, -----\}$

3) Venn diagram



Note:

R: real numbers

Z: integers

$Z^+$: positive integers $\{1, 2, 3, ---$

N : natural numbers $\{0, 1, 2, 3, ----$

$\{\ \}$

$S = \{ \phi, 1, \{1\}, \{1,2\} \}$

انتماء belongs

element $\in$ Set

$\phi \in S$

$1 \in S$

$\{1\} \in S$

$\{1,2\} \in S$

$2 \notin S$

Subset

Set A $\subseteq$ Set B

$\forall x \, (x \in A \longrightarrow x \in B)$ يكون مجموعة عنصر

$\phi \subseteq S$

$\{\phi\} \subseteq S$

$\{1\} \subseteq S$

$\{\{1\}\} \subseteq S$

$\{\{1,2\}\} \subseteq S$

$\{\phi, 1\} \subseteq S$

$\{\{1\}, \{1,2\}\} \subseteq S$

$\vdots$

$\{1, 3, \underline{\underline{2}}\} \subseteq S$

$\{\emptyset, 1, \{1\}\} \subseteq S$

$\{\emptyset, \{1\}, \{1,2\}\} \subseteq S$

$\{\{\underline{\underline{\emptyset}}\}, \{1\}, \{\underline{\underline{2}}\}\} \nsubseteq S$

$\vdots$

$\{\emptyset, 1, \{1\}, \{1,2\}\} \subseteq S$

---

$|S|$ : Cardinality

no of elements $\in S$.

$|S| = 4$

$|\emptyset| = 0$

$|\{\emptyset\}| = 1$

$|\{\ \}| = 1$

$|\{\{1,2,3\}\}| = 1$

$|\{\{1,2\},\{3\}\}| = 2$

$|\{\{\{1,2\},\{3\}\}\}| = 1$

$|\{1,1\}| = 1$

$|\{\{1\},\{1\}\}| = 1$

$|\{1,\{1\}\}| = \cancel{1}\,2$

Notes : $\subseteq$

1) $\emptyset \subseteq S$ any $S$

2) $A \subseteq B \wedge B \subseteq A$ — A, B are sets $\rightarrow$ x $\in$ B) ,

$\Rightarrow A = B$

3) $\subseteq$ Subset — $\{1,2\} \subseteq \{1,2,3\}$

$\subset$ proper subset — $\{1,2,3\} \subseteq \{1,2,3\}$

$\{1,2\} \subset \{1,2,3\}$

$\{1,2,3\} \not\subset \{1,2,3\}$

## Set:

1) $\in$ $\subseteq$

$\subset$

2) $| \ |$

3) Powerset Set of all Subsets

$P(A)$ or $P(A)$

* $A = \{1,2\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

* $A = \{1,2,3\}$

Proper line Subset

$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$

$|A| = n$

$|P(A)| = 2^n$

* $A = \emptyset$

$P(A) = \{\emptyset\}$

* $A = \{\emptyset\}$

$P(A) = \{\emptyset, \{\emptyset\}\}$

4) X : Caretesian poduct

$\underbrace{A \times B}_{\text{Sets}} = \{(x,y) \mid x \in A \land y \in B\}$.

$A = \{1,2,3\}$

$B = \{a,b\}$

$C = \emptyset$ $d = \{5\}$

$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

$\neq$

$B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

$B \times C = \emptyset$

$A \times B \times D = \{(1,a,5), (1,b,5), (2,a,5), (2,b,5), (3,a,5), (3,b,5)\}$

Ex:

$U = \{1, 2, 3, \dots, 10\}$

$A = \{1, 2, 5, 8\}$

$B = \{2, 3, 6\}$

$C = \emptyset$

$D = \{3, 4\}$

$A \cup B = \{1, 2, 5, 8, 3, 6\}$

$A \cup C = \{1, 2, 5, 8\} = A$

$A \cup U = U$

$A \cap B = \{2\}$

$A \cap C = \emptyset$

$A \cap U = A$

$A \cap D = \emptyset$

$A, D$ are disjoint

$A - B = \{1, 5, 8\}$

$B - A = \{3, 6\}$

$A - C = \{1, 2, 5, 8\} = A$

$C - A = \emptyset$

$A - U = \emptyset$

$U - A = \{3, 4, 6, 7, 9, 10\}$

$\bar{A} = \{3, 4, 6, 7, 9, 10\}$

$\bar{B} = \{1, 4, 5, 7, 8, 9, 10\}$

$\bar{C} = U$

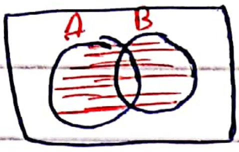
$\bar{U} = \emptyset$

$(\overline{\bar{A}}) = A$

## Operations :

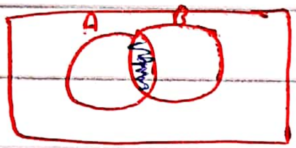
1) $\cup$ union

$$\underbrace{A \cup B}_{\text{Sets}} = \{x \mid x \in A \vee x \in B\}$$



2) Intersection $\cap$

$$\underbrace{A \cap B}_{\text{Sets}} = \{x \mid x \in A \wedge x \in B\}$$

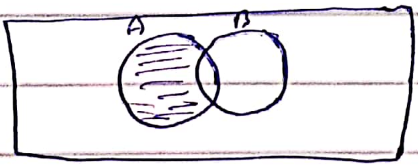


Note :

$$\underset{10}{|A \cup B|} = \underset{6}{|A|} + \underset{\boxed{6}}{|B|} - \underset{2}{|A \cap B|}$$

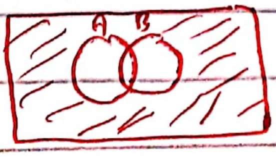
3) Difference : −

$$\underbrace{A - B}_{\text{Sets}} = \{x \mid x \in A \wedge \underset{\neg(x \in B)}{x \notin B}\}$$



4) Complement ¯

$$\bar{A} = \{x \mid x \notin A\}$$



## Sets :-

· Prove 2 sets are equal

- ↳ membership table
- ↳ Venn diagram
- ↳ Set identities (rules)

Ex: $(\overline{A \cup B}) = \overline{A} \cap \overline{B}$

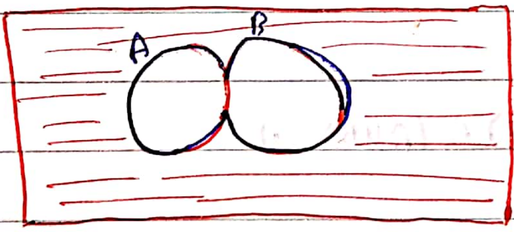
### ① membership table:

=

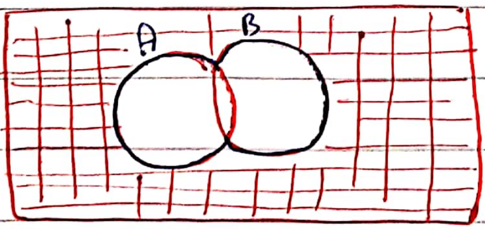
|  |  | (or) | ① |  |  | ② and |
|---|---|---|---|---|---|---|
| $A$ | $B$ | $A \cup B$ | $(\overline{A \cup B})$ | $\overline{A}$ | $\overline{B}$ | $\overline{A} \cap \overline{B}$ |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |

### ② Venn diagram

①



②



## ③ Set Identities (A, B, C : Sets)

1) $A \cup A = A$
2) $A \cap A = A$
3) $A \cup \emptyset = A$
4) $A \cap U = A$
5) $A \cap \emptyset = \emptyset$
6) $A \cup U = U$
7) $\overline{(\overline{A})} = A$
8) $A \cup B = B \cup A$
9) $A \cap B = B \cap A$
10) $A \cup (B \cup C) = (A \cup B) \cup C$
11) $A \cap (B \cap C) = (A \cap B) \cap C$
12) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
13) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
14) $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$
15) $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$

Ex:

$A \cup (A \cap B) = \cancel{B}\ A$

$(A \cup A) \cap (A \cup B)$

~~$A \cap (A \cup B)$~~

$(A \cap U) \cup (A \cap B)$

~~$A \cap (U \cap B)$~~

$A \cap (U \cup B)$

$A \cap U$

$\boxed{A}$

$A \cap (A \cup B) = A$

## Functions:

$F: \underbrace{A \rightarrow B}_{\text{Sets}}$

Such that every element $\in A$
is related to only one
element in set B.

$F: \overset{\text{Domain}}{A} \rightarrow \overset{\text{Codomain}}{B}$

$F(a) = b$

Preimage → $a \in A$

image → $b \in B$

$\{ b's \}$ : range

Ex: $F: \{1, 2, 3\} \rightarrow \{a, b, c, d\}$

$f(1) = a$

$f(2) = c$

$f(3) = d$

Domain $\{1,2,3\}$

Codomain $\{a,b,c,d\}$

Range $\{a,c,d\}$

## Types:-

1) One to one (injective)

different elements in domain
have different images.

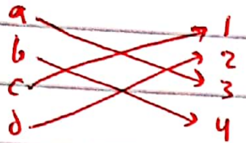
2) Onto (surjective)

Every element in Codomain is image

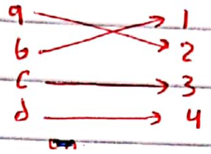
3) Bijective:

One to one and onto

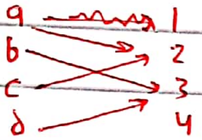
Ex: $f: \{a,b,c,d\} \longrightarrow \{1,2,3,4\}$



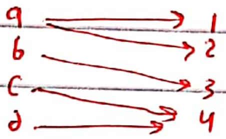
one to one
onto , Bijective



one to one
onto , bijective



not one to one
not onto



not Function

$F: R \longrightarrow R$

$f(x) = x+1$

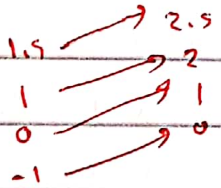
one to one
onto , bijective

⊕ one to one onto

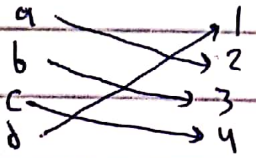
$f: Z \longrightarrow Z \rightarrow$ $= \{1,4,9,16,25\}$ اذا انتاطها

$F(x) = x^2$

Not one to one
not onto



Strictly increasing & decreasing are one to one



one to ~~ont~~ one
not onto

Floor & Ceiling

$\lfloor x \rfloor$ $\lceil x \rceil$

$f : R \rightarrow Z$

| nearest integer $\leq x$ | nearest integer $\geq x$ |
|---|---|
| $\lfloor 3.5 \rfloor = 3$ | $\lceil 3.5 \rceil = 4$ |
| $\lfloor 3.1 \rfloor = 3$ | $\lceil 3.1 \rceil = 4$ |
| $\lfloor 3.9 \rfloor = 3$ | $\lceil 3.9 \rceil = 4$ |
| $\lfloor 3 \rfloor = 3$ | $\lceil 3 \rceil = 3$ |
| $\lfloor -3.5 \rfloor = -4$ | $\lceil -3.5 \rceil = -3$ |

Not one to one

onto

Not bijective

$-\lfloor x \rfloor = \lceil -x \rceil$

$\lfloor x + m \rfloor = \lfloor x \rfloor + m$

## Function operations :-

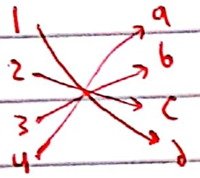
### 1) Inverse :-

$F: A \longrightarrow B$ $\quad\quad$ $F^{-1}: B \rightarrow A$

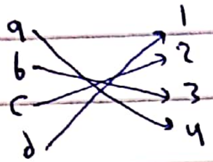
$F(a) = b$ $\quad\quad$ $F^{-1}(b) = a$

Note :- Function to be invertible must be bijective

$F$



$F^{-1}$



Ex. $\quad f: Z \longrightarrow Z$ $\quad\quad$ $f^{-1}: Z \rightarrow Z$

$F(x) = x+1$ $\quad\quad$ $F^{-1}(x) = x-1$

$y = x+1$

$\boxed{x = y-1}$

* $F: R \rightarrow R$

$F(x) = x^2$

Not invertible

* $F Z^+ \longrightarrow \{1, 4, 9, 16, 25, ---\}$

$F(x) = x^2$

$F^{-1}: \{1, 4, 9, 16, 25, ...\} \longrightarrow Z^+$

$F^{-1}(x) = \sqrt{x}$

### 2) Composition :

$o$ after

$F: A \rightarrow B$

$g: B \rightarrow C$

$g \circ f(x) = g(f(x))$

~~Fog (x) = F( [illegible] )~~

$f \circ g(x)$ Not defined

Ex: $f : Z \to Z$ $\quad$ $g : Z \to Z$

$f(x) = 2x + 3$ $\quad$ $g(x) : 3x + 2$

$(f \circ g)(x) = f(g(x))$

$= f(3x+2)$

$= 2(3x+2) + 3$

$= 6x + 7$

$(g \circ f)(x) = g(f(x))$

$= g(2x+3)$

$= 3(2x+3) + 3$

$= 6x + 11$

$f \circ g(x) \neq g \circ f(x)$

☒ $y = 2x + 3$

$x = \frac{y-3}{2}$

$f^{-1} : R \to R$

$f^{-1}(x) : \frac{x-3}{2}$

$(f \circ f^{-1})(x)$

$f(f^{-1}(x))$

$f\left(\frac{x-3}{2}\right)$

$2\left(\frac{x-3}{2}\right) + 3$

$= \boxed{x}$

$(f^{-1} \circ f)(x)$

$f^{-1}(f(x))$

$= f^{-1}(2x+3)$

$= \frac{(2x+3)-3}{2}$

$= \boxed{x}$

# Sequences & Summations

$\Sigma$

Sequences Function $A \to B$ Positions $\{0,1,2,\dots$ $\{1,2,3,\dots$

$1, 1/2, 1/3, 1/4, 1/5 \dots$

Set 1 2 3 4 5 positions

Ex: $a_n = 5^n$

0 1 2

1, 5, 25, 125, ....

Arithmatic sequence ← +6 +6 , 17,

Ex $5, 11, 23, 29, \dots$

0 1 2 ~~4~~3 4

$\xrightarrow{2}$

term to term $a_0 = 5$ . First element

$a_n = a_{n-1} + 6$

nth term $a_n = 5 + 6n$ $n \geq 0$

0 1 2 3 4

×2 ×2

$1, 2, 4, 8, 16, 32, 64, 128, \dots$

geometric (ضرب)

→ term to term

$a_0 = 1$

$a_n = 2 * a_{n-1}$

or $a_{n+1} = 2 \times 2n$

→ nth term ⇒ $a_n = 2^n$ $n \geq 0$

Ex: $1, 7, 25, 79, 241, \dots$

1 2 3

$a_0 = 1$

$a_n = a_{n-1} * 3 + 4$

$a_n = 3^n - 2$ $n \geq 1$

Ex: $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \dots$

n appears n times , $n \geq 0$ , $n \geq 1$

upper bound

$$\sum_{\boxed{i=L}}^{\boxed{u}} \boxed{a_i} \quad \text{index of Sum} \quad = a_L + a_{L+1} + a_{L+2} + \dots - a_u$$

lower bound

$$\sum_{i=3}^{6} \frac{i^2}{2} = \frac{3^2}{2} + \frac{4^2}{2} + \frac{5^2}{2} + \frac{6^2}{2}$$

Rules:

① $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$$\sum_{i=1}^{10} i = 1+2+3+4+5+6+7+8+9+10$$

$$= 55$$

$$= \frac{\overset{5}{\cancel{10}} \times 11}{\cancel{2}} = 55$$

② $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

③ $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

④ $\sum_{i=0}^{n} ar^i$ ?

Ex:

$$\sum_{i=1}^{10} i = 55$$

$$\sum_{i=2}^{10} i = \sum_{i=1}^{10} i - 1$$

$$= \boxed{54}$$

Ex: $\sum_{i=L}^{u} a_i = n$

given

* $\sum_{i=L}^{u} a_i = n$

* $\sum_{i=L+1}^{u} a_i = n - a_L$

* $\sum_{i=L}^{u+1} a_i \cancel{a} = n + a_{u+1}$

* $\sum_{i=L-1}^{u} a_i = n + a_{L-1}$

* $\sum_{i=L+1}^{u} a_i = n - a_L$

(*) $\sum_{i=1}^{11} i = \sum_{i=1}^{10} i + 11$

$= 55 + 11$

* $\sum_{i=L}^{u-1} a_i = n - a_u$

Scanned by CamScanner

$\sum_{i=1}^{9} i = 55 - 10$

$\sum_{i=50}^{100} i$

Ex :

$\sum_{i=1}^{3} 6 = 6(3-1+1)$

$= 6+6+6$

$= \boxed{18}$

Find

$\overset{3i}{\sum_{j=1}^{3}} + \overset{6}{\sum_{j=1}^{3} j}$

$\uparrow$ or

$\sum_{i=1}^{2} \sum_{j=1}^{3} (i+j)$

$= \sum_{i=1}^{2} ((i+1)+(i+2)+(i+3))$

$3 \sum_{i=1}^{2} 3i$

9

$= \sum_{i=1}^{2} (3i+6)$ 12

$= (3+6)+(6+6)$

$= 9+12 = 21$

قوانين $\boxed{7}$ $\sum_{j=1}^{20} \sum_{i=1}^{30} (i+j)$ ?

Ex:

$\boxed{78}$ $\sum_{i=0}^{2} \sum_{j=0}^{3} (2i+3j)$

$\boxed{2}$ $\sum_{i=1}^{3} \sum_{j=0}^{2} i$

$(3 \times 1) + (3 \times 2) + (3 \times 3)$ ✓

$3 + 6 + 9$

$= \boxed{18}$

$\boxed{3}$ $\sum_{i=1}^{10} 3$

$3(10 + 1 - 1)$

$3(10)$

$= 30$

$\boxed{4}$ $\sum_{j=0}^{4} (-2)^j$

$= (-2)^0 + (-2)^1 + (-2)^2 + (-2)^3 + (-2)^4$

$1 - 2 + 4 - 8 + 16$

$\boxed{11}$

$\boxed{5}$ $\sum_{k=0}^{9} \lfloor \sqrt{k} \rfloor$

$= \lfloor \sqrt{0} \rfloor + \lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + \lfloor \sqrt{4} \rfloor + \lfloor \sqrt{5} \rfloor + \lfloor \sqrt{6} \rfloor + \lfloor \sqrt{7} \rfloor + \lfloor \sqrt{8} \rfloor + \lfloor \sqrt{9} \rfloor$

$= 0 + 1 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + 3$

$= \boxed{16}$

$\boxed{6}$ $\sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3$

$= 0 + i^2 + 8i^2 + 27i^2 = 36i^2$

$= (36 \times 0^2) + (36 \times 1^2) + (36 \times 2^2) +$

$= \boxed{180}$

⑦ $\sum_{i=1}^{3} \sum_{j=0}^{2} j$

$$\sum_{j=0}^{2} j = \frac{2(2+1)}{2} = \frac{2\times3}{2} = \frac{6}{2} = 3$$

$$\sum_{i=1}^{3} 3 = 3(3-1+1) = 9$$

$\boxed{8}$ $\sum_{j=5}^{3} j = 0$

~~[illegible]~~

find rule

1 2 3 4 5 6

① 3, 5, 8, 12, 17, 23, 30, 38, ---

$a_n = a_{n-1} + n$

$a_1 = 3$

② 2, 16, 54, 128, 250, 432

$2\times5^3 = 2\times125 = 250$

$2\times6^3 = 2\times216 = 432$

$= 2\times n^3$

③ 2, 3, 7, 25, 121

## Matrix

$$M = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix}$$

$m \times n$

rows ↩ $m$, ↳ columns $n$

Ex:

$$M = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 0 & -1 \end{bmatrix}_{2\times3}$$

$a_{22} = 0$

$a_{13} = 5$

$-1 = a_{23}$

Arithmetic op :-

1) Summation

$C_{m\times n} = A_{m\times n} + B_{m\times n}$

$C_{ij} = a_{ij} + b_{ij}$

Ex:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 4 & 3 & 0 \end{bmatrix}_{2\times3}$$

$$B = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 0 & -1 \end{bmatrix}_{2\times3}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$$

$$D = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}_{2\times2}$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}_{3\times2}$$

$$F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2}$$

- $A + B = \begin{bmatrix} 3 & 3 & 5 \\ 10 & 3 & -1 \end{bmatrix}_{2\times3}$
- $C + D = \begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}_{2\times2}$
- $B + B = \begin{bmatrix} 4 & 2 & 2 \\ 12 & 0 & -2 \end{bmatrix}_{2\times3}$

## 2) Multiplication:

$$C_{m\times n} = A_{m\times k} \underline{\quad \times \quad} B_{k\times n}$$

$$C_{ij} = a_{i1} \times b_{1j} + a_{i2} \times b_{2j} + \cdots \quad a_{ik} \times b_{kj}$$

نفس المثال

$$\begin{array}{cc} A & E \\ \begin{bmatrix} 1 & 2 & 4 \\ 4 & 3 & 0 \end{bmatrix}_{2\times3} \times & \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}_{3\times2} \end{array}$$

$$= \begin{bmatrix} 9 & 6 \\ 4 & 3 \end{bmatrix}_{2\times2}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}_{3\times2} \times \begin{bmatrix} 1 & 2 & 4 \\ 4 & 3 & 0 \end{bmatrix}_{2\times3}$$

$$= \begin{bmatrix} 1 & 2 & 4 \\ 4 & 3 & 0 \\ 6 & 7 & 8 \end{bmatrix}_{3\times3}$$

$$C \times D = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}_{2\times2}$$

$$D \times C = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}_{2\times2}$$

$$D \times F = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}_{2\times2}$$

$$F \times D = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}_{2\times2}$$

## 3) Power:

$$A^{0}_{n\times n} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$ Identity القطر الهر(1) والباقي 0

$A^1_{n\times n} = A$

$A^2_{n\times n} = A \times A$

$\vdots$

$A^n = \underbrace{A \times A \times --- A}_{\text{~~A~~ n times}}$

نفس المثال

$D^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$D^1 = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

$D^2 = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$

$D^3 = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ 10 & 7 \end{bmatrix}$

or $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ 10 & 7 \end{bmatrix}$

4) Transpose:

Note if $A = A^t$

Ex A is Symmetric

$A_{n\times n}$ $a_{ij}$ in $A$

$A_{n\times m}$ $a_{ji}$ in $A^t$

$A^t = \begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 4 & 0 \end{bmatrix}_{3\times 2}$

$D^t = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

C is Symmetric

Ex:

$X = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 7 \end{bmatrix}_{3\times 3}$

اذا زي المراية

بكون Symmetric

$X^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 7 \end{bmatrix}_{3\times 3}$

# Matrices :-

## Boolean operations :- Zero one matrices

### 1) Meet

$C_{m\times n} = A_{m\times n} \text{ meet } B_{m\times n}$

$C_{ij} = a_{ij} \wedge b_{ij}$

### 2) Join

$C_{m\times n} = A_{m\times n} \text{ join } B_{m\times n}$

$C_{ij} = a_{ij} \vee b_{ij}$

Ex:

$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}_{2\times 3}$

$B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{2\times 3}$

$C = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}_{2\times 2}$

$D = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}_{2\times 2}$

$A \text{ join } B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{2\times 3}$

$C \text{ meet } D = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}_{2\times 2}$

### 3) Boolean product $\odot$

$C_{m\times n} = A_{m\times k} \odot B_{k\times n}$

$C_{ij} = (a_{i1} \wedge b_{1j}) \vee (a_{i2} \wedge b_{2j}) \vee \text{-----} (a_{ik} \wedge b_{kj})$

$C \odot D = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}_{2\times 2}$

$D \odot C = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}_{2\times 2}$

$C \odot A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}_{2\times 3}$

### 4) Boolean power

$A_{n\times n}$

$A^{[0]} = I \qquad A^{[2]} = A \odot A$

$A^{[1]} = A \qquad A^{[n]} = \underbrace{A \odot A \odot \cdots\cdots A}_{N \text{ times}}$

$A+B = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ $\quad C^{1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ $\quad C^{[2]} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$C \times D = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ $\quad C^{[0]} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$C^2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

Integers Division

$\rightarrow$ without mod

$\rightarrow$ with mod

Def $|$ divides $\qquad$ $3 \mid 6$

$\qquad$ $6 \nmid 3$

$a/b \rightarrow \frac{b}{a} = k \in \mathbb{Z}$ $\qquad$ $3 \nmid 125$

a: factor of b $\qquad$ $1 \mid 7$

b: multiple of a $\qquad$ $8 \mid 8$

Thm:-

1) If $a \mid b$ then $a \mid bn$, $n \in \mathbb{Z}$

$3 \mid 6$ $\qquad$ proof: Directly

$3 \mid 12$ $\qquad$ Assume $a \mid b \Rightarrow \frac{b}{a} = k$

$3 \mid 18$ $\qquad$ $a \mid bn$ $\qquad$ $\Rightarrow b = ak$

$3 \mid 600$ $\qquad$ $\frac{bn}{a} = \frac{a\boxed{kn}}{a} \in \mathbb{Z} \Longrightarrow a \mid bn$

2) If $a \mid b$ and $a \mid c$ then $a \mid (b+c)$

3) If $a \mid b$ and $b \mid c$ then $a \mid c$

$3 \mid 6$ and $6 \mid 36 \rightarrow 3 \mid 36$

عدد أولي

Primes: P

$P \geq 2$ is prime iff it's only ~~fac~~ factors are 1 and P.

Ex: 2, 3, 5, 7, 11, 13, 17, ....

Merssenne primes: $2^n - 1$

$3 = 2^2 - 1$

$7 = 2^3 - 1$

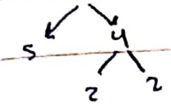
x $15 = 2^4 - 1$

Thm:-

Any composite (not prime) number can be written as product of Primes (uniquely)

$20 = 2.2.5$ $\qquad$ $20 = 2 . 10$

$\qquad\qquad\qquad\qquad\quad 2.5$



Thm:-

If n has no factors $\leq \sqrt{n} \rightarrow$ n is prime

Ex: 101 $\qquad \sqrt{101} \approx 10$

$2 \nmid 101 \qquad 7 \nmid 101$

$3 \nmid 101 \qquad$ 101 is prime

$5 \nmid 101$

indirect

Proof:

LCM $\qquad\qquad$ GCD

Least Common Multiplier $\qquad$ Greatest Common Divisor

Ex:

~~36~~
~~24~~

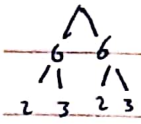
$GCD(36,24) = \boxed{12}$ $\qquad$ $LCM(36,24) = 72$

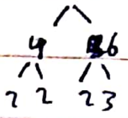
36: 1,2,3,4,9,6,12,18,36 $\qquad$ 36: 36,72,108,---

24: 1,2,3,4,6,8,12,24 $\qquad$ 24: 24,48,72,96

$GCD(x,y) = P_1^{min} . P_2^{min} . P_3^{min} ..... P_n$

$36 = 2^2 3^2$ $\qquad\qquad$ $24 = 2^3 . 3$





$GCD(36,24) = 2^2 . 3^1 = \boxed{12}$

$LCM(x,y) = P_1^{max} . P_2^{MAX} . P_3^{MAX} .--- P_n$

$LCM = 2^3 . 3^2 = 72$

Thm:

$GCD(x,y) \cdot LCM(x,y) = x \cdot y$

Integers and divisions:-

Ex:

$GCD(\underset{2^3}{8}, \underset{3.5}{15}) = 1$

8 & 15 are relatively prime

$GCD(8,15) = 1$ 7, 8, 15

$GCD(8,7) = 1$ are Pairwise relatively

$GCD(7,15) = 1$ prime.

Thm:

$a \in Z$

$d \in Z^+$

$\Rightarrow a = dq + r$

a: divident

d: divisor

q: quotient

r: remainder $0 \leq r < d$.

| Find: | quotient (div) | remainder (mod) |
|---|---|---|
| $\frac{23}{5}$ | 4<br>$23 = 5 \cdot 4 + 3$ | 3 |
| $\frac{77}{7}$ | 11<br>$77 = 7 \cdot 11 + 0$ | 0 |
| $\frac{-12}{8}$ | -2<br>$-12 = 8 \cdot (-2) + 4$ | 4 |
| $-\frac{33}{7}$ | -5<br>$-33 = 7(-5) + 2$ | 2 → دايما موجب |

Def: $a, b \in \mathbb{Z}$
$m \in \mathbb{Z}^+$

$a \equiv b \pmod{m}$ iff $a \bmod m = b \bmod m$

is congruent

$7 \equiv 17$

$7 \bmod 5 = 2$ $\qquad$ $7 \equiv 27 \pmod{5}$

$17 \bmod 5 = 2$ $\qquad$ $17 \equiv 27 \pmod{5}$

اذا ضفت على احد الاعداد اوطرحت احد مضاعفات (m) بطلع Congruent

Thm:

if $a \equiv b \pmod{m}$

$a = b + mk \qquad k \in \mathbb{Z}$

$m \mid (a-b)$

Ex:

$2 \equiv \boxed{8} \pmod{6}$

14
-4

$3 \equiv 9 \pmod{\boxed{6}}$

3
2

العدد لازم يكون ← (mod) الفرق بين العددين

Thm:

if $a \equiv b \pmod{m}$

$c \equiv d \pmod{m}$

then

$(a+c) \equiv (b+d) \pmod{m}$

$(a \cdot c) \equiv (b \cdot d) \pmod{m}$

$2 \equiv 5 \pmod{3}$

$7 \equiv 10 \pmod{3}$

$(2+7) \equiv (5+10) \pmod{3}$ $\qquad$ $(2 \cdot 7) \equiv (5 \cdot 10) \pmod{3}$

$9 = 15 \pmod{3}$ $\qquad$ $14 \equiv 50 \pmod{3}$

Proof by induction:

Ex:

$$\overset{2^0}{1} + \overset{2^1}{2} + \overset{2^2}{4} + \overset{2^3}{8} + \overset{2^4}{16} + \dots 2^n = 2^{n+1} - 1$$

or

$$\sum_{i=0}^{n} 2^i$$

1) Base:

$$2^0 = 2^1 - 1$$

$$1 = 1$$

2) Hypothesis:

Assume $2^0 + 2^1 + \dots 2^n = 2^{n+1} - 1$

3) Step:

prove $\underbrace{2^0 + 2^1 + 2^n + 2^{n+1}}_{2^{n+1}-1} = 2^{n+2} - 1$

$$2^{n+1} - 1 + 2^{n+1} = 2^1 \, 2^{n+1} - 1$$

$$= \boxed{2^{n+2} - 1}$$

⊞ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

1+2+3+4.....n

1) Base: $\sum_{i=1}^{1} i = \frac{1(2)}{2}$

$$1 = 1$$

2) Hypothesis:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

3) Step

prove $\sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2}$

$$\sum_{i=1}^{n+1} i = \underbrace{\sum_{i=1}^{n} i}_{\frac{n(n+1)}{2}} + (n+1)$$

$$\frac{n(n+1)}{2} + (n+1)$$

Steps:

1) Base: prove for first element.

2) Hypothesis: Assume that thm is correct for n.

3) Step: prove for n+1 using step 2.

4) Conclusion: thm is correct ∀n.

$$= \frac{n(n+1) + 2(n+1)}{2}$$

$$= \frac{(n+1)(n+2)}{2}$$

4) Conclusion

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$\forall n$

⊠ $3 \mid (n^3 - n)$ or $(n^3 - n)$ is divisable by 3 $(n \geq 0)$

1) Bases:

$3 \mid (0 - 0)$

$3 \mid 0$

2) Hypothesis

Assume

$3 \mid (n^3 - n)$

3) Step prove

$3 \mid ((n+1)^3 - (n+1))$

$n^3 + 3n^2 + 3n \cancel{+1} - n \cancel{-1}$

$\underline{(n^3 - n)} + (3n^2 + 3n)$

$3 \mid (n^3 - n) + 3 \mid (3n^2 + 3n)$

$\downarrow$

$\frac{3n^2 + 3n}{3} = \frac{\cancel{3}(n^2 + n)}{\cancel{3}}$ ✓

if $a \mid b$ and $a \mid c$ then $a \mid (b + c)$

$3 \mid (n^3 - n + 3n^2 + 3n)$

$\Rightarrow 3 \mid ((n+1)^3 - (n+1))$

4) Conclusion

$3 \mid (n^3 - n)$

$\forall n$

☒ $2^n < n!$ $n \geq 4$

1) Base

$2^4 < 4!$

$16 < 24$

2) Hypothesis

~~prove~~

Assume

$2^n < n!$

3) Step

prove

$2^{n+1} < (n+1)!$

$2 \cdot 2^n < n! \cdot 2$ 5or6--

$\geq 5$

$2^{n+1} < 2n! < (n+1)n!$

$2^{n+1} < 2n! < (n+1)!$

$2^{n+1} < (n+1)!$

4) Conclusion:

$2^n < n!$

$\forall n$

☒ $\boxed{n < 2^n}$

$\boxed{n \geq 1}$

Ex: $\sum_{i=1}^{n}(2i-1)=n^2$

or $1+3+5+7+9 = 5^2$

1) Base:

$\sum_{i=1}^{1}(2i-1)=1^2$

$1=1$

2) Hypothesis:

Assume $\sum_{i=1}^{n}(2i-1)=n^2$

3) Step:

Prove $\sum_{i=1}^{n+1}(2i-1) = (n+1)^2$

$\sum_{i=1}^{n+1}(2i-1)$

$= \sum_{i=1}^{n}(2i-1) + (2(n+1)-1)$

$= n^2+2n+1$

$(n+1)(n+1)$

$= (n+1)^2$

4) Conclusion:

$\sum_{i=1}^{n}(2i-1)=n^2 \quad \forall n$

☒ 1) $4n<2^n$

2) $1\times 2 + 2\times 3 + 3\times 4 + \ldots\ldots n(n+1) = \frac{n(n+1)(n+2)}{3} \quad n>1$

③ $5 \mid (8^n-3^n) \quad n\geq 1$

Relations: $\to$ Binary ↳ (set of pairs)
$\to$ n-ary

① Representation

1) Set builder
2) Listing (Roaster)
3) Table
4) Matrix
5) Graph

Ex: Domain $S = \{1,2,3\}$

$R: S \to S$

$R = \{(x,y) \mid x=y\}$

$R = \{(1,1),(2,2),(3,3)\}$

Table

|  | 1 | 2 | 3 |
|---|---|---|---|
| 1 | ↘ |  |  |
| 2 |  | ↘ |  |
| 3 |  |  | ↘ |

$(1,1) \in R$ $\quad$ $(1,3) \notin R$

$$MR = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{array}_{3\times3}$$

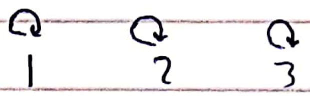
$$MR = \begin{array}{c|ccc} & 2 & 1 & 3 \\ \hline 3 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 \end{array}$$

$1 \longrightarrow 1$

$2 \longrightarrow 2$

$3 \longrightarrow 3$

or



Properties

- Reflexivity
  - reflexive : $\forall x \, (x \in S \longrightarrow (x, x) \in R)$
  - irreflexive $(x, x) \notin R$
- Symmetricity
  - symmetric $\forall x \forall y \, [ (x, y) \in R \longrightarrow (y, x) \in R)$
  - anti symmetric $(x, y) \in R \longrightarrow (y, x) \notin R)$
- Transitivity
  - transitive if $( (x, y) \in R \wedge (y, z) \in R) \longrightarrow (x, z) \in R$

Ex:

Domain $S = \{1, 2, 3\}$

$R_1 = \{(1,1), (2,2), (2,3)\}$

Not reflexive, Not irreflexive, Not Symmetric, Antisymmetric, transitive

$R_2 = \{(2,3), (3,2)\}$

Not reflexive, irreflexive, Symmetric, not Antisymmetric, not transitive

$R_3 = \{(1,2), (2,1), (1,1), (3,1)\}$

Not reflexive, not irreflexive, not Symmetric, not Antisymmetric, not transitive

$R_4 = I_R = \{(1,1), (2,2), (3,3)\}$

Reflexive, not irreflexive, symmetric, Antisymmetric, transitive

$R_5 = \emptyset$

Not reflexive, irreflexive, Symmetric, Antisymmetric, transitive

$$MR_6 = \begin{bmatrix} 1 & 0 & \overset{(1,3)}{1} \\ 0 & 1 & 0 \\ 1 & \underset{(3,2)}{1} & 0 \end{bmatrix}$$

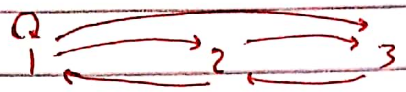
القطر لازم كله 0

لو كل واحد بالمراية بيساوي صفر بتكون Anti

Not Reflexive, not irreflexive, not Symmetric, not Antisymmetric,

Not transitive

$R_7$



عشان ال loop

Not reflexive, Not irreflexive, Not Symmetric, Not antisymmetric, Not transitive

## Relations:

Operations

1) Union $\cup$
2) Intersection $\cap$
3) Difference $-$
4) complement $\bar{\ }$
5) Inverse $\sim$ or $^{-1}$
6) Composition $\circ$ (after)

Ex:

$S = \{1,2\}$ Universal $R = \{(1,1),(1,2),(2,1),(2,2)\}$

$R_1 = \{(1,1),(1,2)\}$ $MR_1 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$R_2 = \{(2,1)\}$ $MR_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$R_3 =$

$MR_u = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$R_1 \cup R_2 = \{(1,1),(1,2),(2,1)\}$

$MR_1 \overset{\vee}{\text{join}} MR_2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$R_1 \cap R_2 = \emptyset$

$MR_1 \overset{\wedge}{\text{meet}} MR_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$R_1 - R_2 = \{(1,1),(1,2)\} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$R_2 - R_1 = \{(2,1)\}$

$\bar{R}_1 = \{(2,1),(2,2)\}$

$R_1^{-1}$ or $R_1^{\sim} = \{(1,1),(2,1)\}$

$R_1^{-1} = MR_1^t = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

عكس الترتيب

$R_1 \circ R_2 = \{(2,1),(2,2)\}$ $MR_2 \odot MR_1 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

$R_2 \circ R_1 = \{(1,1)\}$ $MR_1 \odot MR_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$R_1 \circ R_1 = \{(1,1),(1,2)\}$ $\&=$

$R_1 \circ R_1 \subseteq R_1 \longrightarrow R_1$ is transitive

$|S| = n$

$$\left.\begin{array}{l} R \circ R \\ R \circ R \circ R \\ \underbrace{R \circ R \circ R \circ R \ldots}_{N \text{ times}} \end{array}\right\} \subseteq R \rightarrow R \text{ is transitive}$$

$||$

$M_{R_1}^{[2]} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ العمر ما تطالع (1) جديدة

~~R~~ $R_2 \circ R_2 = \{\}$

Closures:

- reflexive $\overset{\text{Closure}}{\underset{\text{Relation}}{R^{r}}}$
- Symetric $R^{s} = R \cup R^{-1}$
- transitive $R^{t}$ or $R^{*}$

$= R \cup R \circ R \cup \underbrace{R \circ R \circ R}_{\substack{N \text{ times} \\ |S|}}$

$R_1^{r} =$

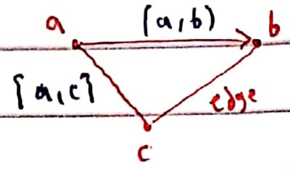
$\{(1,1), (1,2)\} \cup \underbrace{\{(1,1), (2,2)\}}_{\text{Identity}}$

$= \{(1,1), (1,2), (2,2)\}$

$R_1^{s} = \{(1,1), (1,2)\} \cup \{(1,1), (2,1)\}$

$= \{(1,1), (1,2), (2,1)\}$

## Graph:-



Vertex (vertices)
node

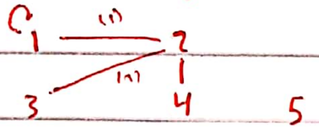
set of vertices          الاتجاه →

G = ({a,b,c} , {(a,b), {a,c}, {b,c}})

G = ({1,2,3,4}, {(1,2), (2,1), {3,4}})

## Terminology

1) Undirected graph (no direction)



- Adjacent: {x,y} : x is adjacent with y.

Ex: 1 is adjacent with 2.

- Incident {x,y} : is incident with x and y

Ex: {3,2} is incident with 3 and 2.

- loop : {x,x} is a loop {1,1} or {1}

- path : Sequence of edges

1,2,3 of length 2

- ~~cycle~~ cycle : path that begins and end at the same node

3,2,3 of length 2

no. of adjacent nodes
↑

deg(1) = 3

deg(2) = 3

deg(3) = 1

deg(4) = 1

deg(5) = 0

Σ deg(v) = 8 even

↳ 2|E|

↳ |E| = 4 edges

Thm:

~~Hand~~

Handshaking thm

$$\sum_{v \in V} \deg(v) = 2|\overset{\text{Set of edges}}{E}|$$

Ex: $|\overset{\text{vertices}}{V}| = 5$

$\deg(v) = 6$

$|E| = ?$

$\sum \deg(v) = 5 * 6 = 30$

$\sum \deg(v) = 2|E|$

$30 = 2|E| \rightarrow |E| = 15$

Given

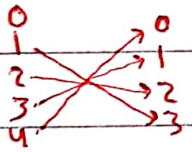
$R : \{0,1,2,3,4\} \rightarrow \{0,1,2,3\}$

$R = \{ (a,b) \mid a+b = 4 \}$

Roaster

$= \{ (1,3), (2,2), (3,1), (4,0) \}$

Graph



Matrix

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0 \end{array} \quad 5 \times 4$$

Properties Not Reflexive

Not irreflixeve

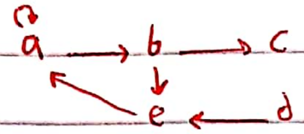
Not ~~Syme~~ Symmetric

Not anti- Symmetric

Not transitive.

## Graph: terminology

Directed



* adjacent  b is adjacent to e

to c

b is adjacent <u>from</u> a

$\overset{out}{deg^+}(a) = 2$ | $\overset{in}{deg^-}(a) = 2$
$deg^+(b) = 2$ | $deg^-(b) = 1$
$deg^+(c) = 0$ | $deg^-(c) = 1$
$deg^+(d) = 1$ | $deg^-(d) = 0$
$deg^+(e) = 1$ | $deg^-(e) = 2$

$\sum_{v \in V} deg^+(v) = \cancel{7}\ 6$ [illegible]

$\sum_{v \in V} deg^-(v) = 6$

$|E| = 6$

Thm:

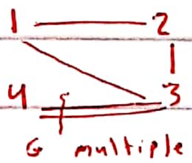
$$\sum_{v \in V} deg^+(v) = \sum_{v \in V} deg^-(v) = |E|$$

## Representation:-

Matrix : Adjucency

ال loop عن القطر

$$MG = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 0 & 2 \\ 4 & 0 & 0 & 2 & 0 \end{array} \quad 4\times4$$



$|V| = 4$

Size of matrix من

$|E| = 6$

$\frac{\sum_{v \in V} deg(v)}{2}$

Symmetric → undirected

بنوجد المجموع بسطر او عامود $deg(3) = \sum$ elements of row + col. او بنحسب جهة وحدة والـ loop مرتين

والـ loop عن ② $= 4$

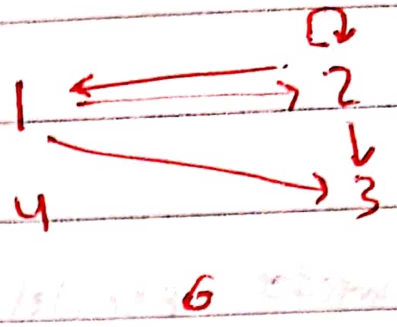
$deg(2) = 4$ (loop = 2)

edge from 1 to 2

بداية السهم ↖ intial

$$MG \quad \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{array} \; \text{terminal} \quad 4 \times 4$$

→ مربعة



$|V| = 4$

$|E| = 5$

(row) $deg^{+}(2) = 3$

(colo.) $deg^{-}(2) = 2$

$R_1 = \{(1,2), (2,3), (3,4)\}$

$R_2 = \{(1,1), (1,2), (2,1), (2,3), (3,1)\}$

$R_1 \cup R_2 = \{(1,2), (2,3), (3,4), (1,1), (2,1), (3,1)\}$

$R_1 \cap R_2 = \{(1,2), (2,3)\}$

$R_1 - R_2 = \{(3,4)\}$

Simple graph → no direction
→ no loops
→ no [illegible] parallel edges

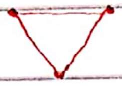
Types:

1) Complete (k)

each node must adjacent with others.


X


K2


K3


K4


K5

| graph | \|V\| | \|E\| | deg(v) | $\sum_{v \in V}$ deg(v) |
|---|---|---|---|---|
| | | | | |

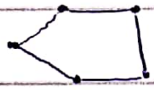
2) Cycle (c)

each node is adjacent with 2 around


X


C3


C4


C5

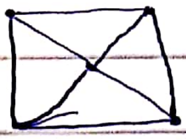

C6

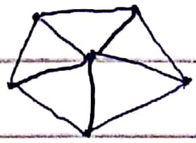
3) Wheel

cycle +1 node inside adjacent to cycle nodes

النقطة الي بالنص مربوطة بـ edges بعدد النقاط


W3


W4

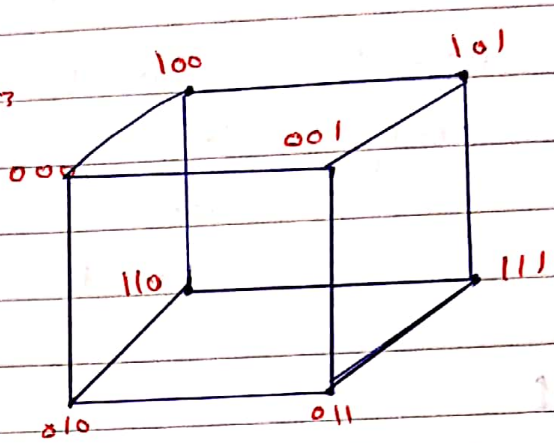

W5

4) n-cubes (Q)

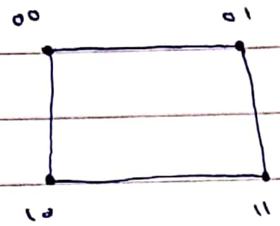
nodes in binary adjacent nodes differ in 1 bit.

Q1

Q3



Q2

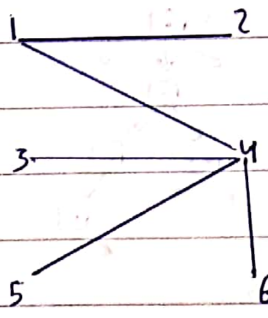


Tree: graph

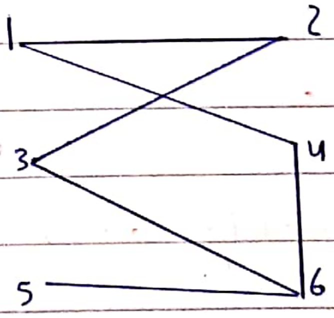
↓

Undirected

- no direction
- no loops
- no simple cycles
- Connected

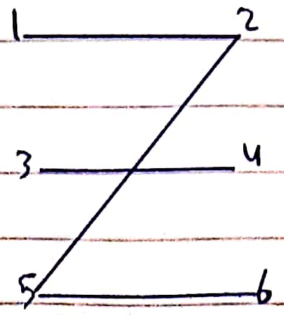


Tree



X Tree simple cycle

X Tree (Not connected)

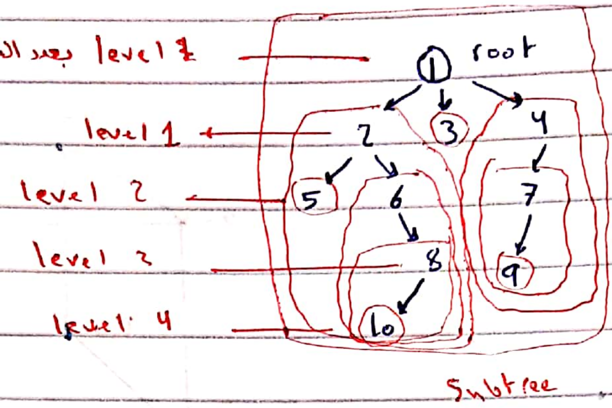


Tree directed

there only <u>one</u> bath From a root to any node.

Heigth:

max path ↳



Terminology:-

root : 1

Parent: of 9 is 7

child: of 1 are 2,3,4

أخوة Siblings: 5,6

اجداد ancestor: of 7 : 4,1

احفاد descendent: of 2 : 5,6,8,10

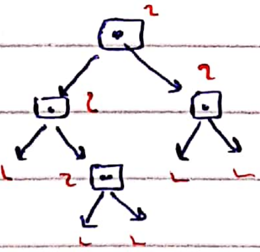
ما بطلع منها اشي leaf (leaves) : 5,3,9,10

الي بطلع منهم تفرع internal : 1,2,4,6,7,8

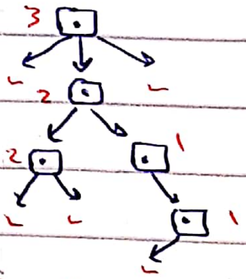
Subtree (مجموعة)

Tree types

→ Size



لانه كلهم نفس عدد التفرع Full binary
Balanced

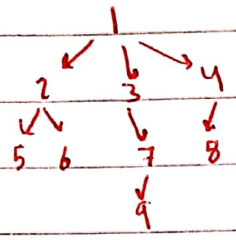


حسب الي تفرع 3-ary
Not Balanced

→ balance → Balanced: difference between leaves ≤ 1

→ not balanced

# Tree traversal



→ Preorder

root, left subtree, right subtree

1, 2, 5, 6, 3, 7, 9, 4, 8

→ inorder

left sub, root, right sub

5, 2, 6, 1, 3, 9, 7, 8, 4

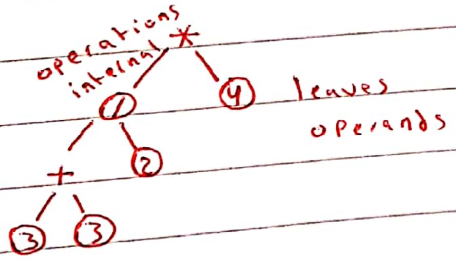
→ Postorder

left sub, right sub, root

5, 6, 2, 9, 7, 3, 8, 4, 1

# Expression tree

(3 + 3) / 2 * 4
① ② ③



inorder : infix

((3 + 3) / 2) * 4) = 12

Postorder : postfix

اول عملية منلاقيها

33 + 2 / 4 *

62 / 4 *

34 *

12

Preorder : prefix

* / + 33 24

* / 62 4

* 34

12