



## ③ Charging RLC - circuits.

### ① Charging - series - RLC - circuit

- $V(t) = V_{\infty} + [A_1 e^{s_1 t} + A_2 e^{s_2 t}] \rightarrow \alpha > \omega_0$  (over-damping)
- $V(t) = V_{\infty} + [A_1 + A_2 t] e^{-\alpha t} \rightarrow \alpha = \omega_0$  (critically-damping)
- $V(t) = V_{\infty} + e^{-\alpha t} [A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)] \rightarrow \alpha < \omega_0$  (under-damping)

$$\left. \begin{aligned} s_1 &= -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\ s_2 &= -\alpha - \sqrt{\alpha^2 - \omega_0^2} \end{aligned} \right\} \begin{aligned} \omega_d &= \sqrt{\omega_0^2 - \alpha^2} \\ I_C^{\omega} &= e \frac{dV(t)}{dt} \end{aligned} \left\{ \begin{aligned} \alpha &= \frac{R}{2L} \\ \omega_0 &= \frac{1}{\sqrt{LC}} \end{aligned} \right.$$

$V(\infty) \rightarrow$  after along time.

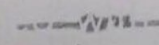
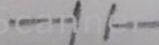
  $\rightarrow$  short circuit.  
  $\rightarrow$  open circuit.

### ② Charging - parallel - RLC - circuit.

- $i(t) = i_{\infty} + [A_1 e^{s_1 t} + A_2 e^{s_2 t}] \rightarrow \alpha > \omega_0$  (over-damping)
- $i(t) = i_{\infty} + [A_1 + A_2 t] e^{-\alpha t} \rightarrow \alpha = \omega_0$  (critically-damping)
- $i(t) = i_{\infty} + [A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)] e^{-\alpha t} \rightarrow \alpha < \omega_0$  (under-damping)

$$\left. \begin{aligned} s_1 &= -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\ s_2 &= -\alpha - \sqrt{\alpha^2 - \omega_0^2} \end{aligned} \right\} \begin{aligned} \omega_d &= \sqrt{\omega_0^2 - \alpha^2} \\ V_L(t) &= L \frac{di(t)}{dt} \end{aligned} \left\{ \begin{aligned} \alpha &= \frac{1}{2RC} \\ \omega_0 &= \frac{1}{\sqrt{LC}} \end{aligned} \right.$$

$i_{\infty} \rightarrow$  after very long time.

  $\rightarrow$  S.C  
  $\rightarrow$  O.C

# CH 8: RLC-CKT's

## (A) Discharging (source free) RLC ckt's.

(1) source free - series - RLC circuit.

-  $i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \rightarrow \alpha > \omega_0$  (over-damping)

-  $i(t) = e^{-\alpha t} (A_1 + A_2 t) \rightarrow \alpha = \omega_0$  (critically-damping)

-  $i(t) = e^{-\alpha t} [A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)] \rightarrow \alpha < \omega_0$  (under-damping)

$$\left. \begin{aligned} s_1 &= -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\ s_2 &= -\alpha - \sqrt{\alpha^2 - \omega_0^2} \end{aligned} \right\} \begin{aligned} \omega_d &= \sqrt{\omega_0^2 - \alpha^2} \\ \frac{di(0)}{dt} &= \frac{-1}{L} (R I_0 + V_0) \end{aligned} \left. \begin{aligned} \alpha &= \frac{R}{2L} \\ \omega_0 &= \frac{1}{\sqrt{LC}} \end{aligned} \right\} \begin{aligned} & \text{حسب} \\ & \text{التاريخ} \\ & \text{والتاريخ} \end{aligned}$$

(2) source free - parallel - RLC - circuit.

-  $v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \rightarrow \alpha > \omega_0$  (over-damping)

-  $v(t) = e^{-\alpha t} (A_1 + A_2 t) \rightarrow \alpha = \omega_0$  (critically-damping)

-  $v(t) = e^{-\alpha t} (A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)) \rightarrow \alpha < \omega_0$  (under-damping)

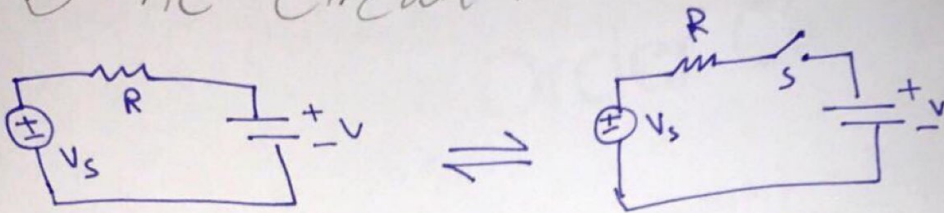
$$\left. \begin{aligned} s_1 &= -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\ s_2 &= -\alpha - \sqrt{\alpha^2 - \omega_0^2} \end{aligned} \right\} \begin{aligned} \omega_d &= \sqrt{\omega_0^2 - \alpha^2} \\ \frac{dv(0)}{dt} &= \frac{-1}{RC} (V_0 + R I_0) \end{aligned} \left. \begin{aligned} \alpha &= \frac{1}{2RC} \\ \omega_0 &= \frac{1}{\sqrt{LC}} \end{aligned} \right\}$$





### ③ # step response #

#### A) RC-circuit.



$$V(t) = V_{\infty} + [V_0 - V_{\infty}] e^{-t/\tau}$$

$V_0$ : initial voltage  $\rightarrow$  before the switch takes action.

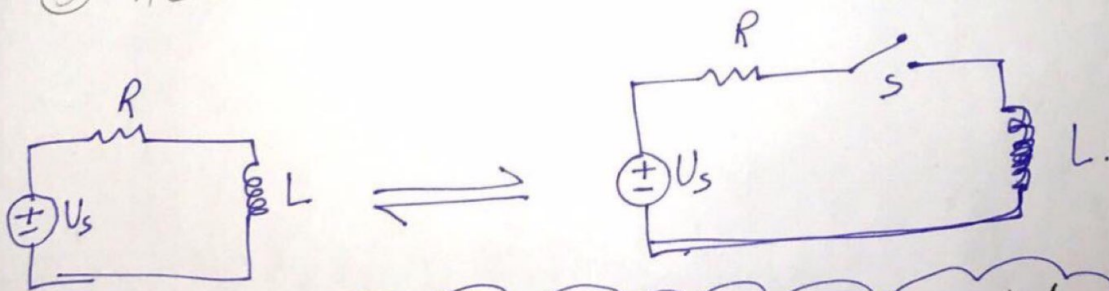
$\tau = RC$   
 $V_{\infty}$ : voltage after long time.  $\rightarrow$  after the switch takes action.

$C \rightarrow$  o.c.  
 $I \rightarrow$  s.c.

\*Tip  $V = V_0 e^{-t/\tau}$

if the switch opened or closed at  $t=0$ .

#### B) RL-circuit.



$$I(t) = I_{\infty} + [I_0 - I_{\infty}] e^{-t/\tau}$$

\*Tip  $V = V_0 e^{-(t-k)/\tau}$   
 if the switch opened or closed at  $t=k$ .

$I_0$ : Initial current.

$\tau = \frac{L}{R}$

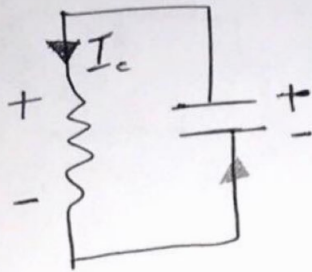
$I_{\infty}$ : current after along time. ( $I_{\text{inductor}} \Rightarrow$  s.c.)



# CH 7 & 8 first order circuits

$$W_C = \frac{1}{2} C V_0^2$$

① source free RC - circuit.



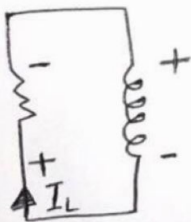
$$V(t) = V_0(t) e^{-t/\tau}$$

$\tau = RC$   $\Rightarrow$  after switches takes action.

\* discharging in opposite direction.

\* DC - conditions  $\Rightarrow$  capacitor  $\approx$  o.c.  
 $I_c = C \frac{dV}{dt}$

② source free RL - circuit.



$$I(t) = I_0(t) e^{-t/\tau}$$

$$\tau = \frac{L}{R} \quad \text{or} \quad R \frac{L}{R_{th}}$$

\* Discharging in the same direction.

\* DC - conditions  $\Rightarrow$  Inductor S.C

$$V_L = L \frac{di}{dt}$$

$$W_L = \frac{1}{2} L I_0^2$$

Conservation of energy dependent source, source free.