29/30 Exellent

الجامعة الأردنية	الامتحان الأول: تفاضل وتكامل 2	الأحد 2018/6/24
اسم الطالب: محمد محمود أبو طبيخة		مدرس المادة: د. مأمون أبو حماد
الرقم الجامعي:		الرقم المتسلسل: 53

Solve questions (1) and (6). __Show your work:__

Q1. Solve

$$\int \frac{\ln(x+1)}{x^2}\,dx$$

5

sol.

$$\int \frac{\ln(x+1)}{x^2}\,dx = -\frac{\ln(x+1)}{x} + \int \frac{dx}{x(x+1)}$$

$u = \ln(x+1)$, $du = \frac{dx}{x+1}$

$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$

$1 = A(x+1) + Bx$

$\boxed{A=1}$

$\boxed{B=-1}$

$u = \ln(x+1)$, $du = \frac{dx}{x+1}$, $dv = x^{-2}$, $v = \frac{-1}{x}$

$$\int \frac{\ln(x+1)}{x^2}\,dx = -\frac{\ln(x+1)}{x} + $$

$$= -\frac{\ln(x+1)}{x} + \ln|x| - \ln|x+1| + C.$$

Q2. Solve

$$\int e^{-2x}\cos 5x\,dx$$

4

$$I = \frac{1}{5}\int e^{-\frac{2}{5}u}\cos u\,du.$$

$u = 5x$

$du = 5dx$, $x = \frac{u}{5}$

$dx = \frac{du}{5}$

$z = \cos u$, $dz = -\sin u\,du$

$dv = e^{-\frac{2}{5}u}du$, $v = \frac{-5}{2}e^{-\frac{2}{5}u}$

$$\frac{1}{5}\int e^{-\frac{2}{5}u}\cos u\,du = \cos u \cdot \frac{-5}{2}e^{-\frac{2}{5}u} - \frac{5}{2}\int e^{-\frac{2}{5}u}\sin u\,du.$$

$z = \sin u$, $dz = \cos u$, $dv = e^{-\frac{2}{5}u}$, $v = \frac{-5}{2}e^{-\frac{2}{5}u}$

$$\frac{1}{5}\int e^{-\frac{2}{5}u}\cos u\,du = \frac{-5}{2}\cos u\, e^{-\frac{2}{5}u} - \frac{5}{2}\left(\frac{-5}{2}\sin u\, e^{-\frac{2}{5}u} + \frac{5}{2}\int e^{-\frac{2}{5}}\cos u\,du\right)$$

$$= \frac{-5}{2}\cos u\, e^{-\frac{2}{5}u} + \frac{25}{4}\sin u\, e^{-\frac{2}{5}u} - \frac{5}{2}\int e^{-\frac{1}{5}u}\cos u\,du.$$

$$\frac{27}{10}\int \cos u \cdot e^{-\frac{2}{5}u}\,du = \frac{-5}{2}\cos u \cdot e^{-\frac{2}{5}u} + \frac{25}{4}\sin u\, e^{-\frac{2}{5}u} + C.$$

Backward

Scanned with CamScanner

$$= \frac{27}{10} \int \cos 5x \, e^{-2x} = \frac{-5}{2} \cos 5x \, e^{-2x} + \frac{25}{4} \sin 5x . e^{-2x} + c.$$

الحمد لله

Scanned with CamScanner

Q3. Solve

$$\int \frac{9x^3 - 3x + 1}{x^3 - x^2} dx$$

5

$$\begin{array}{r} 9 \\ x^3 - x^2 \,\big)\, 9x^3 - 3x + 1 \\ -9x^3 + 9x^2 \\ \hline 9x^2 - 3x + 1 \end{array}$$

$$\int \frac{9x^3 - 3x + 1}{x^3 - x^2} dx = \int 9\,dx + \int \frac{9x^2 - 3x + 1}{x^3 - x^2} dx.$$

$$= \int 9\,dx + \int \frac{9x^2 - 3x + 1}{x^2(x-1)} dx. \rightarrow \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}.$$

$$= \int 9\,dx + \int \frac{2}{x} dx - \int \frac{dx}{x^2} + \int \frac{7}{x-1} dx$$

$$9x^2 - 3x + 1 = Ax(x-1) + B(x-1) + Cx^2$$

$$= 9x + 2\ln|x| + \frac{1}{x} + 7\ln|x-1| + C$$

$\boxed{x=0}$ $1 = -B$ $\boxed{B=-1}$

$\boxed{x=1}$

$\boxed{7 = C}$

$\boxed{A=2}$

Q4. Solve

$$\int \frac{\cos x}{\sin x \cos x + \sin x} dx$$

5

أنظر في الصفحات الفارغة

$$\int \frac{\cos x}{\sin x(\cos x + 1)} dx.$$

$$\int \frac{\cot x}{(\cos x + 1)} \times \frac{\cos x - 1}{\cos x - 1} dx.$$

$$\int \frac{\cot x(\cos x - 1)}{\cos^2 - 1} dx.$$

$$\int \frac{\cot x(\cos x - 1)}{-\sin^2 x}$$

$$-1 \times \int \frac{\cot x}{\sin x} \cdot \frac{\cos x - 1}{\sin x} dx.$$

$$-1 \times \int \frac{\cos x}{\sin^2 x} \cdot \frac{\cos x - 1}{1 - \cos^2 x}$$

$$\frac{\cos x + 1}{(1+\cos x)(1-\cos x)}$$

$$+1 \times \int \cot x.$$

$$\frac{1}{\frac{\cos x}{\sin x}} \times \sin x.$$

$$\frac{\cos x}{\sin x} \times \frac{1}{\sin x}.$$

$$\frac{\cos x}{\sin^2 x}.$$

2

Scanned with CamScanner

Q4.

$$\int \frac{\cos x}{\sin x \cos x + \sin x}\,dx.$$

$$\int \frac{\cos x}{\sin x(\cos x + 1)}\,dx.$$

$u = \tan\frac{x}{2}$

$$\int \frac{\frac{1-u^2}{1+u^2}}{\frac{2u}{1+u^2}\left(\frac{1-u^2}{1+u^2}+1\right)} \cdot \frac{2du}{1+u^2}$$

$$\int \frac{\frac{1-u^2}{1+u^2}}{\frac{2u}{1+u^2}\left(\frac{1-u^2+1+u^2}{1+u^2}\right)} \cdot \frac{2du}{1+u^2}.$$

$$2\int \frac{\frac{1-u^2}{1+u^2}}{\frac{2u}{(1+u^2)^2}} \frac{du}{1+u^2}$$

$$\int \frac{(1-u^2)\cancel{(1+u^2)^2}}{\cancel{2}u\,\cancel{(1+u^2)}} \cdot \frac{\cancel{2}du}{\cancel{1+u^2}}$$

$$\int \frac{1-u^2}{u}\,du.$$

$$\int \frac{1}{u}\,du - \int u\,du.$$

$$\ln|u| - \frac{u^2}{2} + C.$$

$$\ln\left|\tan\frac{x}{2}\right| - \frac{\left(\tan\frac{x}{2}\right)^2}{2} + C.$$

Q5. Find the following integral

$$\int \frac{x+3}{\sqrt{x^2+2x+2}}\,dx$$

5

$\sqrt{(x+1)^2+1}$, $x+1$, 1

$$\int \frac{x+3}{\sqrt{x^2+2x+1-1+2}}\,dx.$$

$x = \tan\theta - 1$

$\frac{x+1}{1} = \tan\theta$

$dx = \sec^2\theta\, d\theta.$

$$\int \frac{x+3}{\sqrt{x^2+2x+1+1}}\,dx$$

$$\int \frac{x+3}{\sqrt{(x+1)^2+1}}\,dx$$

$(x+1)^2+1 = u^2$

$u = \sqrt{(x+1)^2+1}$

$$\int \frac{\tan\theta+2}{\sqrt{\tan^2\theta+1}}\cdot\sec^2\theta\, d\theta.$$

$$\int \frac{\tan\theta+2}{\sec\theta}\cdot\sec^2\theta\, d\theta. = \int \sec\theta\tan\theta\, d\theta + 2\int \sec\theta$$

$= ~~\int\sec\theta\tan\theta~~$

$= \sec\theta + 2\ln|\sec\theta+\tan\theta| + c$

$\sqrt{(x+1)^2+1} + 2\ln\left|\sqrt{(x+1)^2+1} + (x+1)\right| + C$

$= ~~\sec\theta + 2\ln|\sec\theta+\tan\theta|~~$

Q6. Find the following integral (If exist):

$$\int_0^1 \frac{dx}{\sqrt{x}(x+1)}$$

9 $E \to 0^+$

$$\int_0^1 \frac{2u\,du}{u(u^2+1)}$$

$u = \sqrt{x}.$

$u^2 = x$

$2u\,du = dx.$

5

$$= \int_0^1 \frac{2\,du}{\cancel{(u^2+1)}} = \frac{Ax+B}{\cancel{(u^2+1)}}$$

$1 \to 1$

$0 \to 0.$

4

$$2\int_0^1 \frac{du}{u^2+1}$$

$$2\tan^{-1}u\Big|_0^1 = 2\tan^{-1}1 - 2\tan^{-1}0$$

con.

$= 2\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{2}$

3