



$V_{ab} = -V_{ba}$

$V_i = \frac{dw}{dq}$

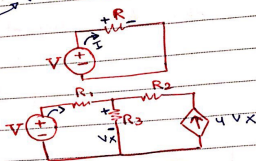
Energy (E)  $\rightarrow$  J  
Power (P)  $\rightarrow$  W

$P = \frac{dw}{dt}$

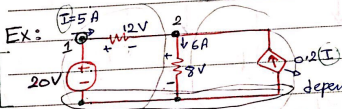
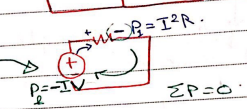
$P = VI$   $\rightarrow V = IR$

$P = I^2 R = \frac{V^2}{R}$   $P = \frac{dE}{dt}$

$P_{in} = P_{out} \rightarrow \sum P = 0$



Power is calculated as  $P = VI$  for a current source. For a dependent current source,  $P = -IV$ .



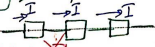
Independent P.S.  
dependent (current supply)

- Branch 3
- Node 3 nodes
- Mesh 2 meshes
- Loop 3 loops



Loop voltage control  
current is  $0.2I$   
control of  $I$

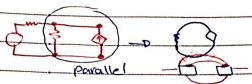
Series  $\rightarrow$  they have same current.



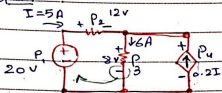
$V_{total} = V_1 + V_2 + V_3$   
 $R_T I = IR_1 + IR_2 + IR_3$   
 $R_T I = I [R_1 + R_2 + R_3]$   
 $R_T = R_1 + R_2 + R_3$



Parallel → they have same voltage.



Ex



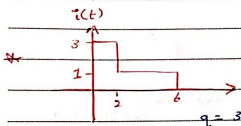
$$P_1 = -VI = -5(20) = -100 \text{ W (produce)}$$

$$P_2 = +VI = 5(12) = +60 \text{ (consumed)}$$

$$P_3 = +VI = 6(8) = +48 \text{ (c)}$$

$$P_4 = -VI = -8(0.2 \times 5) = -8 \text{ W (produce)}$$

$$\Sigma P = 0$$



$$i = \frac{dq}{dt} \Rightarrow q = \int i dt$$

$$q = 3(2) + 1(4) = 10 \text{ C}$$

$$10^{-3} \text{ m}$$

$$10^{-6} \text{ } \mu$$

$$10^{-9} \text{ n}$$

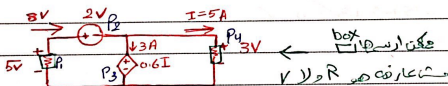
$$10^{-12} \text{ p}$$

$$10^3 \text{ K}$$

$$10^6 \text{ M}$$

$$10^9 \text{ G}$$

$$10^{12} \text{ T}$$



$$P_1 = -VI = -5(8) = -40 \text{ W}$$

$$P_2 = +VI = 2(8) = 16 \text{ W} \quad \Sigma P = 0$$

$$P_3 = +VI = 3(0.6 \times 5) = 9 \text{ W}$$

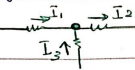
$$P_4 = +VI = 3 \times 5 = 15 \text{ W}$$

**1** \* Kirchof's Current law \* «KCL»

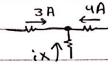
in = out →

Node كحل

$I_1 + I_3 = I_2$   
 $\sum I = 0$



$3 + 4 + ix = 0$



$7 + ix = 0$

$ix = -7$  ✓

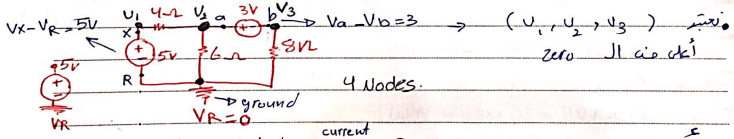
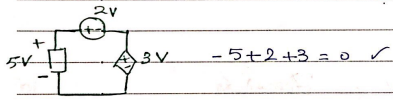
$ix = 7$  ↓ ✓

**2** Kirchof's voltage law \* «KVL»

for any loop, any closed path.

$\sum V = 0$

اذا دخلنا من هنا voltage

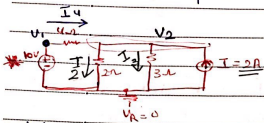
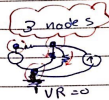
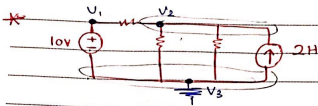


نعتبر (V1, V2, V3) zero الى

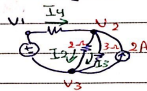
ان مقاومة موجودة بال Branch التيار في كل اتجاه ↓

تغير اتجاه ال current في كل حالة لا يكون direction ال ground node.

(V1, V2, ...) Volt في كل node.



active  
 use node 1 is reference use is 10V  
 voltage is 10V



node 1

(1) \*  $V_1 = 10V$

node 2

(2) \*  $I_4 + 2 = I_2 + I_3$

current is 2A

(3) \*  $I_4 = \frac{V_1 - V_2}{4}$

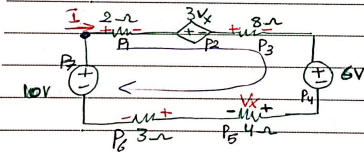
(parallel) ground is 0

(4) \*  $I_2 = \frac{V_2 - V/R}{2}$

$I = \frac{V}{R} = \frac{V_a - V_b}{R}$

(5) \*  $I_3 = \frac{V_2}{3}$

$17 - 13 = 4$



one Mesh (KVL)

current A = current B

By KVL  $\sum V = 0$

$3Vx \rightarrow 3 \times 4I$

$2I + 3Vx + 8I + 6 + 4I + 3I - 10 = 0 \rightarrow 17I + 12I - 10 + 6 = 0$

$Vx = 4I = 4 \times 4$

$29I = 4$

$P_1 = +VI = +\left(\frac{4}{29}\right) \times 2 \quad (C)$

$I = \frac{4}{29} \text{ A}$

$P_2 = +I^2 R = +\left(\frac{4}{29}\right)^2 \times 16 \times 3 \quad (9)$

$\rightarrow$

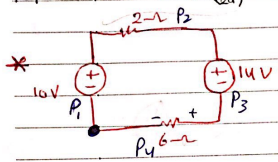
$$P_3 = +I^2 R = \left(\frac{4}{2a}\right)^2 \times 8 \quad (c)$$

$$P_4 = +VI = (6) \left(\frac{4}{2a}\right) \quad (c)$$

$$* P_5 = +I^2 R = \left(\frac{4}{2a}\right)^2 (4) \quad (c)$$

$$P_6 = +I^2 R = \left(\frac{4}{2a}\right)^2 (3) \quad (c)$$

$$P_7 = -VI = -(10) \left(\frac{4}{2a}\right) \quad (p)$$



$$P_1 = VI = (10)(-0.5) = -5W$$

$$P_2 = +I^2 R = +(-0.5)^2 (2) = 0.5W \quad (c)$$

$$P_3 = +VI = (14)(-0.5) = -7W \quad (p)$$

$$P_4 = +I^2 R = +(-0.5)^2 (6) = 1.5W \quad (c)$$

$$\Sigma P = 0$$

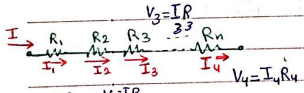
$$-10 + 2I + 14 + 6I = 0$$

$$4 + 8I = 0$$

$$I = -0.5A$$

\* Resistor  $\Omega$   $R(\Omega)$

\* conductance  $\text{mho}$   $G = 1/R$  ( $\frac{-V}{I}$ )



$$V_1 = IR_1 \quad V_2 = IR_2$$

نقطة الاولى عالية الثانية

Series

$$I = I_1 = I_2 = I_3 = \dots = I_N$$

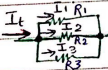
$$V_t = V_1 + V_2 + V_3 + \dots + V_N$$

$$IR = IR_1 + IR_2 + IR_3 + \dots + IR_N$$

$$R_t = R_1 + R_2 + R_3 + \dots + R_N$$

$$\frac{1}{G_t} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \dots + \frac{1}{G_N}$$

Parallel



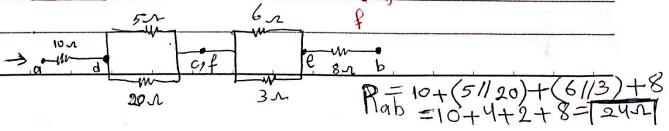
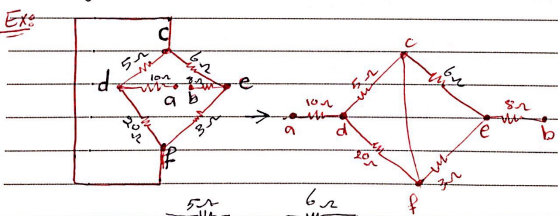
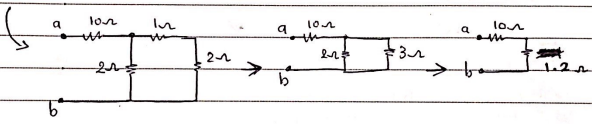
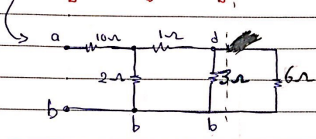
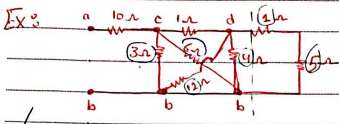
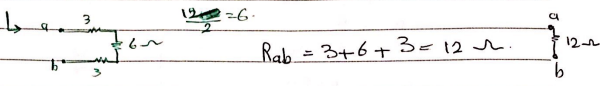
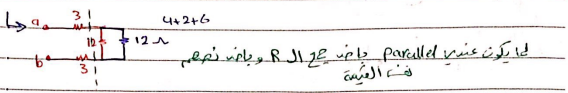
نفس البنية ونفس البنية

$$I_t = I_1 + I_2 + I_3 + \dots + I_N$$

$$\frac{V_t}{R_t} = \frac{V_t}{R_1} + \frac{V_t}{R_2} + \frac{V_t}{R_N}$$

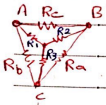
$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_N}$$

$$G_t = G_1 + G_2 + \dots + G_N$$





$Y \leftrightarrow \Delta$



$\Delta \rightarrow Y$

← Subst. plus

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_b R_a}{R_a + R_b + R_c}$$

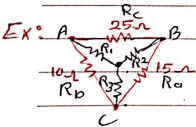
$Y \rightarrow \Delta$

← Subst. ↓

$$R_a = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

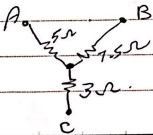
$$R_c = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$



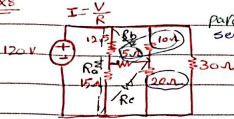
$$R_1 = \frac{25 \times 10}{15 + 10 + 25} = 5 \Omega$$

$$R_2 = \frac{15(25)}{50} = 7.5 \Omega$$

$$R_3 = \frac{150}{50} = 3 \Omega$$



EXS



parallel series

$\Delta$  wappo y nis

$V \rightarrow \Delta$

$$R_a = 75 + \frac{62.5 + 187.5}{5} = \frac{325}{5} = 65 \Omega$$

$$R_b = \frac{325}{15} = 21.6 \Omega$$

$$R_c = \frac{325}{12.5} = 26 \Omega$$



$$R_{eq} = \left[ \left[ (21.6 \parallel 10) + (26 \parallel 20) \right] \parallel 30 \right] \parallel 65 = 9.62 \Omega$$

$$I = \frac{V}{R_{eq}} = \frac{120}{9.62} = 12.49 \text{ A}$$

\* short circuit \* (SC)



(( uab short dize parallel  $\rightarrow$   $R=0$  ni ))  
 (( R del  $\rightarrow$   $\frac{1}{R} \rightarrow \infty$  ))



$$R_{sc} = 0$$

$$V_{sc} = 0$$

$$I_{\infty} = \infty$$

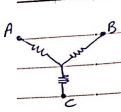
\* open circuit \* (OC)



$$I_{oc} = 0$$

$$V_{oc} = V$$

$$R_{oc} = \frac{V}{I} = \frac{V}{0} = \infty$$



Balanced

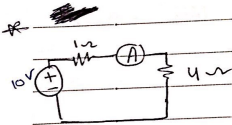
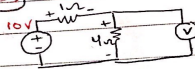
\* Measuring Devices \*

voltage → voltmeter (V)

current → Ammeter (A)

connected in parallel with load  $R_{int} \rightarrow \infty$

Resistance  
Power → connected in series with load



$$I = \frac{10}{5} = 2A$$

①  $R_{int} = 3\Omega$

$$I = \frac{10}{5+3} = 1.25A$$

$R = 0.5\Omega$

$$I = \frac{10}{1+4+0.5} = 1.818A$$

$$I = \frac{10}{4+1} = 2A$$

$$I' = \frac{10}{1+3.3} = 2.3$$

①  $R_{int} = 25\Omega$

$$4 \parallel 25 = \frac{100}{29} \approx 3.3\Omega$$

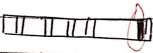
②  $R_{int} = 1000\Omega$

$$1000 \parallel 4 = \frac{4000}{1004} = 3.98\Omega$$

~~$I = \frac{10}{4+1} = 2A$~~   
 $I = \frac{10}{4.9} \approx 2.04A$



circuit like in the picture ...  
 ... (more accurate)



silver  $\pm 10\%$   
 gold  $\pm 5\%$   
 colorless 20%

B Black 0

B Brown 1

R Red 2

O Orange 3

Y Yellow 4

G green 5

B Blue 6

V violet 7

G gray 8

W white 9

red blue green G  
 orange

2630000

... (more accurate)

Ex:

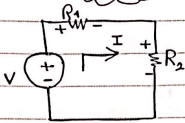
red blue orange



26000  $\rightarrow$  26k $\Omega$

\* voltage divider \*

R in series

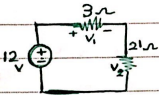


$$\bullet V_1 = \frac{VR_1}{R_1 + R_2}$$

$$\bullet V_2 = \frac{VR_2}{R_1 + R_2}$$

$$\bullet I = \frac{V}{R_1 + R_2}$$

Ex:

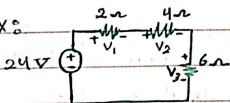


$$\bullet V_1 = \frac{12(3)}{3+21} = \frac{36}{24} = 1.5 \text{ V}$$

$$\bullet V_2 = 12 - 1.5 = 10.5 \text{ V}$$

$$\hookrightarrow \frac{12(21)}{24} = 10.5 \text{ V}$$

Ex:



$$\bullet V_1 = \frac{24(2)}{2+4+6} = 4 \text{ V}$$

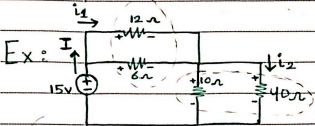
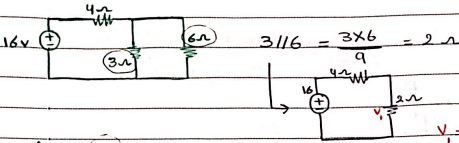
$$\bullet V_2 = \frac{24(4)}{2+4+6} = 8 \text{ V}$$

$$\bullet V_3 = \frac{24(6)}{2+4+6} = 12 \text{ V}$$

$$\hookrightarrow 24 - (8+4) = 12 \text{ V}$$

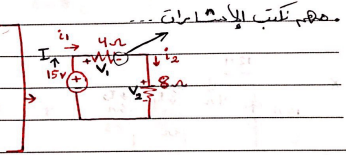


Ex:



•  $12 // 6 \rightarrow \frac{12 \times 6}{12 + 6} = 4 \Omega$

•  $10 // 40 \rightarrow \frac{10 \times 40}{10 + 40} = 8 \Omega$



→  $V_1 = \frac{15(4)}{4+8} = 5 \text{ V}$

•  $i_2 = \frac{V_1}{R_{12}} = \frac{5}{12} \text{ A}$

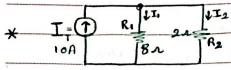
→  $V_2 = \frac{15(8)}{4+8} = \frac{120}{12} = 10 \text{ V}$

•  $i_2 = \frac{V_2}{R} = \frac{10}{40} = \frac{1}{4} \text{ A}$

•  $I_{\text{total}} = \frac{V}{R_{\text{eq}}} = \frac{15}{12} \text{ A}$   
 $\quad \quad \quad \downarrow (8+4)$

\* Current divider \* R in parallel

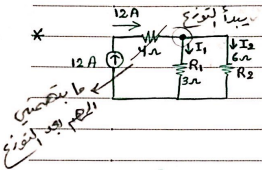
نظريتي basic law بالقياس  
 R ga l'interconnection Current.



$$8 // 2 = \frac{8 \times 2}{8 + 2} = \frac{16}{10}$$

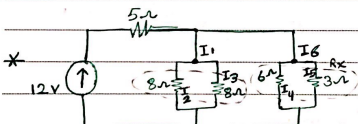
$$I_1 = \frac{I_T \times R_2}{R_1 + R_2} = \frac{10 \times 2}{8 + 2} = \frac{20}{10} = 2A$$

$$I_2 = \frac{I_T \times R_1}{R_1 + R_2} = \frac{10 \times 8}{8 + 2} = \frac{80}{10} = 8A$$



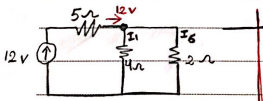
$$I_1 = \frac{12 \times 6}{9} = 8A$$

$$I_2 = \frac{12 \times 3}{9} = 4A$$



$$3 // 6 \Rightarrow R_x = \frac{3 \times 6}{3 + 6} = 2 \Omega$$

$$8 // 8 \Rightarrow R_y = \frac{8 \times 8}{8 + 8} = 4 \Omega$$



$$I_1 = \frac{12 \times 2}{4 + 2} = 4A$$

$$I_6 = \frac{12 \times 4}{6} = 8A$$

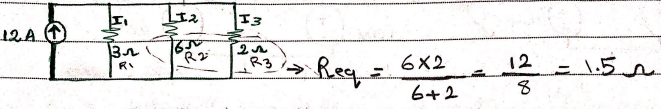
$$I_2 = \frac{I_1 \times 8}{8 + 8} = 2A$$

$$I_4 = \frac{I_6 \times 3}{6 + 3} = \frac{24}{9} A$$

$$I_3 = I_2 = 2A$$

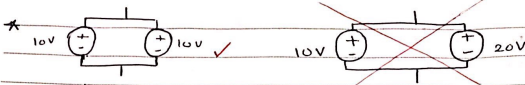
$$I_5 = \frac{8 \times 6}{9} = \frac{48}{9} A$$

Ex 2

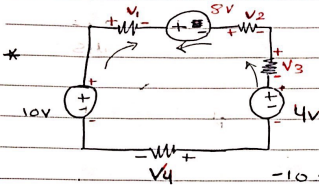


•  $I_1 = \frac{I_T \times R_{eq}}{R_1 + R_{eq}} = \frac{12 \times 1.5}{3 + 1.5} = \frac{18}{4.5} = 4A$

•  $I_{R3} = \frac{12 \times 3}{4.5} = 8A$

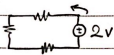


• لا يكون "two voltages in parallel"  
 • لانه يكونوا نفس القطب

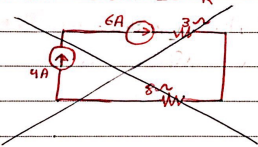


$$-10 + V_1 + 8 + V_2 + V_3 + 4 + V_4 = 0$$

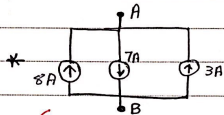
$$V_1 + V_2 + V_3 + V_4 = \boxed{2}$$



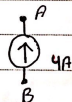
\* Current Sources \*



طابق انه يتالف ال current



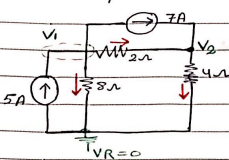
$$8 - 7 + 3 = 4A$$



CH "3"

\* Methods of Analysis \*

• Nodal analysis



لما يكون فيه أكثر من 2 nodes  
الحل بكتابة نودال  
nodal analysis

# equ = # nodes - 1

① select the ~~active~~ <sup>reference</sup> node.

↳ (without the reference)

② select the active nodes and sign ( $V_1, V_2, \dots, V_n$ )  
for each node.

③ Apply KCL

for each active node.

(n-1) equ.

(كل التيارات الداخلة موجبة والخارجة سالبة)  
VR → current ↓ يكون

④ solve the equ.

# node 1

$$I_{in} = I_{out}$$

$$5 = 7 + \frac{V_1 - V_R}{5} + \frac{V_1 - V_2}{2}$$

$$40 = 56 + V_1 + 4V_1 - 4V_2$$

$$\boxed{5V_1 - 4V_2 = -16} \quad \text{①}$$

# node 2

$$7 + \frac{V_1 - V_2}{2} = \frac{V_2}{4}$$

$$28 + 2V_1 - 2V_2 = V_2$$

$$3V_2 - 2V_1 = 28$$

$$\boxed{-2V_1 + 3V_2 = 28} \quad \text{②}$$

\* calculator

• mod → 5 → 1

•  $V_1 = \frac{64}{7}$

• وينظم بالترتيب

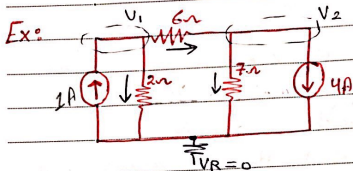
•  $V_2 = \frac{108}{7}$



$$\rightarrow I_{8\Omega} = \frac{V_1}{8} = \frac{64}{7} \times 8 \text{ A}$$

$$I_{2\Omega} = \frac{V_1 - V_2}{2} = \left( \frac{64}{7} - \frac{108}{7} \right) \text{ A}$$

$$I_{4\Omega} = \frac{V_2 - V_R}{4} = \frac{108}{7} \times 4 \text{ A}$$



# Node 1

$$1 = \frac{V_1}{2} + \frac{V_1 - V_2}{6}$$

or we

$$\hookrightarrow 6 = 3V_1 + V_1 - V_2$$

$$\boxed{4V_1 - V_2 = 6} \quad (1)$$

(1+2)

# Node 2

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$$7V_1 - 7V_2 = 168 + 6V_2$$

$$\boxed{7V_1 - 13V_2 = 168} \quad (2)$$

$$V_1 = -2$$

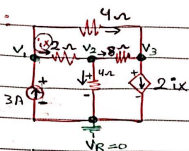
$$V_2 = -14$$

$$\bullet I_{2\Omega} = \frac{V_1}{2} = \frac{-2}{2} = \boxed{-1 \text{ A}}$$

$$\bullet I_{7\Omega} = \frac{V_2}{7} = \boxed{-2 \text{ A}}$$

$$\bullet I_{6\Omega} = \frac{V_1 - V_2}{6} = \frac{-2 - (-14)}{6} = \boxed{2 \text{ A}}$$

Ex:



# node (1)

$$(3 = \frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4}) \quad \times 4$$

$$12 = 2v_1 - 2v_2 + v_1 - v_3$$

$$12 = 3v_1 - 2v_2 - v_3 \quad \text{--- (1)}$$

# node (2)

$$\left( \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2}{4} \right) \times 8$$

$$4v_1 - 4v_2 = v_2 - v_3 + 2v_2$$

$$4v_1 - 7v_2 + v_3 = 0 \quad \text{--- (2)}$$

# node (3)

$$\left( \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = 2 \left( \frac{v_1 - v_2}{2} \right) \right) \times 8$$

$$2v_1 - 2v_3 + v_2 - v_3 = 8v_1 - 8v_2$$

$$0 = 6v_1 - 9v_2 + 3v_3$$

Calculator  $\rightarrow v_1 = 4.8V, v_2 = 2.4V, v_3 = -2.4V$

\* Krammer's Rule \*

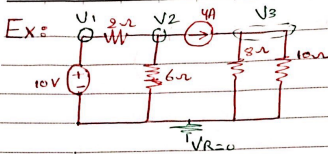
$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ -6 & 9 & -3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\bullet v_1 = \frac{\Delta_1}{\Delta}, v_2 = \frac{\Delta_2}{\Delta}, v_3 = \frac{\Delta_3}{\Delta} \bullet$$

$$\rightarrow P_{3A} = I^2 R = \left( \frac{v_2 - v_3}{8} \right)^2 \times 8 \text{ W} \rightarrow + \left( \frac{2.4 + 2.4}{8} \right)^2 \times 8 \text{ W} \quad (C)$$

$$P_{2ix} = IV = + 2ix(v_3) = +2 \quad (C)$$

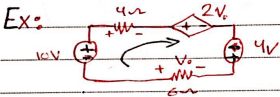
$$P_{3A} = -IV = -3(4.8) \quad (P)$$



\* Node(1):  $V_1 = 10V$

\* Node(2):  $\frac{V_1 - V_2}{2} = 4 + \frac{V_2}{6}$        $V_2 = \sqrt{\frac{6}{4}}$

\* Node(3):  $4 = \frac{V_3}{8} + \frac{V_3}{10}$



(  $V_o = -6I$  )

$-10 + 4I + 2V_o - 4 + 6I = 0$

$-10 + 4I + 2(-6I) + 4 + 6I = 0$

(تأثير المصدر المعتمد فقط)

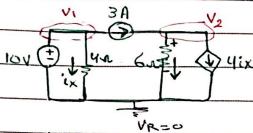
التيار في المقاومة 4Ω

التيار في المقاومة 6Ω

(تأثير المصدر 4V)

## \* Nodal Analysis with Voltage source \*

Ex:



\* Node # 1

$$V_1 = 10V \quad \text{①}$$

\* Node # 2

$$\frac{V_2}{6} + 4i_x = 3 \rightarrow i_x = \frac{V_1}{4} = \frac{10}{4} = 2.5A$$

$$\frac{V_2}{6} + 4(2.5) = 3$$

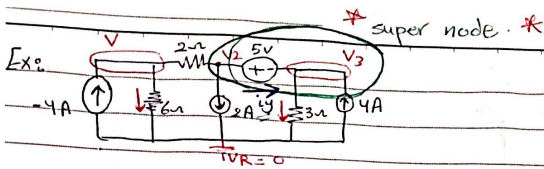
$$\frac{V_2}{6} = -7 \rightarrow V_2 = -42V$$

$$\bullet P_{6\Omega} = I^2 R = \frac{V^2}{R} = \frac{(-42)^2}{6} = \quad \text{W (s)}$$

$$\bullet P_{4i_x} = IV = 4i_x \cdot V_2 = 4(2.5) \times -42 = \quad \text{(p)}$$

$$\bullet P_{3A} = VI = (V_1 - V_2)(3) =$$

$$\bullet P_{10V} = -VI = -10(i_x + 3) = -5.5 \text{ W (p)}$$



Node #1

$$I_{in} = \text{out}$$

$$-4 = \frac{v_1}{6} + \frac{v_1 - v_2}{2}$$

$\times 6$

$$-24 = v_1 + 3v_1 - 3v_2$$

$$4v_1 - 3v_2 = -24 \quad \text{--- (1)}$$

Node  $\rightarrow$  super Node:

$$4 + \frac{v_1 - v_2}{2} = 2 + \frac{v_3}{3}$$

$$24 + 3v_1 - 3v_2 = 12 + 2v_3$$

$$3v_1 - 3v_2 - 2v_3 = -12 \quad \text{--- (2)}$$

$$v_2 - v_3 = 5 \quad \text{--- (3)}$$

$$\rightarrow v_1 = \frac{-54}{11} \text{ V}, \quad v_2 = \frac{16}{19} \text{ V}$$

Find  $i_y$ :

by KCL

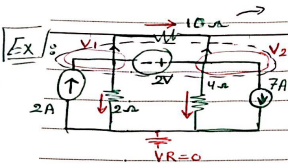
$$v_3 = \frac{-39}{11} \text{ V}$$

$$4 + I_y = \frac{v_3}{3} \rightarrow I_y = -4 + \frac{39}{11} = \underline{\underline{-5.18 \text{ A}}}$$

$$* P_{I_y} = IV = (-5.18) \times 5 = \dots \text{ W (P)}$$



في الـ 10Ω المقاومة  
node الـ 1  
المقاومة 10Ω



2 nodes...  $V$  ...  
بسطها

\* super node:  $I_n = out.$

$$2 = \frac{V_1}{2} + \frac{V_2}{4} + 7$$

$$-5 = \frac{V_1}{2} + \frac{V_2}{4} \quad \times 8$$

$$-20 = 2V_1 + V_2$$

$$-40 = 4V_1 + 2V_2$$

$$-2 = +V_1 - V_2$$

$$-20 = 2V_1 + V_2$$

$$\frac{-22}{3} = \frac{3V_1}{3}$$

$$2 = -V_1 + V_2$$

$$V_1 = \frac{22}{3}$$

$$V_2 - V_1 = 2$$

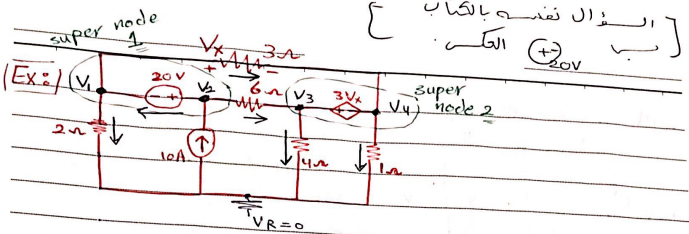
$$V_1 = \frac{-22}{3} \text{ V}, V_2 = \frac{-16}{3} \text{ V}$$

في الـ  $V_1 - V_2$  المقاومة

بسطها اول الاقرب للـ 1

$$I_{10\Omega} = \frac{V_1 - V_2}{10} = \frac{-2}{10} = -0.2 \text{ A}$$

المقاومة 10Ω في الـ  $V$  الـ 1  
بسطها اول الاقرب للـ 1



\* Super node "1".

$$10 = \frac{V_1}{2} + \frac{V_1 - V_4}{3} + \frac{V_2 - V_3}{6} \quad \times 6$$

$$60 = 3V_1 + 2V_1 - 2V_2 + V_2 - V_3$$

$$5V_1 + V_2 - V_3 - 2V_4 = 60 \quad \text{--- ①}$$

\* Super node "2".

$$\frac{V_2 - V_3}{6} + \frac{V_1 - V_4}{3} = \frac{V_3}{4} + \frac{V_4}{1} \quad \times 6$$

$$V_2 - V_3 + 2V_1 - 2V_4 = 1.5V_3 + 6V_4$$

$$2V_1 + V_2 - 2.5V_3 - 8V_4 = 0 \quad \text{②}$$

$V_1 =$

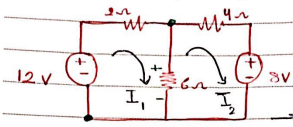
$V_2 =$

$V_3 =$

$$V_2 - V_1 = 20 \quad \text{③}$$

$$V_3 - V_4 = 3V_x \rightarrow \frac{V_3 - V_4}{3} = V_x = V_1 - V_4 \quad \text{④}$$

\* Mesh Analysis \*



1] assign currents  $i_1, i_2, \dots, i_n$

2] apply KVL

3] solve.

2 meshes, 3 loops.

\* mesh #1

$$-12 + 2I_1 + 6(I_1 - I_2) = 0$$

$$8I_1 - 6I_2 = 12 \quad \text{--- ①}$$

$$I_1 = 1.63$$

\* mesh #2

$$I_2 = 0.18$$

$$4I_2 + 8 + 6(I_2 - I_1) = 0$$

$$-6I_1 + 10I_2 = -8 \quad \text{--- ②}$$

ماتریس  
matrix

$$\begin{bmatrix} 8 & -6 \\ -6 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{42}{44} = 1.6A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-8}{44} = 0.18A$$

$$2 \times 2 \qquad \qquad 2 \times 1$$

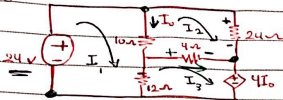
$$\Delta = 80 - 36 = 44$$

$$\Delta_1 = \begin{vmatrix} 12 & -6 \\ -8 & 10 \end{vmatrix} = 120 - 48 = 72$$

$$\Delta_2 = \begin{vmatrix} 8 & 12 \\ -6 & -8 \end{vmatrix} = -64 + 72 = 8$$

$$P_{6\Omega} = I^2 R = (I_1 - I_2)^2 (6) = \dots$$

Ex:



3 meshes:

\* Mesh # 1.

$$-24 + 10(I_1 - I_2) + 12(I_1 - I_3) = 0$$

$$22I_1 - 10I_2 - 12I_3 = 24 \quad \text{--- (1)}$$

\* Mesh # 2.

$$24I_2 + 4(I_2 - I_3) + 10(I_2 - I_1) = 0$$

$$-10I_1 + 28I_2 - 4I_3 = 0 \quad \text{--- (2)}$$

\* Mesh # 3.

$$4I_0 + 12(I_3 - I_1) + 4(I_3 - I_2) = 0$$

but  $I_0 = I_1 - I_2$

$I_0 \downarrow, I_1 \downarrow$   
 $I_1 > I_2$

$$4(I_1 - I_2) + 12(I_3 - I_1) + 4(I_3 - I_2) = 0$$

$$-8I_1 - 8I_2 + 16I_3 = 0 \quad \text{--- (3)}$$

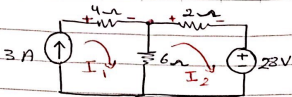
$$I_1 = 2.25A, I_2 = 0.75A, I_3 = 1.5A$$

$$P_{24V} = -I_1 V = -(2.25)(24) = 54W$$

$$P_{4\Omega} = (I_3 - I_2)^2 \times 4 = \quad \quad \quad I_3 > I_2 \quad \quad \quad W \quad (c)$$

$$P_{4V} = +I_0 V = 4I_0 = I_3(4)(I_1 - I_2)$$

Exc



\*  $I_1 = 3A$

--  $I_2$  في اتجاه التيار في المقاومة

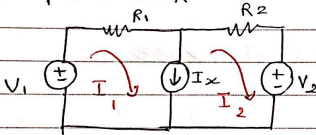
\*  $2I_2 + 28 + 6(I_2 - I_1) = 0$

$8I_2 = -28 + 18$

$8I_2 = -10$

$I_2 = -\frac{10}{8} A$

\* Super Mesh \*



"loop"

$R_1 I_1 + R_2 I_2 + V_2 - V_1 = 0$

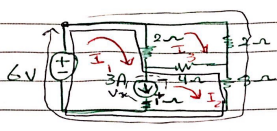
$(I_1 - I_2) = I_x$

(نحو  $I_1 > I_2$  لتيار  $I_x$  في اتجاهه)

.. super mesh ..

### KVL (Mesh analysis)

Ex:



↳ left loop up & right  
↳ left & right

# mesh 3

$$2I_3 + 4(I_3 - I_2) + 2(I_3 - I_1) = 0$$

$$-2I_1 - 4I_2 + 8I_3 = 0 \quad (1)$$

$$* I_1 - I_2 = 3 \quad (2)$$

$$-6 + 2I_3 + 8I_2 = 0$$

$$8I_2 + 2I_3 = 6 \quad (3)$$

$$\begin{cases} I_1 = 3.47 \text{ A} \\ I_2 = 0.47 \text{ A} \\ I_3 = 1.1 \text{ A} \end{cases}$$

•  $P_{3A} = IV$  mesh 2.

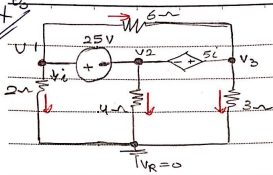
$$4(I_2 - I_3) + 8I_2 + 1(I_2 - I_1) + V_x = 0$$

$$4(0.47 - \cancel{3.47}) + 8(0.47) + 1(0.47 - 3.47) + V_x = 0$$

$$- V_x = 1.49$$

$$\rightarrow P_{3A} = -3 \times 1.49 = -4.47 \text{ W (p)}$$

Ex 2



nodal analysis

$$\bullet \left( \frac{v_1}{2} + \frac{v_2}{4} + \frac{v_3}{3} = 0 \right) \times 12 \quad \textcircled{1} \rightarrow 6v_1 + 3v_2 + 4v_3 = 0$$

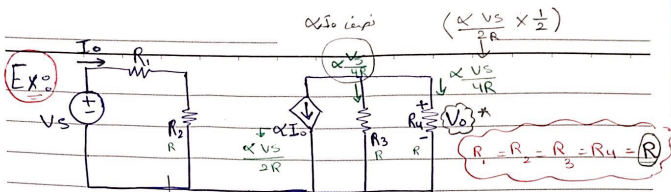
$$\bullet v_1 - v_2 = 25 \quad \textcircled{2}$$

$$\bullet v_3 - v_2 = 5i = 5 \left( \frac{v_1}{2} \right) \quad \textcircled{3}$$

$$\left( v_3 - v_2 = \frac{5v_1}{2} \right) \times 2 \rightarrow 5v_1 + 2v_2 - 2v_3 = 0$$

$$v_1 = 7.6V, \quad v_2 = -17.3V, \quad v_3 = 1.63V.$$





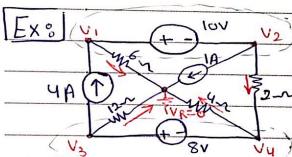
$$I_0 = \frac{V_s}{R_1 + R_2} = \frac{V_s}{2R}$$

$$V_o = R \cdot \frac{\alpha V_s}{4R} = \frac{\alpha V_s}{4}$$

$$\frac{V_o}{V_s} = \frac{\alpha V_s}{4} \times \frac{1}{V_s} = \frac{\alpha}{4}$$

[b]  $\alpha \rightarrow \left| \frac{V_o}{V_s} \right| = 10$

$$\left| \frac{\alpha}{4} \right| = 10 \rightarrow \alpha = 40$$



super node (1)  
 solve --  
 \* super node #1.  
 $(4 - \frac{V_1}{6} + 1 + \frac{V_2 - V_4}{2}) \times 6$   
 $24 = V_1 + 6 + 3V_2 - 3V_4$   
 $18 = V_1 + 3V_2 - 3V_4$  (1)

super node (2)  
 $(\frac{V_2 - V_4}{2} - \frac{V_4}{4} + \frac{V_3}{12} + 4) \times 12$  |  $V_1 - V_2 = 10$  (3)

$6V_2 - 6V_4 = 3V_4 + V_3 + 48$  |  $V_3 - V_4 = 8$  (4)  
 $6V_2 - V_3 - 9V_4 = 48$  (2)

$V_1 = 6V, V_2 = -4V, V_3 = 0, V_4 = -8V$

\* Super position principle \*

① Independent source

⊕ voltage =  $\infty$

⊕ current =  $0\Omega$

each time you should have one source only

$$i_x = i_{x_1} + i_{x_2} + \dots$$

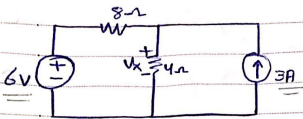
② لا تترك الدارة المفتوحة مع مصدر الجهد

بجانبك د. ص. د.

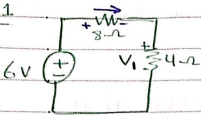
" ارفع ارجلك عن كل خطوة "

③ انتبه لأقطاب التيارات المتعددة

Exo:



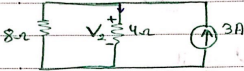
→ \* source 1



$$V_1 = \frac{6(4)}{12} = 2V^+$$

.. voltage divider ..

→ source 2



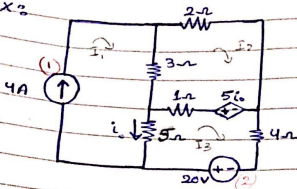
.. current divider ..

$$\bullet I_{4\Omega} = \frac{3(8)}{12} = 2A$$

$$\bullet V_2 = 2 \times 4 = 8V^+$$

$$\bullet V = 8 + 2 = 10V$$

Ex 2



\* Source # 1 \*

$$i_1 = 4A$$

$$i_o = i_1 - i_3$$

$$* 2i_2 - 5i_o + 1(i_2 - i_3) + 3(i_2 - i_1) = 0$$

$$2i_2 - 5(i_1 - i_3) + i_2 - i_3 + 3i_2 - 3i_1 = 0$$

$$2i_2 - 5i_1 + 5i_3 + i_2 - i_3 + 3i_2 - 3i_1 = 0$$

$$-8i_1 + 6i_2 + 4i_3 = 0 \rightarrow \boxed{6i_2 + 4i_3 = 32} \quad (1)$$

$$* 4i_3 + 5(i_3 - i_1) + 1(i_3 - i_2) + 5i_o = 0$$

$$4i_3 + 5i_3 - 5i_1 + i_3 - i_2 + 5(i_1 - i_3) = 0$$

$$4i_3 + 5i_3 - 5(4) + i_3 - i_2 + 20 - 5i_3 = 0$$

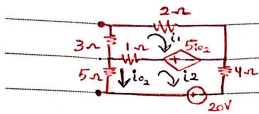
$$\boxed{-i_2 + 5i_3 = 0} \quad (2)$$

$$\boxed{i_o} = i_1 - i_3 = 4 - 0.94 = 3A \downarrow$$

$$i_2 = 4.7A$$

$$i_3 = 0.94A$$

\* Source # 2 \*



$$* 2i_1 + 5i_{02} + 1(i_1 - i_2) + 3i_1 = 0$$

$$i_{02} = -i_2$$

$$2i_1 + 5(-i_2) + i_1 - i_2 + 3i_1 = 0$$

$$2i_1 + 5i_2 + i_1 - i_2 + 3i_1 = 0$$

$$\boxed{6i_1 + 4i_2 = 0} \quad (1)$$

$$* 4i_2 - 20 + 5i_2 + 1(i_2 - i_1) + 5(-i_2)$$

$$\boxed{-i_1 + 5i_2 = 20} \quad (2)$$

$$i_1 = -2.4 \text{ A}$$

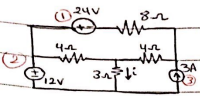
$$i_2 = 3.5 \text{ A}$$

$$\boxed{i_{02}} = -3.5 \text{ A} \downarrow$$

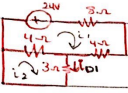
$$3.5 \text{ A} \uparrow \quad \frac{5}{1}$$

$$\rightarrow \boxed{i_0} = i_{01} + i_{02} = 3.1 - 3.5 = -0.4 \text{ A}$$

Ex 10



\* Source # 1



$$* 8i_1 + 4i_1 + 4(i_1 - i_2) + 24 = 0$$

$$12i_1 + 4i_1 - 4i_2 = -24$$

$$\boxed{16i_1 - 4i_2 = -24} \quad (1)$$

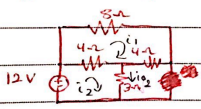
$$* 4(i_2 - i_1) + 3i_2 = 0$$

$$4i_2 - 4i_1 + 3i_2 = 0$$

$$\boxed{-4i_1 + 7i_2 = 0} \quad (2)$$

$$\rightarrow i_1 = -1.75 \text{ A}, i_2 = -1 \text{ A} \quad \boxed{i_1 = i_2 = -1 \text{ A}} \downarrow$$

\* Source # 2



$$* 8i_1 + 4i_1 + 4(i_1 - i_2)$$

$$8i_1 + 4i_1 + 4i_1 - 4i_2 = 0$$

$$\boxed{16i_1 - 4i_2 = 0} \quad (1)$$

$$i_1 = 0.5 \text{ A}$$

$$i_2 = 2 \text{ A}$$

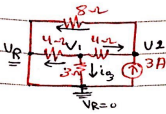
$$i_{02} \downarrow = i_2 = 2 \text{ A}$$

$$* 4(i_2 - i_1) + 3i_2 = 12$$

$$4i_2 - 4i_1 + 3i_2 = 12$$

$$\boxed{-4i_1 + 7i_2 = 12} \quad (2)$$

\* source #3



node #1

$$\left( \frac{V_1}{4} + \frac{V_1 - V_2}{4} + \frac{V_1}{3} = 0 \right) \times 12$$

$$3V_1 + 3V_1 - 3V_2 + 4V_1 = 0$$

$$10V_1 - 3V_2 = 0$$

node #2

$$\left( -3 + \frac{V_2 - V_1}{4} + \frac{V_2}{8} \right) \times 8 = 0$$

$$-24 + 2V_2 - 2V_1 + V_2 = 0$$

$$-2V_1 + 3V_2 = +24$$

$$\cdot V_1 = 3V$$

$$\cdot V_2 = 10V$$

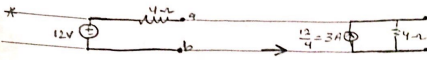
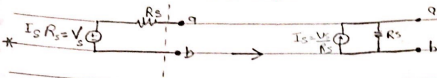
$$\cdot i_{ox} = \frac{V_1}{3} = 1A \downarrow$$

$$\rightarrow i_o = i_{o1} + i_{o2} + i_{o3} \quad *$$

$$= -1 + 2 + 1 = 2A \downarrow$$

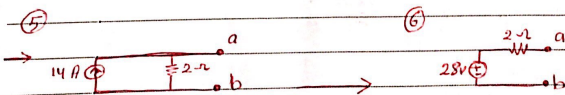
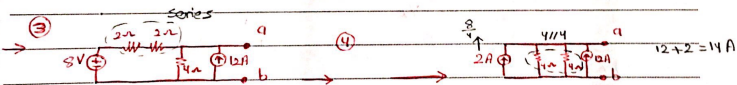
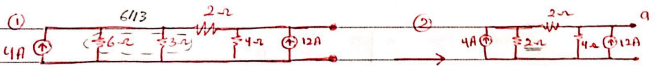


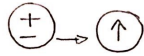
## \* Source Transformation \*



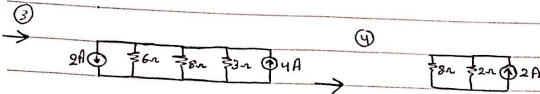
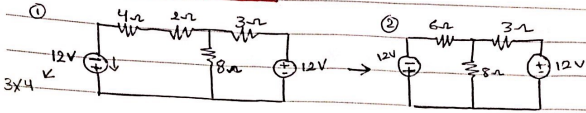
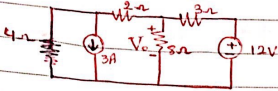
Ex:

- simplify to one current source and one resistance.





Ex: Find  $V_o$  using source transformation

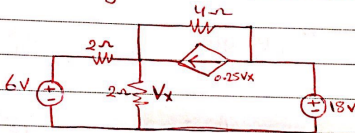


$$I_{8\Omega} = \frac{2(2)}{2+2} = 0.4 \text{ A}$$

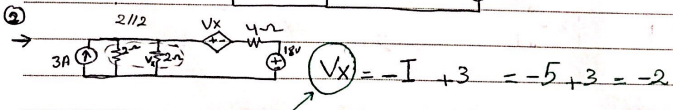
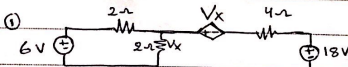
$$V_{8\Omega} = 0.4(8) = 3.2 \text{ V}$$

$$P = -IV = -(2)(3.2) = -6.4 \text{ W}$$

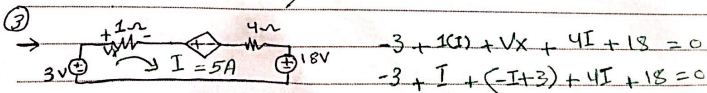
Ex: Find  $V_x$  using source transformation.



$$(0.25V_x) \times 4 = 1V_x$$



$$V_x = -I + 3 = -5 + 3 = -2$$



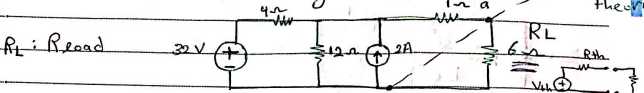
$$-3 + 1(I) + V_x + 4I + 18 = 0$$

$$-3 + I + (-I+3) + 4I + 18 = 0$$

# Thevenin's Theorem

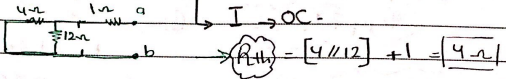
\* Independent source \*

Ex: Find the current through  $6\Omega$  by thevenin's theorem.



① cut the load and assign the terminals as a, b.

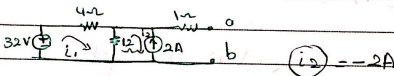
② Kill all indep. source  $\rightarrow V \rightarrow$  SC



$$R_{th} = [4 \parallel 12] + 1 = [4\Omega]$$

③ Reconnect the sources " calculate the voltage between a, b "

$$V_{oc} = V_{th}$$



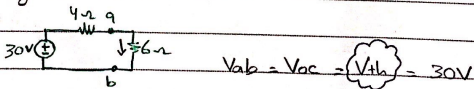
$$-32 + 4i_1 + 12(i_1 - (-2)) = 0$$

$$16i_1 = 32 - 24 = 8 \rightarrow i_1 = 0.5A$$

$$V_{ab} = V_{oc} = V_{th} =$$

$$V_{12\Omega} = 12(i_1 - i_2) = 12(0.5 + 2) = 30V$$

④ Draw thevenin's equivalent " connect the load, do your calculation "

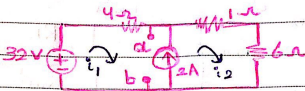


$$I_{6\Omega} = \frac{30}{4+6} = 3A$$

$$P_{6\Omega} = I^2 R = 9 \times 6 = 54W$$

\* Max power  $\rightarrow R_L = R_{th}$

cont: → Ex: Find the current through  $12\Omega$



\* super mesh

$$4i_1 + 1i_2 + 6i_2 - 32 = 0$$

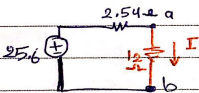
$$4i_1 + 7i_2 = 32 \quad (1)$$

$$i_1 = 1.6A$$

$$-i_1 + i_2 = 2 \quad (2)$$

$$i_2 = 3.6A$$

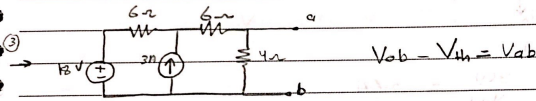
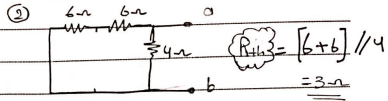
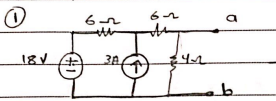
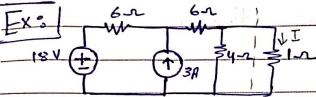
$$R_{th} = [1+6] // 4 = 2.54\Omega$$



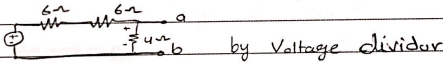
$$\leftarrow V_{ab} = V_{ac} = U_{th}$$

$$= -4(1.6) + 32 = 25.6V$$

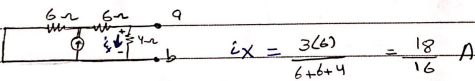
$$I = \frac{25.6}{2.5 + 12}$$



Find  $V_{oc} = V_{th}$  by superposition.

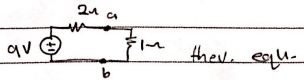


$$V_{ab} = V_{4\Omega} = \frac{18(4)}{6+6+4} = \frac{72}{16} = 4.5V$$



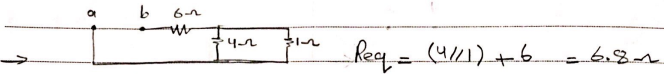
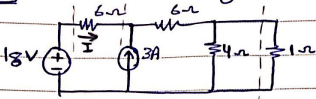
$$V_{4\Omega} = \frac{18}{16} \times 4 = \frac{18}{4} = 4.5V$$

$$V_{oc} = 4.5 + 4.5 = 9V$$



" current is not the main "

Ex: Find I using thevenin's theorem



\*current divider

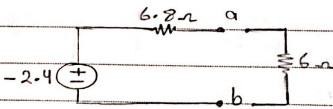
$$I_x = \frac{3(1)}{4+1} = 0.6A$$

$$\bullet V_{6\Omega} = 6 \times 0.6 = 3.6V$$

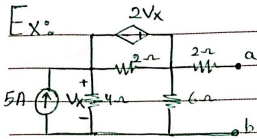
$$\bullet V_{4\Omega} = 4 \times 0.6 = 2.4V$$

$$\bullet V_{ab} = 18 - 2.4 - 3.6 = -2.4$$

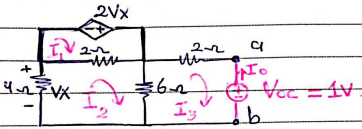
$$\oplus -2.4 \quad \text{or} \quad \ominus +2.4$$



\* Dep + INDep sources \*



- ① cut the load
- ② kill the indep sources.
- ③ connect an excitation source  $\begin{bmatrix} V=1V \\ I=1A \end{bmatrix}$  between a/b.
- ④ calculate  $R_{th} = \frac{V_{oc}}{I_o}$



\* mesh # 1

$$-2V_x + 2[I_1 - I_2] = 0$$

but  $V_x = -4I_2$

$$-2(-4I_2) + 2I_1 - 2I_2 = 0$$

$$2I_1 + 6I_2 = 0 \quad \text{①}$$

$$I_1 = 0.16A$$

\* mesh # 2

$$2(I_2 - I_1) + 6(I_2 - I_3) + 4I_2 = 0$$

$$I_2 = -0.05A$$

$$I_3 = -0.16A$$

$$-2I_1 + 12I_2 - 6I_3 = 0 \quad \text{②}$$

$$\rightarrow I_o = -I_3 = +0.16A$$

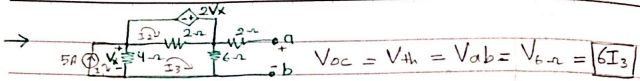
\* mesh # 3

$$2I_3 + 1 + 6(I_3 - I_2) = 0$$

$$-6I_2 + 8I_3 = -1 \quad \text{③}$$

$$\rightarrow R_{th} = \frac{V_{oc}}{I_o} = \frac{1}{0.16} = 6.25 \Omega$$





(m#1)

$$* I_1 = 5A$$

$$(m\#2) \quad -2V_x + 2(I_2 - I_3) = 0$$

$$\rightarrow V_x = 4(I_1 - I_3) = 20 - 4I_3$$

$$-2(20 - 4I_3) + 2I_2 - 2I_3 = 0$$

$$2I_2 + 6I_3 = 40 \quad (1)$$

(m#3)

$$6I_3 + 4(I_3 - I_1) + 2(I_3 - I_2) = 0$$

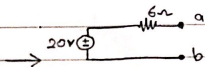
$$-20 + 6I_3 + 4I_3 + 2I_3 - 2I_2 = 0$$

$$-2I_2 + 12I_3 = 20 \quad (2)$$

$$I_2 = 10A$$

$$I_3 = 3.3A$$

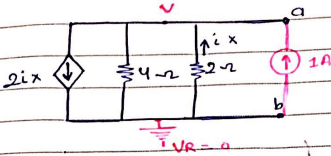
$$\rightarrow V_{ab} = V_{th} = V_{oc} = 6I_3 = 20V$$



$V_{th}$  is left to dep. volt circuit of all sources

\* Dep. Source \*

Ex: Find Thevenin's eqn.



$$V_{ab} = V_{2-\Omega} = I_{2-\Omega}(2)$$

by Nodal analysis --

$$2ix + \frac{V}{4} + \frac{V}{2} = 1$$

$$ix = \frac{-V}{2}$$

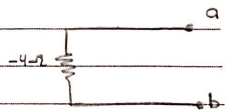
$$2\left(\frac{-V}{2}\right) + \frac{V}{4} + \frac{V}{2} = 1$$

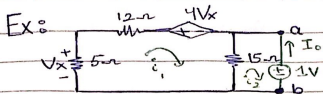
$$-V + \frac{V}{4} + \frac{V}{2} = 1$$

$$-4V + V + 2V = 4$$

$$-V = 4 \rightarrow \boxed{V = -4V} = V_{ab}$$

$$R_{th} = \frac{V_{ab}}{I_{ab}} = \frac{-4}{1} = -4\Omega$$





$$V_x = 5i_1$$

mesh #1

$$(10 + 15 + 5)i_1 - 15i_2 + 4V_x = 0$$

$$30i_1 - 15i_2 + 4(-I_15) = 0$$

$$30i_1 - 15i_2 - 20i_1 = 0$$

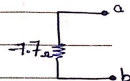
$$10i_1 - 15i_2 = 0 \quad (1)$$

mesh #2

$$15i_2 - 15i_1 + 1 = 0 \quad (2)$$

$$\begin{cases} i_1 = 0.2 \text{ A} \\ i_2 = 0.13 \end{cases}$$

$$i_0 = -i_2 = -0.13 \text{ A}$$

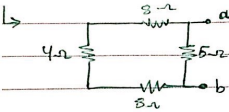
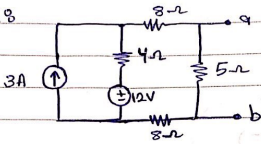


$$R_{th} = \frac{1}{-0.13} = -7.7 \Omega$$

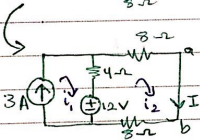
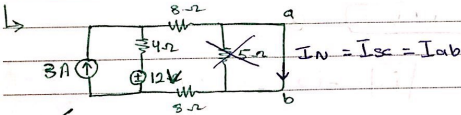
# \* Norton's theorem \*

"short circuit" (1V, 1A)  $\rightarrow$  then up  $\rightarrow$   $\rightarrow$  circuit.

EX8



$$R_N = (4 + 8 + 8) \parallel 5 = \underline{\underline{4\Omega}}$$

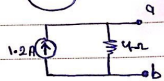


$$i_1 = 3A$$

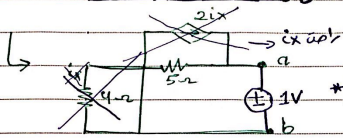
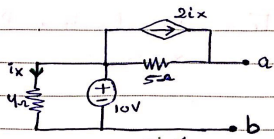
$$8i_2 + 8i_2 - 12 + 4(i_2 - i_1) = 0$$

$$20i_2 = 24$$

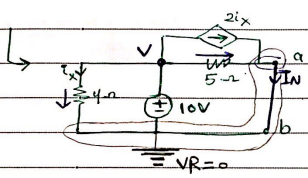
$$i_2 = \frac{24}{20} = 1.2A = \underline{\underline{I_N}}$$



Ex:



\*  $R_N = R_{th} = 5\Omega$

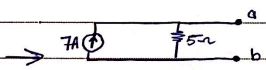


\* Nodal. . .

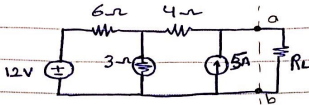
$V = 10V$

$i_x = \frac{V}{R} = \frac{10}{4} = 2.5A$

\*  $I_N = I_{5\Omega} + 2i_x$   
 $= \frac{10}{5} + 2(2.5) = 7A$

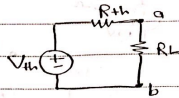


Exo



Find  $R_L$  to transfer

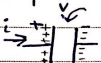
max power



$R_N = R_{th} = R_L$  (max power)

**\* Chapter 6 \***

Capacitors of Inductors - use to store energy -



$A$ : area of the plate

$d$ : distance

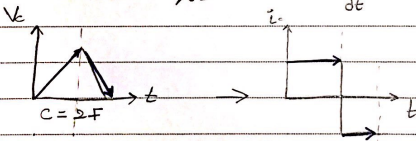
$\epsilon$ : permittivity of the dielectric -

$\epsilon_0$ : " " " air

\*  $C = \frac{\epsilon A}{d}$  \*

\*  $q = CV \rightarrow C = \frac{q}{V} \rightarrow \text{Farad}$

\*  $i = \frac{dq}{dt} \rightarrow i = \frac{d(CV)}{dt} \rightarrow i_c(t) = C \frac{dV_c}{dt}$



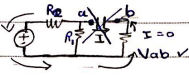
\*  $V_c(t) = \frac{1}{C} \int_{-\infty}^t i_c(t) dt$

\*  $V_c(t) = \frac{1}{C} \int_{t_0}^t i_c(t) dt + V_0$   
initial voltage

\*  $P = VI = VC \frac{dv}{dt}$

\*  $W = \int_{-\infty}^t P \cdot dt = \int CV \frac{dv}{dt} \cdot dt = \frac{1}{2} CV^2 \text{ J}$

\* Capacitor acts as "open" ckt to DC source \*



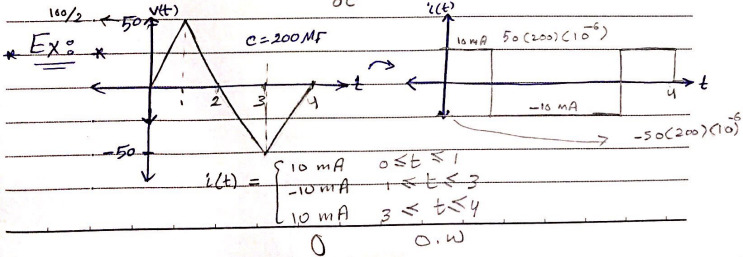
\* The voltage across the capacitor can't change abruptly [suddenly]  $\rightarrow$  chg

\* Ex 6.1 \*  $C = 3 \text{ pF}$   
 $V = 20 \text{ V}$

$q = CV = ? = 60 \text{ pC}$   
 $w = ? = \frac{1}{2} CV^2 = \frac{1}{2} (3)(20)^2 = 600 \text{ pJ}$

\* Ex 2 \*  $C = 5 \text{ MF}$   
 $V(t) = 10 \cos 6000t$

Find  $i(t) = C \frac{dv}{dt} = -0.3 \sin 6000t \text{ A}$

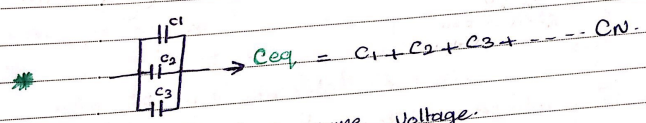




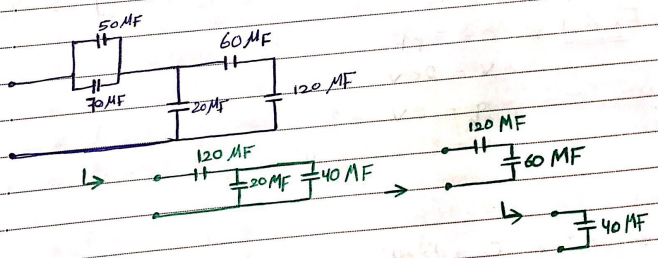
\* Series or parallel capacitors \*



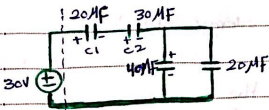
They have same charge.



They have same voltage.



Ex:



voltage across each capacitor



$q = CV \rightarrow 10 \times 30 = 300 \text{ m coulomb}$

at  $C_1 \rightarrow q_1 = 300 \text{ m} \rightarrow V_1 = q/C = 15 \text{ V}$

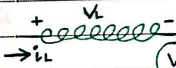
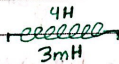
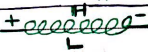
at  $C_2 \rightarrow q_2 = 300 \text{ m} \rightarrow V_2 = q/C = 10 \text{ V}$

at  $(C_3, C_4) \rightarrow V_3 = \frac{300}{60} = 5 \text{ V}$

$q_3 = C_3 V_3 = 40(5) = 200$

open ckt  $\rightarrow$  means the current is zero.

**\* Inductor \***



$V_L = L \frac{di}{dt}$

$\rightarrow$  all the  $\rightarrow$  of turns.

$L = \frac{N^2 \mu A}{l}$

permeability

$l \rightarrow$  length

$A \rightarrow$  cross section Area

$L \rightarrow$  inductance

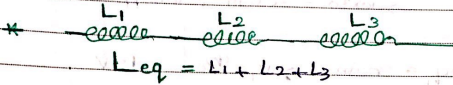
" $\rightarrow$  بولٹی سائیکس"

$i(t) = \frac{1}{L} \int_{t_0}^t V_L(t) dt + I_0$

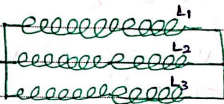
$$\textcircled{W} = \frac{1}{2} L I^2 \quad \underline{J} \rightarrow \text{energy stored.}$$

→ Under the DC conditions the inductor acts as a short ckt.

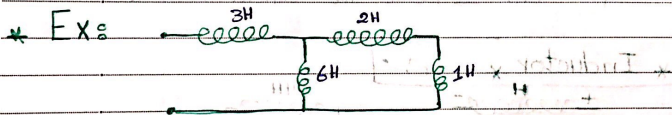
→ The current in the inductor doesn't change suddenly  
 $i_L^-(t_0) = i_L(t_0) = i_L^+(t_0)$



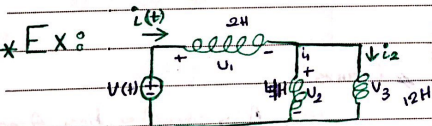
$$L_{eq} = L_1 + L_2 + L_3$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_N}$$



$$* L_{eq} = [(2+1) \parallel 6] + 3 = 5H$$



$$v_2 = v_3$$

$$i(t) = 4(2 - e^{-10t}) \text{ mA}$$

$$i_2(t) = -1 \text{ mA}$$

AC → function of time  
 "current"

→ Find  $i_1(t)$ ,  $v(t)$ ,  $v_1(t)$ ,  $v_2(t)$ ,  $i_1(t)$ ,  $i_2(t)$

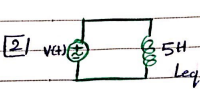
1]  $i_1(0) \rightarrow$  where  $t = \text{Zero}$ .

$$i_1(0) \Big|_{t=0} = 4 \text{ mA}$$

$$i(0) = i_1(0) + i_2(0) \quad \text{KCL}$$

$$i_1(0) = i(0) - i_2(0)$$

$$i_1(0) = 4 - (-1) = 5 \text{ mA}$$

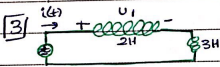


$$v(t) = L_{eq} \frac{di(t)}{dt}$$

$$= 5 \frac{d}{dt} (8 - 4e^{-10t})$$

$$= 5 (40e^{-10t})$$

$$= 200e^{-10t} \text{ mV}$$

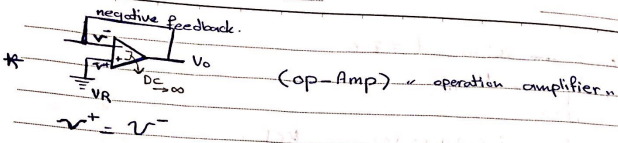


$$v_1(t) = L_1 \frac{di}{dt} = 2(40e^{-10t}) = 80e^{-10t} \text{ mV}$$

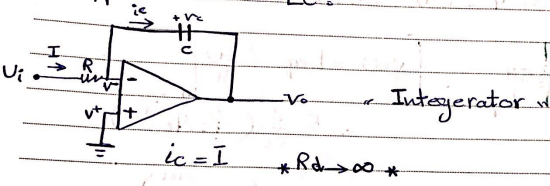
$$v_2 = v(t) - v_1(t) = 3 \frac{di}{dt} = 120e^{-10t} \text{ mV}$$

$$4] \quad i_1(t) = \frac{1}{4} \int_0^t 120 e^{-10t} dt + 5 \rightarrow i_1(0)$$

$$5] \quad i_2(t) = \frac{1}{12} \int_0^t 120 e^{-10t} dt + (-1) \rightarrow i_2(0)$$



\* Application of LC:

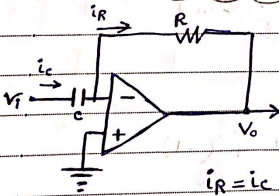


$$V_o = -V_c$$

$$= -\frac{1}{C} \int i_c(t) dt$$

$$= -\frac{1}{C} \int i_c(t) dt = -\frac{1}{C} \int \frac{V_i(t)}{R} dt$$

$$V_o = -\frac{1}{RC} \int V_i(t) dt$$

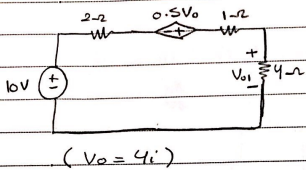
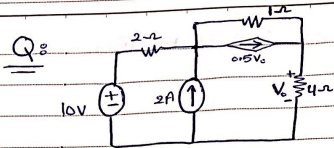


$$V_o = -i_R R$$

$$V_o = -i_c R = -RC \frac{dV_i}{dt}$$

$$V_o = -RC \frac{dV_i}{dt}$$

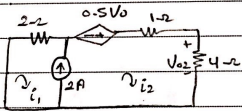
[Inductor]



$$-10 + 2i - 0.5(4i) + 1i + 4i = 0$$

$$5i = 10 \rightarrow i = 2A$$

$$V_{01} = 8V$$



$$2i_1 - 0.5(4i_2) + i_2 + 4i_2 = 0$$

$$2i_1 + 3i_2 = 0$$

$$i_2 - i_1 = 2$$

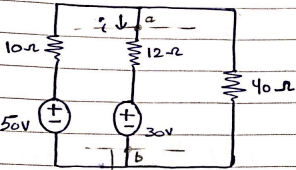
$$5i_2 = 4$$

$$i_2 = 0.8$$

$$V_{02} = 4(0.8) = 3.2V$$

$$* V_0 = 8 + 3.2 = 11.2V$$

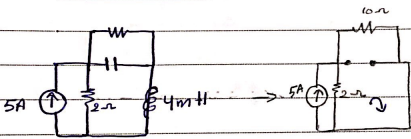
Q:



Find  $i$  by Thev.

--- circuit all

Q:



$$E_L = \frac{1}{2} (I)^2$$

$$\frac{1}{2} C V^2 = \frac{1}{2} L I^2$$

$$\frac{1}{2} C R^2 = \frac{1}{2} L I^2$$

$$R = \sqrt{\frac{L}{C}}$$



\* CH-7 \*

### • First order Circuits

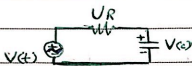
RL  

RC  

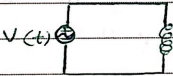
RLC   

∫ CKT with First order differential equations.

$$\frac{dv}{dt}, \frac{di}{dt}$$



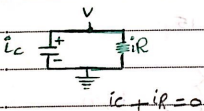
$$\rightarrow -v(t) + v_R + v_C = 0$$
$$-\frac{dv(t)}{dt} + \frac{di(t)}{dt}$$



$$\rightarrow -v(t) + v_C = 0$$
$$-v(t) + \int$$

\* Source free RC CKT

There are No external



$$\frac{C dv}{dt} + \frac{V}{R} = 0$$

$$\frac{C dv}{dt} = -\frac{V}{R} \rightarrow \frac{dv}{dt} = -\frac{1}{CR} dt$$

$$\ln v = -\frac{1}{CR} t + (k) \rightarrow \ln B.$$

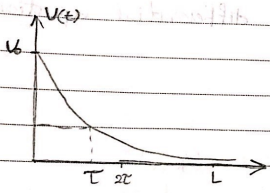
$$\left( \ln v - \ln B = -\frac{1}{CR} t \right)$$



$$\rightarrow \frac{V}{B} = e^{\frac{-1}{CR}t} \rightarrow \left( V = \underbrace{B}_{V_0 \rightarrow \text{initial voltage}} e^{\frac{-1}{CR}t} \right)$$

$$\rightarrow \left[ V = V_0 e^{\frac{-1}{CR}t} \right] \rightarrow \left( V = V_0 e^{\frac{-t}{\tau}} \right)$$

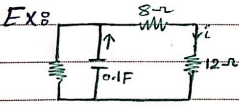
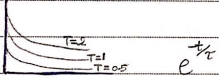
$\tau$ : time constant, Req. C



$$\begin{aligned} e^0 &= 1 \\ e^\infty &= 0 \\ e^{-\infty} &= \infty \end{aligned}$$

$5\tau \approx \infty$   
 الوقت الذي فيه الجهد يصل الى 0.36 من الجهد الابتدائي  
 "completely discharged"

\* كلما كانت الـ  $\tau$  أقل يكون التفريغ أسرع ووصول الـ  $\infty$  في وقت أصغر  
 --- R أكبر  $\tau$  أكبر



$$\begin{aligned} V_C(0) &= 15 \\ V_C(t) \\ V_S(t) \\ i_L(t) \end{aligned} \left. \vphantom{\begin{aligned} V_C(0) \\ V_C(t) \\ V_S(t) \\ i_L(t) \end{aligned}} \right\} \rightarrow t > 0$$

$$\begin{aligned} R_{eq} &= (12+8) // 5 = 4 \Omega \\ \tau &= 4(0.1) = 0.4 \text{ s} \\ V(t) &= V_0 e^{-\frac{t}{\tau}} = 15 e^{-\frac{t}{0.4}} \\ V_C(t) &= 15 e^{-2.5t} \end{aligned}$$

$$\tau = R_{eq} C$$

$$\rightarrow V_s(t) = \frac{12}{8+12} V_c(t) \rightarrow \text{voltage divider}$$

$$= \frac{12}{20} (15) e^{-2.5t} \rightarrow 9e^{-2.5t} \text{ discharge}$$

$$\rightarrow i_s(t) = \frac{V_s(t)}{R} = \frac{9e^{-2.5t}}{12} \text{ A}$$

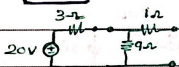
EX:



Ua bika' la.

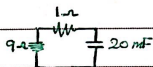
$V_c$  Cuzi pin "switch" alla lb bika'

$t < 0$



$$\cdot V_c(0) = \frac{20 \times 9}{3+9} = \frac{180}{12} = 15V$$

$t > 0$



$$\cdot R_{eq} = 1 + 9 = 10 \Omega$$

$$\cdot \tau = R_{eq} \cdot C = 10 \times 20 \times 10^{-3} = 0.2 \text{ sec.}$$

$$\cdot V_c(t) = 15 e^{-\frac{t}{0.2}} = 15 e^{-5t}$$

\* Free RL Ckts :



$$V_L + V_R = 0$$

$$L \frac{di}{dt} = -iR$$

$$\int \frac{di}{i} = \int \frac{-R}{L} dt \rightarrow \ln(i) = \frac{-R}{L} t + \ln(k)$$

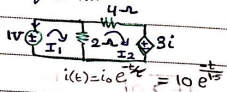
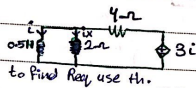
$$\ln(i) - \ln(k) = \frac{-R}{L} t$$

$$\ln\left(\frac{i}{k}\right) = \frac{-R}{L} t \rightarrow \frac{i}{k} = e^{\frac{-R}{L} t}$$

$$i = I_0 e^{\frac{-t}{\tau}} \quad \text{let } \tau = \frac{L}{R_{eq}}$$

Ex:  $i(0) = 10A$

$i(t) =$



\*  $-1 + 2I_1 - 2I_2 = 0$

$2I_1 - 2I_2 = 1 \dots \text{--- (1)}$

\*  $2I_2 - 2I_1 + 4I_2 + (-3I_1) = 0$

$-5I_1 + 6I_2 = 0 \dots \text{--- (2)}$

$I_1 = 3A$

$I_2 = 2.5A$

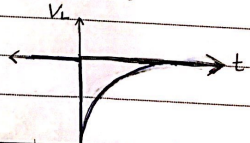
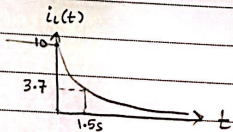
\*  $R_{eq} = \frac{1}{I_1} = \frac{1}{3} \Omega$

\*  $\tau = \frac{L}{R_{eq}} = \frac{0.5}{\frac{1}{3}} = 1.5 s$

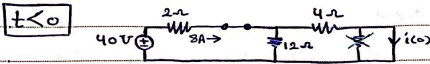
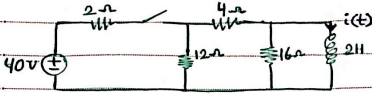
\*  $V_L = L \frac{di}{dt} = V_y$

$V_L = \frac{-5}{1.5} e^{-\frac{t}{1.5}} = V_x$

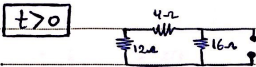
\*  $i_x = \frac{V_x}{2} = \frac{-5}{3} e^{-\frac{t}{1.5}} A$



Ex:



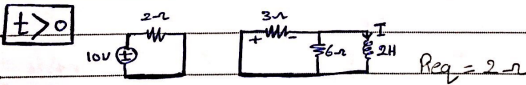
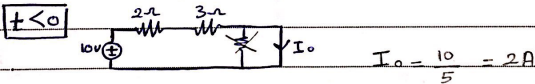
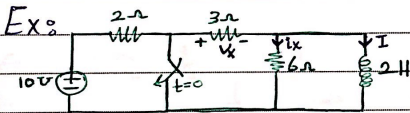
$$* I_0 = \frac{8 \times 12}{16} = 6A$$



$$* R_{eq} = (4+12) // 16 = 8\Omega$$

$$* \tau = \frac{L}{R} = \frac{2}{8} = 0.25 s.$$

$$* i(t) = I_0 e^{-t/\tau} = 6e^{-4t} A.$$



$$\tau = \frac{2}{2} = 1s$$

$$i(t) = 2e^{-t}$$

$$* V_x = -V_L = -L \frac{di}{dt} = -2(2)(-1)e^{-t} = 4e^{-t} V$$

$$R V_{G-n} = U_L = -U_x = -4e^{-t}$$

$$i_x = \frac{U_x}{R} = \frac{-4}{6} e^{-t}$$

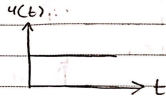
$$\bullet U_x = \begin{cases} 6 & t < 0 \\ 4e^{-t} & t > 0 \end{cases}$$

$$\bullet U_L = \begin{cases} 0 & t < 0 \\ -4e^{-t} & t > 0 \end{cases}$$

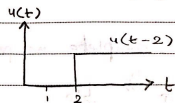
$$\bullet i(t) = \begin{cases} 2 & t < 0 \\ 2e^{-t} & t > 0 \end{cases}$$

\* unit step function \*

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

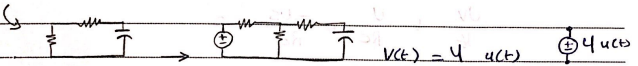
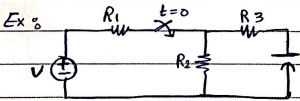
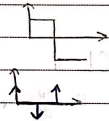
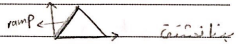


$$u(t) = \begin{cases} 0 & t < 2 \\ 1 & t > 2 \end{cases}$$

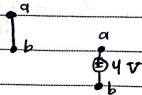


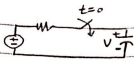
$s(t) = \frac{d u(t)}{dt}$  ↑ unit impulse "impulsi"

$$\text{Ramp } f(t) = \begin{cases} f & t > 0 \\ 0 & t < 0 \end{cases}$$



$$V(t) = \begin{cases} 0 & t < 0 \\ 4 & t > 0 \end{cases}$$

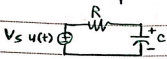




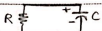
$t < 0$

$$V_C(0)^+ = V_C(0)^- = V_C(0) = U_0$$

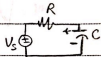
complete response = natural response + forced response



$t < 0$



$t > 0$



~ KCL ~

$$C \frac{dV}{dt} + \frac{U - U_S}{R} = 0$$

$$C \frac{dV}{dt} + \frac{U}{R} - \frac{U_S}{R} = 0$$

$$\frac{dV}{dt} + \frac{U}{RC} + \frac{-U_S}{RC} = 0$$

$$\frac{dV}{dt} + \frac{U}{RC} = \frac{U_S}{RC}$$

$$V(t) = \begin{cases} U_0 & t < 0 \\ U_S + (U_0 - U_S)e^{-\frac{t}{RC}} & t > 0 \end{cases}$$

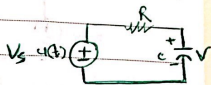




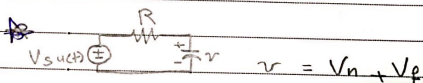
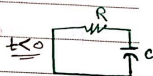
if  $V_0 = 0$

$$V(t) = \begin{cases} 0 & t < 0 \\ V_s(1 - e^{-\frac{t}{\tau}}) & t > 0 \end{cases}$$

$$V(t) = \begin{cases} V_0 & t < 0 \\ V_s + V_0 e^{-\frac{t}{\tau}} - V_s e^{-\frac{t}{\tau}} & t > 0 \end{cases}$$



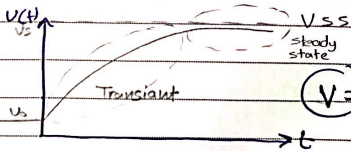
$$V = \underbrace{\left( V_s - V_s e^{-\frac{t}{\tau}} \right)}_{V_s [1 - e^{-\frac{t}{\tau}}] \text{ "Forced" } V_F} + \underbrace{V_0 e^{-\frac{t}{\tau}}}_{\text{nat } V_N}$$



$$V(t) = \begin{cases} V_0 & t < 0 \\ V_s + (V_0 - V_s) e^{-\frac{t}{\tau}} & t > 0 \end{cases}$$

$$= V_s (1 - e^{-\frac{t}{\tau}}) + V_0 e^{-\frac{t}{\tau}}$$

$$= V_s + V_0 e^{-\frac{t}{\tau}} - V_s e^{-\frac{t}{\tau}}$$



$V =$  Transient state + Steady state

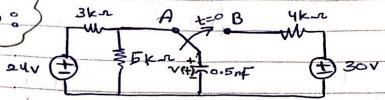
$$V = \underbrace{V(\infty)}_{t > 0} + \left[ \underbrace{V(0)}_{t < 0} - \underbrace{V(\infty)}_{t > 0} \right] e^{-\frac{t}{\tau}} \rightarrow \tau = R_{eq} \cdot C$$



$$v(t) = v_s + [v_0 - v_s] e^{-\frac{t}{\tau}} \rightarrow \begin{matrix} v_s \\ v \\ t \end{matrix}$$

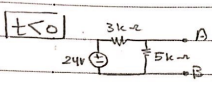
$$i(t) = C \frac{dq}{dt} = C \left[ 0 + \frac{1}{\tau} [v_0 - v_s] \right] e^{-\frac{t}{\tau}}$$

Ex 7-10:

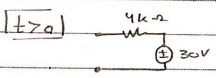


Find  $v(t)$  for  $t > 0$ .

Find  $v(t)$  |  
 $t = 1s$   
 $t = 4s$



$$V_0 = \frac{24(5)}{8} = 15V$$



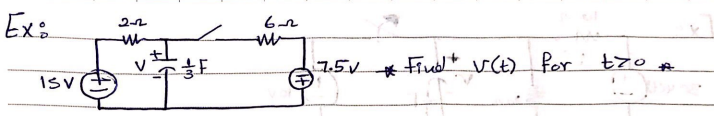
$$V(\infty) = 30V$$

$$R_{eq} = 4k\Omega$$

$$\tau = R_{eq} \cdot C = 4(0.5) = 2s$$

التأخر

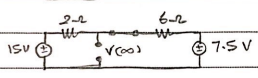
$$v(t) = 30 + (15 - 30) e^{-\frac{t}{2}}$$



$t < 0$



$t > 0$



$$I = \frac{15 - (-7.5)}{8} = \frac{22.5}{8} = 2.8 \text{ A}$$

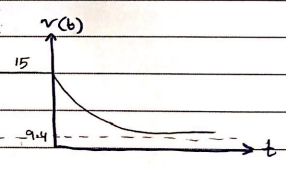
$$V(\infty) = -2(2.8) + 15 = 9.4 \text{ V}$$

$$R_{eq} = 2 \parallel 6 = 1.5 \Omega$$

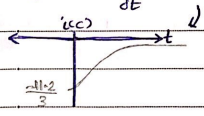
$$\tau = 1.5 \left(\frac{1}{3}\right) = \frac{1}{2} \text{ s}$$

$$* v(t) = 9.4 + (15 - 9.4)e^{-2t}$$

$$= 9.4 + 5.6e^{-2t}$$

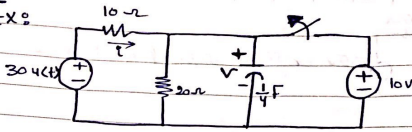


$$* i = C \frac{dv}{dt} = -11.2e^{-2t}$$



7-11

Ex:

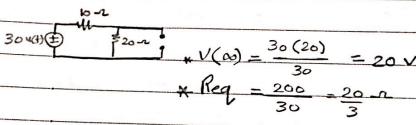


$t < 0$



$V(0) = 10V$

$t > 0$



\*  $V(\infty) = \frac{30(20)}{30} = 20V$

\*  $R_{eq} = \frac{200}{30} = \frac{20}{3} \Omega$

\*  $\tau = \frac{1}{4} \left( \frac{20}{3} \right) = 1.67s = \frac{5}{3}$

\*  $V(t) = V(\infty) + (V(0) - V(\infty)) e^{-\frac{t}{\tau}}$   
 $= 20 + (10 - 20) e^{-\frac{3t}{5}}$   
 $= 20 - 10 e^{-\frac{3t}{5}}$

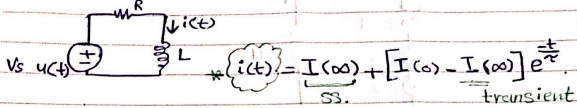
\*  $i = i_R + i_C$

$= \frac{v(t)}{20} + C \frac{dv}{dt} = \frac{(20 - 10 e^{-\frac{3t}{5}})}{20} + \frac{1}{4} \frac{d}{dt} (20 - 10 e^{-\frac{3t}{5}})$

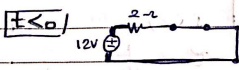
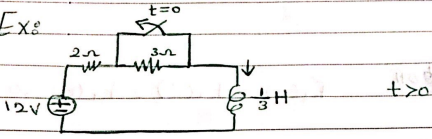
\*  $V(t) = \begin{cases} 10 & t < 0 \\ 20 - 10 e^{-\frac{3t}{5}} & t > 0 \end{cases}$

\*  $i(t) = \begin{cases} -1 & t < 0 \\ \frac{20 - 10 e^{-\frac{3t}{5}}}{20} & t > 0 \end{cases}$

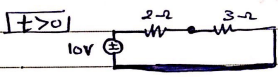
\* Step Response of RL Ckts



Ex:



$I(0^-) = I(0^+) = I(0) = \frac{10}{2} = 5A.$

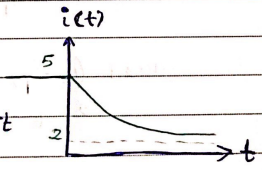


$I(\infty) = \frac{10}{5} = 2A.$

$R_{eq} = 5\Omega$

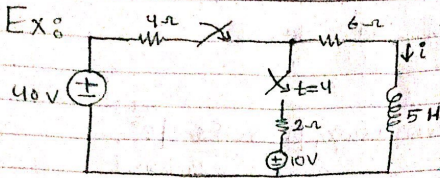
$\tau = \frac{L}{R_{eq}} = \frac{1}{15} s.$

\*  $i_L(t) = 2 + (5-2)e^{-15t} \quad A.$

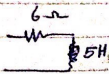


\*  $V_c = c \frac{di}{dt} = 0 + 3(-15) \cdot \frac{1}{3} e^{-15t}$

$\rightarrow = -15e^{-15t}$

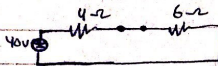


①  $t < 0$



$$I_L(0^-) = I_L(0^+) = I_L(0) = 0$$

②  $4 \leq t < \infty$



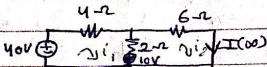
$$I(\infty) = \frac{40}{10} = 4 \text{ A}$$

$$\tau = \frac{L}{R_{eq}} = \frac{5}{10} = \frac{1}{2} \text{ s}$$

$$i_L(t) = 4 + (0 - 4)e^{-2t}$$

$$= 4 - 4e^{-2t}$$

③  $t > 4$



$$-40 + 4i_1 + 2(i_1 - i_2) + 10 = 0$$

$$6i_1 - 2i_2 = 30 \quad \text{--- (1)}$$

$$-10 + 2(i_2 - i_1) + 6i_2 = 0$$

$$-2i_1 + 8i_2 = 10 \quad \text{--- (2)}$$

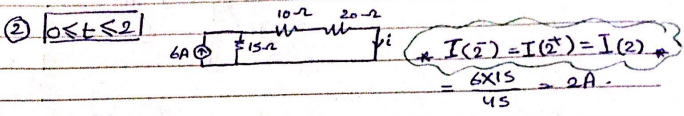
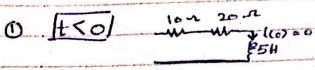
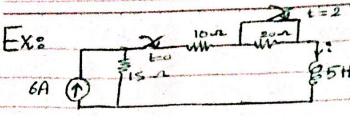
$$i_2 = 2.7 \text{ A} = i(\infty)$$

$$R_{eq} = (4 \parallel 2) + 6 = 7.3$$

$$\tau = \frac{L}{R_{eq}} = \frac{5}{7.3} \text{ s}$$

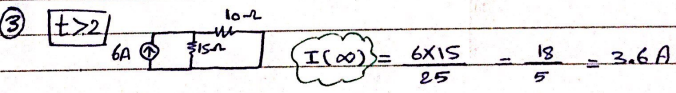
$$i_L(t) = 2.7 + (4 - 2.7)e^{-\frac{2t}{5}}$$

$$i_L(t) = \begin{cases} 0 & t < 0 \\ 4(1 - e^{-\frac{2t}{5}}) & 0 \leq t \leq 4 \\ 2.7 + 1.3e^{-\frac{2(t-4)}{5}} & t > 4 \end{cases}$$



$R_{eq} = 10 + 20 + 15 = 45 \Omega$   
 $\tau = \frac{5}{45} = \frac{1}{9} \text{ s}$

$i(t) = 20 + (0 - 20)e^{-9t}$   
 $\hookrightarrow = 2(1 - e^{-9t}) \text{ A}$



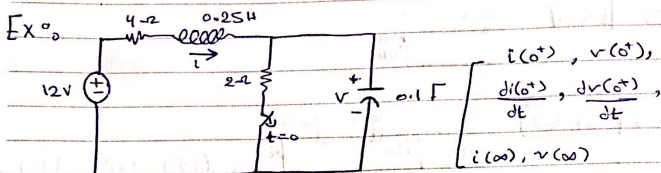
$R_{eq} = 25 \Omega$   
 $\tau = \frac{5}{25} = \frac{1}{5} \text{ s}$

$i_L(t) = 3.6 + (2 - 3.6)e^{-5(t-2)} \text{ A}$

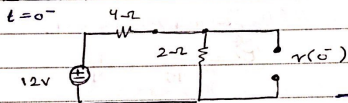


\* CH 8 \*

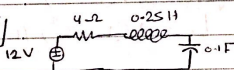
2nd order. CKTs.



$t < 0$



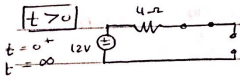
$t = 0$



$$i_L(0^-) = \frac{12}{6} = 2A$$

$$v_c(0^-) = 2 \times 2 = 4V$$

$t > 0$



$$i_L(t) = i_L(0^-) = 2A$$

$$v_c(t) = 4V$$

$$i_c(t) = 2A$$

$$i_c = C \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = \frac{i_c}{C}$$

$$\frac{dv(t^+)}{dt} = \frac{i_c(t^+)}{C} = \frac{2}{0.1} = 20 \text{ V/s}$$

$$\frac{di(\dot{t})}{dt} = \frac{V_L(\dot{t})}{L}$$

$$-12 + 4(2) + V_L(\dot{t}) + V_C(\dot{t}) = 0$$

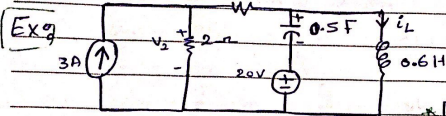
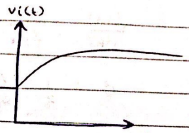
$$-12 + 8 + V_L(\dot{t}) + 4 = 0$$

$$\rightarrow V_L(\dot{t}) = 0$$

$$\rightarrow \frac{di(\dot{t})}{dt} = 0 \text{ A/s}$$

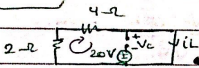
$$i(\infty) = 0$$

$$V(\infty) = 12 \text{ V}$$



$$\left[ i_L(\dot{t}), V_C(\dot{t}), \frac{dV_C(\dot{t})}{dt}, \frac{di_L(\dot{t})}{dt}, \frac{dV_L(\dot{t})}{dt}, i_L(\infty), V_C(\infty), V_R(\infty) \right] = ?$$

$t=0^-$

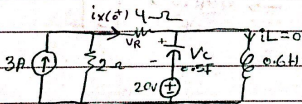


$$i_L(0^-) = 0$$

$$V_C(0^-) = V_C(0) + 20 + 0 + 0 = V_C(0) = -20 \text{ V}$$

$$V_R(0^-) = 0$$

$t=0^+$



$$i_x(0^+) = \frac{3 \times 2}{6} = 1 \text{ A}$$

$$V_{R_x}(0^+) = 1(4) = 4 \text{ V}$$

$$-V_R(0^+) + V_x(0^+) + V_C(0^+) + 20 = 0$$

$$V_L(0^+) = 0$$

$$\frac{di_L(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{0}{0.5} = 0 \text{ A/s}$$

$$\rightarrow V_x(0^+) = V_R(0^+) = 4 \text{ V}$$

$$i_C(0^+) = C \frac{dV_C(0^+)}{dt} = i_C(0^+)$$



Practice  
Prb. 8.2

by KCL

$$i_x(t^+) = i_c(t^+) + i_e(t^+)$$

$$1 = 1$$

$$* \quad 3 = \frac{v_R}{2} + \frac{v_x}{4}$$

$$\frac{d(3)}{dt} = \frac{dv_R}{2dt} + \frac{dv_x}{4dt}$$

$$0 = \frac{1}{2} \frac{dv_R}{dt} + \frac{1}{4} \frac{dv_x}{dt}$$

$$2 \frac{dv_x}{dt} = -4 \frac{dv_R}{dt}$$

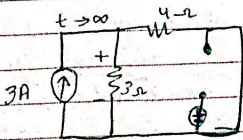
$$\frac{dv_x}{dt} = -2 \frac{dv_R}{dt}$$



$$\frac{-dv_R(t^+)}{dt} + \frac{dv_x(t^+)}{dt} + \frac{dv_c(t^+)}{dt} + 0 = 0$$

$$\rightarrow \frac{-dv_R(t^+)}{dt} + (-2) \frac{dv_R}{dt} = -4$$

$$\rightarrow \frac{-3 dv_R(t^+)}{dt} = -4 \rightarrow \frac{dv_R(t^+)}{dt} = \left[ \frac{4}{3} \right]$$

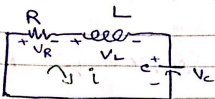


$$i_L(\infty) = 1A$$

$$V_c(\infty) + 20 + -4 + 4$$

$$V_c(\infty) = 20V$$

"Free sources RLC ckt."



$$v_R + v_L + v_C = 0$$

$$Ri + L \frac{di}{dt} + \frac{1}{C} \int i(t) dt = 0$$

$$R \frac{di}{dt} + L \frac{d^2 i}{dt^2} + \frac{1}{C} i = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0$$

$$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$$

$$s_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\star s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \star$$

$$s_2 = \frac{-R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\star s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \star$$

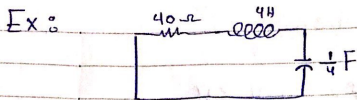
- $\alpha = \frac{R}{2L}$  : damping factor (Neper Freq.) NP/s.
- $\omega_0 = \frac{1}{\sqrt{LC}}$  : resonant Freq (undamped) natural Freq. rad/sec.
- $s_1, s_2$  : Natural Freq. NP/s.
- $\zeta$  :  $\frac{\alpha}{\omega_0}$  damping Ratio.

<p>① IF <math>\alpha &gt; \omega_0</math> overdamped</p> <p>② " <math>\alpha = \omega_0</math> critical damped</p> <p>③ " <math>\alpha &lt; \omega_0</math> Under damped</p>	
--	--

$\rightarrow i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$  ,  $c > \frac{4L}{R_2}$

$c = \frac{4L}{R_2}$  ,  $s_1 = s_2 = -\alpha = -\frac{R}{2L}$   
 $\rightarrow i(t) = (A_2 + A_1 t) e^{-\alpha t}$

$\alpha < \omega_0 \rightarrow \omega_d = \sqrt{\omega_0^2 - \alpha^2}$  damping natural freq.  
 $\rightarrow i(t) = e^{-\alpha t} [\beta_1 \cos \omega_d t + \beta_2 \sin \omega_d t]$



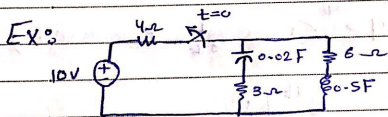
$$\alpha = \frac{R}{2L} = \frac{40}{8} = 5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 1$$

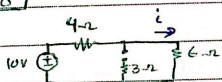
$\alpha > \omega_0$  overdamp.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\begin{cases} s_1 = -0.101 \\ s_2 = -9.899 \end{cases}$$

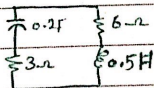


$t = 0^-$



$$i(0^-) = i(0^+) = 1A$$

$t = 0^+$



$$\alpha = \frac{R}{2L} = \frac{4}{1} = 4$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10$$

since  $\omega_0 > \alpha$  underdamped

$$i(t) = e^{-\alpha t} [B_1 \cos \omega_d t + B_2 \sin \omega_d t]$$

$$\bullet \omega d = \sqrt{10^2 - 9^2} = \sqrt{19} = 4.36$$

$$* i(t) = e^{-9t} [B_1 \cos 4.36t + B_2 \sin 4.36t]$$

$$\bullet i(0) = 1 [B_1 \cos 0 + B_2 \sin 0] = 1$$

$$\boxed{B_1 = 1A}$$

$$* v_c(0) = 6V$$

$$6(1) + v_L(0) + 3(1) - 6 = 0$$

$$* v_L(0^+) = -3V$$

$$* \frac{v_L(0)}{L} = \frac{di(0)}{dt} = \frac{-3}{0.5} = \frac{di(0)}{dt} = -6$$

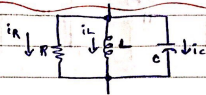
$$\text{under } \left. \frac{di}{dt} \right|_{t=0} = -9 e^{-9t} (1 \cos \omega_d t + B_2 \cos \omega_d t) + e^{-9t} [-\omega_d \sin \omega_d t + B_2 \omega_d \cos \omega_d t]$$

$$= -9(1+0) + (0 + B_2 \omega_d) = -6$$

$$* B_2 = \frac{-6 + 9}{4.36}$$

\* The source free parallel RLC ckt \*

→ KCL →



$$i_R + i_L + i_C = 0$$

$$\frac{V}{R} + \frac{1}{L} \int V(t) dt + C \frac{dV}{dt} = 0$$

$$\left( \frac{dV}{R dt} + \frac{1}{L} V(t) dt + C \frac{d^2 V}{dt^2} = 0 \right) \div C$$

$$\frac{d^2 V}{dt^2} + \frac{1}{RC} \frac{dV}{dt} + \frac{1}{LC} V = 0$$

$$s^2 + \frac{1}{RL} s + \frac{1}{LC} = 0$$

$$\text{let } \alpha = \frac{1}{2RC}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

•  $\alpha > \omega_0 \rightarrow$  overdamping  $v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

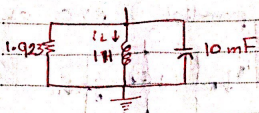
•  $\alpha < \omega_0 \rightarrow$  underdamping  $v(t) = (A_1 + A_2 t) e^{-\alpha t}$

•  $\alpha = \omega_0 \rightarrow$  critical damp  $v(t) = e^{-\alpha t} (\theta_1 \cos \omega_0 t + \theta_2 \sin \omega_0 t)$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$



Ex:



$$V(0) = 5$$

$$i_C(0) = 0$$

Find  $v(t)$  for  $t > 0$

$$\alpha = \frac{1}{RC} = 26, \quad \omega_0 = \frac{1}{\sqrt{LC}} = 10$$

$\alpha > \omega_0$  over damping

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \quad \left\{ \begin{array}{l} V(t) = A_1 e^{-2t} + A_2 e^{-50t} \\ \frac{dV}{dt} = -2A_1 e^{-2t} - 50A_2 e^{-50t} \end{array} \right.$$

$$i_L + i_R + i_C = 0$$

$$i_C = -[i_R + i_L] \rightarrow i_C = -\left(\frac{5}{1.923} + 0\right)$$

$$\frac{dV(t^+)}{dt} = \frac{i_C(t^+)}{C} = \frac{-2.6}{10 \times 10^{-3}} = -260 \text{ V/s.}$$

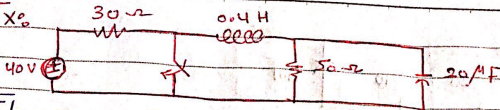
$$V(t^+) = A_1 + A_2 = 5$$

$$\frac{dV(t^+)}{dt} = -2A_1 - 50A_2 = -260$$

$$A_1 = -0.2, \quad A_2 = 5.2$$

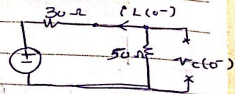
$$V(t) = -0.2 e^{-2t} + 5.2 e^{-50t}$$

Exo



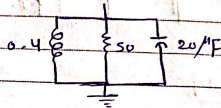
$$t=0^- \cdot v_C(t) = 40 \left( \frac{50}{80} \right) = 25V = v_C(t^+)$$

$$i_L(t^+) = -\frac{40}{80} = -0.5A = i_L(t)$$



$$t=0^+ \cdot \alpha = \frac{1}{2Rc} = 500$$

$$\omega_0 = \frac{1}{\sqrt{Lc}} = 324$$



$\alpha > \omega_0 \rightarrow$  over.

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$= -500 \pm \sqrt{500^2 - 324^2} = -824, -146$$

$$v(t) = A_1 e^{-824t} + A_2 e^{-146t}$$

$$\frac{dv}{dt} = -824 A_1 e^{-824t} - 146 A_2 e^{-146t}$$

$$i_C + i_R + i_L = 0 \rightarrow i_C = -\frac{25}{50} - (-0.5) = 0$$

$$\frac{dv(t^+)}{dt} = 0$$

$$v(0) = 25$$

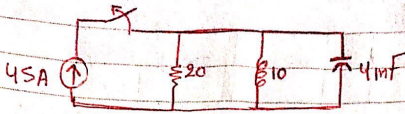
$$A_1 = -5.156V$$

$$A_2 = 30.16V$$

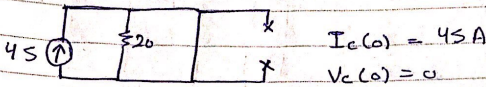
$$\rightarrow v(t) = -5.156 e^{-824t} + 30.16 e^{-146t}$$



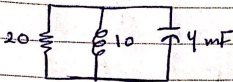
Practice 8.6



$t = 0^-$



$t = 0^+$



$$\alpha = \frac{1}{2RC} = 6.25$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 4 \times 10^{-3}}} = 5$$

$\alpha > \omega_0 \rightarrow$  over.

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -10, -2.5$$

$$v(t) = A_1 e^{-10t} + A_2 e^{-2.5t}$$

$$\frac{dv}{dt} = -10 A_1 e^{-10t} - 2.5 A_2 e^{-2.5t}$$

$$v(0) = A_1 + A_2 = 0$$

$$-10 A_1 - 2.5 A_2 = -112.5$$

$$\begin{cases} A_1 = 150 \\ A_2 = -150 \end{cases}$$

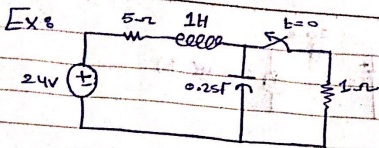
$$v(t) = 150 e^{-10t} - 150 e^{-2.5t}$$

$$v(t) = 150 (e^{-10t} - e^{-2.5t})$$

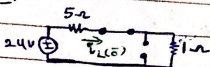
$$i_c + i_R + i_L = 0$$

$$i_c = -i_R - i_L = 0 - 4.5$$

$$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{-4.5}{4 \times 10^{-3}} = -112.5$$

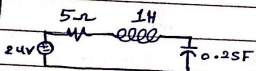


$t = 0^-$



- $i(0^-) = \frac{24}{6} = 4 \text{ A} = i(0^+)$
- $v(0^-) = V_C(0^+) = 4 \text{ V}$

$t = 0^+$



$$-24 + V_R(0^+) + V_L(0^+) + V_C(0^+) = 0$$

$$-24 + 20 + V_L(0^+) + 4 = 0$$

$$\bullet V_L = 0$$

$$\bullet \alpha = \frac{R}{2L} = \frac{5}{2} = 2.5$$

$$\bullet \omega_0 = \frac{1}{\sqrt{LC}} = 2$$

$$\bullet \alpha > \omega_0 \rightarrow \text{over}$$

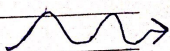
$$\bullet s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \rightarrow s_1 = -1, s_2 = -4$$

$$\bullet V_{ss} = V(\infty) = 24 \text{ V}$$

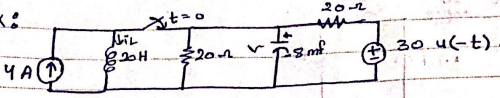
$$\bullet v(t) = V_{ss} + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$= 4$$

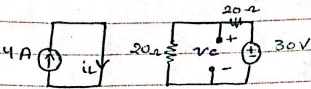
$$\bullet \frac{dv(t)}{dt} = \frac{ic(0^+)}{C} = \frac{4}{0.25} = 16$$



Ex:

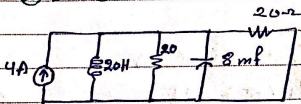


①  $t = 0^-$



- $i_L(0^-) = i_L(0^+) = 4A$ .
- $v_c(0^-) = v_c(0^+) = \frac{30}{40} \cdot 20 = 15V$ .

②  $t = 0^+$



- $\alpha = \frac{1}{2RC} = \frac{1}{2(10)(8 \times 10^{-3})} = 6.25$ .
- $\omega_0 = \frac{1}{\sqrt{LC}} = 2.5$ .
- $\alpha > \omega_0$  "overdamped"
- $s_1, s_2 = \frac{-1}{2C} \pm \sqrt{\frac{1}{4C^2} - \frac{1}{LC}} = -6.25 \pm 5.918$ .

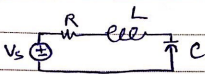
$$i_L(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$= \frac{di_L(0)}{dt} \cdot t + \frac{v_c(0^+)}{L} = \frac{15}{20} t$$

\* step response of ~~series~~ series RLC



①  $t = 0^+$



$$-v_s + v_R + v_L + v_C = 0$$

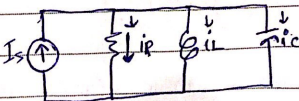
$$-v_s + iR + L \frac{di}{dt} + \frac{1}{C} \int i(t) dt = 0$$

$$-\frac{dv_s}{dt} + R \frac{di}{dt} + L \frac{di}{dt} + \frac{1}{C} i = 0$$

$$v(t) = v_{ss} + v$$

$$= v_s + \left\{ \begin{array}{l} A_1 e^{s_1 t} + A_2 e^{s_2 t} \\ \vdots \end{array} \right.$$

\* step response of parallel RLC \*



$$i(t) = I_s + \left\{ \begin{array}{l} \dots \\ \dots \\ \dots \end{array} \right.$$

Ch-9"

[ lead  $\rightarrow$  متقدم  
lag  $\rightarrow$  متاخر ]

\* Sinusoidal  $\phi$  phasor  
 $\downarrow$   
sin, cos

\* AC: the original change of value  $\phi$  direction

$$x(t) = A \sin(\omega t + \phi)$$

amplitude  $\rightarrow$  angular velocity  $\rightarrow$  phase shift

$$\omega = 2\pi F = \frac{2\pi}{T}$$

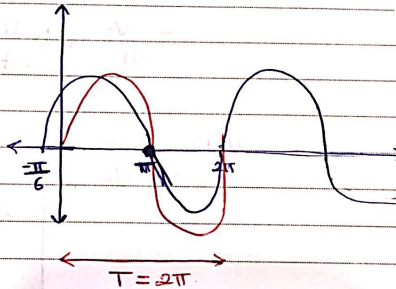
Ex:  $x(t) = 4(\sin(t + 30^\circ))$

\*  $A = 4$

\*  $\omega = 1 \text{ rad/sec}$

\*  $T = \frac{2\pi}{\omega} = 2\pi$

\*  $\phi = 30^\circ$



\*  $\sin x = \cos(x - 90^\circ)$

\*  $-\sin x = \sin(x + 180^\circ) = \cos(x + 90^\circ)$

\*  $-\cos x = \cos(x - 180^\circ)$

\*  $10 \sin(30t - 40) = 10 \cos(30t - 40 - 90)$   
 $30t - 130^\circ$



Exo. Find the phase angle.

$$i_1 = -4 \sin(377t + 25^\circ) = -4 \cos(377t + 25^\circ + 90^\circ) = -4 \cos(377t + 115^\circ)$$

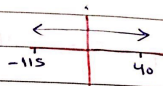
$$i_2 = 5 \cos(377t - 40^\circ)$$

$\rightarrow 115 - (-40) = 155^\circ$   
 phase

•  $\omega = 2\pi f$

•  $f = \frac{\omega}{2\pi} = \frac{377}{2\pi} = 60 \text{ Hz}$

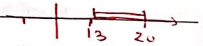
•  $i_1$  leads  $i_2$  by  $115^\circ$ .



Exo.  $i_1 = 2 \sin(10t - 13^\circ)$

$i_2 = 4 \sin(10t - 20^\circ)$

$i_1$  leads  $i_2$  by  $7^\circ$  ( $-13 - (-20) = 7^\circ$ )



~~phasor~~ (polar)

$A \cos(\omega t + \theta) \rightarrow A \angle \theta$

The signal should be in cos form

Exo:  $4 \cos(\omega t) = 4 \angle 0$

$4 \cos(\omega t - 30^\circ) = 4 \angle -30^\circ$

$4 \cos(\omega t + 50^\circ) = 4 \angle 50^\circ$

$-4 \cos(\omega t + 50^\circ) = 4 \cos(\omega t - 130^\circ) = 4 \angle -130^\circ$

$\sin(\omega t) = \cos(\omega t - 90^\circ) = 1 \angle -90^\circ$

$-4 \sin(\omega t + 60^\circ) = 4 \cos(\omega t + 150^\circ) = 4 \angle 150^\circ$

(mode  $\rightarrow$  2  $\rightarrow$  ENG  $\rightarrow$  shift 2  $\rightarrow$  3  $\rightarrow$  = )

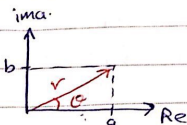
### \* complex numbers \*

\*  $Z = a + jb$

real  $\leftarrow$   $\rightarrow$  imaginary

=  $r \angle \theta$   $\rightarrow$  polar, phasor

complex form, cartesian.



\*  $r = \sqrt{a^2 + b^2}$

\*  $\theta = \tan^{-1} \frac{b}{a}$

\*  $Z = 3 + 4j = r \angle \theta$

$r = \sqrt{3^2 + 4^2} = 5$

$\theta = \tan^{-1} \frac{4}{3} = 53^\circ \rightarrow 5 \angle 53^\circ$

\*  $10 \angle 36.7^\circ$

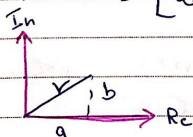
$a = r \cos \theta = 10 \cos 36.7$

$b = r \sin \theta = 10 \sin 36.7$

\*  $\omega = 2\pi f$

\*  $f = \frac{1}{T}$  Hz

$A \cos [\omega t + \theta] \rightarrow A \angle \theta$



$a = r \cos \theta$

$b = r \sin \theta$

$Z = a + jb$

$Z = r \angle \theta$

$r = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1} \frac{b}{a}$



- $a + jb \rightarrow r \angle \theta = r e^{j\theta}$

- $r e^{j\theta} \rightarrow r \cos \theta + j r \sin \theta$

- $1 e^{j\theta} \rightarrow \cos \theta + j \sin \theta$

Exo

- \*  $1 e^{-j30} = \cos 30 - j \sin 30 = 1 \angle -30^\circ$   
 $= \sqrt{3}/2 - j1/2$

- \*  $5 \angle 36^\circ$

$(4 + j3) \times (5 \angle 60^\circ) \rightarrow 25 \angle 96.7^\circ$

$\frac{3 + j6}{8 \angle 53^\circ + (17 + j40)} = 0.3 \angle 3.7^\circ$

- \*  $r(t) = A \cos(\omega t + \theta)$

$V = A \angle \theta$

$\frac{dr(t)}{dt} = \frac{d}{dt} [V(t)]$

phasor =  $j \omega V$



$\frac{di}{dt} = j \omega I$

$\frac{d^2 i}{dt^2} = j \omega j \omega I$

$(j \cdot j = -1)$

$\frac{d^2 i}{dt^2} = -\omega^2 I$

$$\star \int i(t) dt = \frac{I}{j\omega}$$

$$\star \int v(t) dt = \frac{V}{j\omega}$$


~~Impedance~~ Impedance.  
3 types of load.

1) Resistance  R

2) Inductor  L


3) capacitor  C


(polar) phasor  $j\omega$  Impedance  $j\omega L$

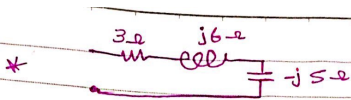
$j\omega L$  R 

$$Z_L = j\omega L \quad (-r)$$

$$Z_C = \frac{-j}{\omega C}$$

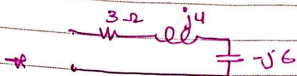
Ex<sup>o</sup>   $\rightarrow j\omega L = j 30 \Omega$   
 $\omega = 10 \text{ rad/sec}$   $0 + j 3 = 3 \angle 90^\circ$

$0.5 \text{ F}$   
  
 $\omega = 4 \text{ rad/sec}$   $\rightarrow \frac{-j}{\omega C} = -j 0.5$   
 $-j 0.5 = 0.5 \angle -90^\circ$



$$Z_{eq} = 3 + j6 - j5 = 3 + j1 = \sqrt{10} \angle \oplus 18^\circ$$

Inductive



$$Z_{eq} = 3 + j4 - j6 = 3 - j2 = \sqrt{13} \angle \ominus 33^\circ$$

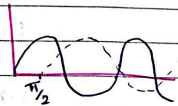
capacitive

\* Resistive  $\Rightarrow$   $\phi = 0^\circ$

\*  $V_L = L \frac{di}{dt}$

\*  $V_L = j\omega L I$

\*  $\frac{V}{I} = Z = j\omega L = \omega L \angle 90^\circ$



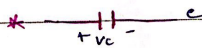
• In the Inductor  $V$  leads  $I$  by  $90^\circ$ .

• In a Capacitor  $I$  leads  $V$ .

\* R (Resistive load) \*

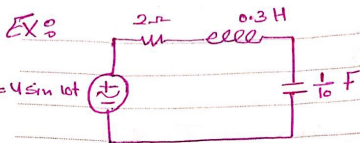
$$R = \frac{V}{I} \rightarrow R \angle 0^\circ$$

$I, V$  Inphase.



$$\frac{V_C}{I_C} = \frac{1}{\omega C} \angle -90^\circ$$

$$V = \frac{1}{C} \int i \rightarrow V = \frac{1}{C} \cdot \frac{1}{j\omega} I = \frac{V_C}{I_C} = Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

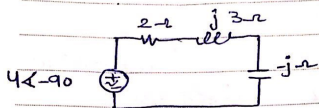


$$v(t) = 4 \sin 10t$$



$$v(t) = 4 \cos (10t - 90^\circ)$$

Find  $V_R(t)$ ,  $V_L(t)$ ,  $V_C(t)$ .



$$V_R = \frac{(4\angle -90^\circ)(2)}{2 + j3 - j}$$

$$V_L = \frac{(4\angle -90^\circ)(j3)}{2 + j3 - j} = 4.24 \angle -45^\circ \text{ V.}$$

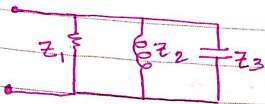
$$V_C = \frac{(4\angle -90^\circ)(-j)}{2 + j3 - j} = 1.41 \angle 135^\circ \text{ V.}$$

- $V_R(t) = 2.8 \cos (10t - 135^\circ)$
- $V_L(t) = 4.24 \cos (10t - 45^\circ)$
- $V_C(t) = 1.4 \cos (10t + 135^\circ)$

$$I = I_C = C \frac{dv_C}{dt}, \quad \omega = \frac{1}{2} \text{ cv}^2 \quad (t=3)$$

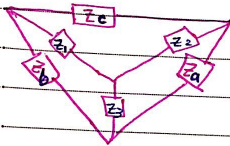
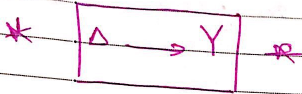
\* Admittance  $Y$

$$Y = \frac{1}{Z} \quad \text{or} \quad \text{moe}$$



$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

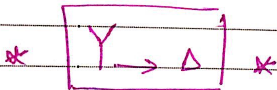
$$Y_T = Y_1 + Y_2 + Y_3$$



$$\bullet \quad Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$\bullet \quad Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

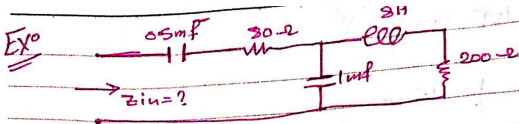
$$\bullet \quad Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$



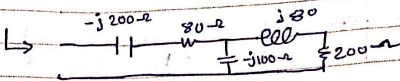
$$\bullet \quad Z_a = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_1}$$

$$\bullet \quad Z_b = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_2}$$

$$\bullet \quad Z_c = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_3}$$

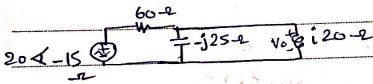
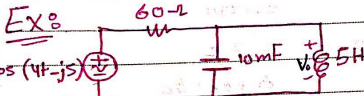


$$\omega = 10 \text{ rad/sec}$$



$$\bullet \frac{(j80 + 200) * (-j100)}{j80 + 200 - j100} + 80 - j200 = Z_{in}$$

$$Z_{in} = 129.52 - j292 \text{ } \Omega$$



$$j20 \parallel (-j25) = j100$$

$$V_0 = \frac{(20 \angle -15^\circ)(j100)}{60 + j100} = 17.15 \angle 15.96^\circ$$

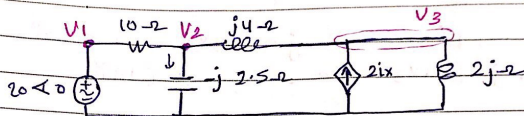
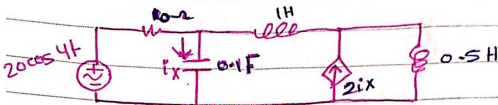
$$\bullet v(t) = 17.15 \cos(4t + 15.96^\circ) \text{ V}$$

\* Ch 10 \*

Steady state Analysis.

\* Nodal analysis

Ex: Find  $i_x$  in time DC.



node #1

$$V_1 = 20 \angle 0$$

node #2

$$\frac{V_1 - V_2}{10} = \frac{V_2}{-j2.5} + \frac{V_2 - V_3}{j4}$$

$$0.1V_1 - 0.1V_2 = j0.4V_2 - j0.25V_2 + j0.25V_3$$

$$2 \angle 0 - 0.1V_2 - j0.4V_2 + j0.25V_2 - j0.25V_3 = 0$$

$$(-0.1 - 0.15j)V_2 - j0.25V_3 = 2 \angle 188$$

node #3

$$I_x = \frac{V_2}{-j0.25}$$

$$\frac{V_2 - V_3}{j4} + 2I_x = \frac{V_3}{j2}$$





$$\rightarrow \frac{V_2 - V_3}{j4} + 2 \left[ \frac{V_2}{-j2.5} \right] = \frac{V_2}{j2}$$

$$-j0.25V_2 + j0.25V_3 + 0.8jV_2 = -j0.5V_3$$

$$j0.55V_2 + j0.75V_3 = 0$$

$$(V_2) = \frac{-j0.75V_3}{j0.55} = 1.3V_3$$

sub in node 2

$$(-0.1 - 0.15j)(-1.3V_2) - j0.25V_3 = 2 \angle 180^\circ$$

$$0.13V_3 + j0.19V_3 - j0.25V_3 = 2 \angle 180^\circ$$

$$(0.18 - j0.06)V_3 = 2 \angle 180^\circ$$

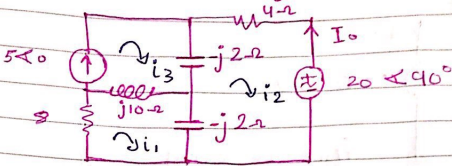
$$V_3 = \frac{2 \angle 180^\circ}{0.18 - j0.06}$$

$$= 13.96 \angle -155^\circ \text{ V}$$

$$V_2 = -1.3V_3 = 18 \angle 25^\circ \text{ V}$$

$$* I_X = \frac{V_2}{-j2.5} = \frac{18 \angle 25^\circ}{-j2.5} = 7.2 \angle 155^\circ \text{ A}$$

\* Mesh Analysis \*



• mesh # 3

$$I_3 = 5 \angle 0^\circ$$

• mesh # 2

$$-j2 [I_2 - I_1] - j2 [I_2 - I_3] + 4I_2 + 20 \angle 90^\circ = 0$$

$$j2I_1 + [4 - j4]I_2 = 20 \angle -90^\circ$$

$$j2I_1 + [4 - j4]I_2 + j2I_3 = 20 \angle -90^\circ$$

$$\left[ \frac{(j2)15 \angle 0^\circ}{2 \angle 90^\circ} \right] = 10 \angle 90^\circ$$

$$\begin{cases} A \angle \theta_1, B \angle \theta_2 \\ |AB| \angle \theta_1 + \theta_2 \end{cases}$$

• mesh # 1

$$8I_1 + j10(I_1 - I_3) - j2[I_1 - I_2] = 0$$

$5 \angle 0^\circ$

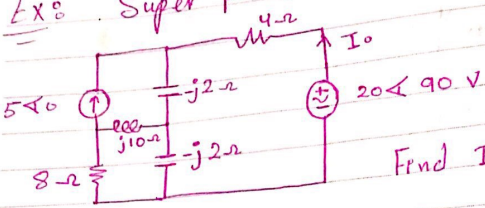
$$[8 + j8]I_1 + j2I_2 = 50 \angle 90^\circ$$

Cramer's Rule :

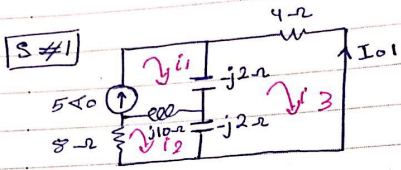
$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j50 \end{bmatrix} \quad \begin{matrix} I_1 = \frac{\Delta_1}{\Delta} \\ I_2 = \frac{\Delta_2}{\Delta} \end{matrix}$$

$$I_0 = -I_2$$

Ex: Super position.



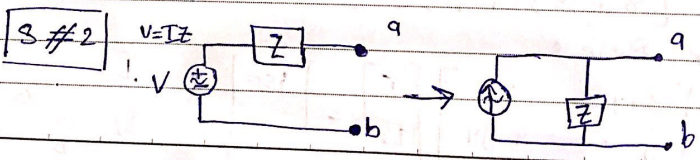
Find  $I_o$  using S.P



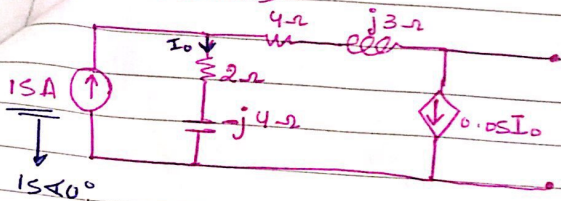
- Mesh # 1  
 $I_1 = 5∠0$
- Mesh # 2  
 $(8+j8)I_2 - j2I_3 = -j50$
- Mesh # 3  
 $j2I_2 + (4-j4)I_3 = -j16$

Kremmer's Rule :

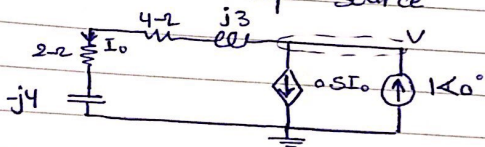
$$\begin{bmatrix} 8+8j & j2 \\ j2 & 4-4j \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -j50 \\ -j16 \end{bmatrix}$$



Thevenins



① kill indep. source



$$1 \angle 0^\circ = 0.5 I_0 + \frac{V}{4 + j3 - j3}$$

$$1 \angle 0^\circ = \frac{1.5V}{(6-j)}$$

$$V = 1(6-j) = \frac{6.08}{1.5} \angle -9.4^\circ \text{ V}$$

$$Z_{th} = \frac{V}{1 \angle 0^\circ} = 4.05 \angle -9.4^\circ \Omega$$

$$V_{th} = V_{ab} =$$

$$1.5 \angle 0^\circ = I_0 + 0.5 I_0 = 1.5$$

$$I_0 = 1.0 \angle 0^\circ \text{ A}$$

$$V_{ab} = -[4 + j3](0.5 I_0) + (2 - j4) I_0$$