* Introduction:

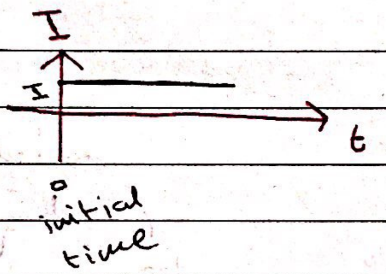
* Electric circuit (ckt):

- an interconnection of electric devices (elements).

- to transfer energy from one point to another.

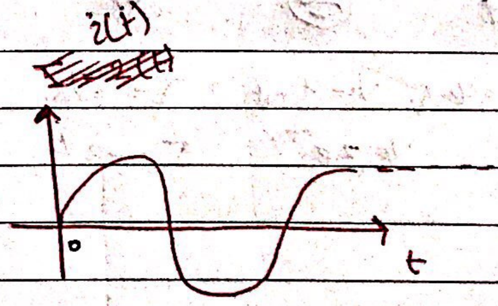
* Two parts of circuits → DC - part
→ AC - part

* D-C (ckt):
Direct current circuit



* constant in value and direction.

* A-c (ckt):
alternating current circuit



⇒ the current is time-varying.

1 Electric charge:

Basic quantity in electricity

* unit: Coulomb "C"

* atom → electrons: (e) $(-1.6 \times 10^{-19}\ C)$
→ protons: (p) $(1.6 \times 10^{-19}\ C)$

Q) How many (e) in 1 C?

an: $1/e = 6.24 \times 10^{18}$ electrons.

- "C" is a large unit.

2 Electric current:

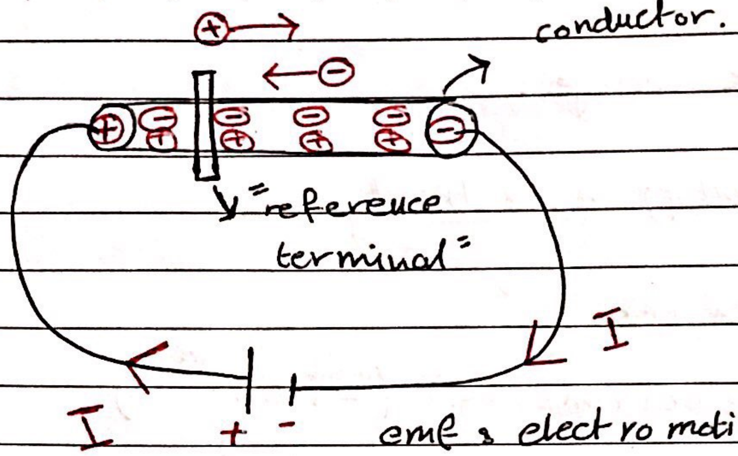
* i or $i(t)$: time-varrying current.

* I: constant current.

unit: Ampere "A"

$1\ A = 1\ C/sec.$

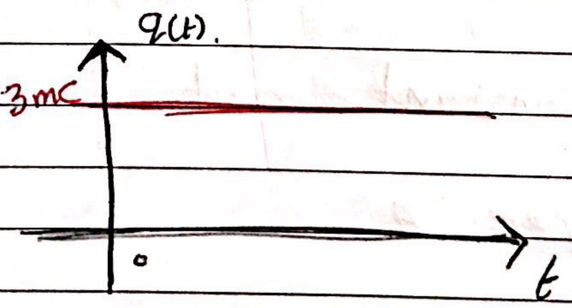
Def: The current means charges in motion (flow of charges).



emf : electro motive force.

* current direction is the same as direction of +ve charges.

* $q(t)$ = the net (total) charges that has passed the refernce terminal since the initial time $t=0$ in a specific direction.



Ex:

Sol.

$q(0) = 3$ mc

$q(1) = 0$

$q(2) = 0$

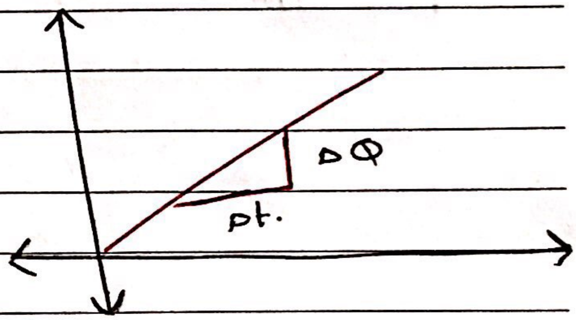
$\vdots$

$$\boxed{i(t) = \frac{dq(t)}{d(t)}}$$

Five Apple

Ex: The current is constant (I) only when $q(t)$ is linear function of time

$$\boxed{I = \frac{\Delta Q}{\Delta t}}$$



• $i = \frac{dq}{dt} \Rightarrow \int_{q(t_1)}^{q(t_2)} dq = \int_{t_1}^{t_2} i\,dt \Rightarrow \boxed{\underbrace{q(t_2) - q(t_1)}_{} = \int_{t_1}^{t_2} i(t)\,dt}$

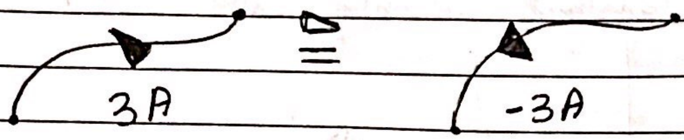
the total net charge flow between $t = t_1$ & $t = t_2$.

From (*) : $q(t) - q(t_0) = \int_{t_0}^{t} i(t')\,dt'$

$\Rightarrow q(t) = \int_{t_0}^{t} i(t')\,dt' + q(t_0)$

t_0 and $q(t_0)$ should be given.

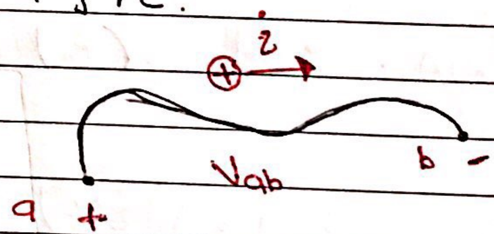
• Current direction: current → numerical value (either +ve or -ve). → Direction.



3 Voltage (potential difference)

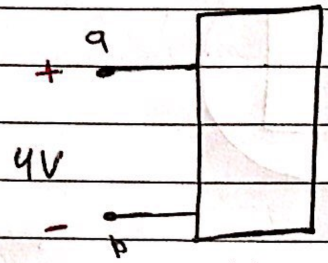
Def: The amout of energy (w) needed to move +1 C from the "+ve" labled terminal to the "-ve" labled terminal.

$\boxed{V = \frac{dw}{dq}}$ unit: 1 V = 1 J/C.



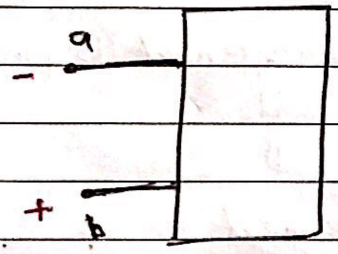
Voltage V
- → numerical value
  +ve or -ve
- → polarity.

e.g:



$V_{ab} = V_a - V_b = 4\,V.$

$\Downarrow$



4 Power: $P(t)$, P, unit "W"

Def: the time-rate of energy absorbed by an element.

$$\boxed{P = \frac{dw}{dt}}$$

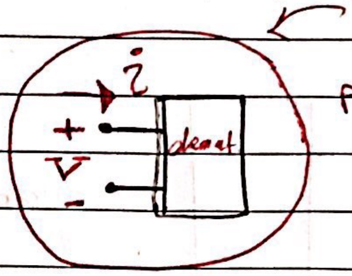
$1\,W = 1\,J/sec.$

$$P = \frac{dw}{dt} \times \frac{dq}{dq} = \frac{dw}{dq} \cdot \frac{dq}{dt} = Vi$$

$$P = V i$$

$P = Vi$

power absorbed by an element.



passive sign convention

- absorbed → passive
- generate → active.

step 1: use the passive sign convention.

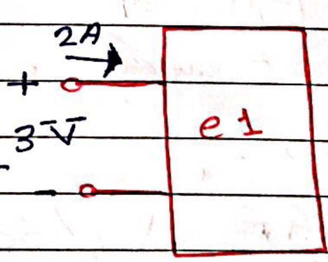
step 2: $p = iv$

→ +ve : the element is passive and absorbed.

→ -ve : the element is active and generate.

Ex: Determine the power absorbed. ~~or generated~~. ~~or generated~~ / or generated by each element.

(1)



sol.

$P = iV = (2)(3) = 6\,W$

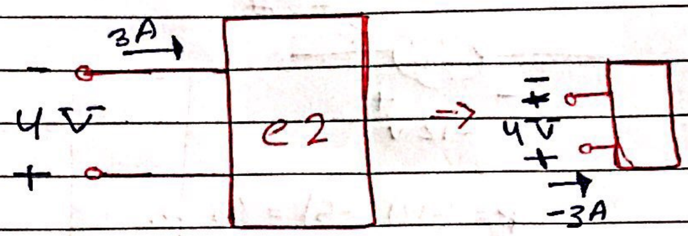
passive element.

- absorbed 6 W

OR

- generated -6 W.

(2)



$p = i(V) = (-3)(4) = -12$ W

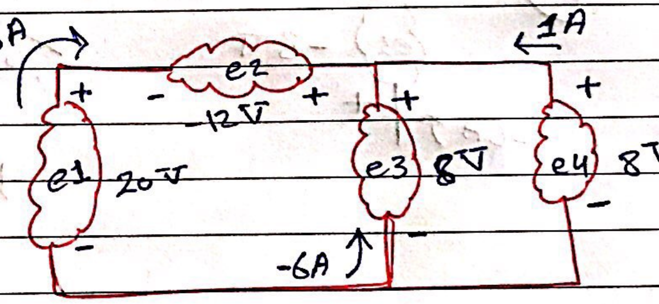
-absorbed -12 W

$\triangleq$

-generated 12 W.

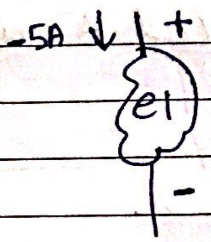
active element

Ex:



Determine power absorbed by elements?

Sol. e1



$p = -100$ W

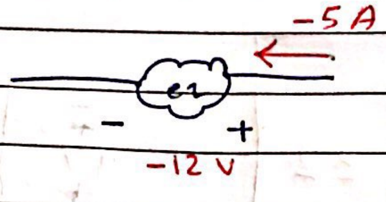
absorbed -100 W

OR

✓generated 100 W.

active

e2



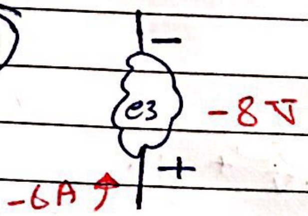
$p = (-12)(-5) = 60$ w.

✓ absorbed 60 w

$\triangleq$ generated -60 w

passive

e3



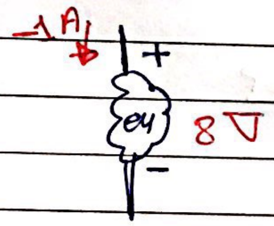
$p = (-6)(-8) = 48$ W

✓ absorbed 48 watt

passive

$\triangleq$ generated -48 watt.

e4



$p = (-1)(8) = -8$ watt

absorbed. -8 watt

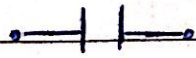
✓ $\triangleq$ generated 8 watt.

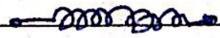
active

$$\sum P^{ob} = \sum P^{gen.}$$

* Passive elements :-

1) Resistors : 

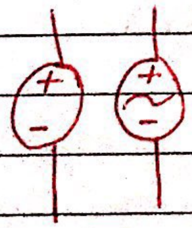
2) capacitors : 

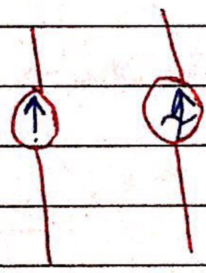
3) inductors : 

* power → +ve → absorbed.

→ -ve → generated (active element.)

* These sources are independent :

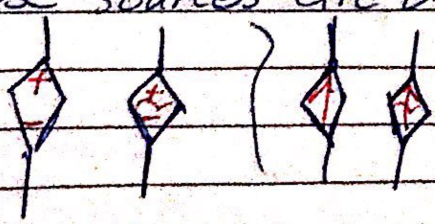




current source

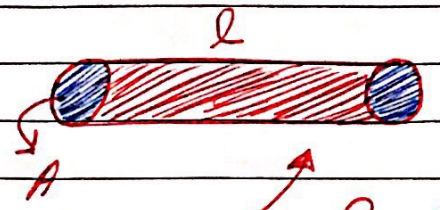
Voltage source

* These sources are dependent :



Chapter 2 # Resistance & Resistors.

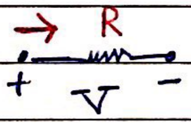
* Resistance : a material properity to resist the current passing



ρ = resistivity.

$$\boxed{R_{(\Omega)} = \frac{\rho_{\Omega . m}\, l_{(m)}}{A_{m^2}}}$$

* Resistor :



Basic passive element.

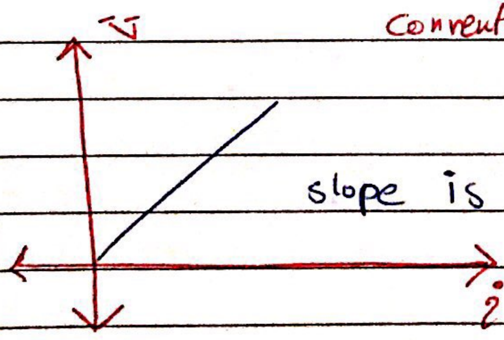
ohm's law

$$\boxed{V = iR}$$

R is constant value

= "ثابته بالداره الكهربائية"

* يجب ان تكون المقاومه بال "passive sign convention"



slope is equal to R because $R = \frac{V}{i}$

$0 \leq R \leq \infty$

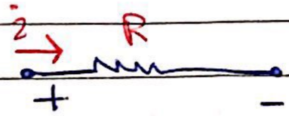
short circuit ($R = \frac{V}{i} \Rightarrow V=0$)

open circuit $R = \frac{V}{i} \rightarrow i = 0$

Five Apple

Scanned with CamScanner

* power absorbed (dissipated) by resistors



$P = V \times i$

$P = i^2 \times R$

$P = \frac{V^2}{R}$

conductance # (G)

$G = \frac{1}{R} \Rightarrow (\Omega)^{-1}, S^{-1}$

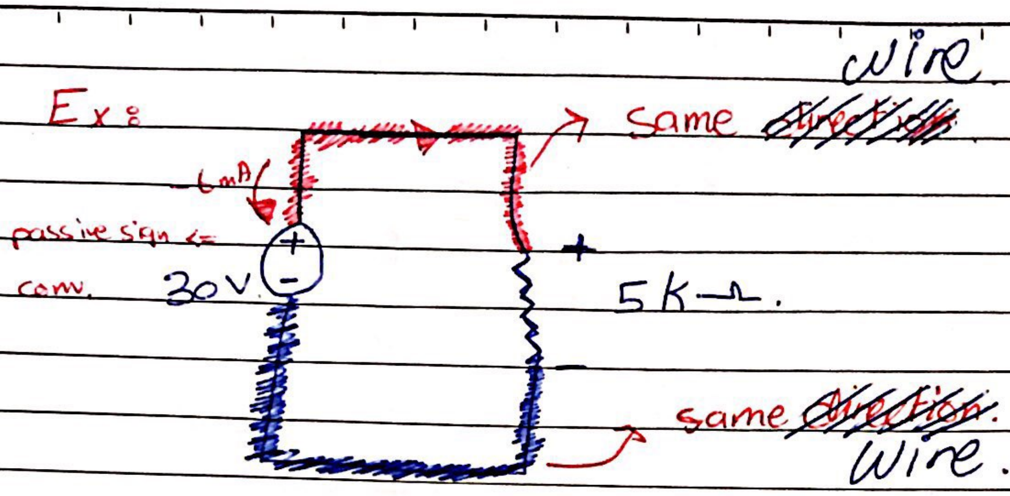
Ex8

$\stackrel{\Delta}{=}$

because, $G = \frac{1}{R} = \frac{1}{10 \times 10^3} = 0.1 \, mS$.

$V = i \times R \stackrel{\Delta}{=} V = \frac{i}{G} \stackrel{\Delta}{=} G = \frac{i}{V} \stackrel{\Delta}{=} i = GV.$

Ex:



Find: 1) current 2) G 3) power dissipated by 5 kΩ.

4) power generated by the voltage source

Sol.

1) $V = iR = 30 = i * 5\,10^{3} \Rightarrow i = 6\,mA$.

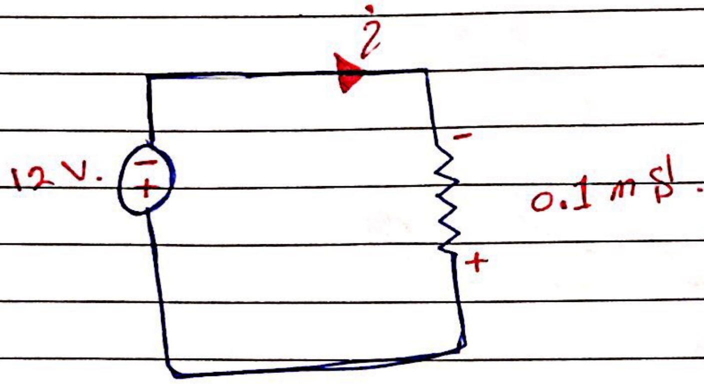
2) $G = \frac{1}{R} = \frac{1}{5\,10^{3}} = 0.2\,m\Omega^{-1}$ OR mS.

3) $P = i^{2} R = 36\,10^{-6} * 5\,10^{3} = 180\,mW$

4). $p = iV = -6\,10^{-3} * 30 = -180\,W$ obsorbed.

So, it generates $+180\,W$.

Ex: Find the current:

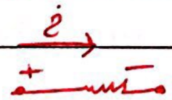


sol.

$$R = \frac{1}{G} = \frac{1}{0.1\times10^{-3}} = 10\,k\Omega.$$

$$i = \frac{V}{R} = \frac{12}{10k\Omega} = 1.2\,mA.$$

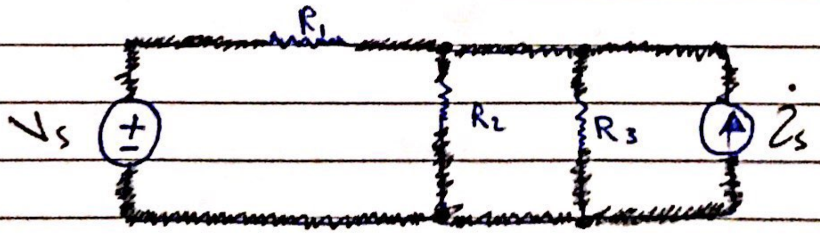
$$i = -1.2\,mA$$



* Branches: Nodes, loops: a closed path starts from a node and back to the same node.

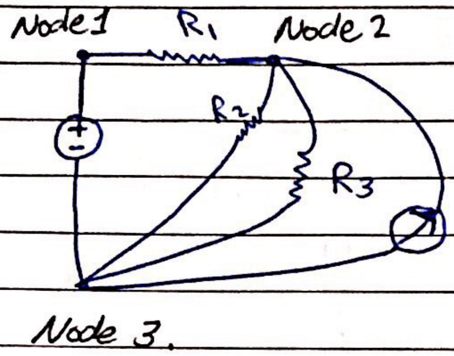
Nodes: The point of connection of 2 or more branches.

Ex:



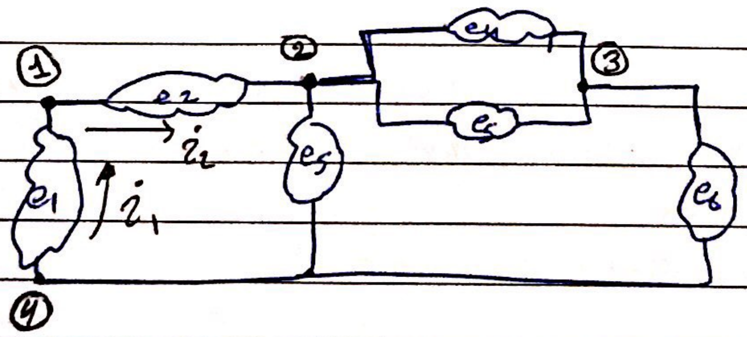
(6 Loops)

branch : a single element connected between 2 nodes.



Element in series :- exclusivley share a single node and consequently carry the same current.

Elements in parrallel :- are connected to the same two nodes and consquently have the same voltage.



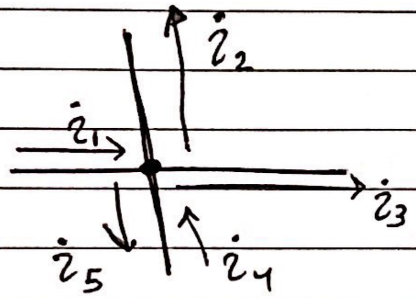
e_1 & e_2 are in series ($i_1 = i_2$).

e_4 & e_5 are in parrallel ($V_4 = V_5$).

Kirchoff's laws:

KCL (Current law) | KVL (voltage law).

1. $\boxed{1}$ KCL: The algebric sum of current entering or leaving a node is zero.



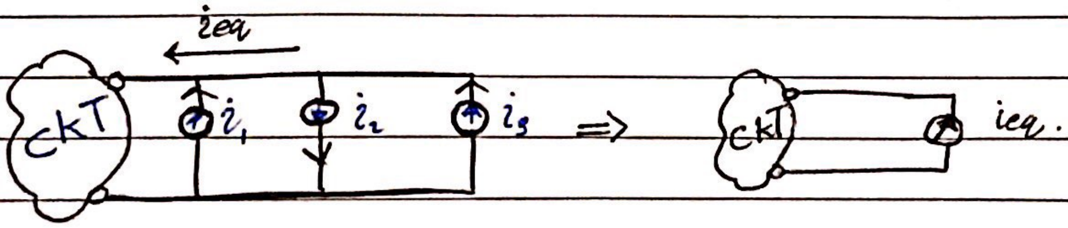
$$i_1 - i_2 - i_3 + i_4 - i_5 = 0$$
(entering).

$$-i_1 + i_2 + i_3 - i_4 + i_5 = 0$$
(leaving).

* $i_1 + i_4 = i_5 + i_2 + i_3$

"مجموع التيارات الداخلة يساوي مجموع التيارات الخارجة"

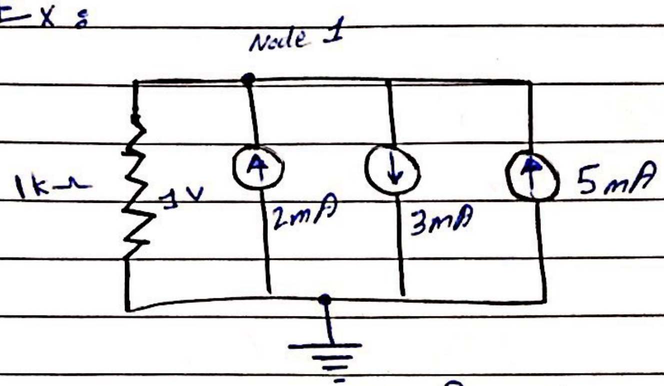
- Current source in parrallel:



KCl: $i_1 + i_3 = i_2 + i_{eq}$

$$i_{eq} = i_1 + i_3 - i_2$$

Ex :



ref. node.

apply KCL at node (1) :-

$$2mA + 5mA = 3mA + i$$

$$i = (2 + 5 - 3)\,mA$$

$$i = 4mA$$

$$V = iR$$

$$= 4mA * 1k\Omega$$

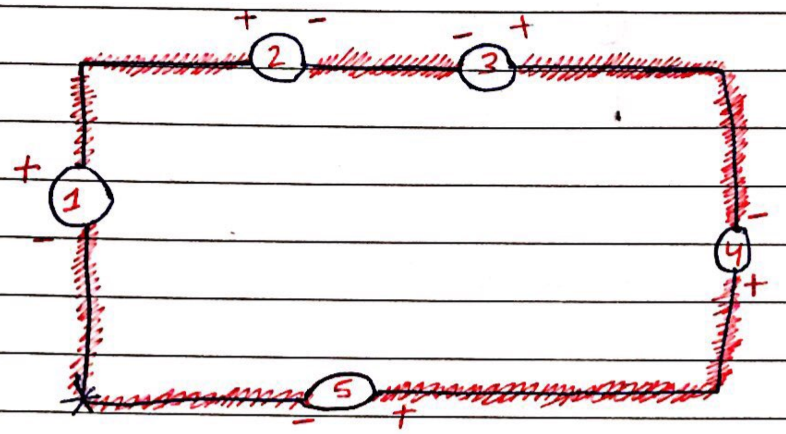
$$V = -4\,Volt.$$

~~[2] Kirchhoff's voltage (KVL).~~

~~- The algebric sum of all voltages in a closed path (Loop) is zero.~~

[2] Kirchhoff's voltage law (KVL):

The algebric sum of all voltages in a closed path (loop) is zero.



5 nodes
1 loop.

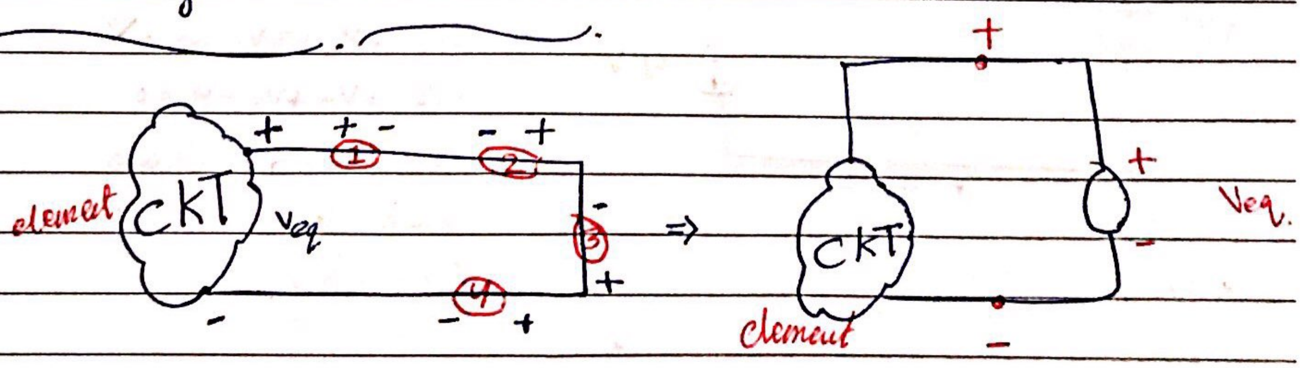
$V_{xx} = \text{zero}.$

$V_1 - V_2 + V_3 + V_4 - V_5 = 0.$

OR

$-V_1 + V_2 - V_3 - V_4 + V_5 = 0.$

* voltage source in series:



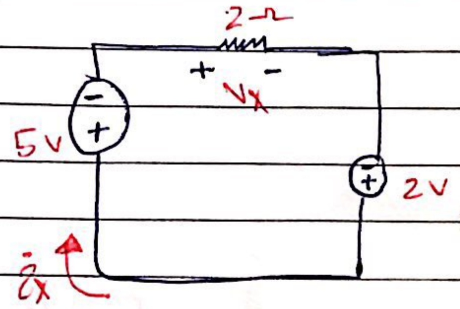
$KVL = -V_q + V_1 - V_2 - V_3 + V_4 = 0.$

$V_{eq} = V_1 - V_2 - V_3 + V_4$

Five Apple

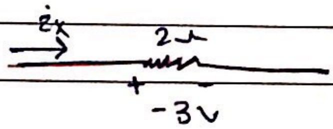
Ex: Find i_x, V_x:

*Use KVL in single loop.



KVL $\Rightarrow$ $5V + V_x - 2V = 0$.

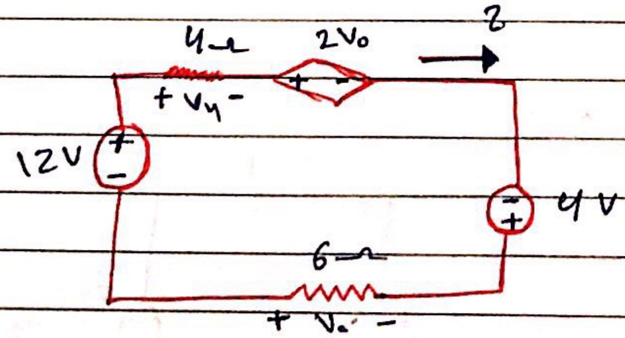
$V_x = -3V$.



$i_x = \frac{V}{R} = \frac{-3}{2} = -1.5\ A$.

Ex: Find V_o, i, Power generated by 4V

* Use KVL in single Loop.



KVL:

$-12 + V_4 + 2V_o - 4 - \cancel{V_o} = 0$.

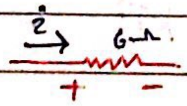
$-12 + V_4 + V_o - 4 = 0$

$-12 + 4i - 6i - 4 = 0$.

$-2i = 16$

$i = -8\ A$

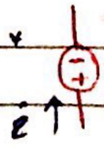
* V_o $\Rightarrow$



$V_o = iR$

$= -(-8)(6) = 48\ V$.

*

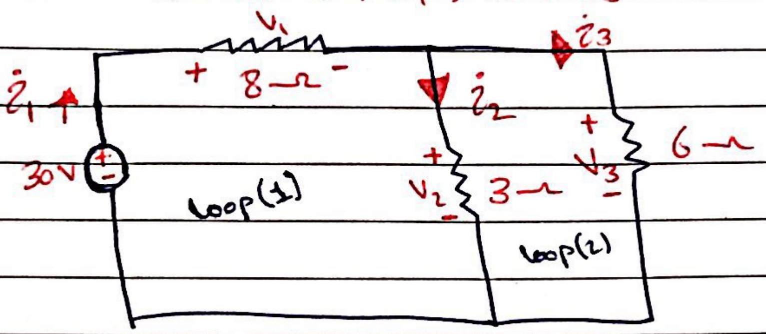


$P = iV$

$= -(-8)(4) = 32$ Volt.

so, generated -32 Volt.

Ex: Find $V_1, V_2, V_3, i_1, i_2, i_3$.



Sol.

$-30V + V_1 + V_2 = 0$ (KVL loop 1).

$30 = 8i_1 + 3i_2$ —— ①

(KCL Loop 2)

$-i_1 + i_2 + i_3 = 0$

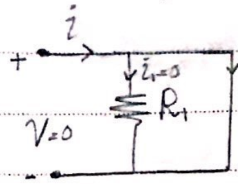
(KVL Loop 3) $-V_2 + V_3 = 0$.

$-3i_2 + 6i_3 = 0$ —— ②

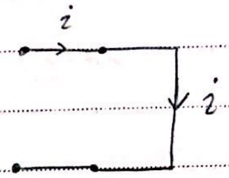
حل معادلات.

/ /

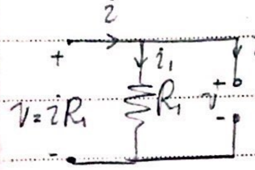
Ex :- → Some important cases:-



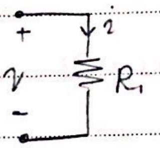
$R_2 = 0\Omega$ (S.C) ⇒



⇒ (S.C)

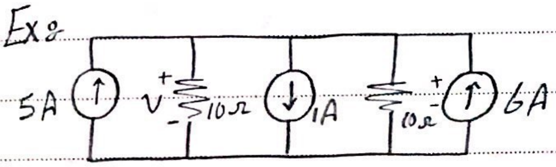


$R_2 = \infty$ (o.c) ⇒



⇒ (O.C)

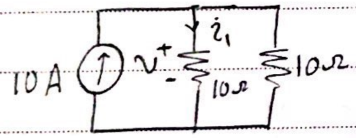
Ex :



find V ?

⇓

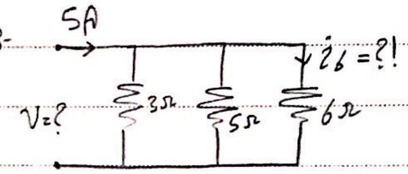
$6 + 5 - 1 = 10\ A$



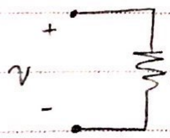
$i_1 = 10 * \frac{10}{20} = 5\ A$, $V = Ri = 50V$

Power in 6A → $P = Vi$ ⇒ $-6 * 50 = -300$ ⇒ generated.

Ex :-



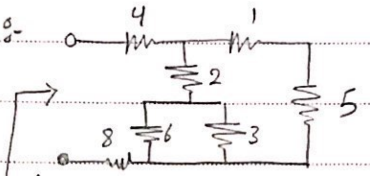
find V, i_6 ?!



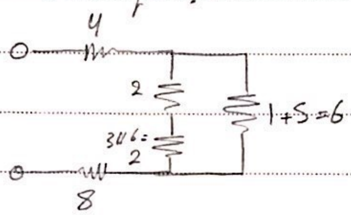
$R_{eq} = 3 \| 5 \| 6 = \frac{10}{7}\ \Omega$, $V = 5 R_{eq} = \frac{50}{7}$ Volt

$i_6 = \frac{\frac{50}{7}}{6} = \frac{50}{42}\ A$ OR $i_6 = 5 * \frac{\frac{1}{6}}{\frac{1}{3} + \frac{1}{5} + \frac{1}{6}} = \frac{50}{42}\ A$

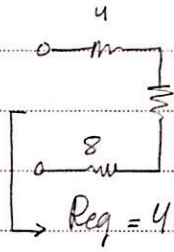
Ex:-



find R_{eq} = ?!



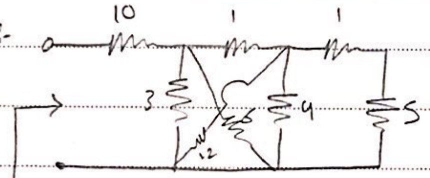
$\Rightarrow$



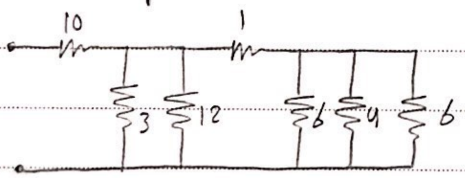
$4 \parallel 6 = \frac{24}{10} = 2.4$

$R_{eq} = 4 + 2.4 + 8 = 14.4\ \Omega$

Ex:-



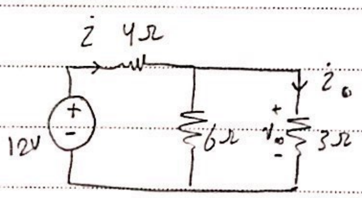
Find R_{eq} = ?!



$R_{eq} = \left(\left((6 \parallel 4 \parallel 12) + 1\right) \parallel 6 \parallel 3\right) + 10$

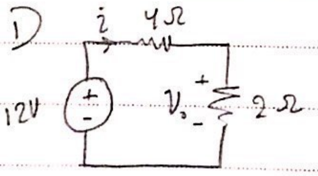
$= 11.2\ \Omega.$

Ex:-



find i, i_o, v_o ?!

1)



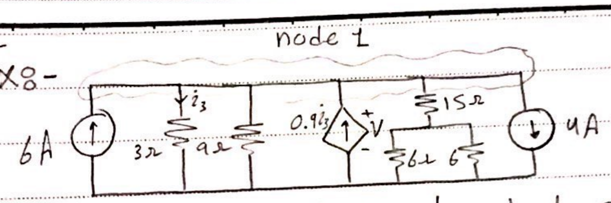
$-12 + 4i + 2i = 0$

$i = 2A$, $v_o = 2 * 2 = 4$ Volt.

$i_o = \frac{4}{3}$ A

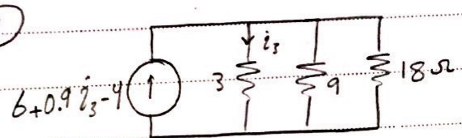
/ /

Ex:-



find v & Power of the dependent Source.

1)



$$i_3 = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{9}+\frac{1}{18}} * (6+0.9\,i_3-4)$$

2) kcl at Node 1

$$6+0.9\,i_3 = i_3 + \frac{v}{9} + \frac{v}{18} + 4 \rightarrow ①$$

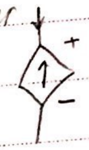
$$\frac{v}{3} = i_3 \rightarrow ②$$

Sub ② into ①:-

$$6+0.9\,\frac{v}{3} = \frac{v}{3} + \frac{v}{9} + \frac{v}{18} + 4 \rightarrow (*)$$

$$v = 10 \text{ Volt}$$

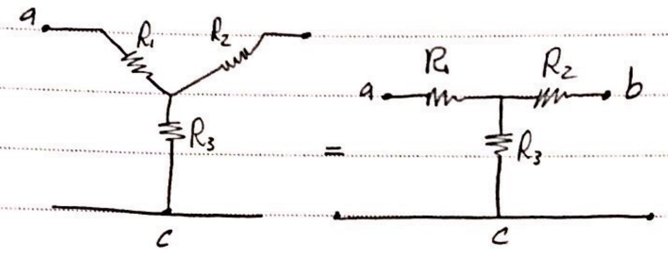
power



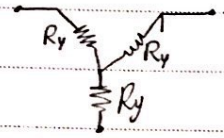
$v=10$, $0.9\,i_3 = 3A$

* Wye & Delta Network :-

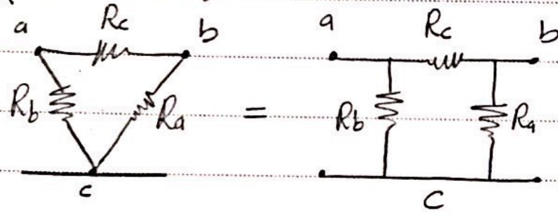
Wye :-



Balanced Wye :- $R_1 = R_2 = R_3 = R_y$

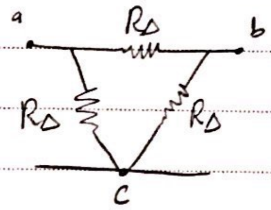


* Delta Network :-

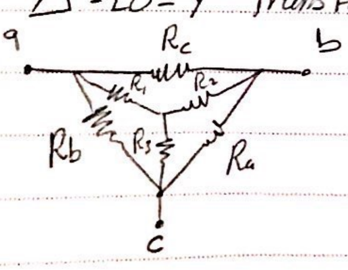


Balanced Delta :-

$R_a = R_b = R_c \triangleq R_\Delta$



Δ - to - Y Transformation



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

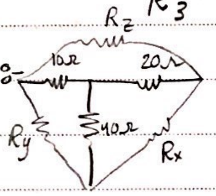
Y-to-Δ Transformation.

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Ex:-

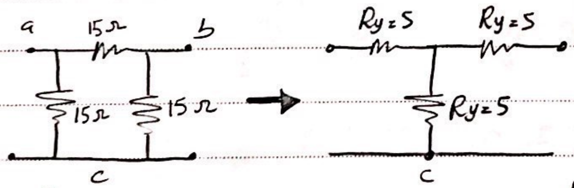


⟹ Transform the Y-Network to the Δ-Network.

$$R_z = \frac{(10)(20) + (20)(40) + (40)(10)}{40}$$

$$R_y = \frac{11}{20} \quad , \quad R_x = \frac{11}{10}$$

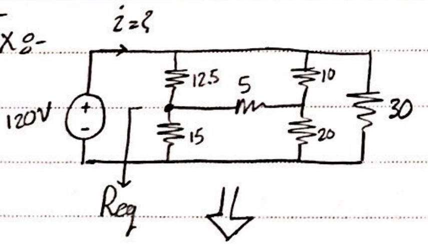
Ex:-



$R_\Delta = 15\Omega$

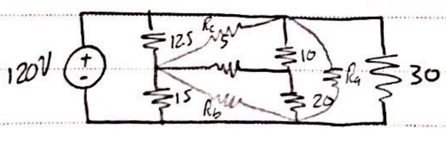
$$R_y = \frac{R_\Delta}{3} = 5\Omega.$$

Ex:-



find Req and i ?

⇓

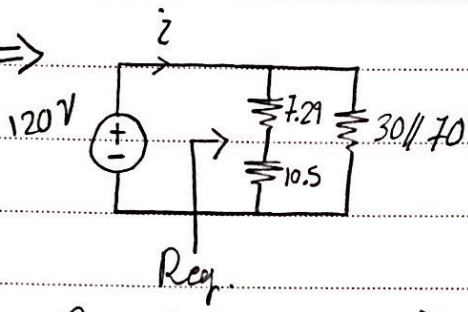


Y → Δ → $R_a = \frac{350}{5} = 70\Omega$

$$R_b = \frac{350}{10} = 35\Omega$$

$$R_c = \frac{350}{20} = 17.5\Omega$$

$\Longrightarrow$



$$R_{eq} = (7.292 + 10.5) \,//\, 21$$
$$= 9.632\ \Omega$$
$$i = \frac{120}{9.632} = 12.46\ A$$

/ /

* CH_3 :- Nodal & Mesh Analysis :-

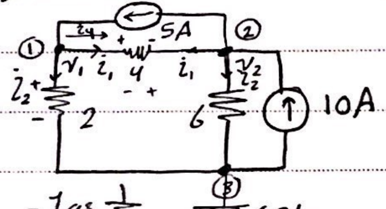
\# Nodal Analysis :-

To find the node Voltages →

Based on kcl →

Case I :- CKTs with no voltage sources.

Ex → Calcute the node Voltages using the Nodal analysis.



find i_4 = ?! → $i_4 = \frac{V_1 - V_2}{4} = \frac{13.3 - 20}{3}$

$\boxed{= -1.66A}$

3 Nodes < 1 as ⏚ ; 2 nodes = V_1, V_2

$0 = V_3$

kcl at node 1 →

$$5 = i_1 + i_2$$

$$5 = \frac{V_1 - V_2}{4} + \frac{V_1 - \overset{0}{V_3}}{2} \Rightarrow 5 = \frac{V_1}{4} - \frac{V_2}{4} + \frac{2\times V_1}{2\times 2}$$

$$5 = \frac{3V_1}{4} - \frac{V_2}{4} \Rightarrow (0.75 V_1 - 0.25 V_2 = 5) \rightarrow ①$$

kcl at node 2 →

$$10 = 5 + i_1 + i_2$$

$$10 = 5 + \frac{V_2 - V_1}{4} + \frac{V_2}{6}$$

$$-0.25 V_1 + \left(\frac{1}{4} + \frac{1}{6}\right) V_2 = 5$$

$$(-0.25 V_1 + 0.42 V_2 = 5) \rightarrow ②$$

Solving ① & ②

$V_1 = 40/3$ V

$V_2 = 20$ V

* Mesh Analysis :-

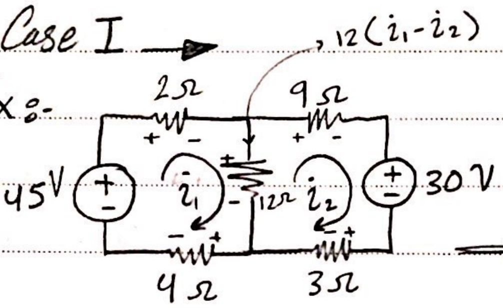
- Use the mesh currents as CKT variable *
- Mesh :- aloop the does not contain any other loops. within it.
- Mesh analysis :- apply KVL

Case I :- CKT's with no current sources.

Case II :- CKT's with current sources (we use Supermesh)

Case I →

Ex :-



⟹ Polirty نفرض الـ

Find the mesh currents :-

- find i_1 & i_2 :-

KVL at mesh 1 →

$-45 + 2i_1 + 12(i_1 - i_2) + 4i_1 = 0$

$2i_1 + 12i_1 - 12i_2 + 4i_1 = 45$

$(18i_1 - 12i_2 = 45)$ ①

KVL at mesh 2 →

$-12(i_1 - i_2) + 9i_2 + 30 + 3i_2 = 0$

$+12i_2 + 12i_1 + 9i_2 + 3i_2 = -30$ →

$12i_1 = -30$

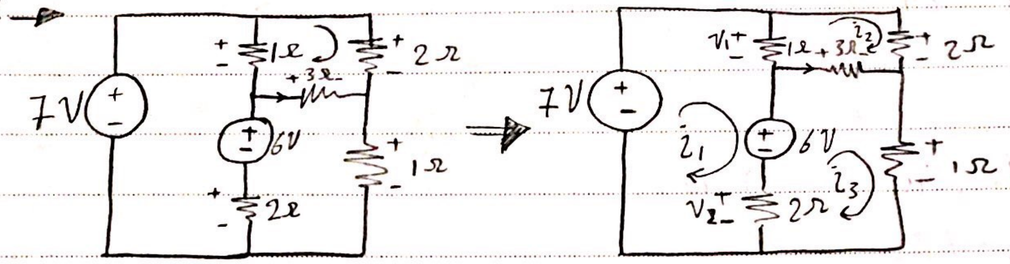
$i_1 =$

Scanned with CamScanner

* Mesh Analysis :-

Case I :- No Current sources in CkT.

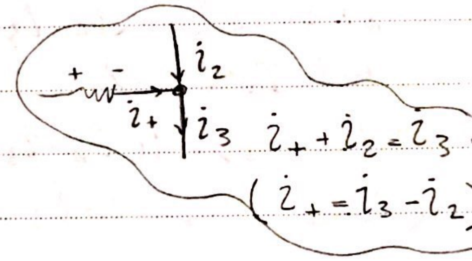
Ex →



mesh ① →

$$-7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3) = 0$$

$$(3i_1 - i_2 - 2i_3 = 1) \text{ ①}$$



mesh ② →

$$-1(i_1 - i_2) + 2i_2 - 3(i_3 - i_2) = 0$$

$$(-i_1 + 6i_2 - 3i_3 = 0) \text{ ②}$$

mesh ③ →

$$-2(i_1 - i_3) - 6 + 3(i_3 - i_2) + 1i_3 = 0$$

$$(-2i_1 - 3i_2 + 6i_3 = 6) \text{ ③}$$

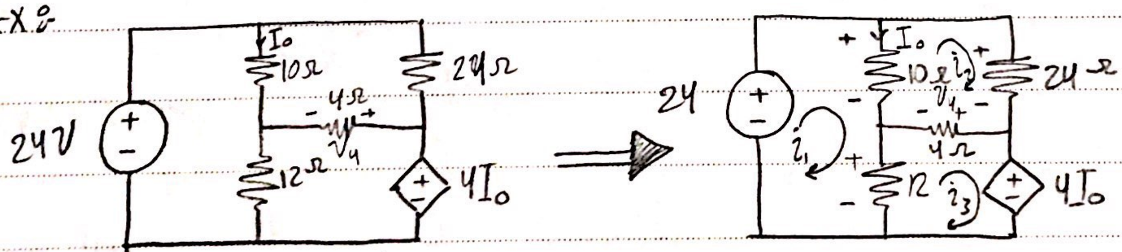
Solving ① & ② & 3 :-

$i_1 = 3A$

$i_2 = 2A$

$i_3 = 3A$

Ex :-



find V_4 using Mesh analysis :-

Mesh ① :-

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

$$(22 i_1 - 10 i_2 - 12 i_3 = 24) ①$$

Mesh ② :-

$$-10(i_1 - i_2) + 24 i_2 + 4(i_2 - i_3) = 0$$

$$(-10 i_1 + 38 i_2 - 4 i_3 = 0) ②$$

Mesh ③ :-

$$-12(i_1 - i_3) - 4(i_2 - i_3) + 4 I_o = 0$$

but $I_o = i_1 - i_2$

$$-12(i_1 - i_3) - 4(i_2 - i_3) + 4(i_1 - i_2) = 0$$

$$(-8 i_1 - 8 i_2 + 16 i_3 = 0) ③$$

Solve ① & ② & ③ :-

$i_1 = 2.25A$

$i_2 = 0.75A$

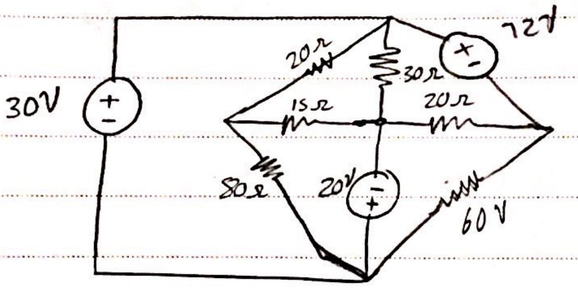
$i_3 = 1.5A$

$V_4 = 4(i_2 - i_3)$

$= 4(-0.75)$

$= -3V$

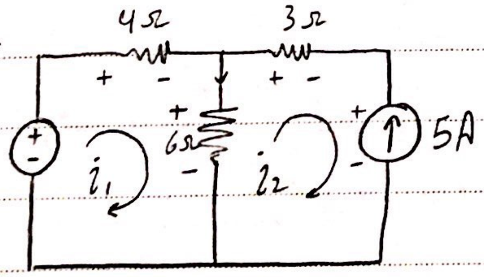
Ex: find the mesh currents:-



* Case II :- CKTs with current sources.

↳ case II-(a):- when the current source exists only in one mesh

Ex:-



mesh ②: $i_2 = -5A$

KVL at mesh ①: $\overset{i_2}{\uparrow}$

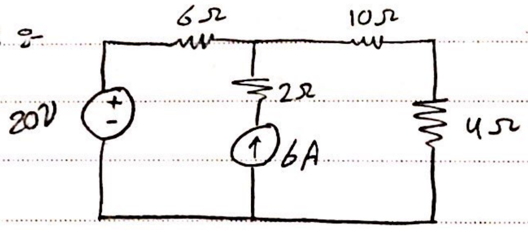
$-10 + 4i_1 + 6(i_1 - (-5)) = 0$

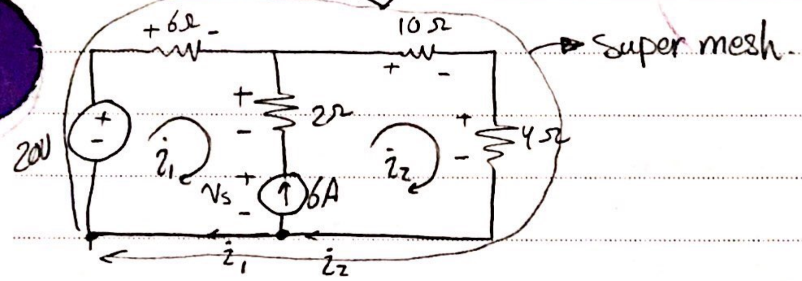
$10i_1 - 10 + 30 = 0$

$(i_1 = -2A)$

* Case II - (b) :- the Current Source exist between two meshes :-

Ex :- find i_1 & i_2 using mesh analysis →





KVL at the Super node :-

$-20 + 6i_1 + 10i_2 + 4i_2 = 0$

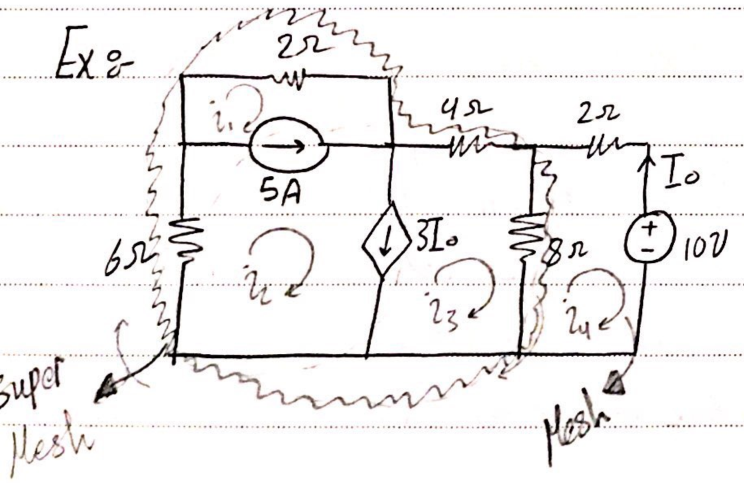
$(6i_1 + 14i_2 = 20)$ ①

kcl at the current Source.

$i_2 - i_1 = 6$

$(-i_1 + i_2 = 6)$ ②

Ex :-



find the mesh currents :-

* Super mesh →

$$+6i_2 + 2i_1 + 4i_3 + 8(i_3 - i_4) = 0$$

$$(i_1 + 3i_2 + 6i_3 - 4i_4 = 0) \quad (1)$$

* Mesh →

$$-8(i_3 - i_4) + 2i_4 + 10 = 0$$

$$(-8i_3 + 10i_4 = -10) \quad (2)$$

$$i_2 - i_1 = 5A \rightarrow (-i_1 + i_2 = 5) \quad (3)$$

$$i_2 - i_3 = 3I_o \rightarrow i_2 - i_3 = 3(-i_4)$$

$$(i_2 - i_3 + 3i_4 = 0) \quad (4)$$

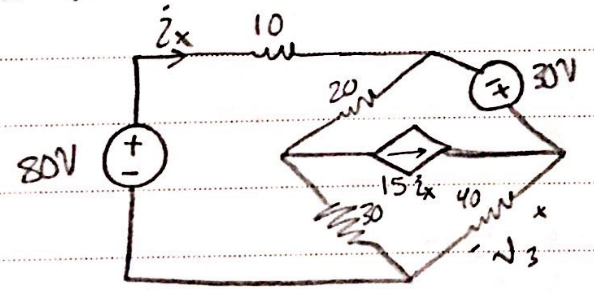
$$\begin{bmatrix} 1 & 3 & 6 & -4 \\ 0 & 0 & -6 & 10 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \\ 5 \\ 0 \end{bmatrix}$$

Solving 1, 2, 3, 4

$i_1 = 7.5A$, $i_3 = 3.93A$

$i_2 = -2.5$, $i_4 = 2.143A$

Practice 4.10



Find :- V_3 using → Nodal analysis
→ Mesh analysis.

Solution → $i_1 = 87/149$

$i_2 = 917/149$

$i_3 = 388/149$

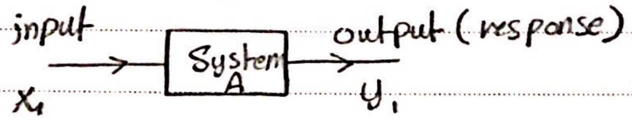
$V_3 = 40i_3 = 104.16V$

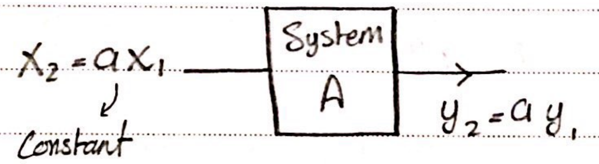
* Chapter 4 →
(Circuits Theorems)

- Linearty Property :-

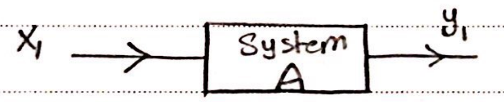
A system is said to be Linear System if it satifies the following two properties :-

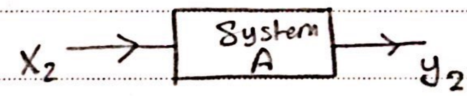
1. Scaling property :-

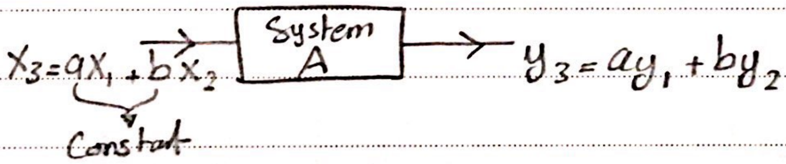




2. Additivity property :-

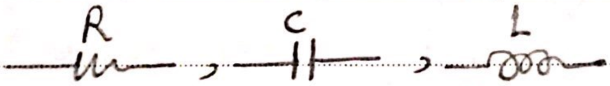






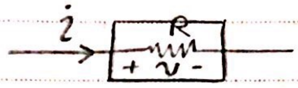
? when the electric Circuit is linear ?

Ans → when all of its elements are linear.

① the passive elements :-  are linear elements.

② The independant Source :- are linear.

- Check the resistor:



$i_1 \Rightarrow v_1 = R i_1$

$i_2 = a i_1 \Rightarrow v_2 = R i_2$

$= R(a i_1)$

$= a R i_1$

$= a v_1$

③ The dependant Sources:-

$i_s = k_1 v_x$ ✓
OR
$i_s = k_2 i_x$

Linear

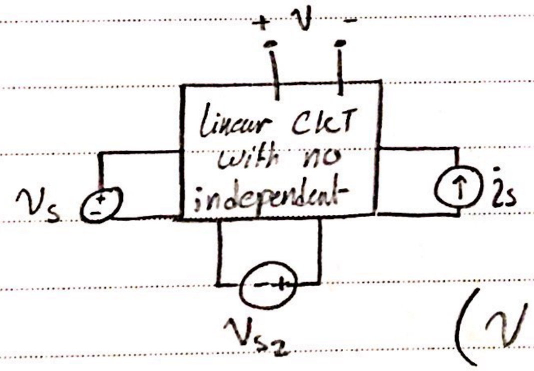
$v_s = k_1 v_x$ ✓
OR
$v_s = k_2 i_x$

Linear

$i_s = k_1 v_x^2$ ✗
OR
$i_s = k_1 i_x v_x$

Non-linear.

1 Superposition:-



find v using superposition and Find P.

$$(v = v' + v'' + v''')$$

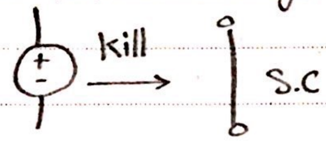
The value of v when v_{s2} & i_2 are killed (off)
v', P'

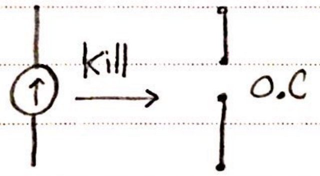
The value of v when v_s & i_s are killed
v'', P''

The value of v when v_s & v_{s2} are killed
v''', P'''

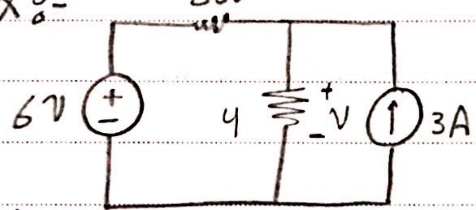
$$(i = i' + i'' + i''')$$

• To kill a Voltage Source replace it by a (O-S).C





Ex:-



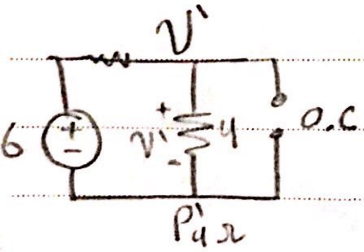
find v & $P_{4\Omega}$ using ① Nodal Analysis
② Super position.

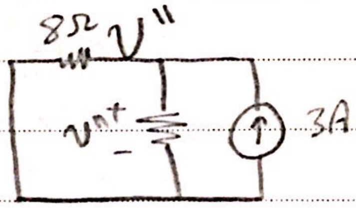
Sol → ①

$V = 10$, $P_{4\Omega} = \frac{V^2}{4} = \frac{100}{4} = 25W$

② using super position

$V = V' + V''$





$V = V' + V''$

$= 2 + 8 = 10V$

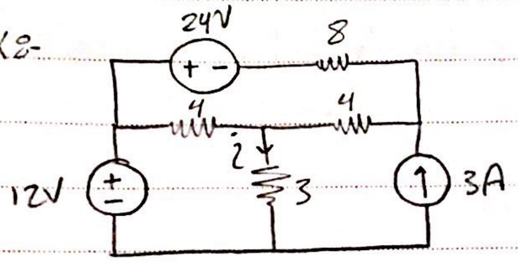
$P_{4\Omega} = \frac{V^2}{4} = 25W$

$V'' = (3)(8 \parallel 4)$

$= 8V$

$V' = \frac{4}{4+8} 6 = 2V$

Ex:-

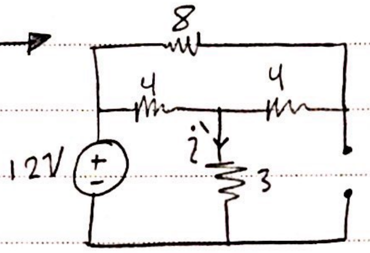


find i using the super position method:-

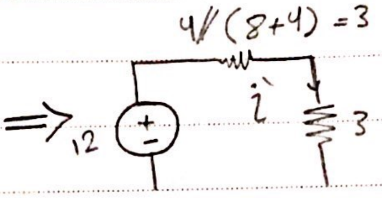
$$i = i' + i'' + i'''$$

Sol →

i' →



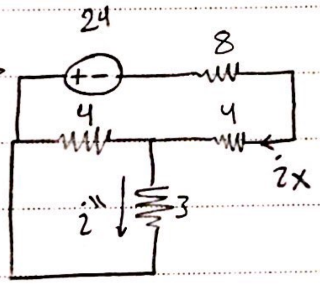
⇒



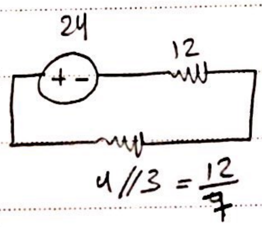
$$i' = \frac{12}{3+3} = 2A$$

open CKT بـ 3A , Short Circuit بـ 24 Voltage Sor لـ, بعمل #

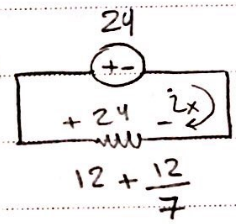
i'' →



⇒



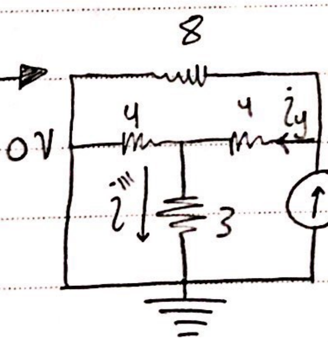
⇒



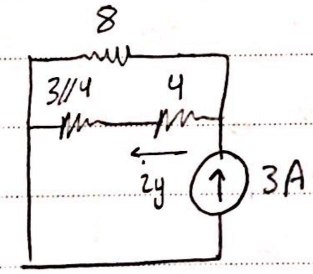
$$i_x = \frac{-24}{12 + \frac{12}{7}} = -1.75 \qquad i'' = \frac{4}{4+3}\, i_x$$

$$i'' = \frac{4}{7} * -1.75 = -1A$$

i''' →



⇒



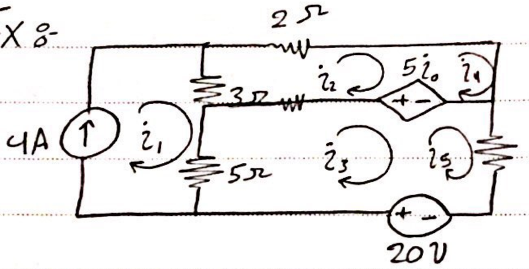
$$i_y = \frac{8}{4//3+8}$$

$$i''' = \frac{v}{4+3}\, i_y = 1A$$

$$i = i' + i'' + i'''$$

$$i = 2 + -1 + 1 = \boxed{2}$$

Ex :-



$*$ Note : we don't Kill the dependend Sources.

— find, using Superposition. i_o

$$i_o = i_o' + i_o''$$

i_o' :- mesh analysis

$$i_1 = 4$$

mesh ② $\quad 6i_2 + 4i_3 = 32 \rightarrow ①$

mesh ③ $\quad -i_2 + 5i_3 = 0 \rightarrow ②$

Solve :- ① & ② $\quad i_2 = 80/17$

$$i_3 = 16/17$$

$$i_o' = i_1 - i_3$$

$$52/17 \text{ A}$$

i_o'' mesh analysis

mesh ④

$$6i_4 + 4i_5 = 0 \rightarrow ①$$

mesh ⑤

$$-i_4 + 5i_5 = 20 \rightarrow ②$$

Solving ① & ②

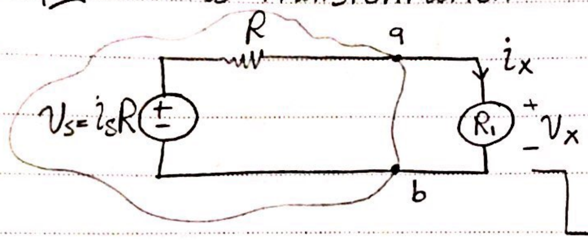
$$i_4 = -40/17$$

$$i_5 = 60/17$$

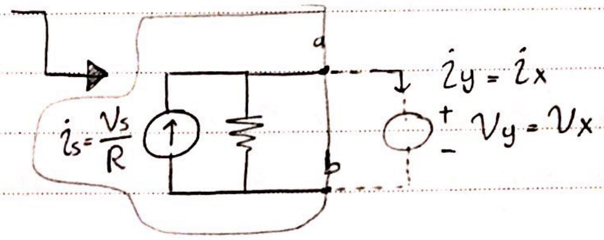
$$i'' = -5i_5 = \frac{-60}{17} \quad , \quad i = \frac{52}{17} + \frac{-60}{17} = \frac{-8}{17} \text{ A}$$

$$i_o = \frac{52}{17} - \frac{8}{17} = 2.58 \text{ A}$$

2 Source Transformation.



$i_s = \frac{V_s}{R}$ الشرط ان يكون



$$V_x = \frac{R_1}{R_1 + R} V_s$$

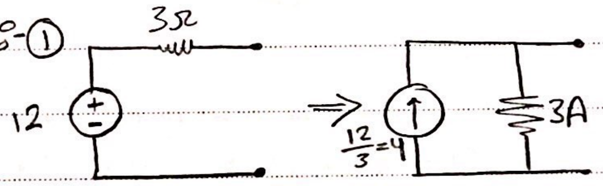
$$i_x = \frac{V_s}{R_1 + R}$$

$$i_y = \frac{R}{R + R_1} i_s$$

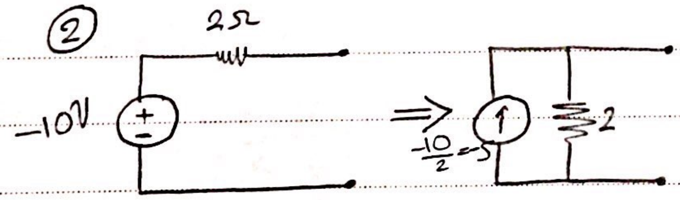
$$= \frac{R}{R_1 + R} \cdot \frac{V_s}{R} = \frac{V_s}{R + R_1} = i_x$$

$$V_y = i_y R_1 = \frac{V_s R_1}{R + R_1} = V_x$$

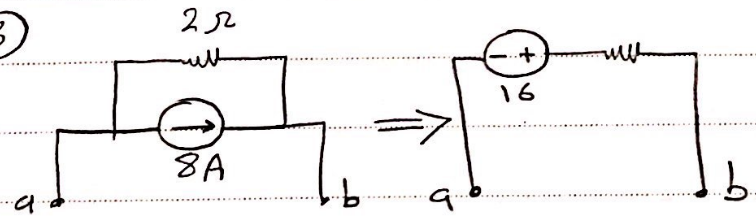
Ex:- ①



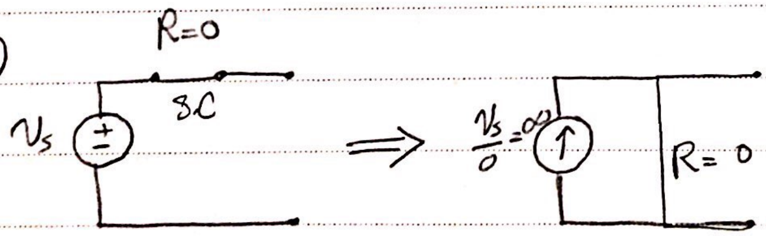
②



③

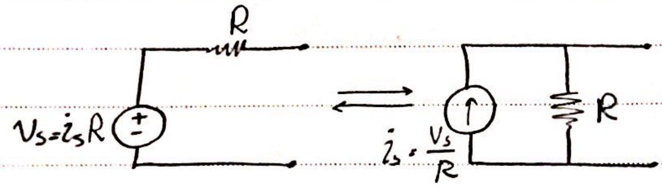


④



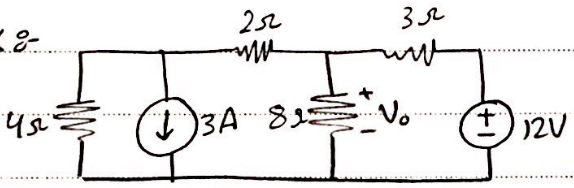
* لا يمكن استخدام التحويل اذا كانت R=0 او R=∞ .

Source Transformation :-



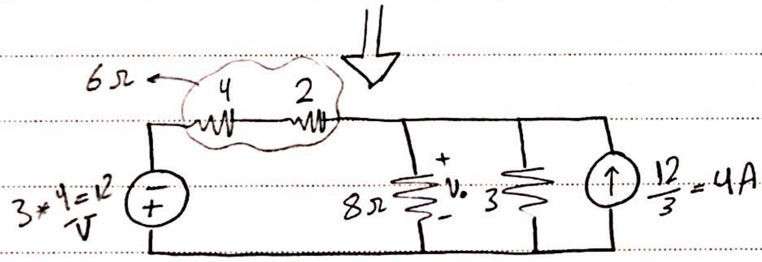
Source transformation helps in Simplifying :-
The ckt to be single -Node CKT OR Single - Loop.

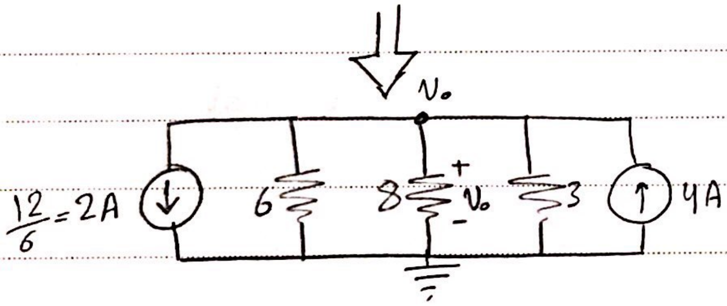
Ex :-



find V_o using source transformation

Sol →



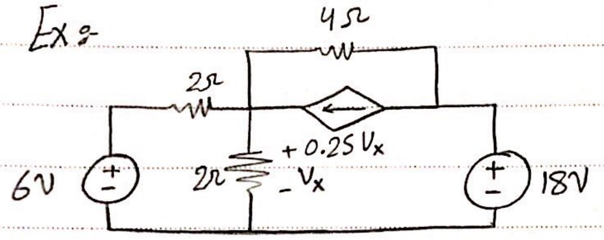


kcl at the node :-

$$-2 - \frac{V_o}{6} - \frac{V_o}{8} - \frac{V_o}{3} + 4 = 0$$

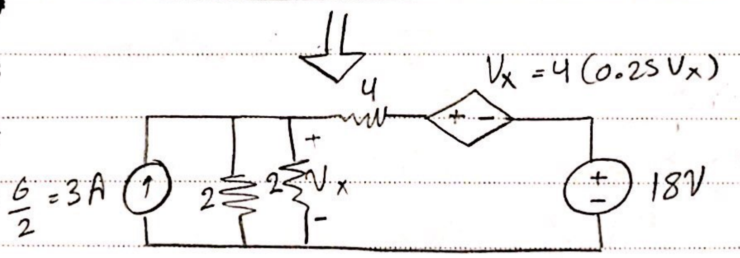
$$V_o = 3.2\ V$$

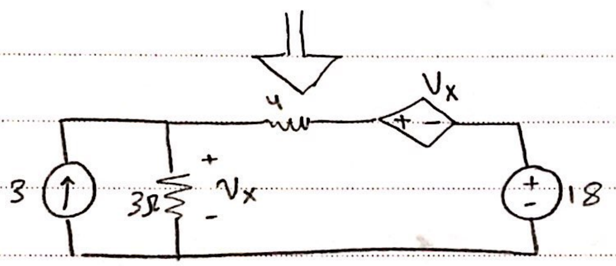
Ex:

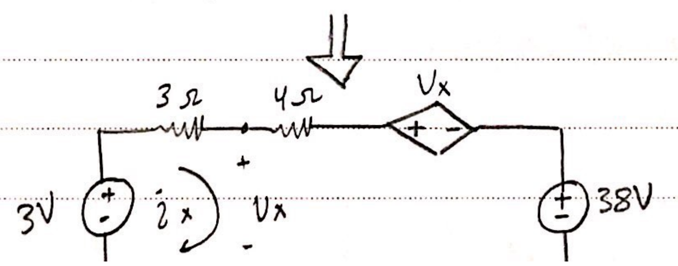


find V_x using source transformation.

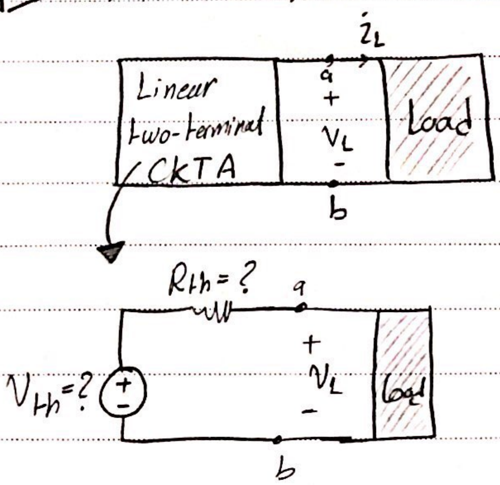
$V_x = 4(0.25 V_x)$





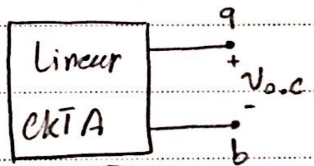


3 Thevenin Theorem :-



- How to find V_{th}?

ans :-



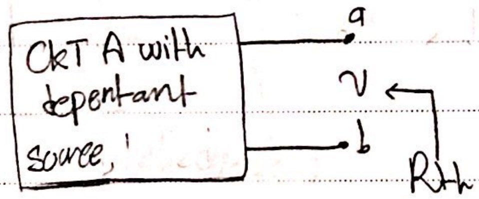
find $V_{o.c}$ using CkT analysis

$$V_{th} = V_{o.c}$$

- How to find R_{th}?

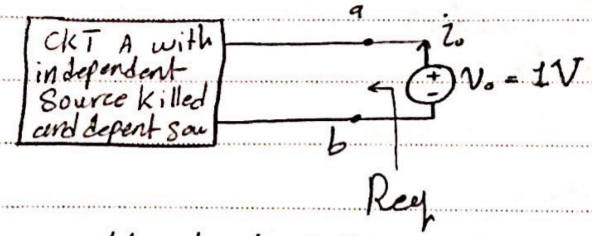
Case I :- CkT A has independant

Linear CkT A



$$R_{th} = R_{eq}$$

Case II :- CkT A has independant & dependent sources :-



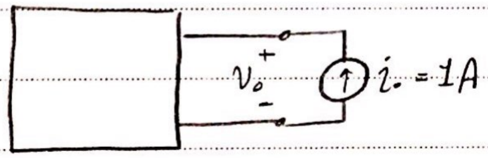
- add a test Voltage Source v_o (usually V)
using CkT analysis find ?

$$R_{th} = R_{eq} = \frac{v_o}{i_o}$$

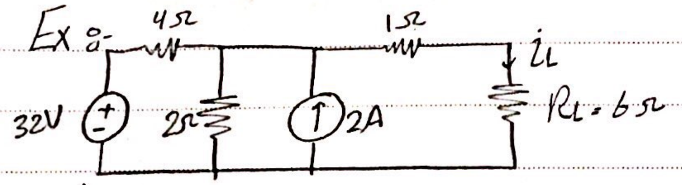
OR :- add test Current Source $i_o = 1A$

find v_o

$$R_{th} = \frac{v_o}{i_o}$$

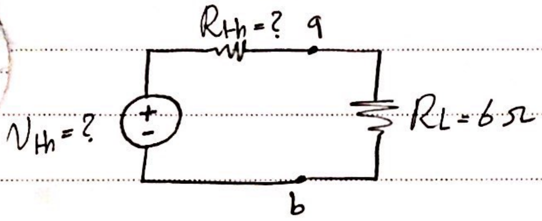


Ex :-

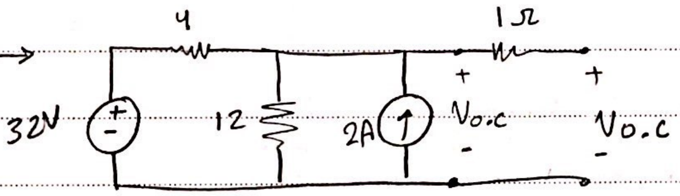


find the thevenin equivalent CKT seen by R_L
Then find i_L and power dissipated by R_L

Sol →



V_{th} →



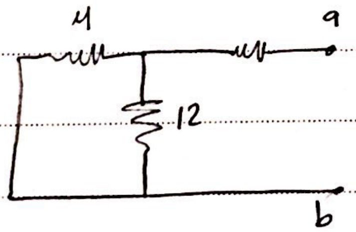
using nodal analysis :-
kcl at node ② →

$$\frac{V_{o.c} - 32}{4} + \frac{V_{o.c}}{12} = 2$$

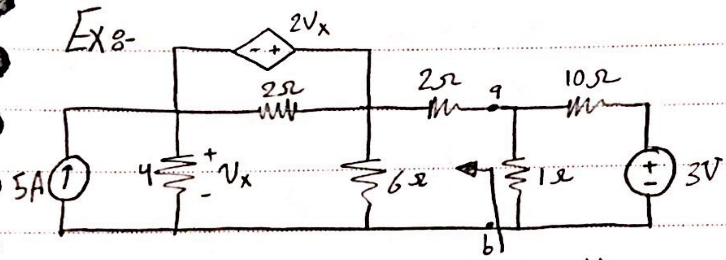
$V_{o.c} = 30\ V$

$V_{th} = 30V$

R_{th} → kill all independent sources in CKT :-

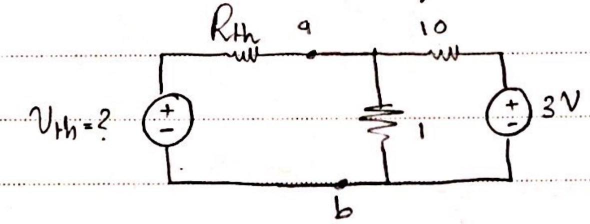


Ex:-

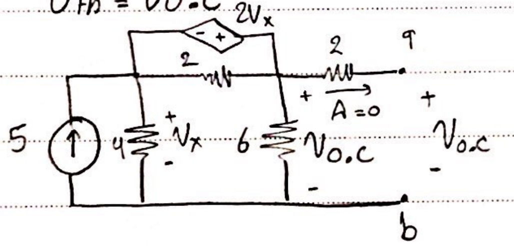


Determine the

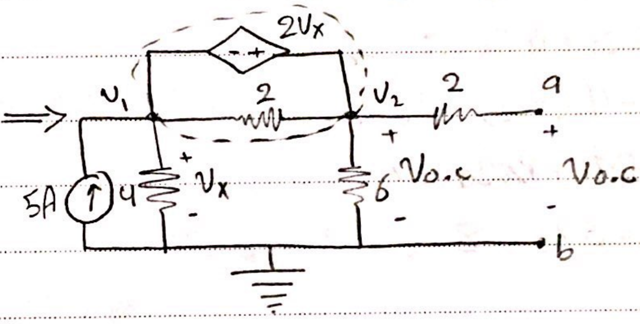
thevenin equivalant Ckt ?



$V_{th} = V_{o.c}$



$\Longrightarrow$



Kcl at the Supernode:-

$$5 = \frac{V_1}{4} + \frac{V_2}{6} \longrightarrow (1)$$

$$V_2 - V_1 = 2V_x \quad , \quad V_x = V_1 \longrightarrow (2)$$

$$-3V_1 + V_2 = 0 \longrightarrow (2)$$

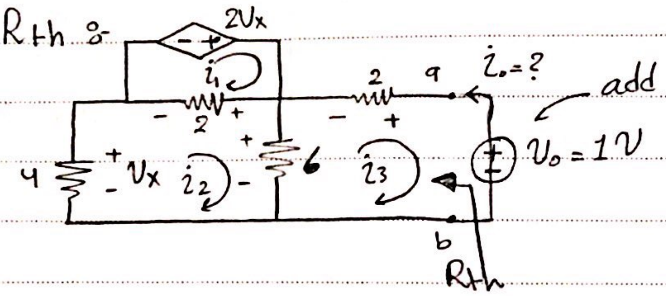
Solving (1) & (2)

$V_1 = 20/3\,V$

$V_2 = 20V$

$\Rightarrow V_{o.c} = V_2 = 20V$

find R_{th} :-



$$R_{th} = \frac{V_o}{i_o} = \frac{1}{i_o}$$?

find i_o using ckt analysis

use mesh analysis →

mesh ① → $2(i_1 - i_2) - 2V_x = 0$, $V_x = -4i_2$

$\Rightarrow 2(i_1 - i_2) - 2(-4i_2) = 0$

$2i_1 + 6i_2 = 0$ → ①

mesh ② → $-4(-i_2) - 2(i_1 - i_2) + 6(i_2 - i_3) = 0$

$-2i_1 + 12i_2 - 6i_3 = 0$ → ②

mesh ③ → $-6i_2 + 8i_3 = -1$ → ③

Solving ① & ② & ③ :-

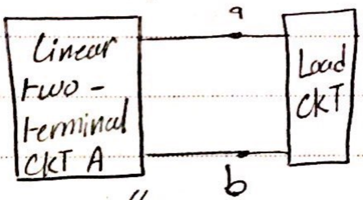
$i_1 = \frac{1}{6} A$, $i_2 = -1/18 A$, $i_3 = -1/6 A$

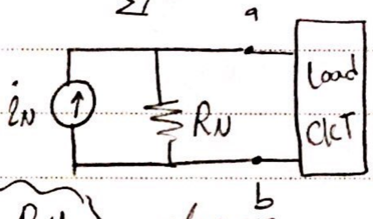
$i_o = -i_3 = 1/6 A$

$$R_{th} = \frac{V_o}{i_o} = \frac{1}{1/6} = 6$$

Scanned with CamScanner

4 Norton Equivalent CKT :-

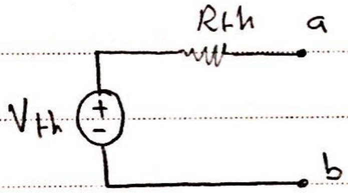




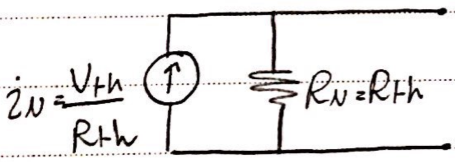
$R_N = R_{th}$ → always.

Method ① :-

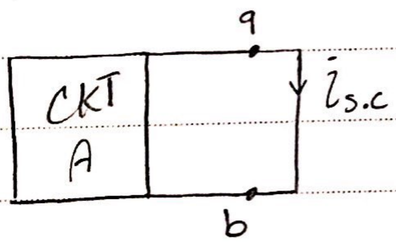
Find the Thevenien CKT :-



Then by source Transformation →



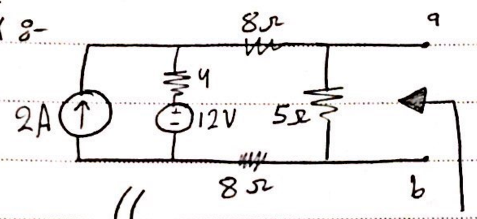
Method ② :-

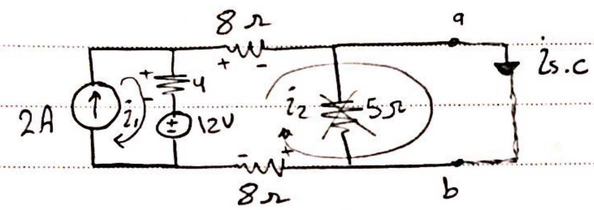


$i_N = i_{s.c}$

$R_N \equiv R_{th}$ ← same steps we follow to find R_{th}.

Ex :-





sol → ($i_N = i_{s.c}$)

using mesh analysis :-

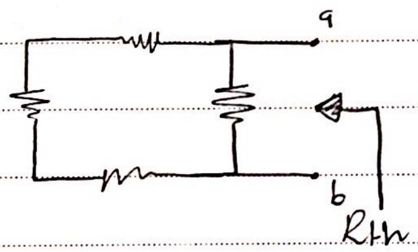
$i_1 = 2A$ $-12 - 4(2 - i_2) + 8 i_2 + 8 i_2 = 0$

$20 i_2 = 20$

$i_2 = 1$

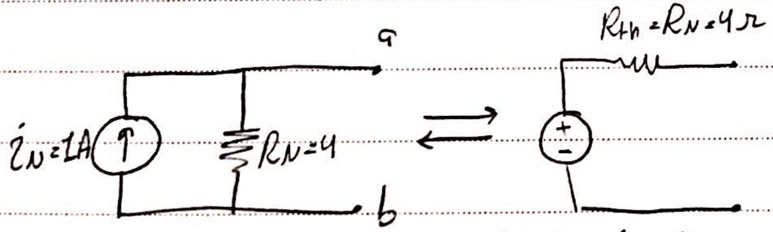
$i_{s.c} = i_2 = 1A$

$R_N = R_{th}$



$= (8 + 4 + 8) \,//\, 5$

$= 4\Omega$

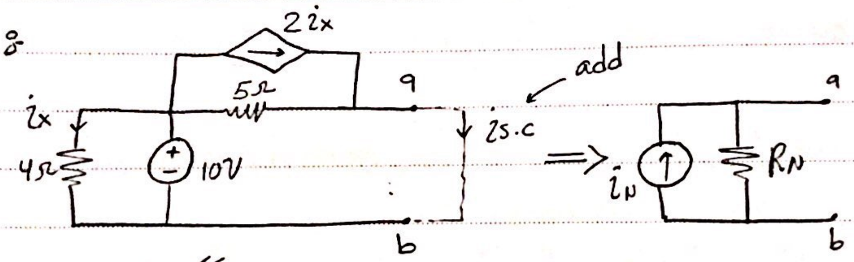


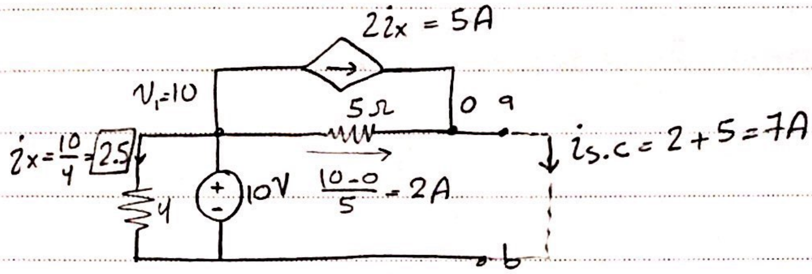
$V_{th} = i_N R_N$

$= 4V$

Scanned with CamScanner

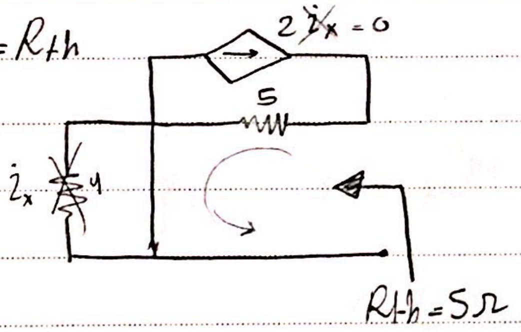
Ex :



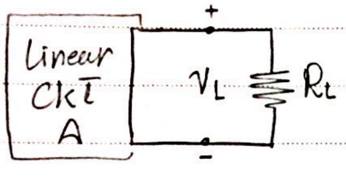


Sol $\rightarrow$ $i_{s.c} = i_N = \boxed{7A}$

$R_N = R_{th}$



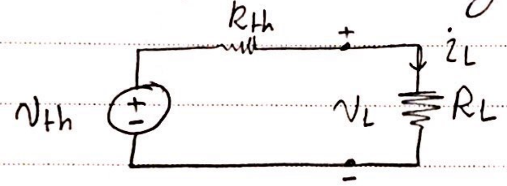
* Maximum Power Transfer :-



What is the value of R_L such that it will absorb the maximum Power :-

$P_L = i_L^2 R_L$ find R_L when $P_L = P_L^{max}$

Replace CKT A by its thevenin equivalent :-



$$P_L = i_L^2 R_L = \left(\frac{v_{th}}{R_{th}+R_L}\right)^2 R_L$$

To find R_L that maximum P_L :

$$\frac{\partial P_L}{\partial R_L} = \frac{(R_{th}+R_L)^2 v_{th} - v_{th}^2 R_L 2 (R_{th}+R_L)}{(R_{th}+R_L)^4} = 0$$

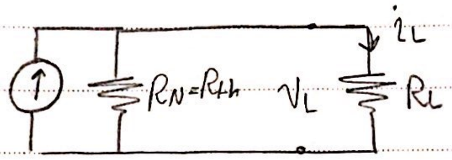
$$(R_{th}+R_L)^2 v_{th}^2 - v_{th}^2 R_L 2 (R_{th}+R_L) = 0$$

$$(R_{th}+R_L) - 2R_L = 0$$

$$R_{th} - R_L = 0 \longrightarrow R_L = R_{th}$$

$$P_L^{max} = \frac{v_{th}^2 R_{th}}{(R_{th}+R_{max})} \Longrightarrow P_L^{max} = \frac{v_{th}^2}{4R_{th}}$$

if we replace CKT A by its Norton equivelant :

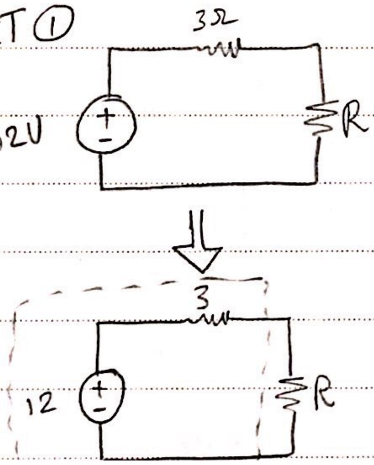


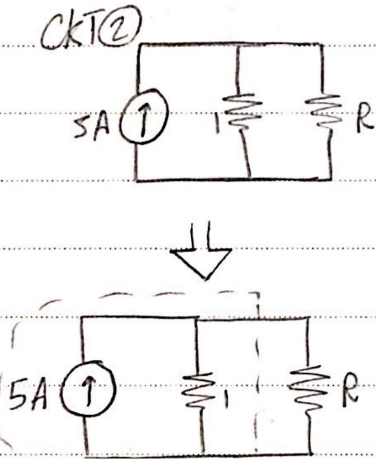
① $P_L = i_L^2 R_L = \left(\dfrac{R_N \, i_N}{R_N + R_N} \right) R_L$

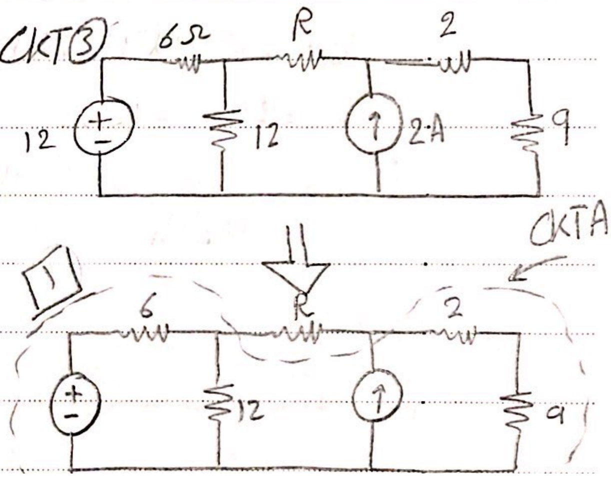
$$\frac{dP_L}{dR_L} = \ldots = 0 \longrightarrow R_L = R_N = R_{th}$$

$$P_L^{max} = \frac{i_N^2 \, R_N}{4}$$

Ex:- for each of following CkTs determine the value of R such that it will absorb the maximum Power and then find that max power:







$V_{th} = V_{o.c} = V_a - V_b$

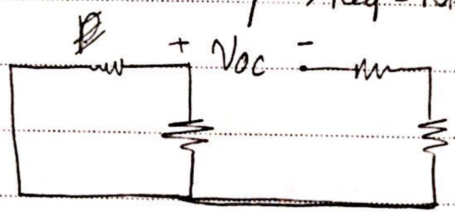
$V_a = \frac{12}{12+6}\, 12 = 8V$

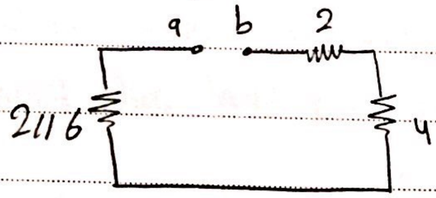
$V_b = 2(2+9) = 22V$

$V_{th} = 8 - 22 = -14V$

$\rightarrow R_{eq} = R_{th}$

$\boxed{2}$



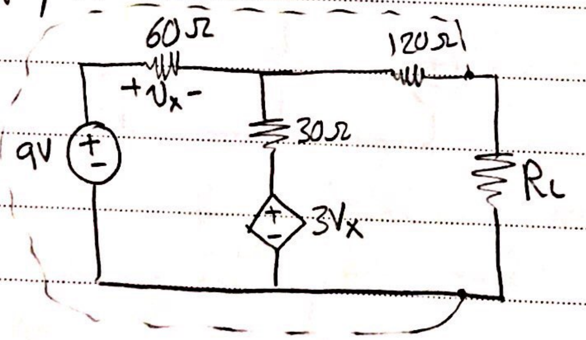


$R_{th} = 4+2+9 = 15\Omega$

$R = R_{th} = 15\Omega$

$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{(-14)^2}{(4)(5)}$

* practice :-



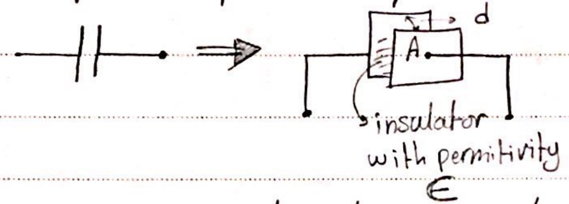
find R_L to absorb the max Power.

* CH6 : Capacitors and Inductors.

passive elements $\begin{cases} \rightarrow \text{resistor} \\ \rightarrow \text{Capacitors} \\ \rightarrow \text{inductors.} \end{cases}$ $\Big\}$ Store energy.

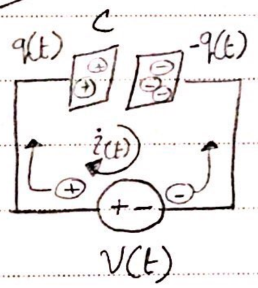
[1] Capacitors :-

- passive element stores energy as electric field. (charges)
- Two-parallel plates Capacitor.



The Capacitor has Capacitance. C

$$C = \epsilon \frac{A}{d}$$

\# 

V↑ , q↑

V↓ , q↓

$q(t) \propto V(t)$

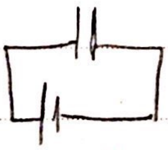
$$q(t) = C\ V(t)$$

amont of charge standred at C.

Voltage difference across C

* If $V(t) = V$ (DC Voltage)

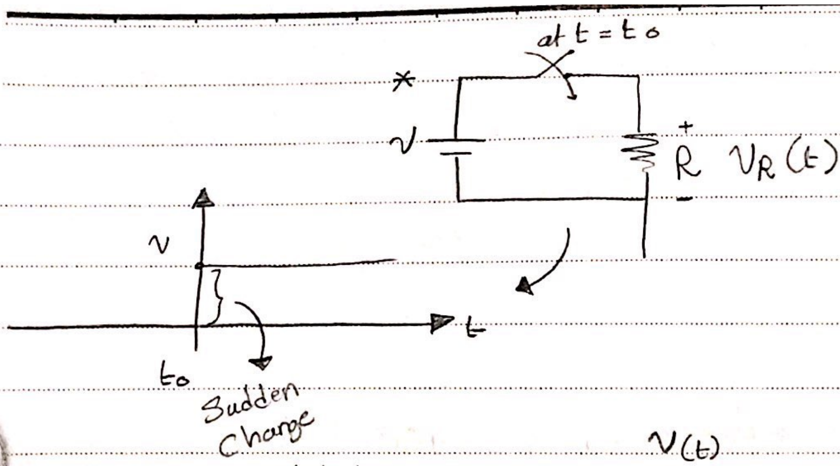
$q(t) = Q = CV$

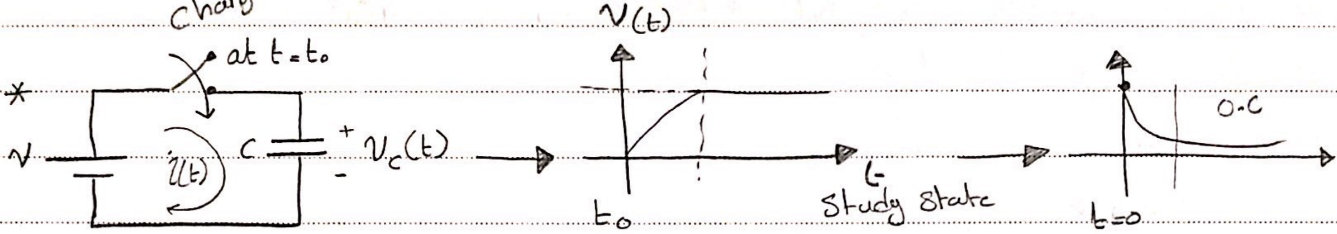


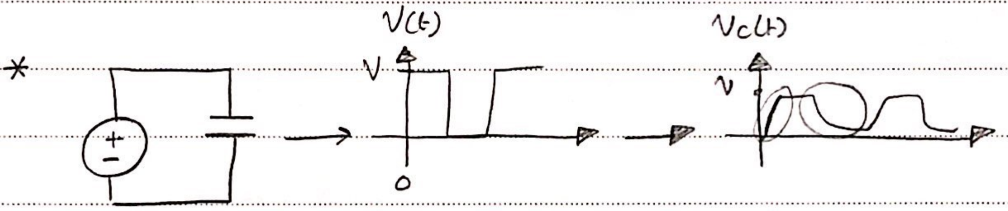
$i = 0A$

—||— $\triangleq$ O.C

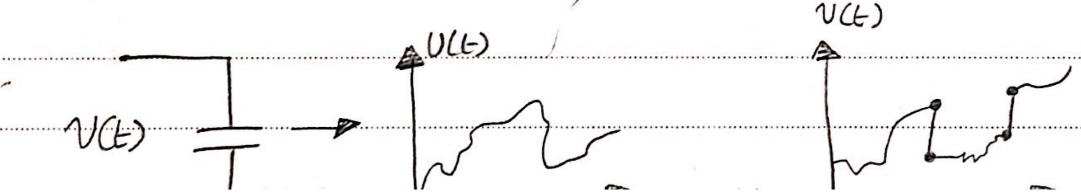
at the Steady- $\Big\{$ Note: The Capacitor is always O.C in DC-CKTs ← DC Voltage Source with $V(t) = V$







Note :- The capacitor resists the Sudden charge of V.
$V_c(t)$:- Contineous function of t.



* Voltage - Current relationship in Capacitors :-

$$q(t) = C\,v(t)$$

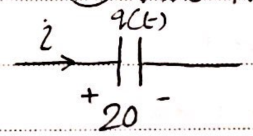
$$\frac{dq}{dt} = C\,\frac{dv(t)}{dt}$$

$$\Rightarrow i(t) = C\,\frac{dv(t)}{dt}$$

$v(t)$, $i(t)$ → Both have Same sign (charging), opposite sign (discharging)

Ex :- ⓐ Calculate the charge stored on a 3 pF capacitor with 20V across it. ⓑ find the current.

Sol →



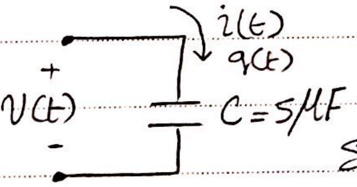
ⓐ $q(t) = C\,v(t)$

$= (3\ pF)(20\ V)$

$= 60\ pC = Q$

ⓑ $i = 0$ (in Steady State)

Ex :-



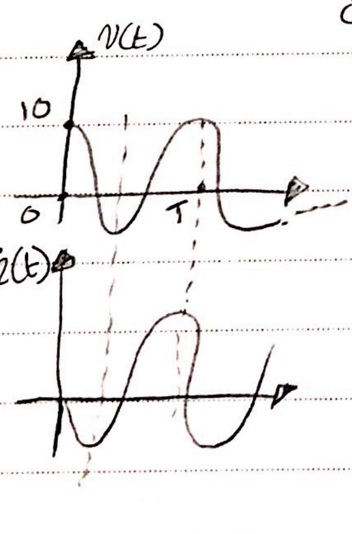
$v(t) = 10 \cos(6000\,t)\ V$

find $q(t)$ & $i(t)$

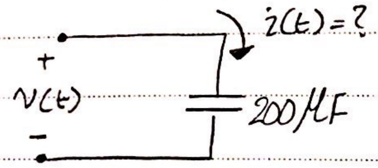
Sol → $q(t) = C\,v(t) = 50 \cos(6000\,t) = \mu C$

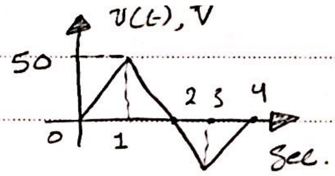
$i(t) = C\,\frac{dv}{dt} = -(5 * 10^{-6})(10)(6000) \sin(6000)\,A$

$= -0.3 \sin(6000\,t)\ A$



Ex:-

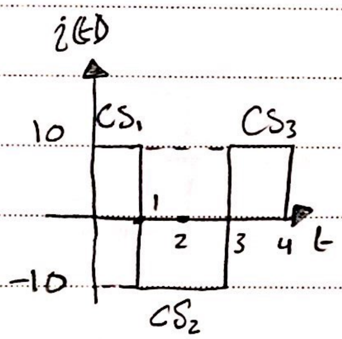




Sol:- $i(t) = C \dfrac{dv(t)}{dt}$

$$v(t) = \begin{cases} 50t, & 0<t<1 \\ 100-50t, & 1<t<3 \\ -200+50t, & 3>t<4 \end{cases}$$

$$i(t) = C\dfrac{dv(t)}{dt} = \begin{cases} (200*10^{-6})(50) = 10\,mA, & 0<t<1 \\ (200*10^{-6})(-50) = -10\,mA, & 1<t<3 \\ 10\,mA, & 3<t<4 \end{cases}$$



* Find $v(t)$ from $i(t)$:-

$$i(t) = C\frac{dv}{dt}\, t \Rightarrow \frac{1}{C}\int_{t_0}^{t} \frac{1}{C}\, i(t)\, dt = \int_{v(t_0)}^{v(t)} dV(t)$$

$$\Rightarrow v(t) - v(t_0) = \frac{1}{C}\int_{t_0}^{t} i(t)\, dt$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\, dt + v(t_0) \quad (*)$$

t_0 :- intial time

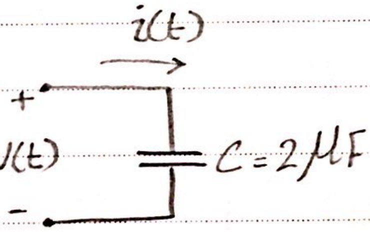
$v(t_0)$ = Capacitor Voltage at $t = t_0$ } always given

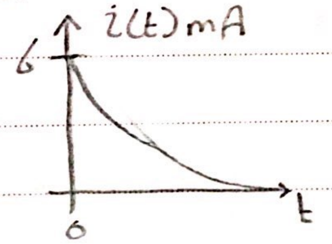
$$v(t_0) = \frac{q(t_0)}{C} \rightarrow \text{intial stored charge}$$

Ex:- Let $i(t) = 6e^{-300t}$

Find $v(t) = ?!$

Such that $v(0) = 0\ V$





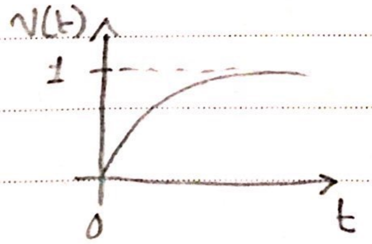
using (*)

$$v(t) = \frac{1}{2*10^{-6}}\int_{0}^{t} 6e^{-300t} * 10^{-3}\, dt + 0$$

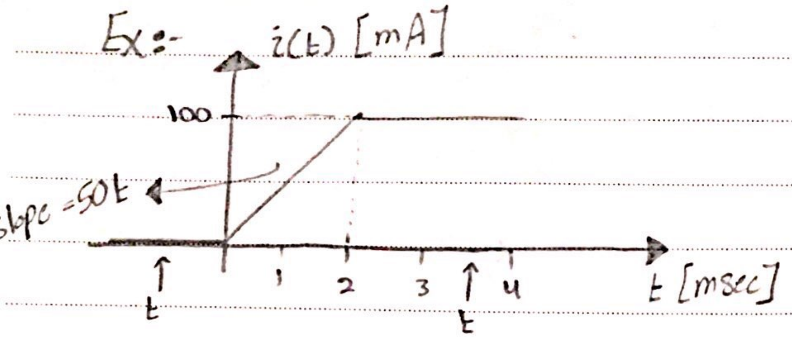
$$\Rightarrow \frac{\cancel{10^{-3}}}{\cancel{2}*10^{-6}}\left(\frac{\cancel{6}e^{-3000t}\Big|_0^t}{-\cancel{3000}}\right)$$

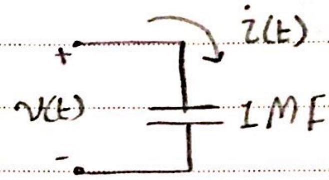
$$= -\left[e^{-3000t} - 1\right]$$

$$= 1 - e^{-3000t}$$



Ex:-





find $v(t)$? $\quad v(-\infty) = 0\ V$

Case I :- $t \leq 0$, $v(t) = \frac{1}{10^{-3}} \int_{-\infty}^{t} 0 \, dt + \cancel{v(-\infty)}^{0}$

$$v(t) = 0\,V$$

Case II :- $0 < t \leq 2$

$v(t) = \frac{1}{10^{-3}} \int_{0}^{t} 50t \, dt + v(0)$, $v(0) = 0 \rightarrow$ من الي قبلها

$$v(t) = \frac{1}{10^{-3}} \int_{0}^{t} 50t \, dt + \cancel{v(0)}^{0}$$

$$= 10^{+3} * \frac{50t^2}{2}\Big]_0^t = 25t^2 * 10^{+3}\ V$$

$$v(2) = 25 * (2*10^{-2})^2 * 10^{+3}$$

$$= 0.1\,V$$

Case III :- $2 < t$

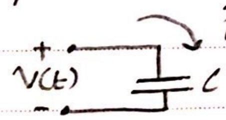
$$v(t) = \frac{1}{10^{-3}} \int_{2\,m\,Sec}^{t} 100 * 10^{-3} \, dt + v(2\,Sec)$$

$$= \frac{1}{10^{-3}} * 100 * 10^{-3} (t - 2*10^{-3}) + 0.1\,V$$

$$v(t) = \begin{cases} 0 , & t \leq 0 \\ 25t^2 * 10^{-3} , & 0 < t \leq 2 \\ 100(t - 0.02) + 0.1 , & 2 < t \end{cases} \longrightarrow q(3) = 10^{-3}(100(3*10^{-3} - 0.02) +$$

* Energy Stored in the Capacitor :-

$P(t) = V(t)\, i(t)$



$w(t) = ?$

$q(t) = V(t)C$

$P(t) = \frac{dw(t)}{dt}$

$w(t) = \int_{t_0}^{t} P(t)\, dt + w(t_0)$ → intial Stored energy

$= \int_{t_0}^{t} V(t)\, i(t)\, dt + w(t_0)$

$= \int_{V(t_0)}^{V(t)} C V'(t) \frac{dv(t)}{dt}\, dt + w(t_0)$

$\Longrightarrow w(t) = \int_{V(t_0)}^{V(t)} C V' \, dV' + w(t_0)$

$w(t) = \left(C \frac{V'^2}{2} \Big]_{V(t_0)}^{V(t)} \right) + w(t_0)$

$w(t) = \frac{1}{2} C V^2(t) - \frac{1}{2} C V^2(t_0) + w(t_0)$

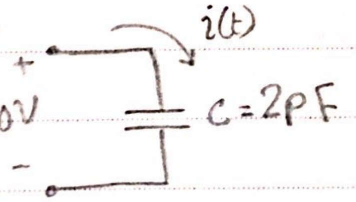
$w(t) = \frac{1}{2} C V^2(t)$ "J"

* $P(t) = \frac{dw(t)}{dt}$ "Watt"

$q(t) = C\, V(t) \Longrightarrow V(t) = \frac{q(t)}{C}$

$w(t) = \frac{q^2(t)}{2C} \Longrightarrow *$

Ex :-



① $q(t) = CV(t) = 20 * 3pF = 60\ pC$

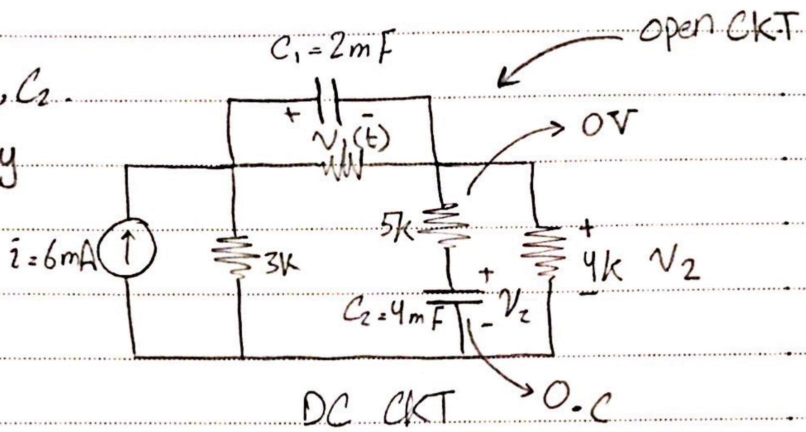
② $i(t) = C \frac{dv}{dt} = 0A$ (—||— $\triangleq$ OC)

③ $w(t) = \frac{1}{2} C V^2(t) = \frac{1}{2} (20)^2 (3pF) = 600\, P$

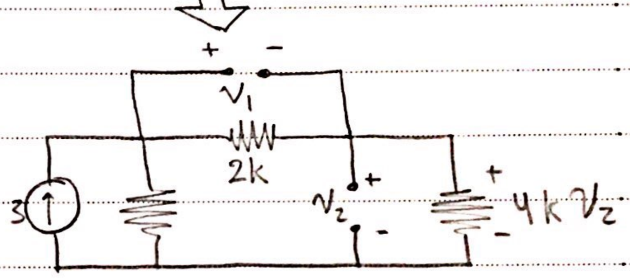
④ $p(t) = \frac{dw}{dt} = 0$ Watt

* Ex:-

1. find the energy storead in C_1, C_2.
2. find the energy dissapated by the $3k\Omega$ resistor.



$$i_x = \frac{3k}{3k+6k} * 6mA$$

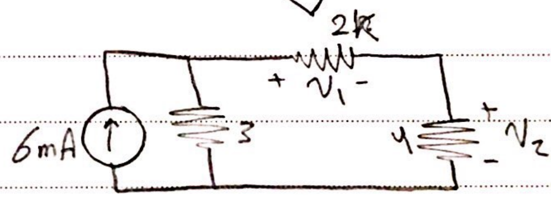


$i_x = 2mA$

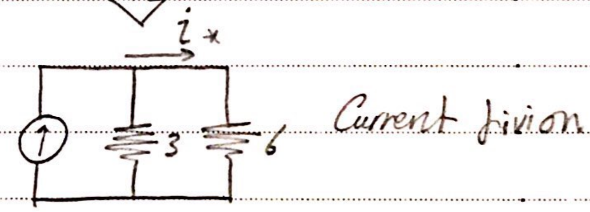
$V_1 = 2m\,(2k\Omega) = 4V$

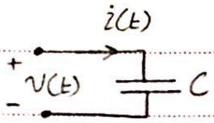
$V_2 = 2Am\,(4k\Omega) = 8V$

$w_1(t) = \frac{1}{2} * 2Fm * 4^2 = 16\,mJ$



$w_2(t) = \frac{1}{2} C_2 V_2^2 = \frac{1}{2} * 4mF\, 8^2 =$
128 mJ.





$q(t) = C\,v(t)$

$i(t) = C\,\dfrac{dv(t)}{\partial t}$

DC - Ckt :- $i(t) = 0$

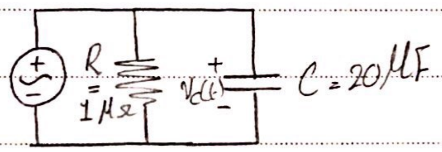
$—||— \trianglerighteq$ O.C

(steady state)

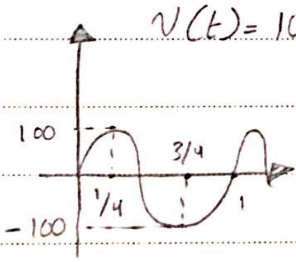
$w_C(t) = \frac{1}{2} C v^2(t)$

$= \dfrac{q^2(t)}{2C}$ "J"

* practice :-



$v(t) = 100 \sin(2\pi t)$ V

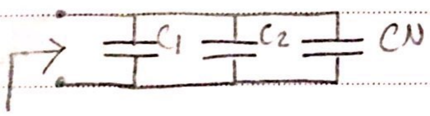


find ⓐ $W_c(t) = \ldots ?$ $(0.1 \sin^2(2\pi t)\ J)$

ⓑ $W_R(t) = \ldots ??$

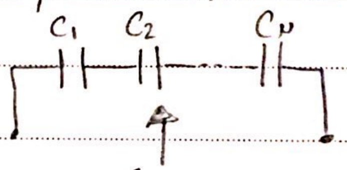
$P_R(t) = i^2 R = \dfrac{V^2}{R}$

- Capacitors in parallel :-



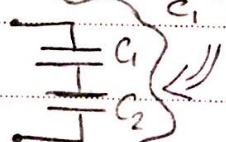
$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$.

- Capacitors in Series :-

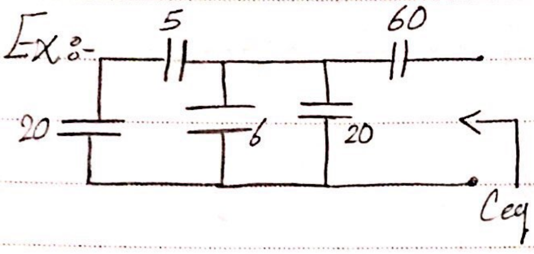


$C_{eq} = \dfrac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_N}}$

$C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}$ ⟶



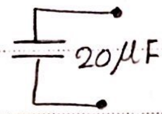
Ex :-



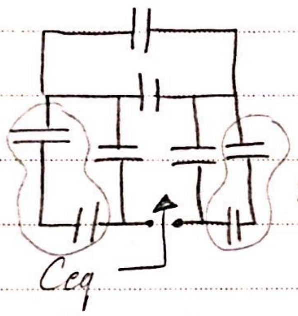
Sol → $\frac{(5)(20)}{5+20} = 4\mu F$

$4\mu F + 6\mu F + 20\mu F = 30\mu F$

$\frac{(30)\mu\,(60)\mu}{30\mu + 60\mu} = 20\mu F$ →

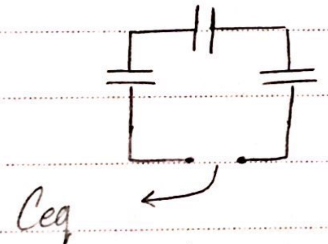


Ex :-



all Capacitores are 4 µF

Sol → $C_{eq} = 2.1818\ \mu F$



$C_{eq} = \frac{1}{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}} = 2.1818$

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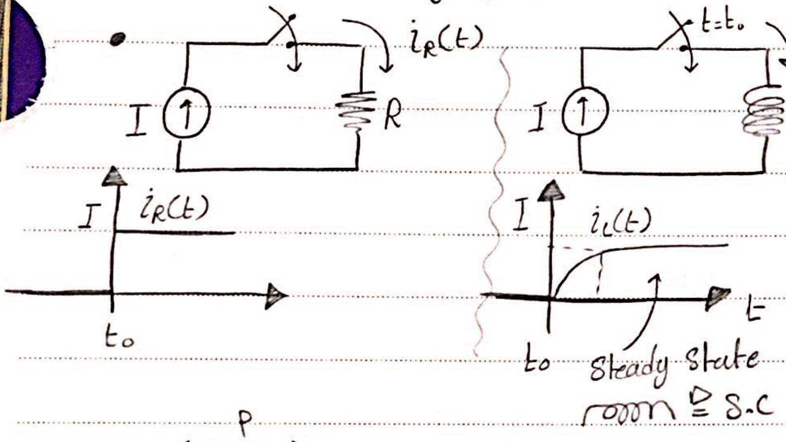
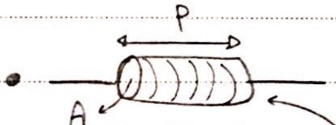
2 Inductors

Symbol →

- passive element.
- Store energy as magnetic field.
- The inductor has "inductance" L with unit "Henry" (H)
- 

- 

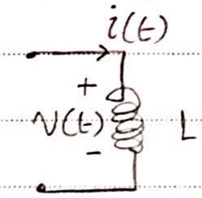
Core material with "Permeability μ"

$$L = \frac{N^2 \mu A}{l} \quad [H]$$

m

- Current - Voltage - relationship.

$$v(t) = L \frac{di(t)}{dt}$$



if $i(t) = I$ (DC CKT)

$v(t) = 0\,V$, i.e ≘ S.C in DC

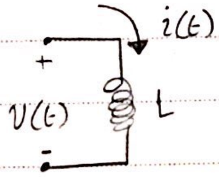
$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\, d(t) + i(t_0)$$

t_0 : inital time.

$i(t_0)$:- intial Current.

- energy stored in the inductor :-

$$w(t) = \frac{1}{2} L\, i^2(t)$$

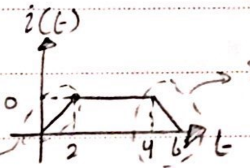


prof :- $w(t) = \int_{t_0}^{t} P(t)\, dt + w(t_0)$

$$= \int_{t_0}^{t} L \frac{di(t)}{\cancel{dt}} (i)\, \cancel{dt}$$

$$= \int_{2(t_0)}^{2}$$

Ex :-



slope = $-5 * 10^3$

[m Sec]

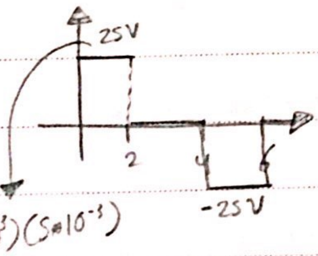
slope = $5 * 10^3$

find $v(1\,m\,Sec) = ? \Rightarrow 25V$

$v(3\,m\,Sec) = ? \Rightarrow 0$

$v(5\,m\,Sec) = ? \Rightarrow -25V$

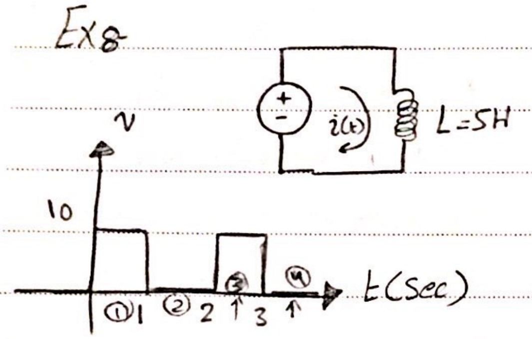
Sol :- $V(t) = L \frac{di}{dt}$



$(5*10^3)(5*10^{-3})$

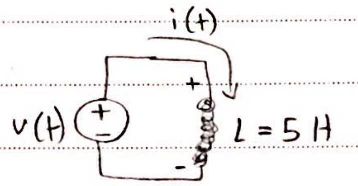
$= 25V$

Ex8



find $i(t)$?!

$i(0) = -1$ A



sol.

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$

$$0 < t \leq 1 \ ; \ i(t) = \frac{1}{5}\int_{0}^{t} 10\,dt + -1 \Rightarrow \frac{1}{5}(10t) - 1 \Rightarrow 2t - 1 \ \text{A}$$

$$1 \leq t \leq 2 : i(t) = \cancel{\frac{1}{5}\int_{1}^{t_0} 0\,dt} + i(1)$$

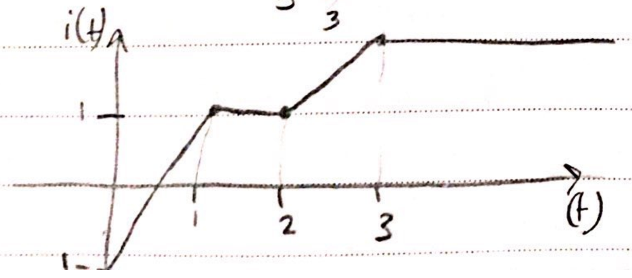
$$\Rightarrow i(1) = (2)(1) - 1 = 1 \text{A}$$

$$2 < t \leq 3 : i(t) = \frac{1}{5}\int_{2}^{t} 10\,dt + i(2)$$

$$= 2(t-2) + 1$$

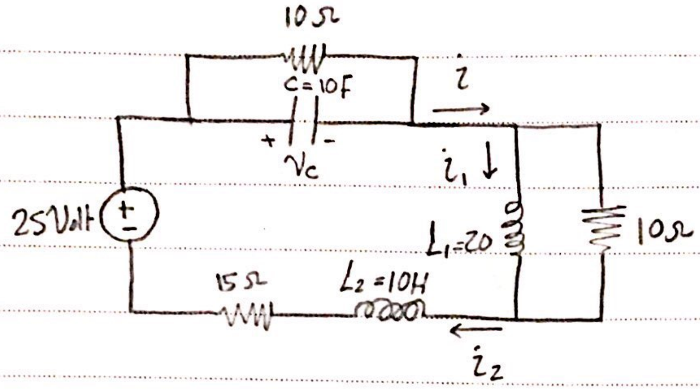
$$= 2t - 3 \ \text{A}$$

$$3 < t : i(t) = \cancel{\frac{1}{5}\int_{3}^{t} 0\,dt} + i(3) = i(3) = (2)(3) - 3 = 3 \ \text{A}$$



Scanned with CamScanner

Ex :-



Find the W_L, W_{L_1}, W_{L_2} under the steady state Condition.

Sol :-

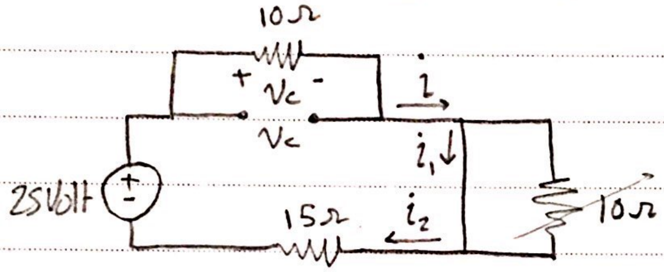
DC - CKT → ⊣⊢ = O.C

ᴗᴗᴗᴗ = S.C

$W_C = \frac{1}{2} C V^2$

$W_{L_1} = \frac{1}{2} L_1 i_1^2$

$W_{L_2} = \frac{1}{2} L_2 i_2^2$



$$V_c = \frac{10}{10+15} * 25 = 10\,\text{Volt}$$

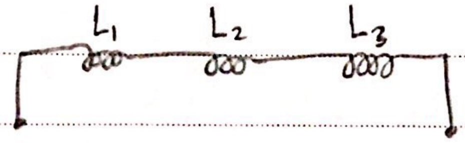
$$i_1 = i_2 = \frac{25}{10+15} = 1\,A$$

$$W_L = \frac{1}{2}(10)(10)^2 = 500\,J$$

$$W_{L_1} = \frac{1}{2}(20)(1)^2 = 10\,J$$

$$W_{L_2} = \frac{1}{2}(10)(1)^2 = 5\,J$$

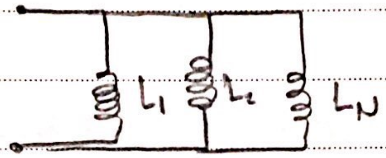
* Series Induction :-



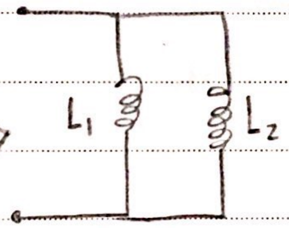
$$L_{eq} = L_1 + L_2 + \cdots + L_N$$

* parallel Induction

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}}$$

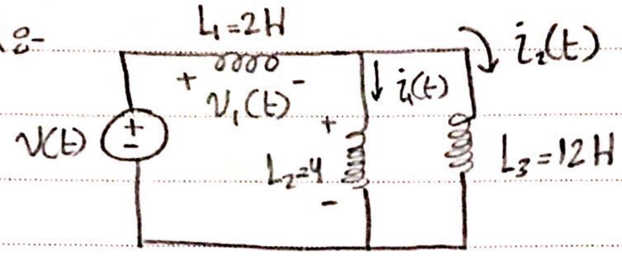


*



$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

* Ex:-



Let $i(t) = 8 - 4e^{-10t}$ mA

$i_2(0) = -1$ mA

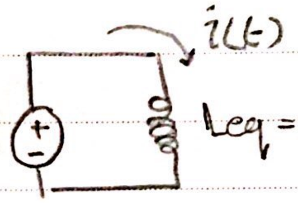
find $i_1(0)$, $v(t)$, $v_1(t)$, $v_2(t)$, $i_1(t)$, $i_2(t)$

[a] $i(t) = i_1(t) + i_2(t)$

$i(0) = i_1(0) + i_2(0)$

$4 - (-1) = 5$ mA

[b]



$L_{eq} = L_2 // L_3 + L_1$

$= 5H$

$v(t) = L_{eq} \dfrac{di(t)}{dt} = 5(-4)(-10)$

$= 200e^{-10t}$ mV.

$v_1(t) = L_1 \dfrac{di(t)}{dt} = 2(-4)(-10)e^{-10t} = 80e^{-10t}$ mV

$v_2(t) = v(t) - v_1(t) = 120e^{-10t}$ mV

$i_1(t) = \dfrac{1}{L_2} \int_0^t v_2(t)\, dt + i_1(0) = \dfrac{1}{4} \int_0^t 120^{-10t}\, dt + 5$ mA

$30 \left(\dfrac{e^{-10t}}{-10} \right]_0^t + 5 \rightarrow 8 - 3e^{-10t}$ mA

$i_2(t) = i(t) - i_1(t) = 8 - 4e^{-10t} - 8 + 3e^{-10t} = -e^{-10t}$ mA

CH7 :- 1st order CKT's
(RC and RL CKTs)

Excitation → [RC or RL CKT] → Response.

Two Types of Excitation

1. Intial stored energy
(Source - free CKTs)
"intial Conditions"

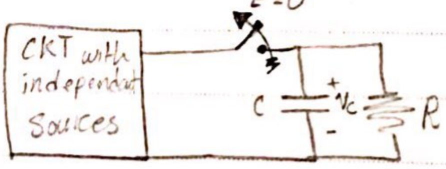
Intial energy → [Source - free RC or RL CKT] → Natural Responde

no independunt Sources.

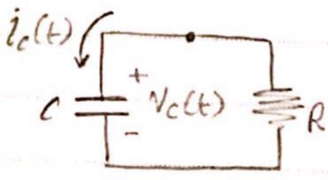
2. Driven CKTs :-

Indepent Sources → [RC or RL CK] → Forced response

(a) Source - Free RC CKT



at $t > 0$



$\Rightarrow$ $\left.\begin{array}{l} V_c(0) = V_o \\ W_c(0) = \frac{1}{2} C V_o^2 \end{array}\right\}$ intial Conditions obtain form $t < 0$ CKT

* find $V_c(t)$, $t>0$ Natural response

$$i_c(t) + i_R(t) = 0 \quad \circledast$$

$$C\,\frac{V_c(t)}{dt} + \frac{1}{R} V_c(t) = 0$$ ← 1st order Differ. equations.

$\Longrightarrow$ proposed Solution :- $V_c(t) = Ae^{Bt}$ $\circledast\circledast$

find A & B

$$V_c(0) = Ae^{B(0)} = V_0$$

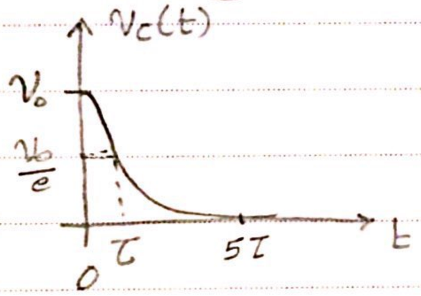
$$\Longrightarrow \boxed{A = V_0}$$

Sub $\circledast\circledast$ into $\circledast$:-

$$C\left[ABe^{Bt}\right] + \frac{1}{R}Ae^{Bt} = 0$$

$$\Longrightarrow CB + \frac{1}{R} = 0 \Longrightarrow B = -1/RC$$

$$\Longrightarrow V_c(t) = V_0 e^{-t/RC} \quad t>0$$



The ⊣⊢ is dischaging

$V_c(\infty) = 0$, $W_c(\infty) = 0$ J

- Time Constant τ :-

$$V_c(\tau) = \frac{V_0}{e}$$

$$V_0 e^{-\tau/RC} = V_0 e^{-1}$$

$$\rightarrow \tau/RC = 1 \rightarrow \boxed{\tau = RC}$$

$V_c(5\tau) \simeq 0\,V$

$W_c(5\tau) \simeq 0$ J

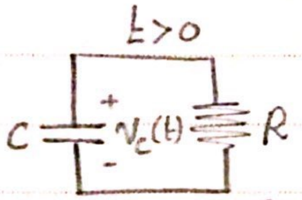
. Energy Dissipated by R:- $w_R(t)$, $t>0$

$$P_R(t) = v_R(t)\, i_R(t)$$
$$= \frac{v_R^2(t)}{R} = \frac{v_C^2(t)}{R} = \frac{V_0^2}{R} e^{-2t/RC}$$

$$w(t) = \int_0^t P_R(\lambda)\, d\lambda + \overset{0}{\cancel{w_R(0)}}$$

$$\int_0^t \frac{V_0^2}{R} \cdot e^{-2\lambda/RC}\, d\lambda = \frac{V_0^2}{R} \frac{e^{-2\lambda/RC}}{-2/RC}\Bigg]_0^t = \frac{-1}{2} V_0^2 \left[e^{-2t/RC} - 1\right]$$

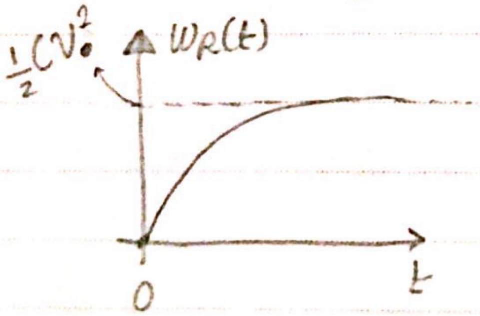
$$= \frac{1}{2} V_0^2 \left[1 - e^{-2t/RC}\right] \text{J},\ t>0$$

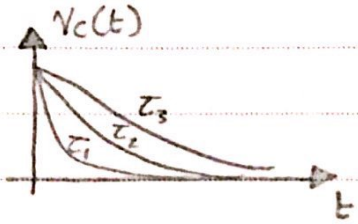


$$= -1/2\, C V_0^2 \left[e^{-2t/RC} - 1\right]$$

$$= 1/2\, C V_0^2 \left[1 - e^{-2t/RC}\right] \text{J},\ t>0$$

$$w_R(0) = 0 \text{ J}$$
$$w_R(\infty) = \boxed{1/2\, C V_0^2}$$

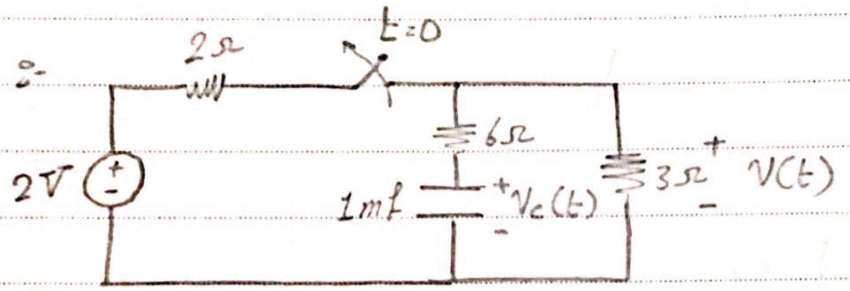




$\tau_1 < \tau_2 < \tau_3$

$\tau = RC$

Ex :-



Find (a) $W_c(0)$

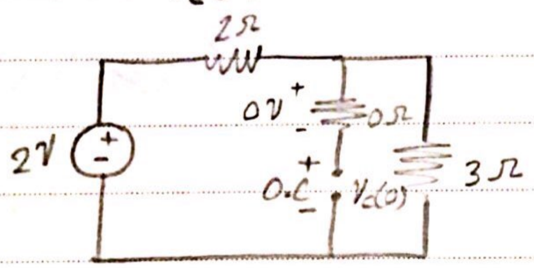
(b) $V_c(t)$, $t>0$

(c) $V(t)$, $t>0$

Sol →

(a) $W_c(0) = \frac{1}{2} C V_c^2(0)$

find $V_c(0)$ from $t<0$ CKT :-

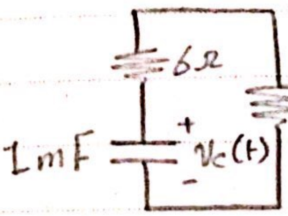


$V_c(0) = \frac{3}{3+2}(2V) = 1.2V$

$W_c(0) = (1/2)(1 * 10^{-5})(1.2)^2$

$= 0.72 \ mJ$

(b) $t>0$



$$V_c(t) = V_c(0)\, e^{-t/\tau}$$

$$V_c(0) = 1.2\ V$$

$$\tau = R_{eq}C = (3+6)(1\times10^{-3})$$

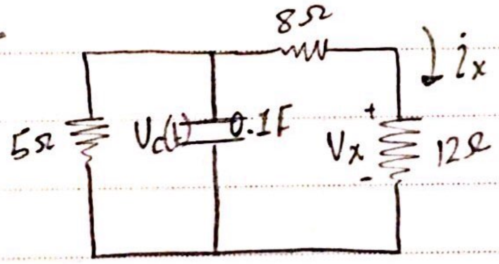
$$= 9\times10^{-3}\ sec$$

$$V_c(t) = 1.2\, e^{-\frac{1000}{9}t}\ ,\ t>0$$

(c) $V(t) = \frac{3}{3+6} V_c(t)$

$$= \frac{1.2}{3} e^{-\frac{1000}{9}t} \quad t>0$$

Ex:-



$V_c(0) = 15V$
find $V_x(t)$, $i_x(t)$, $w_{12}(t)$ $t>0$

$t>0$, $V_c(t) = V_c(0)\, e^{-t/RC}$

$V_c(t) = 15 * e^{-t/0.4}$, $t>0$

1) $V_x(t) = V_c(t) * \frac{12}{20}$

$$V_x(t) = 15 e^{-t/4} * \frac{12}{20} = 9 e^{-t/0.4}\ V\ ,\ t>0$$

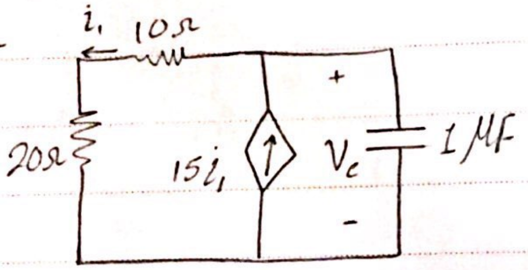
2) $i_x(t) = \frac{V_x(t)}{12} = \frac{9 e^{-t/0.4}}{12}$ V

3) $w_{12}(t) = \int_0^t \frac{V_x^2(\lambda)}{12} d\lambda$

?!

/ /

Ex:-



If $V_c(0) = 2V$

find $V_c(t)$, $t > 0$

Sol →

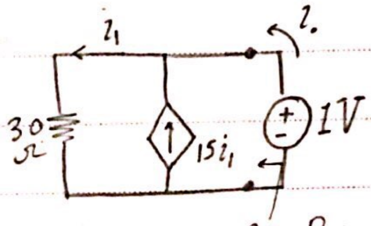
$$V_c(t) = V_c(0)\, e^{-t/\tau}$$
$$= 2\, e^{-t/\tau}\ V$$

$\tau = R_{eq} C$

??

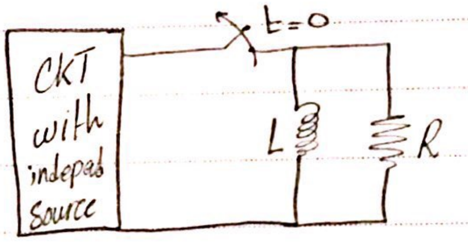
$$R_{eq} = \frac{1}{i_o}$$

تأكد

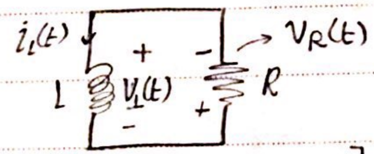


$R_{eq} = R_{th}$

(*) Source - Free RL CKT :-



$t > 0$ ⟹



intial Condition → $i_L(0) = I_o$

intial stored energy → $W_L(0) = \frac{1}{2} L I_o^2$

} From $t<0$ CKT

- The naturel response is $i_L(t)$:-

$$V_L(t) + V_R(t) = 0$$

$$L \frac{d i_L(t)}{dt} + R\, i_L(t) = 0 \quad (*)$$

1-st-order diff equation.

proposed solution :- $i_L(t) = Ae^{Bt}$ — ⊛⊛

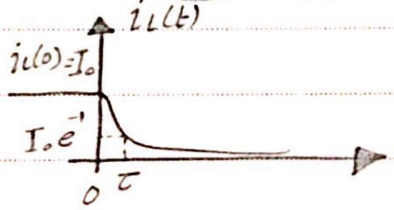
find A & B ?

$i_L(0) = I_o = Ae^{B(0)} \Rightarrow A = I_o$

Subst ⊛⊛ into ⊛ :-

$L\,ABe^{Bt} + R\,Ae^{Bt} = 0 \longrightarrow$ Sub ⊛⊛ into ⊛

$\Rightarrow i_L(t) = i_L(0)e^{-\frac{R}{L}t}$, $t>0$



$i_L(\tau) = I_o e^{-1} = I_o e^{-\frac{R}{L}\tau}$

$1 = \frac{R}{L}\tau \Rightarrow \tau = L/R$

$i_L(t) = I_o e^{-t/\tau}$

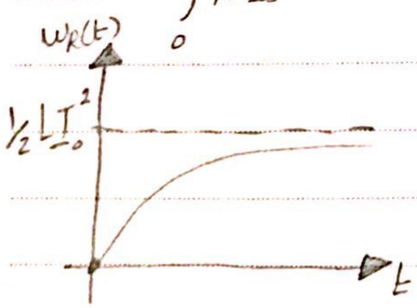
$I_o = i_L(0)$ A, $t>0$, $i_L(5\tau) \simeq 0$

$\tau = L/R$

* energy dissipated by R :-

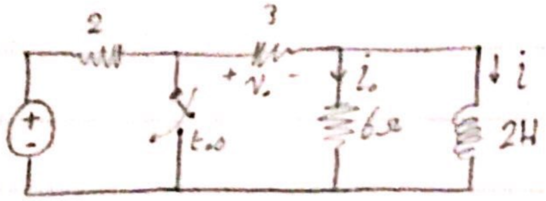
$$w_R(t) = \int_0^t i_L^2(\lambda)\,R\,d\lambda + \cancel{w_R(0)}^{\,0}$$

$$= \int_0^t R I_o^2 e^{-2t/\tau}\,d\lambda = \text{----} = \tfrac{1}{2} L I_o^2\left[1 - e^{-2t/\tau}\right] , t>0$$



$w_R(5\tau) \simeq \frac{1}{2} L I_o^2$

Ex:



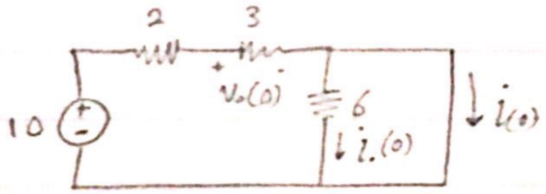
Find v_o, i_o, i for $t>0$

Sol →

$$i(t) = I_o e^{-t/\tau}$$

$I_o = i(0)$

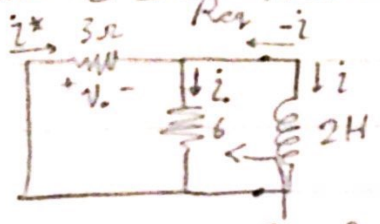
From $t<0$



$$i(0) = \frac{10}{2+3} = 2A = I_o$$

$$i_o(0) = 0\,A, \quad V_o(0) = \frac{3}{3+2} 10 = 6V$$

Find $\tau = \frac{L}{R_{eq}}$ from $t>0$



$R_{eq} = R_{th} = 6 \parallel 3 = 2$

$$\tau = \frac{2}{2} = 1 \text{ Sec}$$

$$i(t) = 2e^{-t}$$

$$- i_o(t) = \frac{3}{3+6} * -i = -\frac{6}{9} e^{-t} A$$

$$V_o(t) = -V_L(t) = -L\frac{di(t)}{dt} = -2\left[-2e^{-t}\right] = 4e^{-t}$$

طريقة ثانية

$$- V_o(t) = 3i^*$$

$$= 3(i + i_o)$$

$$= 3\left[2e^{-t} + \frac{-6}{9}e^{-t}\right] = 3\left[2 - \frac{6}{9}\right]e^{-t}\,V = 4e^{-t}\,V$$

* Note :-

$i(0^-) = 2$

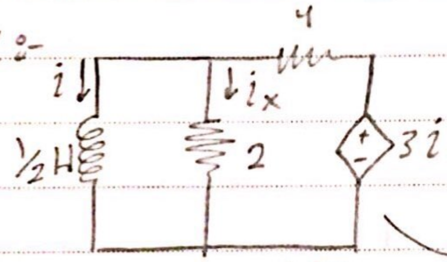
$i_o(0^-) = 0$

$i_o(0^+) = -6/9$ $\neq$

$V_o(0^-) = 6\,V$

$V_o(0^+) = 4\,V$

Ex :-



If $i(0) = 10A$ find $i(t)$ & $i_x(t)$ for $t > 0$

[illegible] بطلع $I_x(t)$

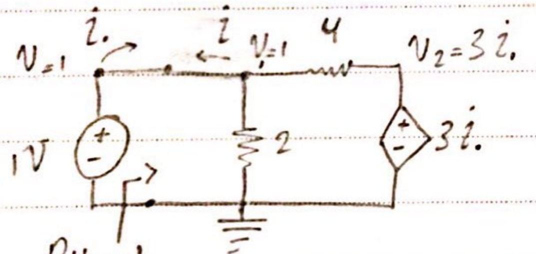
$i(t) = i(0)\,e^{-t/\tau}$

$= 10\,e^{-t/\tau}$

$i(t) = i(0)\,e^{-t/\tau}$

$= 10\,e^{-t/\tau}$

$\tau = \frac{L}{R_{th}}$



$R_{th} = \frac{1}{i_o}$

KCL at Node ① :-

$i_o = 1/2 + \frac{1 - 3i_o}{4}$

$i_x(t) = 1/2$

$i_o - 3/4\, i_o = 1/2 + 1/4$

$i_o = \frac{3/4}{1/4}$, $i_o = 3$, $R_{th} = \frac{1}{3}$

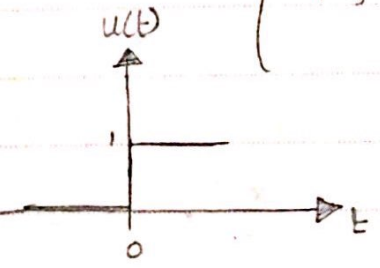
$R_{th} = 1/3$, $\tau = 3/2$ sec , $i(t) = 10\,e^{\frac{2t}{3}}$

* Step Response of an RC CKT:-

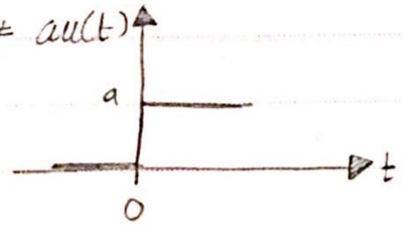
Singularity (switching) function

- Unit step function:-

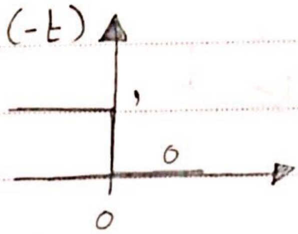
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$



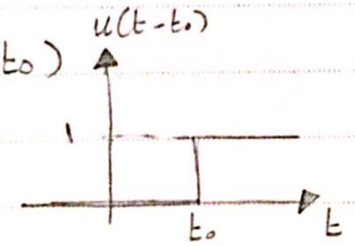
$a\,u(t)$



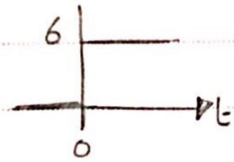
$u(-t)$



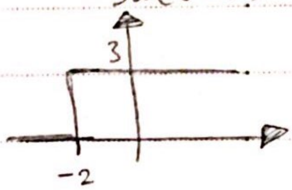
$u(t-t_0)$



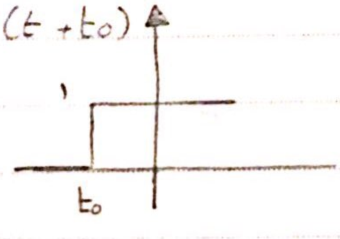
Ex:- $6u(t)$



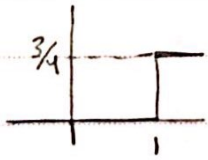
$3u(t+2)$



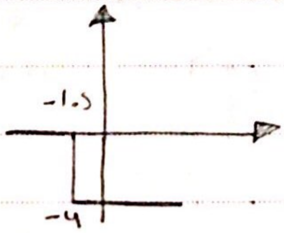
$u(t+t_0)$



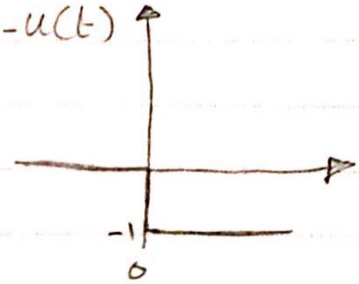
$3/4\,u(t-1)$



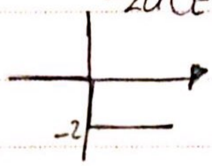
$-4u(t+1.5)$

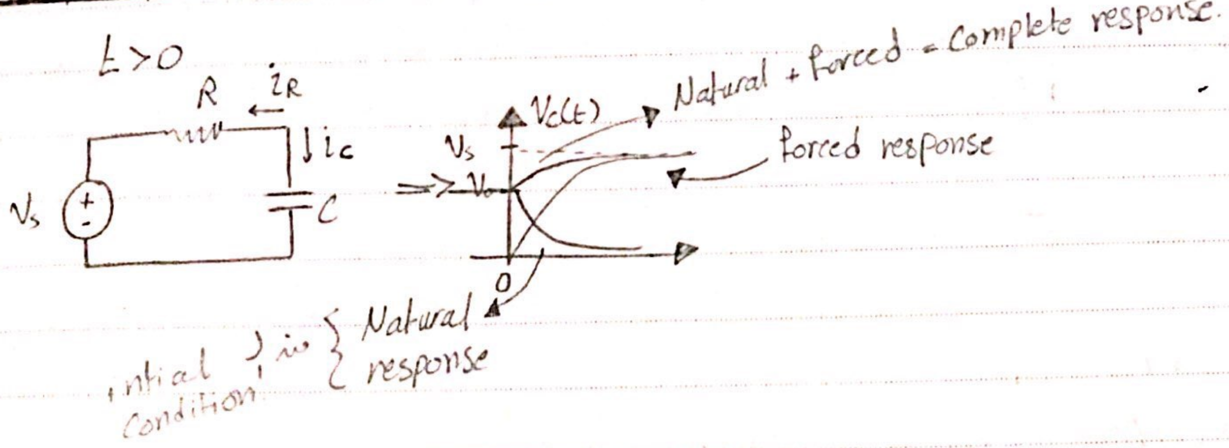


$-u(t)$



$-2u(t)$





Find the Complete response $V_c(t)$?

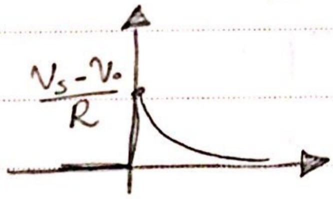
$$V_c(t) = \begin{cases} V_o, & t<0 \\ V_s + [V_o - V_s]e^{-t/\tau}, & t>0 \end{cases}$$

$\tau = RC$

$$V_c(0^-) = V_o$$
$$V_c(0^+) = V_o$$
$$V_c(\infty) = V_s$$

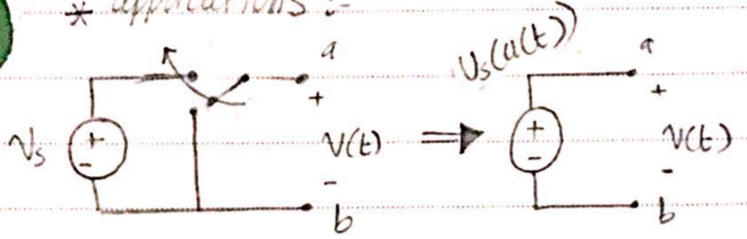
$\dashv\vdash \triangleq O.C$

$$i_c(t) = C\frac{dV_c}{dt} = \begin{cases} 0, & t<0 \\ -\frac{C[V_o - V_s]}{\tau}e^{-t/\tau}, & t>0 \end{cases} = \begin{cases} 0, & t<0 \\ \frac{[V_s - V_o]}{R}e^{-t/\tau}, & t>0 \end{cases}$$

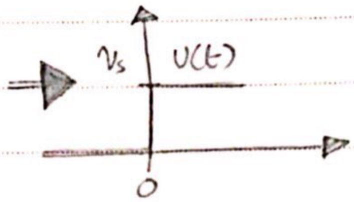


$$i_c(0^-) = 0.$$
$$i_c(0^+) = \frac{V_s - V_o}{R}$$

* applications :-

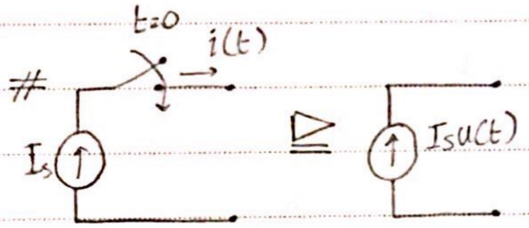


$$v(t) = \begin{cases} 0, & t<0 \\ V_s, & t>0 \end{cases}$$



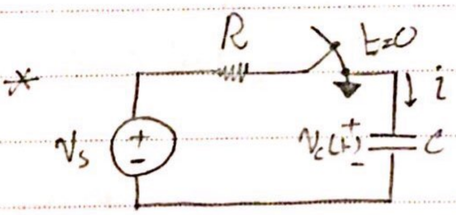
$= V_s u(t)$

#



$$i(t) = \begin{cases} 0, & t<0 \\ I_s, & t>0 \end{cases}$$

*

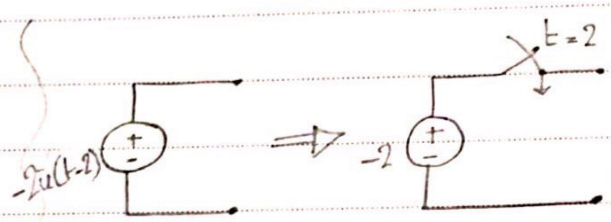


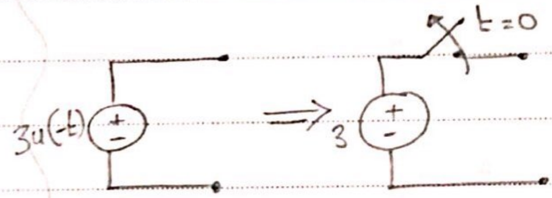
t<0

- $V_c(0^-) = V_c(0^+) = V_c(0) = V_0$
- $W_c(0) = \frac{1}{2} C V_0^2$

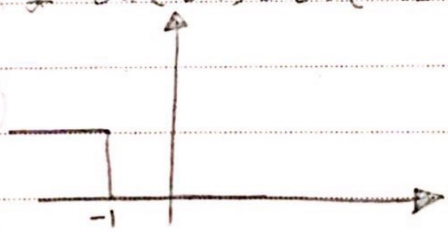
→ initial Conditions.

$V_0 < V_s$

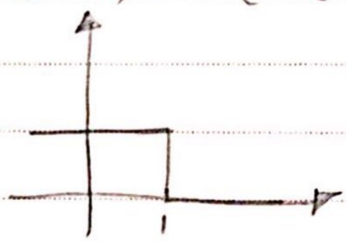




* $3u(-t-1) = 3u(-(t+1))$



* $u(-t+1) = u(-1(t-1))$



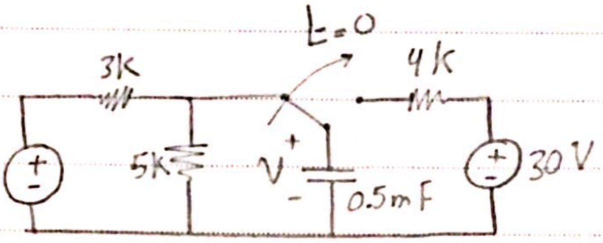
* To find the complete response in RC or in RL CkTs :-

$$f(t) = f(\infty) + [f(0^+) - f(\infty)] e^{-t/\tau}, \quad t > 0$$

$\left.\begin{array}{l} f(0^+) \\ f(\infty) \end{array}\right\}$ $t>0$ CkT , τ from $t>0$ CkT

$f(0^+) = f(0^-)$
only for capacitor voltage or inductor current.

Ex:-

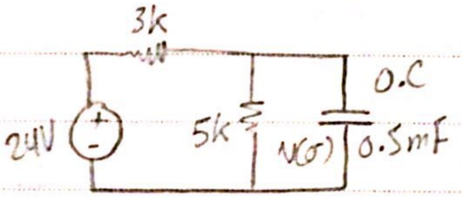


Find a) V for all t   b) energy stored at the capacitor

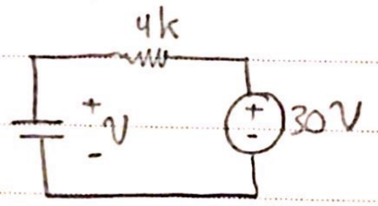
Sol →

$$V(t) = \begin{cases} V(0^-) \text{ (from } t<0 \text{ CkT)}, & t<0 \\ V(\infty) + [V(0^+) - V(\infty)] e^{-t/\tau}, & t>0 \end{cases}$$

$t<0$



$t>0$



$V(0^-) = \frac{5k}{5k+3k} * 24$

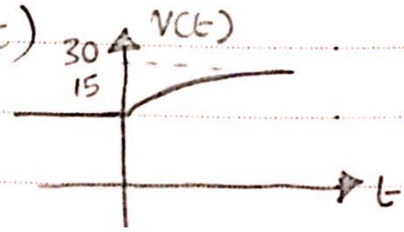
$= \frac{5}{8} 24 = (15\ V)$

- $V(0^+) = V(0^-) = (15\ V)$
- $V(\infty) = 30\ V$

$\tau = RC = (4k) * (1/2\ mF)$

$(\tau = 2\ \text{Sec})$

$$V(t) = \begin{cases} 15, & t<0 \\ 30 + [15 - 30] e^{-t/2}, & t>0 \end{cases}$$

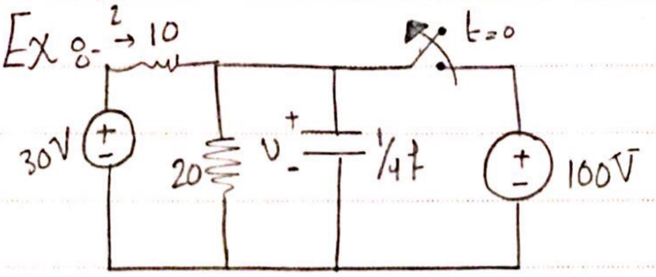


* Step - Response of RC CKT

The Complete response

$$f(t) = \begin{cases} f(0^-) \,, & t<0 \\ f(0) + [f(0^+) - f(\infty)]\, e^{-t/c} \,, & t>0 \end{cases} \qquad \tau = RC$$

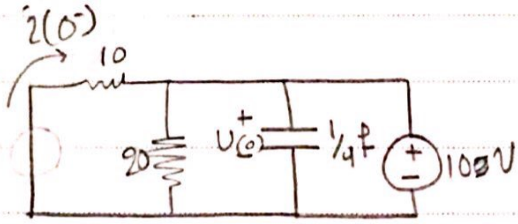
Ex :-



find the Complete response :-

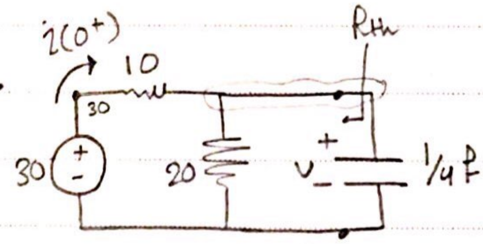
Sol →

$t<0$ →



$(V(0^-) = 10 \text{ Volt})$, $i(0^-) = \frac{-10\,V}{10\,\Omega} = -1\,A$

$t>0$ →



$$i(0^+) = \frac{30 - V(0)}{10} = 2A$$

$$= 2A$$

$$V(\infty) = 30 * \frac{20}{30} = 20 \text{ Volt}$$

$$R_{Th} = (20 \parallel 10) = 6.66$$

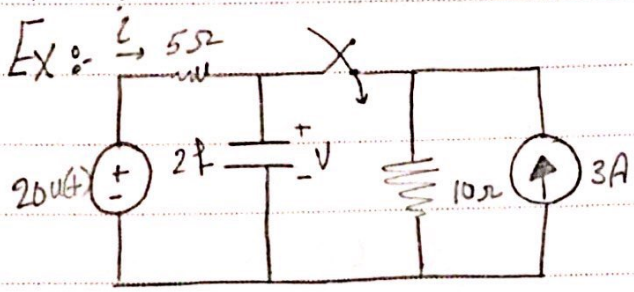
$$V(t) \begin{cases} 10 \,, & t<0 \\ 20 + [20 - 20]\, e^{-t/\tau} \end{cases}$$

V(t) $\tau = R_{Th} C$

$$\tau = 6.66 * 0.25 = 5/3$$

$$V(t) = \begin{cases} 10 \,, & t<0 \\ 20 - 10 e^{-3t/5} \,, & t>0 \end{cases} \qquad i(t) = \begin{cases} -1 \,, & t<0 \\ 1 + e^{-3t/5} \,, & t>0 \end{cases}$$

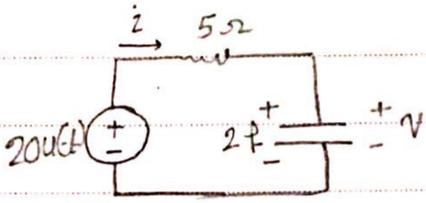
Ex :-



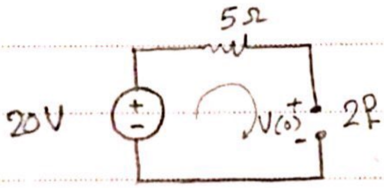
Find v(t) & i(t) for all t ?

Sol →

t < 0



$\dashv\vdash \triangleq$ O.C

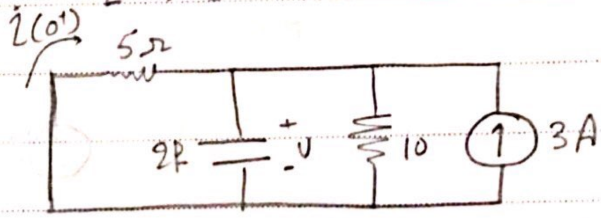


~~-20+5~~

$V(0^-) = 20\ V$

$i(0^-) = 0\ A$

t > 0



$V(0^+) = V(0^-) = 20$ Volt

$i(0^+) = \frac{-20}{5} = -4\ A$

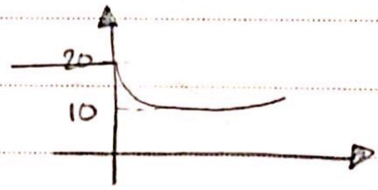
$V(\infty) = (10 // 5) * 3 = 10$

$i(\infty) = 3 * \frac{10}{15} = -2\ A$

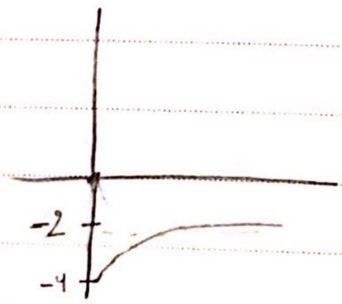
$\tau = R_{th} C$

$= (5//10)\left(\frac{2}{10}\right) = \frac{2}{3}$ sec

$$v(t) = \begin{cases} 20, & t<0 \\ 10 + 10e^{-3t/2}, & t>0 \end{cases}$$

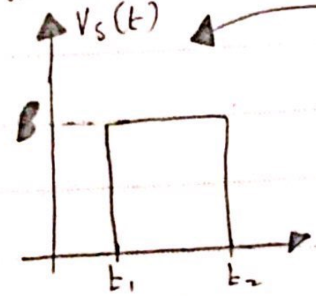


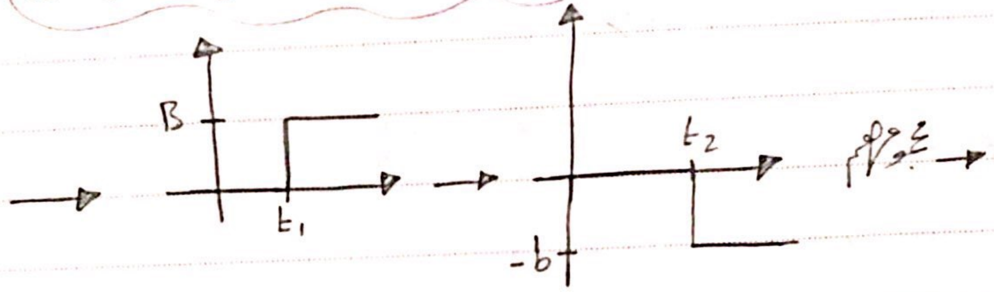
$$i(t) = \begin{cases} 0, & t<0 \\ -2 - 6e^{-3/2\, t}, & t>0 \end{cases}$$

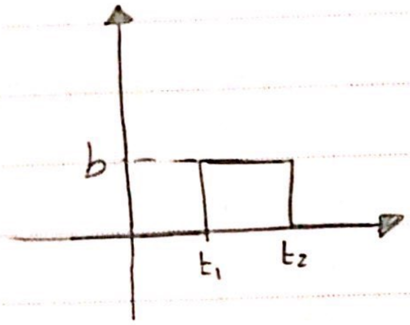


* rect - function :-

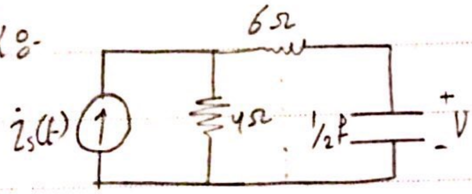
$V_s(t) = B[u(t-t_1) - u(t-t_2)]$

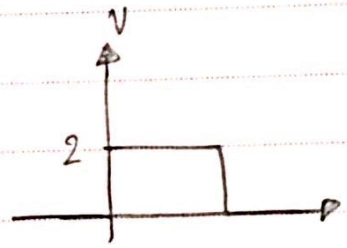






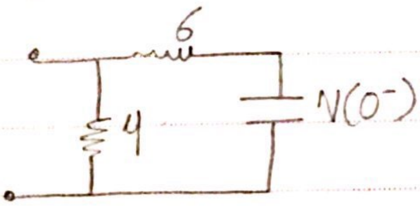
Ex :-



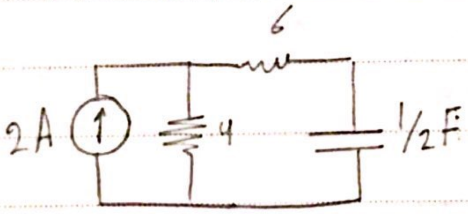


assume $V(0) = 0$
find V as a function of t

$t < 0$



$t > 0$



$V(0^+) = V(0^-) = 0$

$V(\infty) = (2A)(4\Omega) = 8$ Volt

$\tau = (6+4)\frac{1}{2} = 5$

$$V(t) = \begin{cases} 0, & t < 0 \\ 8 - 8e^{-t/5}, & 0 < t < 1 \\ V(\infty) + [V(1^+) - V(\infty)], & t > 1 \end{cases}$$

$t < 1$

$V(1^-) = 8 - 8e^{-1/5} = 1.45V$

$t > 1$

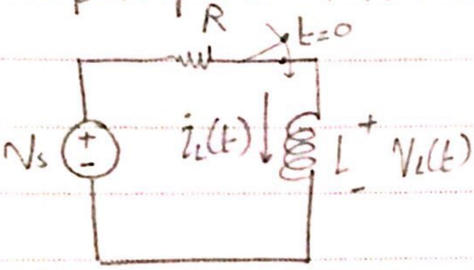
$V(1^+) = V(1^-) = 1.45V$

$\tau = 5$ sec

$\Rightarrow$

$$V(t) = \begin{cases} 0, & t<0 \\ 8-8e^{-t/5}, & 0<t<1 \\ 45\, e^{-(t-1)/5}, & t>1 \end{cases}$$

* Step response of RL CKT



$t<0, \quad i_L(0^-) = i_L(0^+) = I_o$

$$w_L(0) = \frac{1}{2} L I_o^2$$

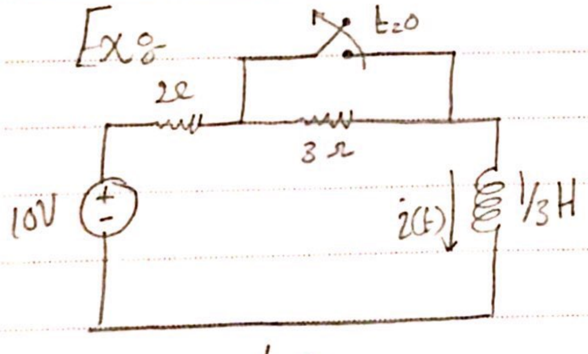
$t>0$

$$-V_s + Ri + L\frac{di(t)}{dt} = 0$$

$$i_L(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$$

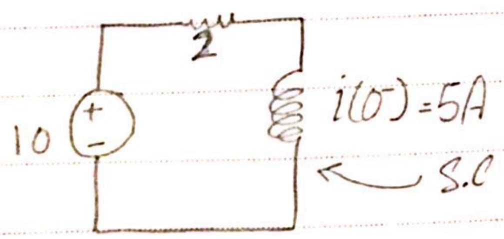
$$V_L(t) = L\frac{di(t)}{dt} \qquad V_L(0^+) \neq V_L(0^-)$$

Ex:

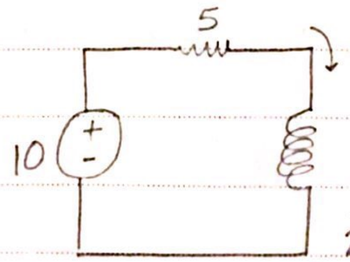


find i(t) for all time

$t<0$



$t>0$



$i(0^+) = ?$

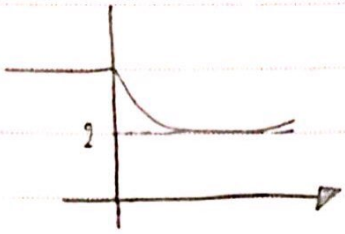
$i(\infty) = ?$

$i(0^+) = i(0^-) = 5A$

$i(\infty) = \frac{10}{5} = 2A$

$\tau = \frac{L}{R_{th}} = \frac{1/3}{5} = 1/15 \Rightarrow$

$$i(t) = \begin{cases} i(0^-), & t<0 \\ i(\infty)+[i(0^-)-i(\infty)]e^{-t/\tau}, & t>0 \end{cases} \Rightarrow \begin{cases} 5, & t<0 \\ 2+3e^{-15t}, & t>0 \end{cases}$$

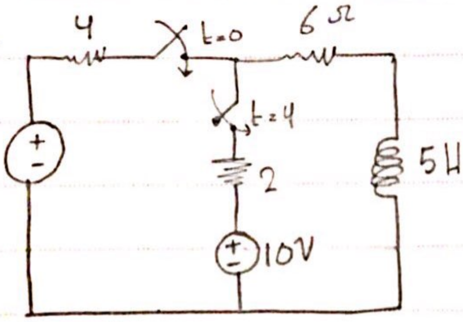


$$V(t) = \begin{cases} 0, & t<0 \\ \frac{-45e^{-15t}}{3}, & t>0 \end{cases}$$

$V(0^+) = -15V$

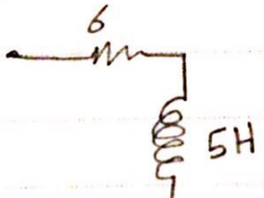
$V(\infty) = 0V$

Ex:-



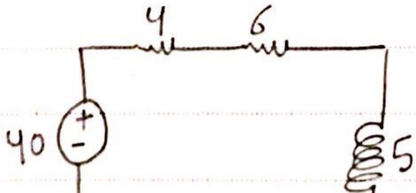
$$i(t) = \begin{cases} i(0^-), & t<0 \\ i(\infty)+[i(0^-)-i(\infty)]e^{-t/\tau}, & 0<t<4 \\ i(\infty)+[i(4^-)-i(\infty)]e^{-(t-4)/\tau}, & 4<t \end{cases}$$

$t<0$



$i(0^-) = 0A$

$0<t<4$



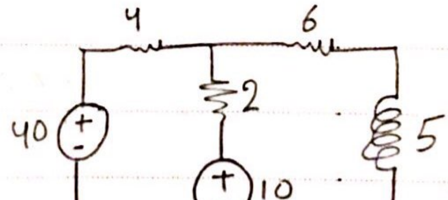
$i(0^+) = i(0^-) = 0A$

$i(\infty) = \frac{40}{10} = 4A$

$\tau = \frac{L}{R_{th}} = \frac{5}{10} = \frac{1}{2}$

$i(t) = 4-4e^{-2t}, 0<t<4$

$t>4$



$i(4^+) = i(4^-) = 4-4e^{-2t}$

$= 4[1-e^{-8}]$

$= 3.99A$

$i(\infty) = 2.7$

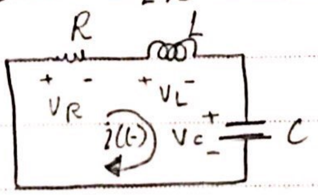
$\tau = \frac{L}{R_{..}} = \frac{15}{..}$ Sec

* CH8 :- 2nd-order RLC CkTs :-

- Source-Free Series RLC CkTs :- $t>0$

intial Conditions

- $i(0^+) = i(0^-) = I_0$
- $V_c(0^+) = V_c(0^-) = V_0$



The natural response $i(t)$ oR $V_c(t)$

Find $i(t)$?

KVL :- $V_R(t) + V_L(t) + V_C(t) = 0$

$$Ri + L\frac{di}{dt} + \frac{1}{c}\int_0^t i(t)\,dt + V(0) = 0$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{c}i(t) = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

Define $\boxed{\alpha = \frac{R}{2L}}$ damping factor.

$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$ resonant frequency rad/Sec.

$$f_0 = \frac{\omega_0}{2\pi}$$

$$\frac{d^2i}{dt^2} + 2\alpha\frac{di}{dt} + \omega_0^2 i = 0 \quad (*)$$

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Solve fun ⊛ to find $i(t)$?

propose $i(t) = Ae^{st}$ — ⊛⊛

Substitut ⊛⊛ into ⊛:-

$\rightarrow (As^2e^{st} + 2\alpha Ase^{st} + \omega_o^2)$

Two roots :-

$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2}$

$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$

$i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$ ✓

works for ① $\alpha > \omega_o$

② $\alpha < \omega_o$

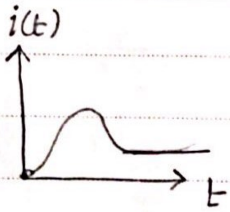
For ③ $\alpha = \omega_o$

$i(t) = (A_2 + A_1 t)e^{st}$

$\boxed{1}$ $\alpha > \omega_o$:- overdumped CKT

$$\Rightarrow \frac{R}{2L} > \frac{1}{\sqrt{LC}} \Rightarrow C > \frac{4L}{R^2}$$

$$\left.\begin{aligned} S_1 &= -\alpha + \sqrt{\alpha^2 - \omega_o^2} \\ S_2 &= -\alpha - \sqrt{\alpha^2 - \omega_o^2} \end{aligned}\right\}$$ Two different negative and real roots.



$i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$

Find :- A_1 & A_2 from $i(0^+) = i(0^-)$

$\frac{di}{dt}(0^+)$ from 0^+ CKT

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/ /

$\boxed{2}$ $\alpha < \omega_0$:- Underdamped CKT

$\hookrightarrow C < \frac{4L}{R^2}$

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$= -\alpha \pm \sqrt{-(\omega_0^2 - \alpha^2)}$$

$$= -\alpha \pm \sqrt{-1}\,\sqrt{\omega_0^2 - \alpha^2}$$

$$S_{1,2} = -\alpha \pm j\omega_d \quad , \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$ ← damping frequency

$$i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

$$= A_1 e^{-(\alpha - j\omega_d)t} + A_2 e^{-(\alpha + j\omega_d)t}$$

$$= A_1 e^{-\alpha t} \cdot e^{j\omega_d t} + A_2 e^{-\alpha t} \cdot e^{-j\omega_d t}$$

$$= e^{-\alpha t}\left[A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}\right]$$

$$= e^{-\alpha t}\left[A_1 \cos(\omega_d t) + jA_1 \sin(\omega_d t) + A_2 \cos(\omega_d t) + jA_2 \sin(-\omega_d t)\right]$$

$$= e^{-\alpha t}\left[\underbrace{(A_1 + A_2)}_{B_1} \cos(\omega_d t) + j\underbrace{(A_1 - A_2)}_{B_2} \sin(\omega_d t)\right]$$

$$i(t) = e^{-\alpha t}\left[B_1 \cos(\omega_d t) + jB_2 \sin(\omega_d t)\right]$$

- $i(0^+)$
- $\frac{di}{dt}(0^+)$

تعويض (زي)

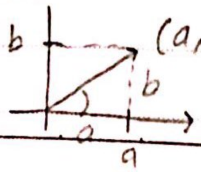
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$\sin(-\theta) = -\sin(\theta)$$

$$a + jb = \sqrt{a^2 + b^2} \, , \, \tan^{-1}\left(\frac{b}{a}\right)$$

$$i(t) = \sqrt{B_1^2 + B_2^2}\, e^{-\alpha t} \cos\left(\omega_d t + \tan^{-1}\left(\frac{B_1}{B_2}\right)\right)$$



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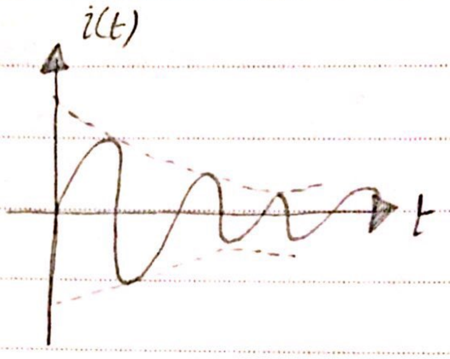
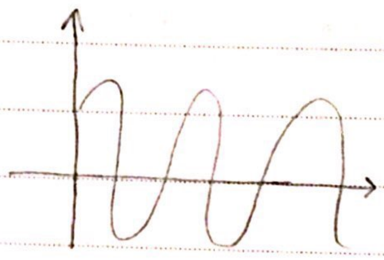


Special Case : $R=0 \Rightarrow \alpha = 0$

$i(t) = B_1 \cos(\omega_d t) + j B_2 \sin(\omega_d t)$



osillation ✓

damping ✗

$\boxed{3}$ $\alpha = \omega_0$

$S_1 = -\alpha$

$S_2 = -\alpha$

$S_1 = S_2 = -\alpha$

$i(t) = (A_2 + A_1 t) e^{-\alpha t}$

from $i(0^+)$ ✓

$\frac{di}{dt}(0^+)$ ✓

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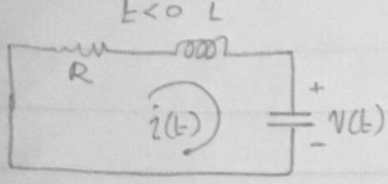
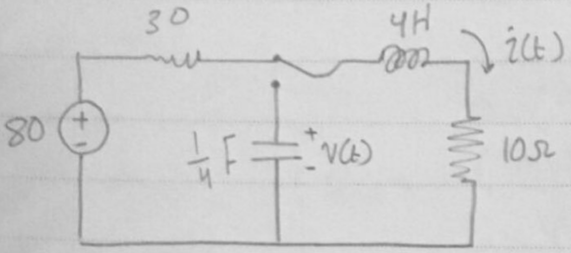
* Source-Free series RLC CkT:-



$\alpha = ...$

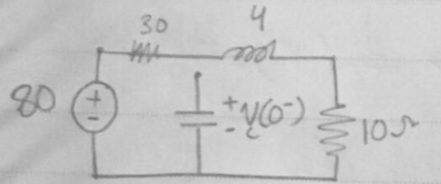
$\omega_0 = ...$

Ex:-



find $v(t)$ and $V_c(t)$ for $t>0$

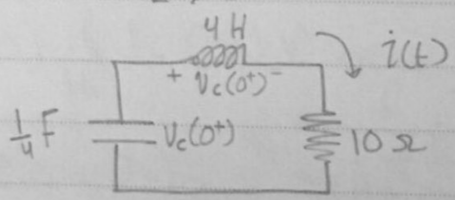
t<0



$i(0^-) = \frac{80}{30+10} = 2A$

$V(0^-) = 0V$

t>0



$\alpha = \frac{R}{2L} = (1.25)$ neper/sec

$\omega_0 = \frac{1}{\sqrt{LC}} = 1$ rad/sec

$\alpha > \omega_0$ → over damped series RLC CkT

$i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$

$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -0.5$

$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -2$

$i(t) = A_1 e^{t/2} + A_2 e^{-2t}$, $t>0$

$V_c(t) = A_3 e^{-0.5t} + A_4 e^{-2t}$

⟹

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Find A_1 & A_2 $\to i(0^+)$, $\to \frac{di}{dt}(0^+)$

$i(0^+) = i(0^-) = 2A$

$i(0) = A_1 + A_2 = 2 \longrightarrow ①$

$\frac{di}{dt}(0^+) = \longrightarrow$ $V_L = L\frac{di}{dt}$ لازم نجيب معادلة

KVL: $-V_c(0^+) + V_L(0^+) + 10\,i(0^+) = 0$

$V_L(0^+) = V_c(0^+) - 10\,i(0^+)$

$= 0 - (10)(2)\,V - 20V$

$V_L(t) = L\frac{di}{dt} \longrightarrow V_L(0^+) - L\frac{di}{dt}(0^+) \longrightarrow \frac{di}{dt}(0^+) = \frac{-20}{4} = -5$

$\frac{di}{dt} = -0.5A_1e^{-0.5t} - 2A_2e^{-2t} \longrightarrow (-5 = -0.5A_1 - 2A_2)$ ②

solving ① & ② $A_1 = -2/3$

$i(t) = -2/3\,e^{-0.5t} + \frac{8}{3}e^{-2t},\ t > 0$

$-i(0^+) = V_c\frac{dv}{dt}(0^+) \xrightarrow{OR} V_c(t) = \frac{1}{c}\int^{t} i(\tau)\,d\tau + V(0)$

$= -\frac{1}{c}\int_0 \left(\frac{-2}{3}e^{-0.5\tau} + \frac{8}{3}e^{-2\tau}\right)dt + V(0)$

$V_{10}(t) = 10\,i(t)$, $V_c(t) = L\frac{di}{dt}$

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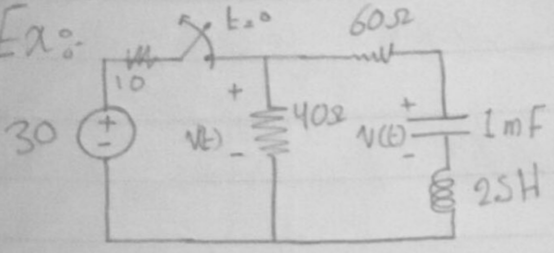
Ex:

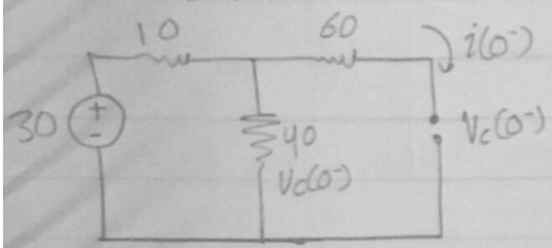


find $v(t)$ for $t>0$

Sol →

find $i(t)$ then $v(t) = -40\,i(t)$

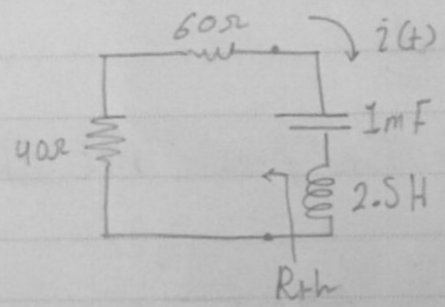
$t<0$



$i(0^-) = 0A$

$$V_c(0^-) = \frac{40}{40+10}\,30 = 24\,V$$

$t>0$



$$\alpha = \frac{R}{2L} = \frac{100}{2\left(\frac{5}{2}\right)} = 20 \text{ Neper/sec}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 20 \text{ rad/sec}$$

$\alpha = \omega_0 \longrightarrow$ Critially damped ckT

$$i(t) = (A_2 + A_1 t)e^{-\alpha t}$$
$$= (A_2 + A_1 t)e^{-20t}\,A \;,\; t>0$$

find

$A_1 \quad i(0^+)$

$A_2 \quad \frac{di}{dt}(0^+)$

$i(0^+) = i(0^-) = 0 = A_2 \longrightarrow A_2 = 0$

$i(t) = A_1 t e^{-2t}$

$\frac{di}{dt} = ? \longrightarrow$ KVL: $40i(0^+) + 60\,i(0^+) + V_L(0^+) + V_c(0^+) = 0$

$0 + 0 + V_L(0^+) + 24 = 0$

$V_L(0^+) = -24\,V \quad \Rightarrow$

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$$V_L(0^+) = L\frac{di}{dt}(0^+) \longrightarrow \frac{di}{dt}(0^+) = \frac{-24}{\frac{5}{2}} = -9.6\,A$$

$$\frac{di}{dt} = A_1 t(-20e^{-20t}) + A_1 e^{-20t} \longrightarrow \boxed{-9.6 A_1}$$

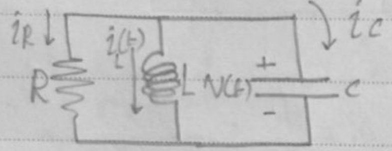
$$i(t) = -9.6\,t\,e^{-20t}\ A \quad , t > 0$$

$$v(t) = 40(-9.6te^{-20t})$$

$$\tau = \frac{1}{20}$$

* Source-Free parallel RLC CKT :-

$t > 0$



initial Conditions :

$v_C(0^+) = v_C(0^-) = V_0$

$i_L(0^+) = i(0^-) = I_0$

natural response is $v_C(t)$ or $i_L(t)$

$$\text{kcl} \longrightarrow i_R + i_L + i_C = 0$$

$$\frac{v}{R} + \frac{1}{L}\int v(t)\,dt + i(0) + C\frac{dv}{dt} = 0$$

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\left(\frac{dv}{dt}\right) + \frac{1}{L}v = 0$$

$$\frac{dv^2}{dt^2} + \frac{1}{R}\left(\frac{dv}{dt}\right) + \frac{1}{LC}v = 0$$

$$\frac{dv^2}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = 0$$

$$\alpha = \frac{1}{2RC} \quad , \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

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$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$
$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

Cases :-

Case ① $\alpha > \omega_0$:- arerdamped

S_1, S_2 :- differenal real negative roots

$V(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$

$V_c(0^+)$

$\frac{dV}{dt}(0^+)$

Case ② under damped

S_1, S_2 : Complex roots

$\omega_0 \sqrt{\omega_0^2 - \alpha^2}$

$V(t) = e^{-\alpha t}\left[B_1 \cos(\omega t) + j B_2 \sin(\omega t)\right]$

$v(0^+), \frac{dv}{dt}(0^+)$

Case ③ Ciritically dampad

$S_1 = S_2 = \alpha$

$V(t) = (A_2 + A_1 t) e^{-\alpha t}$

$A_2 \& A_1$ = frem initial Conditional

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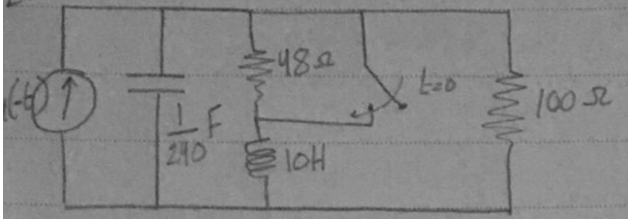
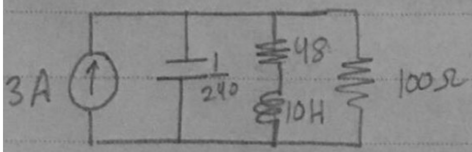
Ex:



find $v(t)$ & $i(t)$ $t>0$

Sol →

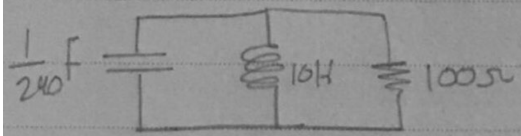
$t<0$



$V(0^-) = (3)(48 // 100) = 97.3\ V$

$i(0^-) = \frac{100}{100+48}(3) = 2.027\ A$

$t>0$



Source free parallel RLC CKT

$\alpha = \frac{1}{2RC} = 1.2$ Neper/sec

$\omega_d = \frac{1}{\sqrt{LC}} = 4.9$ rad/sec.

$\alpha < \omega_o$:- Under damped Case.

$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 4.75$ rad/sec.

$V(t) = e^{-\alpha t}\left[B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)\right]$

find B_1 & B_2 using $V(0^+)$ & $\frac{dv}{dt}(0^+)$

$V(0^+) = V(0^-) = 97.3 = 1\left[B_1(1) + B_2(0)\right]$

$B_1 = 97.3 \to (1)$

$i_C(0^+) + i_L(0^+) + i_R(0^+) = 0$ in Zero CKT

$\frac{1}{240}\frac{dv(0^+)}{dt} + \frac{97.3}{100} = 0$, $\frac{dv(0^+)}{dt} = -720$

$\frac{dv}{dt} = e^{-\alpha t}\left[B_1 \omega_d \sin(\omega_d t) + B_2 \omega_d \cos(\omega_d t)\right] +$
$-\alpha e^{-\alpha t}\left[B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)\right]$

⟹

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$-720 = B_2 \omega j - \alpha B_1$

$-720 = -1.2\, B_1 + 4.7 B_2 = -720 \longrightarrow (2)$

$B_2 = -127$

$\Longrightarrow v(t) = e^{-1.2t}\left[97.3 \cos(4.7t) - 127 \sin(4.7t) \right] V. \quad t > 0$

$i(t) = \frac{1}{L} \int_0^t v(\tau)\, d\tau + i(0)$

$= \frac{1}{10}$ ___

OR $\quad i(t) = e^{-\alpha t}\left[B_3 \cos(\omega_j t) + B_4 (\sin(\omega_j t) \right]$

B_3 & B_4 $\quad i(0^+)$? $\frac{di}{dt}(0^+)$

$i(0^+) = i(0^-) = 2.027 \triangleq B_3$

$V_L(0^+) = L \frac{di}{dt}(0^+) \longrightarrow \frac{di(0^+)}{dt} = \frac{97.3}{10} = 9.73$

$9.73 = B_4 \omega d - \alpha B_3$

$B_4 = 2.4$

* step response RLC CKT $<$ Series / parellel

- The $t > 0$ ckT has independent Dc Soures
- the response = The steby state response + The transienl response

$f(t) = f(\infty) + f(t), \quad t > 0$

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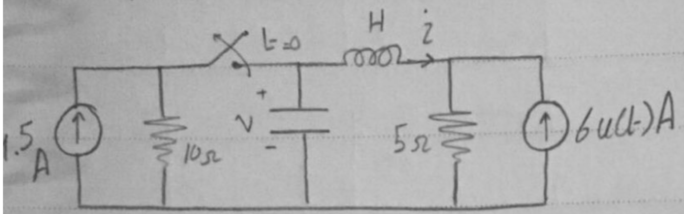
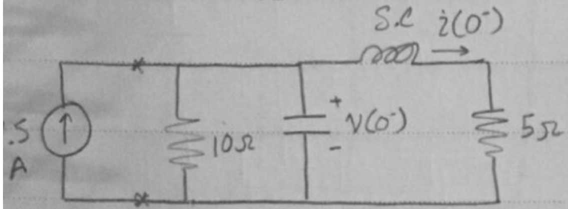
Ex:- Prob 8:33 :-



Find v(t) and i(t) :-

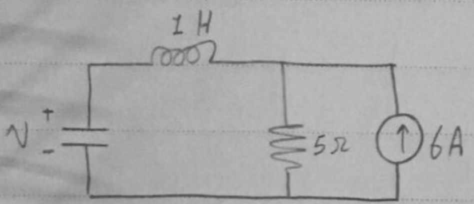
Sol :- t<0



$$v(0^-) = (4.5)(10 // 5) = 15\,V$$

$$i(0^-) = \frac{15}{5} = 3\,A$$

t>0



step-response series RLC ckT

to determine kill all independant sources.

$$\alpha = \frac{R}{2L} = 2.5 \text{ nep/sec}$$

$$w_o = \frac{1}{\sqrt{LC}} = 0.5 \text{ rad/sec}$$

$\alpha > w_o \rightarrow$ over dumped

let $f(t) = v(t)$

$$v(t) = v(\infty) + v_t(t)$$

$$= v(\infty) + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

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$V(\infty) = (5)(6) = 30V$

$\uparrow$

$t \to \infty$

$-||-$: O.C

$-\!\!\!\text{ooo}\!\!\!-$ = S.C

$S_1 = -\alpha + \sqrt{\alpha^2 - w_0^2} = -4.95$

$S_2 = -\alpha - \sqrt{\alpha^2 - w_0^2} = -0.0505$

$V(t) = 30 + A_1 e^{-4.95t} + A_2 e^{-0.0505}$, $t > 0$

Find A_1 & A_2 using . $v(0^+)$?

. $\frac{dv}{dt}(0^+)$?

$v(0^+) = v(0^-) = 15\,V$

$15 = 30 + A_1 + A_2$

$\Rightarrow A_1 + A_2 = -15$ ①

$-i(t) = 4 \frac{dv}{dt}$, $t > 0$

$-i(0^+) = 4 \frac{dv}{dt}(0^+)$

$\Rightarrow \frac{dv}{dt}(0^+) = \frac{-i(0^+)}{4} = -\frac{3}{4}$

$\frac{dv}{dt}(0^+) = -4.95 A_1 e^{-4.95t} - 0.0505 A_2 e^{0.0505t}$

$\frac{-3}{4} = -4.95 A_1 - 0.0505 A_2$ ②

Solving for ① & ② : $A_1 = 0.3077$

$A_2 = -15.3076$

$\rightarrow V(t) = \begin{cases} 15 & , t < 0 \\ 30 + 0.3077 e^{-4.95t} - 15.3076 e^{-0.0505t} & , t > 0 \end{cases}$

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* AC Circuits :-

  - Complex number

• Complex number →

$x^2 = -1 \implies x = \sqrt{-1} = j$

$j^2 = -1 \;,\; j^3 = j \cdot j \cdot j = -j$

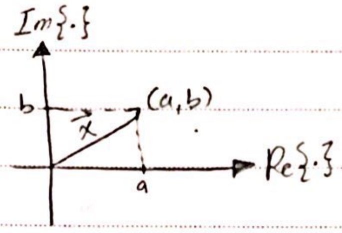
$j^3 = -1 \;,\; j^5 = -j$

[1] In rectangular form

$\vec{x} = a + jb$

$a = Re\{\vec{x}\}$

$b = Im\{b\}$

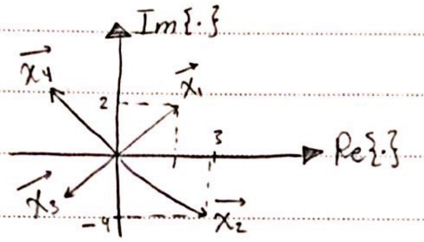


$\vec{x_1} = 1 + j2$

$\vec{x_2} = 3 - j4$

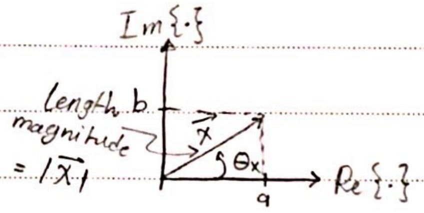
$\vec{x_3} = -1 - j2$

$\vec{x_4} = -2 + j3$



[2] Exponential form

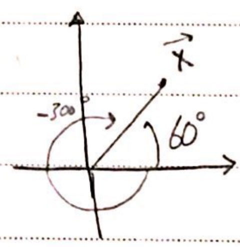
$\vec{x} = |\vec{x}| e^{j\theta_x}$



[3] Polar form

$\vec{x} = |\vec{x}| \angle \theta_x$

Ex: $\vec{x} = 10 e^{j60^\circ} = 10 \angle 60^\circ = 10 \angle -300^\circ$



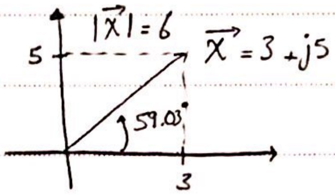
F $* \ \vec{x} = a + jb \longrightarrow \vec{x} = |\vec{x}| e^{j\theta_x}$

$$|\vec{x}| = \sqrt{a^2 + b^2}$$

$$\theta_x = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\vec{x} = 3 + j5 \triangleq \sqrt{3^2 + 5^2}$$

$$= 6 \angle 59.03^\circ$$

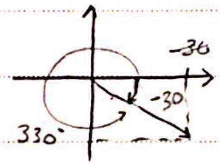


$* \ \vec{x} = |\vec{x}| \ \theta_x \longrightarrow \vec{x} = \underbrace{|\vec{x}| \cos(\theta_x)}_{a} + j \, |\vec{x}| \sin(\theta_x)$

Ex :- $\vec{x} = 18 \angle -30^\circ = 18 \angle 330$

$$\triangleq 18 \cos(-30^\circ) + j(18) \sin(-30^\circ)$$

$$= 15.58 - j9$$



18.7

$$\vec{x}_1 = e^{j\theta} \triangleq \cos(\theta) + j \sin(\theta) \longrightarrow ①$$

$$\vec{x}_2 = e^{-j\theta} = \cos(-\theta) + j(-\theta)$$

$$= \cos(\theta) - j \sin(\theta) \longrightarrow ②$$

$$① + ② \longrightarrow \cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{}$$

$$① - ② \longrightarrow \sin(\theta) = \frac{e^{j\theta 2} - e^{-j\theta}}{2j}$$

$* \ \vec{x}_1 = a + jb \triangleq |\vec{x}_1| \ \theta_{x_1}$

$\vec{x}_2 = c + jd \triangleq |\vec{x}_2| \ \theta_{x_2}$

$$\vec{x}_1 = -a - jb \triangleq |\vec{x}| \ \theta_{x_1} \pm 180^\circ$$

Ex :- $\vec{z} = 30 \angle -40^\circ$

$$-\vec{z} = -(30 \angle -40^\circ)$$

$$= 30 \angle (-40 \pm 180^\circ)$$

$$= 30 \angle 140^\circ$$

$$= 30 \angle -220^\circ$$

* Conjugate of $\vec{x}_1 \triangleq \vec{x}_2^*$

$\vec{x}_1^* = a - jb$

$= |\vec{x}_1| - \theta_{x_1}$

Ex:- $\vec{Z}_1 = 2 + j3$

$\vec{Z}_2 = -2 - j4$

$\vec{Z}_3 = 50 * 220^\circ$

$\vec{Z}_4 = 10 * -170^\circ$

Sol →

$\vec{Z}_1^* = 2 - j3$

$\vec{Z}_2^* = -2 + j4$

$\vec{Z}_3^* = 50 \angle -220^\circ = 50 - 140^\circ$

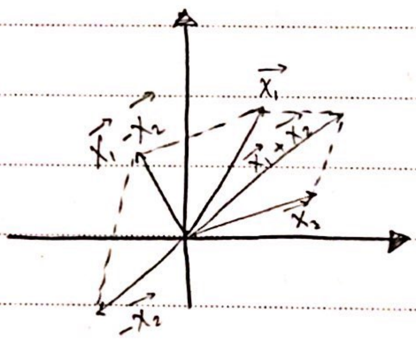
$\vec{Z}_4^* = 10 \angle 170^\circ$

* Sum and difference of two complex numbers:-

$\vec{x}_1 + \vec{x}_2 = (a + jb) + (c + jd)$

$= (a + c) + j(b + d)$

$\vec{x}_1 - \vec{x}_2 = (ac) + j(b - d)$



Ex:- $\vec{x}_1 = 3 + j4$

$\vec{x}_2 = -2 - j3$

$\vec{x}_1 + \vec{x}_2 = 1 + j \triangleq \sqrt{2} * \tan^{-1}(1)$

$\vec{x}_1 - \vec{x}_2 = 5 + j7 = \sqrt{25 + 49} * \tan^{-1}(7/5)$

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* multiplication:-

$\vec{x_1} \cdot \vec{x_2} = (a+jb)(c+jd)$

$= ac + jad + jbc - bd$

$= (ac-bd) + j(ad+bc)$

$\vec{x_1} \cdot \vec{x_2} = \left(|\vec{x_1}| \theta_{x_1}\right)\left(|\vec{x_2}| \theta_{x_2}\right)$

$= |\vec{x_1}||\vec{x_2}|(\theta_{x_1} + \theta_{x_2})$

Ex:- $\vec{Z} = 3 \angle 73°$, $\vec{y} = 6 \angle 12°$

$\vec{Z}\vec{Y} = 18 \angle 85°$

* Division:-

$\frac{\vec{x_1}}{\vec{x_2}} = \frac{a+jb}{c+jd} \cdot \frac{\vec{x_2}^*}{\vec{x_2}^*} = \frac{(a+jb)(c-jd)}{\vec{x_2} \cdot \vec{x_2}^*} = \frac{(a+jb)(c-jd)}{|\vec{x}|^2}$

$= \frac{(ac-bd)+j(bc-ad)}{c^2+d^2}$

$\frac{\vec{x_1}}{\vec{x_2}} = \frac{|\vec{x_1}| \theta_{x_1}}{|\vec{x_2}| \theta_{x_2}} = \frac{|\vec{x_1}|}{|\vec{x_2}|} (\theta_{x_1} - \theta_{x_2})$

Ex:- $\frac{30 \angle 12°}{5 \angle 3°} = 6 \angle 9°$

Notes:- • $\vec{x} + \vec{x}^* = (a+jb) + (a-jb)$

$= 2a$

• $\vec{x} - \vec{x}^* = 2jb$

• $\vec{x} \cdot \vec{x}^* = |\vec{x}|^2$

• $(\vec{x}^*) = \vec{x}$

• $\frac{\vec{x}}{\vec{x}^*} = \frac{|\vec{x}| \theta_x}{|\vec{x}| \theta_x} = 1 \angle 2\theta = e^{j2\theta_x}$

- $\vec{Z} = 1\angle 90^\circ \triangleq e^{j90^\circ}$

  $\triangleq 1\cos(90^\circ) + j(1)\sin(90^\circ)$

  $\triangleq j$

$j \triangleq 1\angle 90^\circ = e^{j90^\circ}$

$-j \triangleq 1\angle -90^\circ = e^{-j90^\circ}$

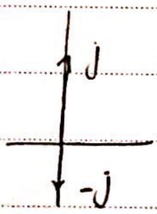
Ex: • $\vec{Z} = 5\angle 0^\circ$

$= 5\angle 36^\circ$

$= 5\angle -36^\circ$

$= 5 + j0$

$= 5$

- 

$\vec{Z} = 5\angle \pm 180^\circ$

$= -5 + j0$

* AC Circuits :-

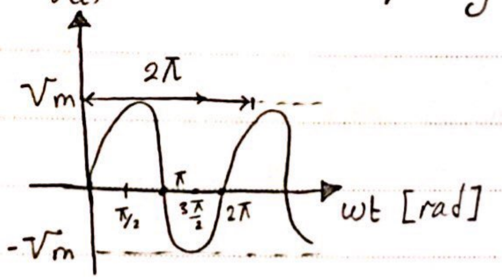
- Sinusoids Characteristics :-

$v(t) = V_m \sin(\omega t)$

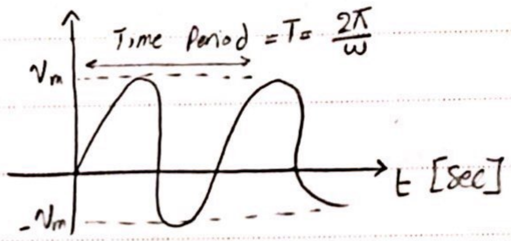
V_m → peak value or amplitude

ωt → argument (angle) [rad]

ω : radian frequency



بنقسم ع ω ⟹



frequency ⟹ $f = \frac{1}{T}$ [Hz] # number of cycles per sec.

$$\omega = \frac{2\pi}{T} \Rightarrow \omega = 2\pi f$$

Ex:- $v(t) = 20 \sin(1000t)$

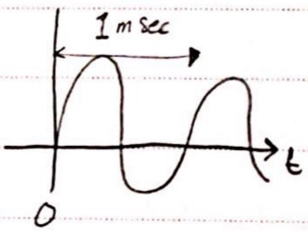
find f and T

Sol → $f = \frac{\omega}{2\pi} = \frac{1000}{2\pi} = \dots$

Ex:- $v(t) = 10 \sin(2000\pi t)$, find f & T

$f = \frac{\omega}{2\pi} = \frac{2000\pi}{2\pi} = 1000$ Hz

$T = \frac{1}{f} = 1$ m sec



Ex:- $v(t) = 220 \cos(100\pi t)$

$f = \frac{\omega}{2\pi} = 50$ Hz

* more general case :-

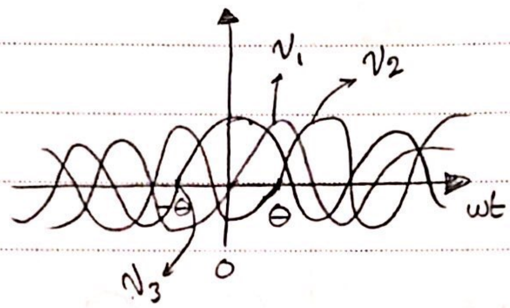
$v(t) = V_m \sin(\omega t + \theta)$

argument or angle → $\omega t + \theta$; θ → "Phase"

If $\theta = 0 \longrightarrow v_1(t)$

If $\theta < 0 \Rightarrow v_2(t)$

If $\theta > 0 \longrightarrow v_3(t)$



Lagging & Leading :-

$v_3(t)$ leads $v_1(t)$ by θ

$v_2(t)$ lags $v_1(t)$ by θ

$v_1(t)$ lags $v_2(t)$ by $-\theta = (2\pi - \theta)$

$v_2(t)$ leads $v_1(t)$ by $-\theta$

v_3 leads v_2 by 2θ

To Compare two Signals :-

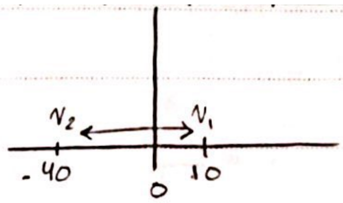
1. Both must be cosine OR sine.
2. Both must have +Ve amplitude
3. Both must have same "ω"

* Note →

$$-\sin(\omega t) = \sin(\omega t \pm 180^\circ)$$
$$-\cos(\omega t) = \cos(\omega t \pm 180^\circ)$$
$$\mp \sin(\omega t) = \cos(\omega t \pm 90^\circ)$$
$$\pm \cos(\omega t) = \sin(\omega t \pm 90^\circ)$$

Ex:- $v_1(t) = 10 \cos(1000\pi t - 10^\circ)$

$v_2(t) = 2 \cos(1000\pi t + 40^\circ)$



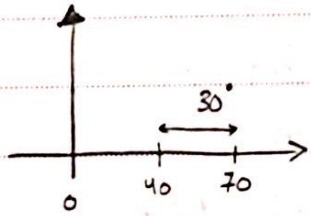
- v_2 leads v_1 by 50° (-310)

v_1 lags v_2 by 50°

v_2 lags v_1 by -50° (310°)

Ex:- $v_1(t) = -2 \cos(100\pi t + 20^\circ)$

$v_2(t) = 12 \sin(100\pi t - 40^\circ)$



Sol $\rightarrow$ $v_1 = 2 \sin(100\pi t + 20^\circ - 90^\circ)$

$v_1(t) = 2 \sin(100\pi t - 70^\circ)$

* phasors:

Convert the Sinusoids from time-domain to phasor domain (complex number)

$v(t) = V_m \cos(\omega t + \theta)$

let $Z(t) = V_m e^{j(\omega t + \theta)}$

$= V_m \angle \omega t + \theta$

$= V_m \cos(\omega t + \theta) + j V_m \sin(\omega t + \theta)$

هاد المهم

$v(t) = Re\{Z(t)\} = Re\{V_m e^{j(\omega t + \theta)}\} = Re\{V_m e^{j\theta} \cdot e^{j\omega t}\}$

* Time-domain $\longrightarrow$ phasor domain
* $v(t) = V_m \cos(\omega t + \theta) \longrightarrow \vec{V} = V_m \angle \theta$
* CkT analysis using differetial equ $\longrightarrow$ CkT analysis using algebric eq
* very complicated analysis.

/ /

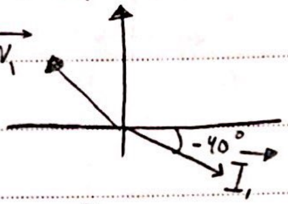
Ex:- $i_1(t) = 6\cos(50t - 40^\circ) \Rightarrow \vec{I_1} = 6\angle -40$ A

$v_1(t) = -4\sin(30t + 50^\circ)$

$= 4\cos(30t + 50^\circ + 90^\circ)$

$= 4\cos(30t + 140^\circ)$

$\Rightarrow \vec{v_1} = 4\angle 140^\circ$ V



$\vec{I} = 30\angle -20^\circ \rightarrow i(t) = 30\cos(\omega t - 20^\circ)$

$\vec{I} = -3 + j4$ A

$= \sqrt{9+16}\ \angle \tan^{-1}(4/-3)$

$= 5\angle 126.87^\circ \rightarrow i(t) = 5\cos(\omega t + 126.87^\circ)$.

Find $\rightarrow$ $i(t) = \underbrace{4\cos(\omega t + 30^\circ)}_{i_1(t)} + \underbrace{5\sin(\omega t - 20^\circ)}_{i_2(t)}$

$i_1(t) \rightarrow \vec{I_1} = 4\angle 30^\circ$

$i_2(t) = 5\cos(\omega t - 20^\circ - 90^\circ)$

$\rightarrow \vec{I_2} = 5\angle -110^\circ$

\# $\vec{I} = \vec{I_1} + \vec{I_2} = 4\angle 30^\circ + 5\angle -110^\circ$

$= 3.218\angle -56.97^\circ$

$i(t) = 3.1218\cos(\omega t - 56.97)$

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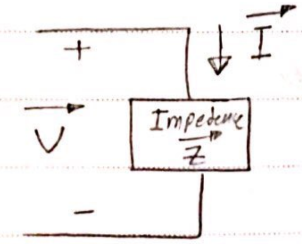
* Phasors :-

$v(t) = V_m \cos(\omega t + \theta) \rightarrow \vec{V} = V_m \angle\theta$

$i(t) = I_m \cos(\omega t + \theta) \rightarrow \vec{I} = I_m \angle\theta$

* Impedence :- $[\Omega]$

$\vec{Z} = \dfrac{\vec{V}}{\vec{I}}$



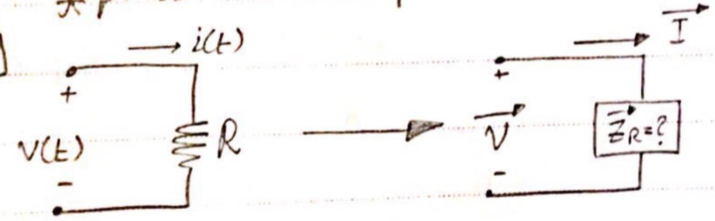
$\vec{Z} = R + jX$

resistor absorb energy

reactance "store element"

* phasor relationships for CKT elements :-

1



time-domain

$v(t) = V_m \cos(\omega t + \theta_v) \Longrightarrow \vec{V} = V_m \angle\theta_v$ V

$i(t) = I_m \cos(\omega t + \theta_i)$ $\quad \vec{I} = I_m \angle\theta_i$ A

$v(t) = R\,i(t)$

$= R\,I_m \cos(\omega t + \theta_i) \rightarrow \vec{V} = V_m \angle\theta_v$

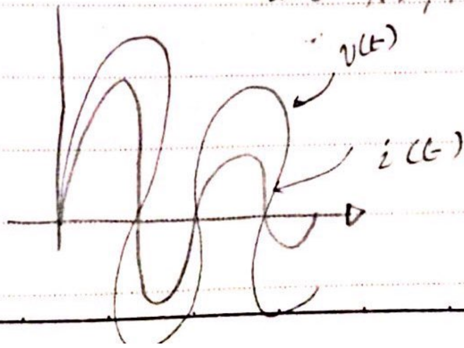
$= R\,I_m \angle\theta_i$

$\rightarrow V_m = R\,I_m \rightarrow v(t)$ & $i(t)$ for ——— are in phase.

$\vec{V} = R\,I_m \angle\theta_i$

$\vec{V} = R\,\vec{I}$

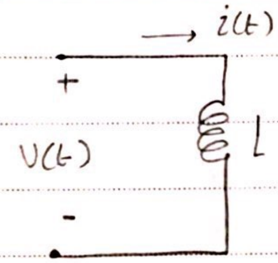
$\boxed{\vec{Z}_R = R}$

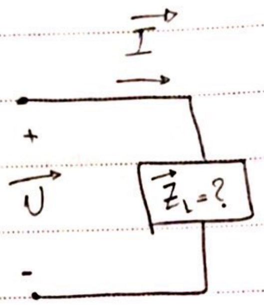


in phase

No phase shift

2 Inductors :-





$v(t) = V_m \cos(\omega t + \theta_v) \Longrightarrow \vec{V} = V_m \angle \theta_v$

$i(t) = I_m \cos(\omega t + \theta_i) \qquad \vec{I} = I_m \angle \theta_i$

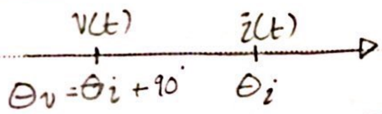
$v(t) = L \dfrac{di(t)}{\partial t}$

$= -L\, I_m\, \omega \sin(\omega t + \theta_i)$

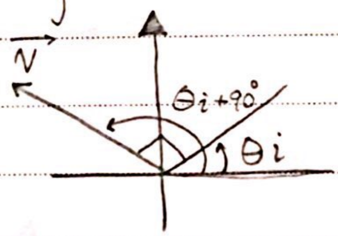
$= \omega L\, I_m \cos(\omega t + \theta_i + 90^\circ) \longrightarrow \vec{V} = \omega L\, I_m \angle \theta_i + 90^\circ$

$V_m = \omega L\, I_m$

$\theta_v = \theta_i + 90^\circ$



$i(t)$ lags $v(t)$ by 90°



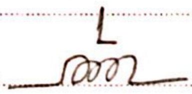
$\vec{V} = \omega L\, I_m \angle \theta_i + 90^\circ$

$\vec{V} = (\omega L \angle 90^\circ)(I_m \angle \theta_i)$

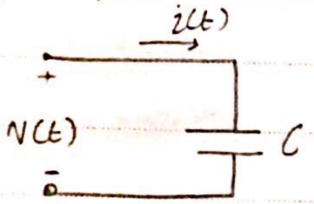
$\vec{V} = j\omega L\, \vec{I}$

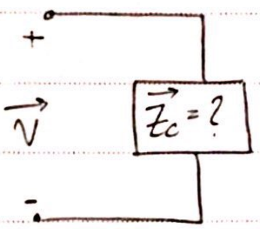
$\vec{Z}_L$

$\Longrightarrow$



3 Capacitor :-





$v(t) = V_m \cos(\omega t + \theta_v)$ $\Longrightarrow$

$i(t) = I_m \cos(\omega t + \theta_i)$

$i(t) = C \dfrac{dv}{dt}$

$\vec{V} = V_m \angle \theta_v$

$\vec{I} = I_m \angle \theta_i$

$\Rightarrow i(t) = -C V_m \omega \sin(\omega t + \theta_v)$

$= \omega C V_m \cos(\omega t + \theta_v + 90^\circ) \longrightarrow \vec{I} = \omega C V_m \angle \theta_v + 90^\circ$

$I_m = \omega C V_m$

$\theta_i = \theta_v + 90^\circ$

i(t) leads v(t) by 90°

The $-||-$ is Called leading element.

$\vec{I} = (\omega C \angle 90^\circ)(V_m \angle \theta_v)$

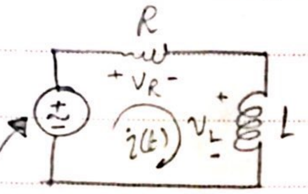
$\vec{I} = j\omega C \vec{V}$

$\vec{V} = \underbrace{\dfrac{1}{j\omega C}}_{\vec{Z_c}} \vec{I}$

$\vec{Z_c} = \dfrac{1}{j\omega C} = -j\dfrac{1}{\omega C} = \dfrac{1}{\omega C}\angle -90^\circ$

$\overset{C}{-||-} \rightarrow -||- \; \dfrac{1}{\omega C}\angle -90^\circ$

Ex: find $i(t)$ →



$v(t) = V_m \cos(\omega t + \theta)$

Sol → $-v(t) + v_R(t) + v_L(t) = 0$

$-v(t) + R\,i(t) + L\frac{di}{dt} = 0$

$\Rightarrow L\frac{di}{dt} + Ri = V_m \cos(\omega t + \theta)$

we propose → $i(t) = I_m \cos(\omega t + \phi)$?

$\Rightarrow -L I_m \omega \sin(\omega t + \theta) + R I_m \cos(\omega t + \theta) = V_m \cos(\omega t + \theta) \Rightarrow$

$\boxed{i(t) \text{ leads } v(t) \text{ by } 90^\circ}$

using $A\cos\alpha + B\sin\alpha = \sqrt{A^2+B^2}\cos\left(\alpha + \tan^{-1}\left(\frac{-B}{A}\right)\right)$

The —||— is called leading element.

$\sqrt{R^2 I_m^2 + \omega^2 L^2 I_m^2}\cos\left(\omega t + \theta + \tan^{-1}\left(\frac{\omega L I_m}{R I_m}\right)\right) = V_m \cos(\omega t + \theta)$

$I_m \sqrt{R^2+\omega^2 L^2}\cos\left(\omega t + \theta + \tan^{-1}\left(\frac{\omega L}{R}\right)\right) = V_m \cos(\omega t + \theta)$

$\rightarrow I_m = \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \rightarrow i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t + \theta - \tan^{-1}\left(\frac{\omega L}{j^2}\right)\right)$

$\theta_i = \theta - \tan^{-1}\left(\frac{V_L}{R}\right)$

OR →

$j\omega L = \omega L \angle 90^\circ \rightarrow \vec{I} = \frac{\vec{V}}{R + j\omega L} = \frac{V_m\angle\theta}{\sqrt{R^2+\omega^2L^2}\angle\tan^{-1}\left(\frac{\omega L}{R}\right)}$

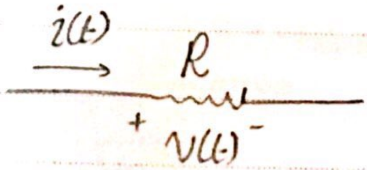
$= \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\angle\theta - \tan^{-1}\left(\frac{\omega L}{R}\right) \rightarrow$

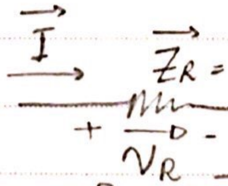
$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2 j^2}}\cos\left(\omega t + \theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$

1 Time-Domain $\longrightarrow$ phasor Domain

$v(t) = V_m \cos(\omega t + \theta)$ $\qquad \vec{V} = V_m \angle \theta^\circ$

$i(t) = I_m \cos(\omega t + \phi)$ $\qquad \vec{I} = I_m \angle \phi^\circ$

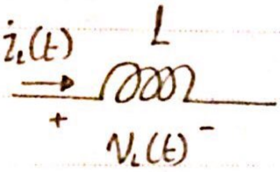


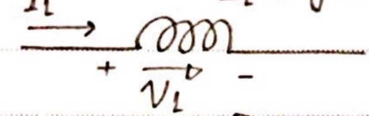


$$\vec{Z}_R = \frac{\vec{V}}{\vec{I}}$$

$$\vec{V}_R = R\,\vec{I}_R$$

2





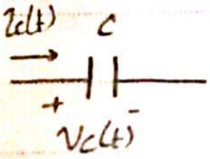
$$\vec{Z}_L = j\omega L = \omega L \angle 90^\circ$$

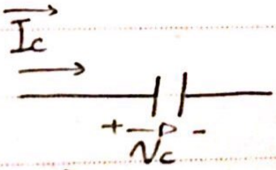
$$\vec{V}_L = \vec{Z}_L \vec{I}_L$$
$$= (j\omega L)\vec{I}_L$$

$\vec{I}_L$ lags $\vec{V}_L$ by 90°

 lagging element.

3





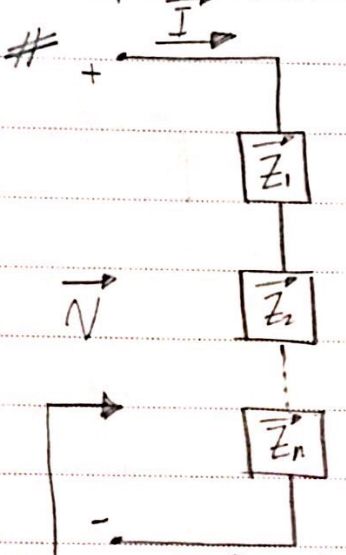
$$\vec{Z}_c = \frac{1}{j\omega C} = -j\frac{1}{\omega C} = \frac{1}{\omega C}\angle 90^\circ$$

$$\vec{V}_c = \vec{Z}_c \vec{I}_c$$

I_c leads $\vec{V}_c$ by 90°

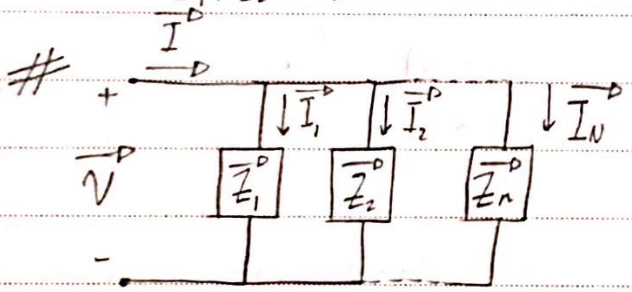
 is called leading element.

- Impedences in Series & Voltage division:

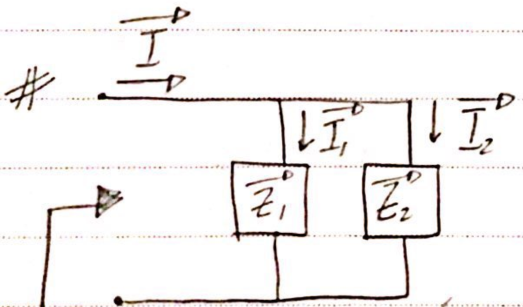


$$Z_{eq} = \vec{Z}_1 + \vec{Z}_2 + \dots \vec{Z}_n$$

$$\vec{V}_n = \frac{\vec{Z}_n}{\vec{Z}_1 + \vec{Z}_2 - \vec{Z}_n} \vec{V}$$

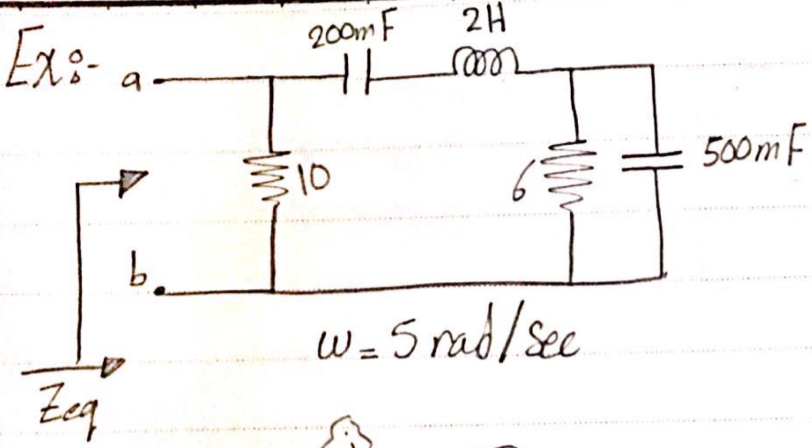


$$\vec{I}_N = \frac{\frac{1}{\vec{Z}_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \dots \frac{1}{Z_n}} \vec{I}$$ (Current division)



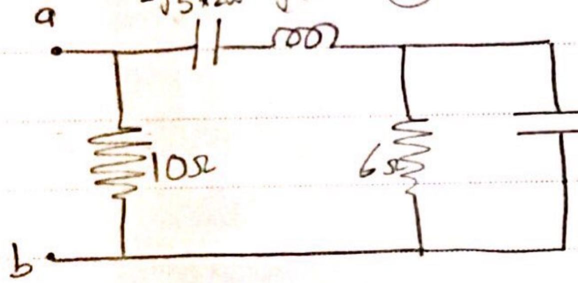
$$Z_{eq} = \frac{\vec{Z}_1 \cdot \vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \quad \Bigg\{ \quad \vec{I}_1 = \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{I}$$

Ex:- find $\vec{Z}_{eq}$ = ?



$\omega = 5$ rad/sec

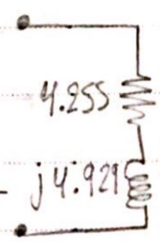
Sol



• $(-j0.4)//6\Omega = \frac{-j0.4*6}{-j0.4+6} = \frac{-j2.4}{6-j0.4} = 0.02655-j0.3985\,\Omega \cong 0.39911\angle -86.18$

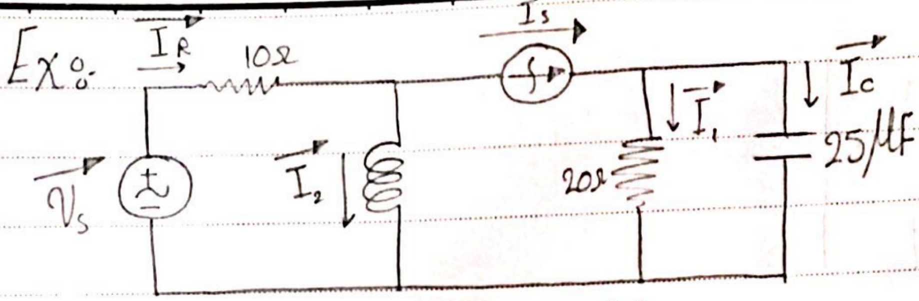
• $-j + j10 + 0.02655 - j0.3982 = 0.02655 + j8.602\,\Omega$

$Z_{eq} = (10)//(0.02655+j8.602) = \frac{0.2655+j86.02}{10.02655+j8.602} = \underbrace{4.255}_{R} + j\underbrace{4.929}_{I}$



$j4.929 \to \omega L_{eq} \Rightarrow L_{eq} = \frac{4.929}{\omega}$

Ex:



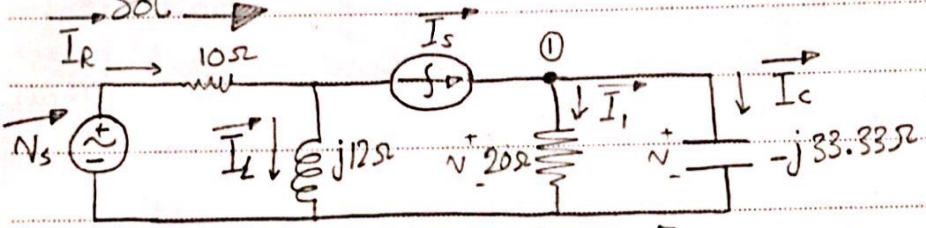
$\omega = 1200$ rad/sec

$\vec{I_C} = 1.2\angle 28^\circ$ A

$\vec{I_L} = 3\angle 53^\circ$ A

find $\vec{I_s}$, $\vec{V_s}$, $i_R(t)$?

Sol.



at ① → 1) $\vec{I_s} = \vec{I_1} + \vec{I_C}$

$$\vec{I_s} = \frac{\vec{I_C}(-j33.33)}{20} + \vec{I_C}$$

$$\vec{I_s} = \frac{(1.2\angle 28^\circ)(-j33.33)}{20} + 1.2\angle 28^\circ$$

$$\vec{I_s} = 1.99\angle -62^\circ + 1.2\angle 28^\circ$$

$$= 2.33\angle -31^\circ \text{ A}$$

2) $\vec{I_R} = \vec{I_L} + \vec{I_s}$

$$= 3\angle 53^\circ + 2.33\angle -31^\circ$$

$$= 3.99\angle 17.42^\circ \longrightarrow i_R(t) = 3.99\cos(1200t + 17.42^\circ)\text{ A}$$

3) $\vec{V_s} = ?$

$$-V_s + 10\vec{I_R} + \vec{I_L}(j12) = 0$$

$$\vec{V_s} = 10(3.99\angle 17.42^\circ) + (3\angle 53^\circ)(12\angle 90^\circ)$$

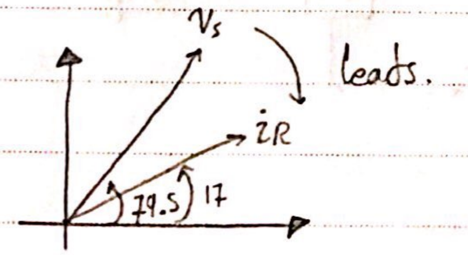
(يتبع) ⟹

/ /

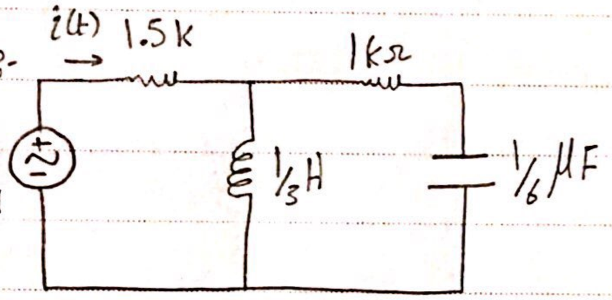
$$\vec{V_s} = 39.9\angle 17.42^\circ + 36\angle 143^\circ$$
$$= 39.9\angle 74.5\ V$$

$$V_s(t) = 39.9\cos(1200t + 74.5)\ V$$

$V_s(t)$ leads $i_R(t)$ $\Longrightarrow$



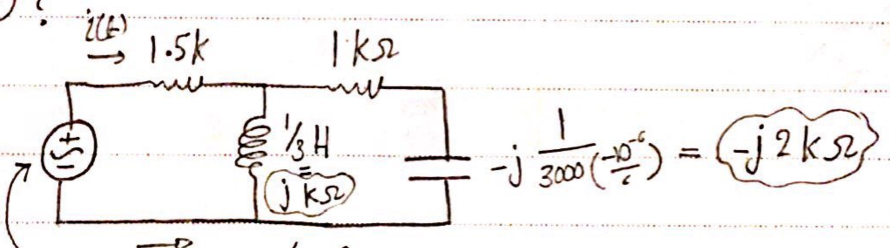
Ex:-



$V_s(t) = 40\sin(3000t)\ V$

find $i(t)$?

Sol $\rightarrow$



$$V_s(t) = \vec{V} = 40\angle -90^\circ$$

$$\vec{Z_{eq}} = \left[(1-j2)//j\right] + 1.5 = \frac{(1-j2)(j)}{(1-j2)+j} + 1.5 = 2 - j1.5\,k$$

$$\vec{I} = \frac{\vec{V}}{\vec{Z_{eq}}} = \frac{40\angle -90^\circ}{(2-j1.5)*10^3} = 16\angle -53^\circ\ mA$$

$$i(t) = 16\cos(3000t - 53^\circ)\ mA$$

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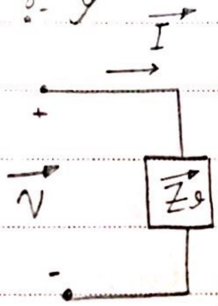
* Admittance :- $\vec{Y}$



or $\vec{Y}_s$

$$\vec{Z} = \frac{\vec{V}}{\vec{I}}$$

$$\vec{Y} = \frac{\vec{I}}{\vec{V}} = \frac{1}{\vec{Z}}$$

$$\vec{Z} = R + jX \; [\Omega]$$

$$\vec{Y} = G + jB \; (S)$$

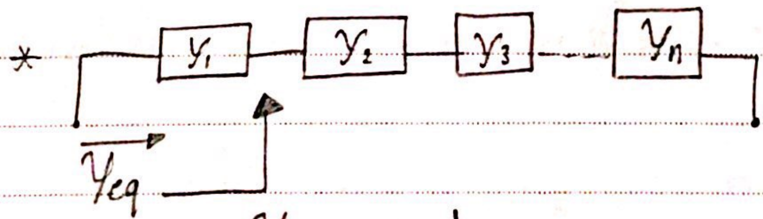
Condehntoree ↲ ↳ Susbtance

$$\vec{Y} = \frac{1}{\vec{Z}} = \frac{1}{R+jX} * \frac{R-jX}{R-jX}$$

$$\frac{R-jX}{R^2+X^2} = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2}$$

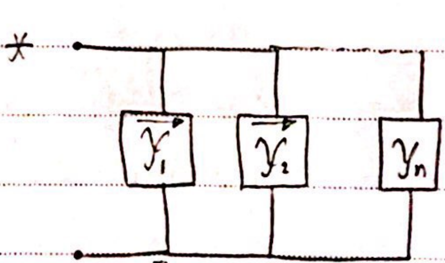
$$G = \frac{R}{R^2+X^2} \quad , \quad B = \frac{-X}{R^2+X^2}$$

*



$$Y_{eq} = \frac{1}{\frac{1}{Y_1} + \frac{1}{Y_2} + \frac{1}{Y_n}}$$

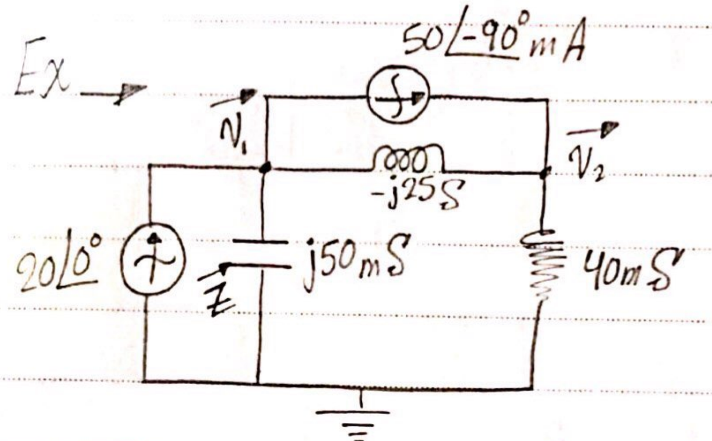
*



$$\vec{Y}_{eq} = \vec{Y}_1 + \vec{Y}_2 + \dots \vec{Y}_n$$

+ Nodel & Mesh Analysis :-

Ex →



Find $\vec{V_1}$ & $\vec{V_2}$ using Nodel Analysis

Node ①:- $20\angle0^\circ = \vec{V_1}(j50S) + (\vec{V_1} - \vec{V_2})(-j25S) + 50\angle-90^\circ A$

$j50\vec{V_1} - j25\vec{V_1} + j25\vec{V_2} = 20\angle0^\circ - 50\angle-90^\circ$

$\rightarrow$ $j25\vec{V_1} + j25\vec{V_2} = 20 + j50$ ①

Node ②:- $50\angle-40^\circ mA = (\vec{V_2} - \vec{V_1})(-j25\,mS) + \vec{V_2}\,40\,mS$

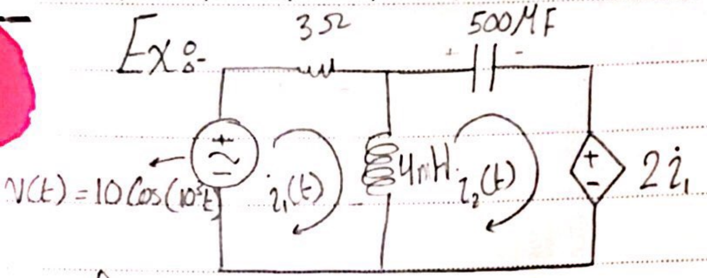
$j25\vec{V_1} + (40 - j25)\vec{V_2} = 50\angle-90^\circ$ ②

Solving →

$\vec{V_1} = 1.062\angle23.3^\circ V$

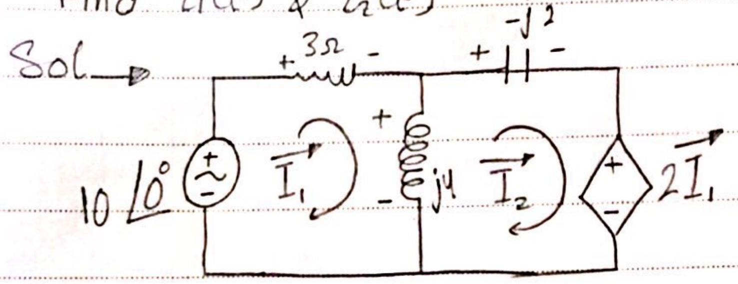
$\vec{V_2} = 1.593\angle-80^\circ V$

Ex:-



Find $i_1(t)$ & $i_2(t)$

Sol →



mesh 1 → $-10 + 3\vec{I}_1 + j4(\vec{I}_1 + \vec{I}_2) = 0$

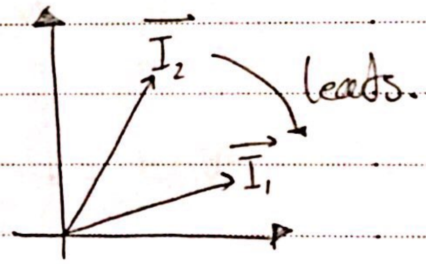
$(3+j4)\vec{I}_1 - j4\vec{I}_2 = 10$ → ①

mesh 2 → $-j4(\vec{I}_1 - \vec{I}_2) + (-j2\vec{I}_2) + 2\vec{I}_1 = 0$

$(2-j4)\vec{I}_1 + j2\vec{I}_2 = 0$ → ②

Solving ① & ② → $\vec{I}_1 = 1.24 \angle 29.7° \, A$

$\vec{I}_2 = 2.77 \angle 56.3° \, A$

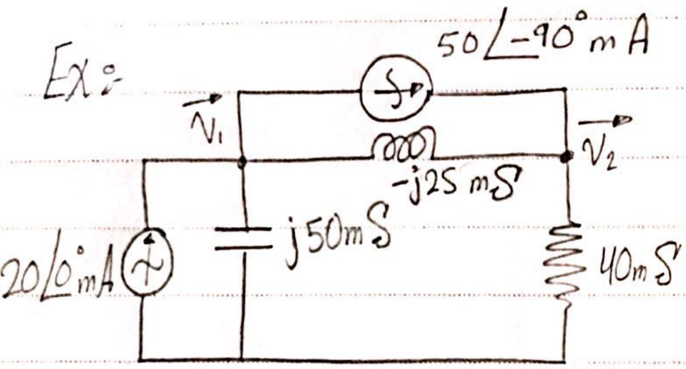


$i_1(t) = 1.24 \cos(1000t + 29.7) \, A$

$i_2(t) = 2.77 \cos(1000t + 56.3°) \, A$

* Super position:-

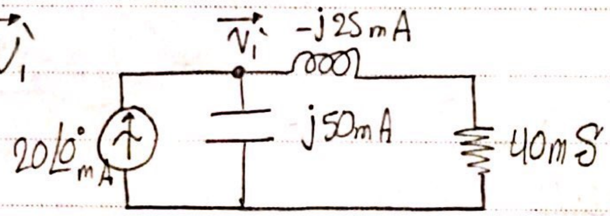
Ex:-

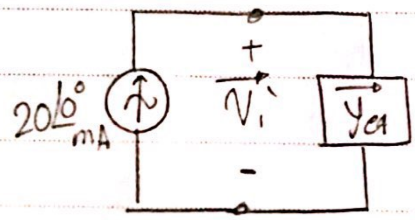


Find $\vec{V}_1$ using the superposition.

Sol →

find $\vec{V}_1'$



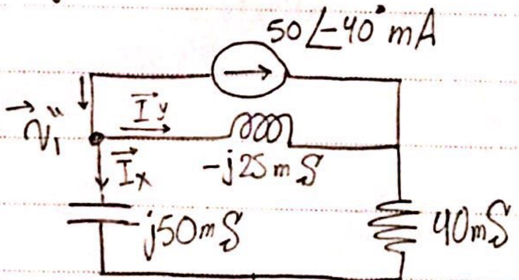


$$\vec{Y}_{eq} = \frac{(-j25m)(40m)}{(40-j25)m} + j\,50\,mS$$

$$= \frac{-j}{40-j25} + j\,50 * 10^{-3} = 0.011 + j\,0.032$$

$$\vec{V}_1' = \frac{20\angle 0^\circ}{0.011 + j0.032}\,10^{-3} = 0.195 - j0.556\ V$$

find $\vec{V}_1''$



$$\vec{I}_x = \frac{\frac{1}{-j25mS}}{\frac{1}{j25mS} + \frac{1}{40mS} + \frac{1}{j50mS}} (-50\angle -90^\circ)$$

$$= \frac{j40}{j40 + 25 + j20} (50\angle 90^\circ)$$

$$V_1'' = \vec{I}_x * (j50m)$$ ___

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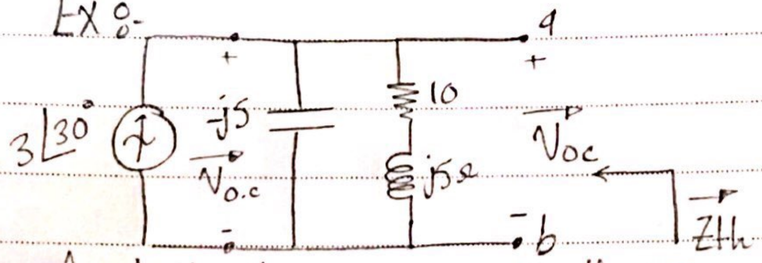
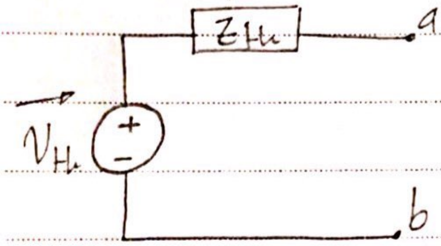
Ex :-



find the thevenin equivalent CKT
Seen by points a & b



$$\vec{Z}_{th} = (10+j5) \,//\, -j5 = \frac{(10+j5)(-j5)}{10+j5-j5} = \frac{25-j50}{10}$$

$$= 2.5 - j5 \ \Omega$$

$$V_{th} = \vec{V}_{o.c} = (3\angle 30^\circ)(2.5-j5)$$

$$= 16.77 \angle -33.43$$