24

الجامعة الأردنية	الامتحان الثاني: تفاضل وتكامل 3	الخميس 2018/11/22
اسم الطالب: محمد أبو حليمة الرقم الجامعي		مدرس المادة: د. حسن النجار وقت المحاضرة: ١٣ – ١

يتكون الامتحان من 7 أسئلة في 3 صفحات

[1] Let $\bar{r}(t) = \langle e^t \cos t,\ e^t \sin t,\ e^t \rangle$.

(a) (2 marks) Sketch the graph of $\bar{r}(t)$.

$x = e^t \cos t.$

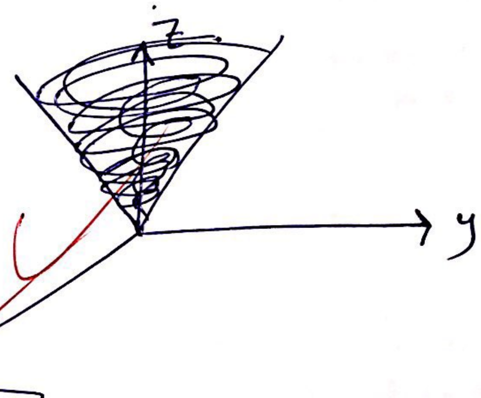
$y = e^t \sin t.$

$z = e^t.$

$e^{2t}\cos^2 t + e^{2t}\sin^2 t = e^t$

$x^2 + y^2 = z^2$

helix on the cone $x^2+y^2=z^2$



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(b) (3 marks) Find parametric equations for the tangent line to the curve of $\bar{r}(t)$ at the point $(-\pi, 0, \pi)$.

$(1,0,1)$

$x = 1 + t$

$y = t$

$z = 1 + t.$

3

$\cancel{V_L = \bar{r}(t) = \langle e^t \cdot \sin t}$

$V_L = \bar{r}'(t)$

$\bar{r}(t)\hat{i} = e^t \cancel{\sin t} + e^t \cos t.$

$\bar{r}(t)\hat{j} = e^t \cos t + \cancel{e^t \sin t}.$

$\bar{r}(t)\hat{k} = e^t.$

$t =$

$1 = e^t \cos t.$

$\boxed{t = 0}$

$t=0 \Rightarrow \quad V_L = \langle 1, 1, 1 \rangle$

(c) (3 marks) Find the arc length for $\bar{r}(t)$ where $0 \le t \le \pi$.

$\vec{r}'(t) = \langle e^t \sin t + e^t \cos t, e^t \cos t + e^t \sin t, e^t \rangle.$

$L = \int_0^\pi |\bar{r}'(t)|\, dt.$

3

$\cancel{L = \int_0^\pi \sqrt{(e^t \sin t)^2 + (e^t \cos t)^2 + (e^t \cos t + e^t \sin t)^2 + e}}$

$L = \int_0^\pi \sqrt{(e^t \sin t + e^t \cos t)^2 + (e^t \cos t + e^t \sin t)^2 + e^{2t}}\ dt.$

$L = \int_0^\pi \sqrt{e^{2t}\sin^2 t + (\cancel{-2e^{2t}\cos t \sin t}) + e^{2t}\cos^2 t + e^{2t}\cos^2 t + \cancel{2e^{2t}\cos t \sin t} + e^{2t}\sin^2 t + e^{2t}}\ dt$

$= \int_0^\pi \sqrt{2e^{2t}\sin^2 t + 2e^{2t}\cos^2 t + e^{2t}}\ dt \Rightarrow \int_0^\pi \sqrt{2e^{2t} + e^{2t}}\ dt =$

$\int_0^\pi \sqrt{3}\, e^t\, dt$

$= \sqrt{3}e^\pi - \sqrt{3}e^0$

$= \sqrt{3}e^\pi - \sqrt{3}$

$= \sqrt{3}(e^\pi - 1).$

[2] (2 marks) Show that if $|\bar{r}(t)| = c$ is constant, then $\boxed{\bar{r}(t) \perp \bar{r}'(t)}$

$r(t) \cdot r'(t) = 0.$

$|r(t)|^2 = r(t) \cdot r(t) = c^2.$

$r'(t) \cdot r(t) + r(t) \cdot r'(t) = 2c$

$2(r'(t) \cdot r(t)) = 2c.$

$r'(t) \cdot r(t) = c$

$|r(t)|^2 = c^2.$

$(r(t) \cdot r(t))' = (c^2)' = 0$

$r(t) \cdot r'(t) + r(t) \cdot r'(t) = 2c.$

$2\, r(t) \cdot r'(t) = 2c$ $\boxed{r(t) \cdot r'(t) = c}$

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2

[3] (3 marks) Find the curvature for $\bar{r}(t) = \langle t, t^3, t^2 \rangle$ at the point $(1, 1, 1)$.

$t = 1$

$$K(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3} = \frac{\sqrt{76}}{(\sqrt{14})^3}$$

$r'(t) = \langle 1, 3t^2, 2t \rangle.$

$r'(1) = \langle 1, 3, 2 \rangle.$

$r''(t) = \langle 0, 6t, 2 \rangle.$

$r''(1) = \langle 0, 6, 2 \rangle.$

$|r'| = \sqrt{1+9+4} = \sqrt{14}$

$r'(t) \times r''(t)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 2 \\ 0 & 6 & 2 \end{vmatrix}$$

$r' \times r'' = \langle -6, 2, 6 \rangle \Leftarrow$

$|r' \times r''| = \sqrt{36+4+36} = \sqrt{76}$

3

[4] Let $f(x, y) = \sqrt{36 - 9x^2 - 4y^2}$

(a) (3 marks) Find and sketch the domain of $f(x, y)$.

$36 - 9x^2 - 4y^2 \geq 0$

$36 \geq 9x^2 + 4y^2$

$1 \geq \frac{x^2}{4} + \frac{y^2}{9} \Rightarrow D_f = \{(x,y) : 1 \geq \frac{x^2}{4} + \frac{y^2}{9}\}$

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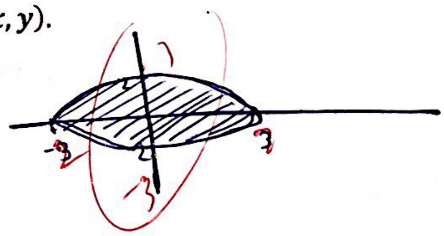
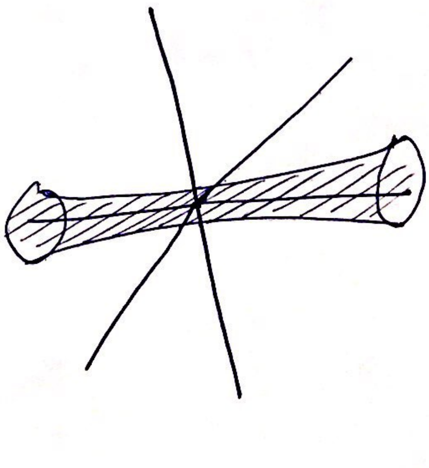


(b) (2 marks) sketch the surface $z = f(x, y)$.

$z = \sqrt{36 - 9x^2 + 4y^2}.$

$z^2 = 36 - 9x^2 + 4y^2.$

$9x^2 - 4y^2 + z^2 = 36 \rightsquigarrow$ heypepocoid of one sheet along y axis.



[5] Find the following limits (if they exist)

(a) (3 marks) $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6} = \frac{0}{0}$.

1. along x-axis $\to \boxed{y=0}$

$\lim_{x\to 0} \frac{x(0)^3}{x^2+(0)^6} = \frac{0}{x^2} = 0.$

3

2. along $\boxed{y=x}$

$\lim_{x,y\to(0,0)} \frac{xy^3}{x^2+y^6} = \lim_{x\to 0} \frac{x^4}{x^2+x^6} = \lim_{x\to 0}\frac{x^4}{x^2(1+x^4)} = 0.$

3. along $y=x^2$

$\lim_{x\to 0} \frac{x^7}{x^2+x^{12}} = 0,$

4. $x=y^3$

$\lim_{y\to 0} \frac{y^3 \cdot y^3}{y^6+y^6} = \frac{y^6}{2y^6} = \frac{1}{2}$

$\therefore \lim_{(x,y)\to(0,0)}$ d.n.e

~~we will solve it using polar cord.~~

~~$x=r\cos\theta$, $y=r\sin\theta$ | $\lim_{r\to 0^+} \frac{r\cos\theta \cdot r^3\sin^3\theta}{r^2\cos^2\theta + r^6\sin^6\theta}$~~

~~$\lim_{r\to 0^+} \frac{r^4\sin^3\theta\cos\theta}{r^2(\cos^2\theta + r^4\sin^6\theta)} = 0.$~~

(b) (3 marks) $\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2}$

1

$\lim_{r\to 0^+} \frac{e^{-(r^2\cos^2\theta)-(r^2\sin^2\theta)}-1}{r^2\cos^2\theta + r^2\sin^2\theta}$

$\lim_{r\to 0^+} \frac{e^{-r^2\cos^2\theta - r^2\sin^2\theta}-1}{r^2}$

$\lim_{r\to 0^+} \frac{e^{-r^2}-1}{r^2} = \frac{\frac{1}{e^{r^2}}-1}{r^2} \Leftarrow$

$\Rightarrow \frac{1-e^{r^2}}{r^2(e^{r^2})} = 0.$ ~~exist~~

[6] (2 marks) Given that $f(x,y)$ is differentiable with $f(2,5)=6$, $f_x(2,5)=1, f_y(2,5)=-1$. Use a linear approximation to estimate $f(2.2,\ 4.9)$

2

$f(x,y) \approx L(x,y) = f(a,b) + f_x(a,b)\cdot(x-a) + f_y(a,b)\cdot(y-b).$

$= 6 + 1(x-2) + (-1)\cdot(y-5)$

$= 6 + x - 2 - y + 5$

$\boxed{L(x,y) = x-y+9}$

$f(2.2, 4.9) \approx L(2.2, 4.9)$

$= 2.2 - 4.9 + 9$

$= -2.7 + 9 = 6.3.$

[7] (4 marks) If $z=f(x,y)$, $x=r^2+s^2$, $y=2rs$. Find $\frac{\partial^2 z}{\partial r\partial s}$

$\frac{\partial}{\partial r}\left(\frac{\partial z}{\partial s}\right)$

$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}.$

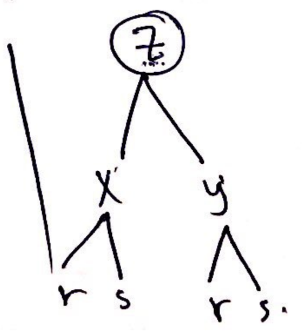
3.5

$= f_x \cdot 2s + f_y \cdot 2r$

$\left(\frac{\partial z}{\partial s}\right) = 2s f_x + 2r f_y.$

$x = r^2+s^2$

$y = 2rs.$



Continoue...

$$\frac{\partial z}{\partial r}\left(\frac{\partial z}{\partial s}\right) = \cancel{2f_y = 2\frac{dz}{dy}}.$$

$$= \cancel{\frac{dz}{dx}\cdot\frac{dx}{dr} + \frac{dz}{dy}\cdot\frac{dy}{dr}}$$

$$= \cancel{f_x \cdot 2r + f_y \cdot 2s}.$$

$$= 2r.f_x + 2s f_y.$$

$$= \cancel{(2r f_x + 2s f_y)}$$

$$= \cancel{2r f_x +}$$

$$\frac{\partial z}{\partial r}\left(\frac{\partial z}{\partial s}\right) = \frac{dz}{dr}\left(2s.\frac{dz}{dx} + 2r.\frac{dz}{dy}\right)$$

$$= (2r f_x + 2s f_y)(2s f_x + 2r f_y).$$

$$= 4rs f_x^2 + 4r^2 f_x f_y + 4s^2 f_x . f_y + 4rs f_y^2$$

$$= 4rs.\frac{d^2z}{dx^2} + 4r^2\frac{d^2z}{dydx} + 4s^2\frac{d^2z}{dydx} + 4rs\frac{d^2z}{dy^2}.$$

$+ 2f_y$