$\frac{15}{30}$

الجامعة الأردنية	الامتحان الثاني: تفاضل وتكامل 2	الأحد 2018/7/22
اسم الطالب....		مدرس المادة:.
الرقم الجامعي:		الرقم المتسلسل:...

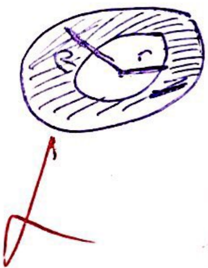
Solve questions (1) to (5). <u>Show your work:</u>

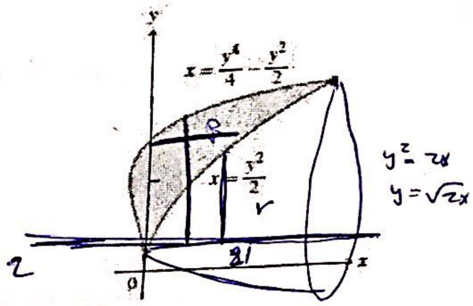
Q1. Set up the integral to find the following (Do not evaluate integrals):

a- The volume of the solids generated by revolving the shaded region about the line $y = 2$. (3 points)

$r = y - 2 \rightarrow r = \sqrt{2x} - 2$

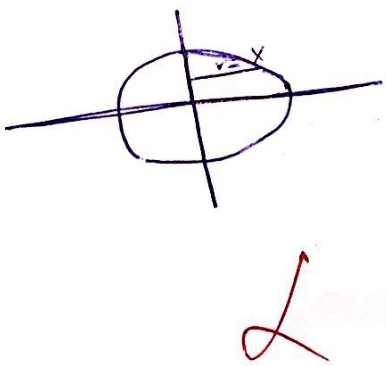
$R = y - 2 \rightarrow R =$





b- The surface area of the solid generated by revolving $x^{2/3} + y^{2/3} = a^{2/3}$ in the first quadrant about x-axis.

(3 points)



$r = x$

$x^2 = \left(\sqrt[3]{a^2} + \sqrt[3]{y^2}\right)^3$

$x = \left(\sqrt[3]{a^2} + \sqrt[3]{y^2}\right)^{3/2}$

c- The arc length of the curve $y = \sin x - x\cos x$, $0 \le x \le \pi$. (2 points) ✓

$\frac{dy}{dx} = \cos x - (x \cdot -\sin x + \cos x \cdot 1)$

$= \cos x - (-x\sin x + \cos x)$

$= \cos x + x\sin x - \cos x$

$\left(\frac{dy}{dx}\right)^2 = (x\sin x)^2$

$\left(\frac{dy}{dx}\right)^2 = x^2\sin^2 x$

$\sqrt{\left(\frac{dy}{dx}\right)^2 + 1} = \sqrt{x^2\sin^2 x + 1} \rightarrow$

$= \int_0^{\pi} \sqrt{x^2\sin^2 x + 1}\, dx.$

2

Q2. Find a horizontal line $y = k$ that divides the area between $y = x^2$ and $y = 4$ into two equal parts. (5 points)

$x = \sqrt{y}$

① $\cancel{\int_0^4 \sqrt{y}\, dy} = \cancel{\int_0^4 \sqrt{y}\, dy.}$

① $\int_0^2 4 - x^2\, dx.$ ✓

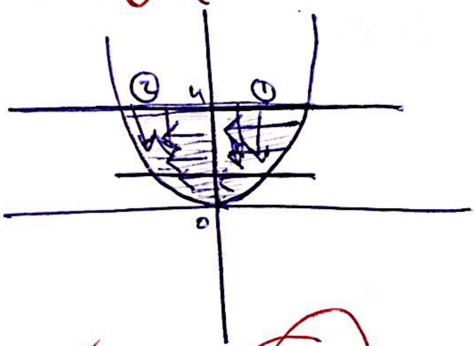
② $\cancel{\int_{-2}^0 4 - x^2\, dx.}$

$\cancel{4 - \frac{x^3}{3}\Big|_0^2 = 4 - \frac{x^3}{3}\Big|_{-2}^0}$

$\cancel{4 - \frac{8}{3} - (4 - 0) = k - 0 - (k - \frac{8}{3}).}$

$\cancel{4 - \frac{8}{3} =}$

ما شاء عليك



① $\int_0^{\sqrt{k}} k - x^2\, dx = \int_{-2}^{0} k - x^2\, dx.$

$kx - \frac{x^3}{3}\Big|_0^2 = kx - \frac{x^3}{3}\Big|_{-\sqrt{k}}^0$

$\cancel{k - \frac{8}{3}(k - 0)}$

$\cancel{k(2) - \frac{8}{3} - (k(0) - 0) = k(0) - 0 - \left(k(-2) - \frac{(-2)^3}{3}\right)}$

$2k - \cancel{\frac{8}{3}} = \cancel{+2k} \cancel{+ \frac{8}{3}}.$

2

Q4. Find the limit of the sequence $\left\{\sqrt{n}\left(\sqrt{n+5}-\sqrt{n}\right)\right\}_{n=1}^{\infty}$. (4 points)

$\lim_{n\to\infty} \sqrt{n}\left(\sqrt{n+5}-\sqrt{n}\right)$

$\lim_{n\to\infty} \sqrt{n}\sqrt{n+5} - n$

$\lim_{n\to\infty} \sqrt{n^2+5n} - n \times \frac{\sqrt{n^2+5}+n}{\sqrt{n^2+5}+n}$

$\lim_{n\to\infty} \frac{\cancel{n^2}+5n-\cancel{n^2}}{\sqrt{n^2+5n}+n} = \lim_{n\to\infty} \frac{5n}{\sqrt{n^2+5n}+n} = \frac{\infty}{\infty}$ Lopitol $\lim_{n\to\infty} \frac{5}{\frac{2n+5+2\sqrt{n^2+5n}}{2\sqrt{n^2+5n}}}$

$\lim_{n\to\infty} \frac{5}{\frac{2n+5}{2\sqrt{n^2+5n}}+1}$

3

Q5. Which of the following series converge and which diverge? (Give reasons for your answers)

a- $\sum_{n=1}^{\infty} \frac{1}{(\ln(n+2))(\ln(n+1))} \ln\left(\frac{n+1}{n+2}\right)$ (4 points)

1

$\sum_{n=1}^{\infty} \frac{\ln(n+1) - \ln(n+2)}{\ln(n+2)\ln(n+1)} \Rightarrow \sum_{n=1}^{\infty} \frac{\ln(n+1)}{\ln(n+2)\ln(n+1)} - \frac{\ln(n+2)}{\ln(n+2)\ln(n+1)}$

$\lim_{n\to\infty} \frac{1}{\ln 3} - \frac{1}{\ln(n+1)}$ $\qquad \sum_{n=1}^{\infty} \frac{1}{\ln(n+2)} - \frac{1}{\ln(n+1)}$

$\frac{1}{\ln 3} - \lim_{n\to\infty} \frac{1}{\ln(n+1)} = \frac{1}{\infty} = 0$ $\qquad s_n = \frac{1}{\ln 3} - \frac{1}{\ln(n+1)}$

$\frac{1}{\ln 3} - 0 = \boxed{\frac{1}{\ln 3}}$ $\therefore$ conv.

by telescoping.

b- $\sum_{n=1}^{\infty} \frac{3^n}{n+1}$ (3 points)

3

$\lim_{n\to\infty} \frac{3^n}{n+1} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{3^n \times 1 \cdot \ln 3}{1} = \frac{\infty}{1} = \infty \neq 0$ $\therefore$

div - by div - test.

Q5 $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

$\lim_{a \to \infty} \int_2^a \frac{1}{n \ln n} \, dn.$ ~~$= 1 + 8$~~

$\cancel{z = \frac{1}{\ln n}}$

$dv = \frac{1}{n} \to n^{-1}$

$dz = \frac{-1 * \frac{1}{n}}{(\ln n)^2}$

$v = \ln n.$

$dz = \frac{-1}{n(\ln n)^2}$

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✓

c- $\sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}}$ (3 points)

$$e^{2n}+1 > e^{2n}$$

$$\frac{e^n}{e^{2n}+1} < \frac{e^n}{e^{2n}}$$

3

$$\underbrace{\frac{e^n}{e^{2n}+1}}_{\text{conv. by}} < \underbrace{\frac{1}{e^n}}_{} \rightarrow \text{conv. by g-series}$$

Limit comparison test.

$$\left(\frac{1}{e}\right)^n$$

$$\left|\frac{1}{e}\right| < 1.$$

$$\frac{1}{2\ln 2} + \frac{1}{3\ln 3} + \frac{1}{4\ln 4}$$

d- $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ $\Rightarrow$ $\Rightarrow$ $\frac{A}{n} + \frac{B}{\ln n}$ (3 points)

في الخلف محلولة

1

$$\boxed{B=1}$$

$$\frac{1}{n\ln n} < \frac{1}{n}$$

Sol.

$$\frac{1}{n\ln n}$$

$$\ln n > 1$$

$$n\ln n > n$$

$$\frac{1}{n\ln n} < \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{n}{\ln n} \frac{\infty}{\infty}$$

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$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$.

$\cancel{b_n = \frac{1}{n}}$ ~~div.~~

~~by p series test.~~

$\cancel{\lim_{n\to\infty} \frac{\frac{1}{n \ln n}}{\frac{1}{n}}}$

$\cancel{\lim_{n\to\infty} \frac{n}{\ln n \cdot n} = \frac{1}{\infty} = 0}$ ~~test fail.~~

$\cancel{b_n = \frac{1}{\ln n}}$. $\ln n < n$

$\cancel{\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\frac{1}{n \ln n}}{\frac{1}{\ln n}} = \frac{1}{n} = \frac{1}{\infty} = 0}$

$\frac{1}{\ln n} > \frac{1}{n}$ div

$\frac{1}{\ln n} > 1$

$\int_2^{\infty} \frac{1}{n \ln n}$

$\lim_{a\to\infty} \int_2^{a} \frac{1}{n \ln n} dn. \Rightarrow \int_2^{a} \frac{1}{n \ln n} = \cancel{\frac{A}{n}} + \cancel{\frac{B}{\ln n}}$.

$1 = A(\ln n) + Bn$