$\frac{23}{30}$

The University of Jordan

Department of Mathematics, Math 202, First Exam

46 Name:. Number: Section: 8-9 (46)

Q1) (14 points) Fill in the blank

$\frac{11}{14}$

1. An integrating factor of the D.E.l. $dx + \underbrace{\left(\frac{x}{y} - \sin y\right)}_{N} dy = 0$ is y

$\frac{\partial M}{\partial y} = 0, \frac{\partial N}{\partial x} = \frac{1}{y} \quad R(x) = \frac{0 - \frac{1}{y}}{\frac{x}{y} - \sin y}, \ f(x) = \int e^{\frac{1}{y}}$

2. If $(2xy^m + 2y)dx + (2x^2y + 2x)dy = 0$ is exact, then $m =$ 2

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow 2x \cdot m \cdot y^{m-1} + 2 = 4yx + 2 \Rightarrow m \cdot y^{m-1} = 2y$

3. A fourth-order linear homogeneous D.E. with solution
$y(x) = c_1e^{-3x} + c_2e^{-x} + c_3e^{3x} + c_4xe^{3x}$. is $y^{(4)} - 2y''' - 15y'' + 6y' + 18y = 0$

$r_1 = -3, r_2 = -1, r_3 = 3, r_4 = 3 \mid (r+3)(r+1)(r-3)(r-3) = 0$

$(r+3)(r+1)(r^2 - 6r + 6) \Rightarrow (r+3)(r^3 - 6r^2 + 6r + r^2 - 6r + 6) \Rightarrow (r^4 - 6r^3 + 6r^2 + r^3 - 6r^2 + 6r + 3r^3 - 18r^2 + 18r + 3r^2 - 18r + 18)$

$\Rightarrow r^4 - 2r^3 - 15r^2 + 6r + 18$

4. Let $r^2 + 2r + 2 = 0$ be the characteristic equation of 2^{nd} —order linear D.E with constant coefficients then we can write the solution as $y = c_1 e^{-x}\cos x + c_2 e^{-x}\sin x$

$4 - 4 \times 1 \times 2 = -4 \quad r = \frac{-2 \pm \sqrt{-4}}{2} \quad y = c_1 e^{-x}\cos x + c_2 e^{-x}\sin x$

$r = -1 \pm i$

-1

5. If $y + x - xy' = 0, y(1) = 1$, then $y(2) =$ $\ln 2 + \frac{1}{2}$

$-xy' + y = -x \quad e^{\int \frac{1}{x} dx} \quad y = \int e^{\int \frac{1}{x} dx} \cdot 1 dx + c \quad \Rightarrow \frac{1}{2} \cdot y = \ln 2 + 1 \Rightarrow y = \ln 2 + \frac{1}{2}$

$y' - \frac{1}{x}y = 1$

$x^{-1} \cdot y = \int x^{-1} dx + c \Rightarrow x^{-1} y = \ln x + c \Rightarrow \frac{1}{1} \cdot 1 = 0 + c \Rightarrow c = 1$

6. If $W(1, g(x)) = 4$, then $g(x) =$ $4x$

$\begin{vmatrix} 1 & g(x) \\ 0 & g'(x) \end{vmatrix} = g'(x) - 0 = \int 4 \, dx$

$4x$

7. The general solution of $y''' + \frac{1}{x^2}y' - \frac{1}{x^3}y = 0$ is $y = c_1x + c_2x\ln x + c_3x\ln^2 x$

$x^3y''' + xy' - y = 0 \quad r(r-1)(r-2) + r - 1 = 0$

$r(r-1)(r-2) + (r-1) = 0$

$(r-1)(r(r-2) + 1) = 0$

$(r-1)(r^2 - 2r + 1) = 0$

$(r-1)(r-1)(r-1) = 0$

$y = c_1x + c_2x\ln x + c_3x\ln^2 x$

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Q2) (5 points) Solve the differential equation $xy' - y = x^3y^3$, $y(1) = 1$.

$y' - \frac{1}{x} \cdot y = x^2 \cdot y^3 \dots (*)$

Let: $u = y^{1-3}$

$u = y^{-2}$

$u' = -2y^{-3} \cdot y'$

Multiply $(*)$ by $(-2y^{-3})$

$\Rightarrow -2y^{-3} \cdot y' + \frac{2}{x} y^{-2} = -2x^2$

$\Rightarrow u' + \underbrace{\frac{2}{x}}_{P(x)} \cdot u = \underbrace{-2x^2}_{f(x)}$

$e^{\int \frac{2}{x} \cdot dx} \cdot u = \int e^{\int \frac{2}{x} \cdot dx} \cdot -2x^2 \cdot dx + c$

$e^{2\ln x} \cdot u = \int e^{2\ln x} \cdot -2x^2 \cdot dx + c$

$x^2 \cdot u = \int -2x^4 \cdot dx + c$

$x^2 \cdot u = \frac{-2x^5}{5} + c$

$\Rightarrow u = \frac{1}{x^2}\left(\frac{-2x^5}{5} + c\right)$

$y^{-2} = \frac{1}{x^2}\left(\frac{-2x^5}{5} + c\right)$

$1 = \frac{1}{1}\left(\frac{-2}{5} + c\right)$

$\boxed{c = \frac{2}{5}}$

$\Rightarrow y^2 = \frac{x^2}{\frac{-2x^5}{5} + \frac{2}{5}}$

$y = \pm \sqrt{\frac{x^2}{\frac{-2x^5}{5} + \frac{2}{5}}}$

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Q3) (4 points) Find the form of the particular solution to: $y'' + 4y' + 4 = x^2e^{-2x}$, if the undetermined coefficients method is to be used.

y_h: $y'' + 4y' + 4 = 0$

$16 - 4 \times 1 \times 4$

0

$r^2 + 4r + 4 = 0$

$(r+2)(r+2) = 0$

$r = -2, -2$

$y_h = c_1 e^{-2x} + c_2 e^{-2x} \cdot x$

$y_p = e^{-2x}(a_2x^2 + a_1x + a_0)\, x$

$y_p = e^{-2x}(a_2x^3 + a_1x^2 + a_0x)$

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$y_g = c_1 e^{-2x} + c_2 e^{-2x} \cdot x + e^{-2x}(a_2x^2 + a_1x + a_0)x$

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Q3) (7 points) Find the general solution to: Find gen sol (using var. of para)

$$\frac{xy''}{x} - \frac{2y'}{x} = \frac{3x^{-1}}{x}. \quad (x > 0)$$

$$y'' - \frac{2}{x} \cdot y' = \frac{3}{x^2}$$

y_h: $r(r-1) - 2r = 0$

$r^2 - r - 2r = 0$

$r^2 - 3r = 0$

$r(r-3) = 0$

$r = 0, 3$

$y_h = c_1 x^0 + c_2 x^3$

$= c_1 + c_2 x^3$

$$W[1, x^3] = \begin{vmatrix} 1 & x^3 \\ 0 & 3x^2 \end{vmatrix} = 3x^2 - 0$$

$$y_p = -1 \cdot \int \frac{\cancel{x^3} \cdot \frac{3}{\cancel{x^2}}}{3x^2} \cdot dx + x^3 \cdot \int \frac{1 \cdot \frac{3}{x^2}}{3x^2} \cdot dx$$

$$= -\int \frac{1}{x} \cdot dx + x^3 \cdot \int \frac{1}{x^4} \cdot dx$$

$$= -\ln|x| + x^3 \cdot \frac{x^{-3}}{-3}$$

$$= -\ln|x| - \frac{1}{3}$$

$$y_g = c_1 + c_2 x^3 - \ln|x| - \frac{1}{3}$$

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