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الجامعة الأردنية	0301201 تفاضل وتكامل 3	الامتحان الأول: 2018/10/21
اسم الطالب: محمد سعود أبو طلية		مدرس المادة: د. حسن النجار
الرقم الجامعي:		وقت المحاضرة: ~~...~~ 12-1

يتكون الامتحان من 5 أسئلة في 3 صفحات

[1] Consider the lines:

$L_1: x = 1 + 6t;\ \ y = 3 - 7t;\ \ z = 2 + t$

$L_2: x = 10 + 3s;\ \ y = 6 + s;\ \ z = 14 + 4s$

a) (4 marks) Find the point of intersection between the two lines L_1 and L_2.

① $1 + 6t = 10 + 3s$

$-3 \times$ ② $3 - 7t = 6 + s$

③ $2 + t = 14 + 4s$

$1 + 6t = 10 + \cancel{3s}$

$-9 + 21t = -18 - \cancel{3s}$

$-8 + 27t = -8$

$\boxed{t = 0}$

$1 = \cancel{10} + 3s \Rightarrow 3s = -9 \Rightarrow \boxed{s = -3}$

sub $t=0$ in ①②③

$2 = 14 - 12$

$\boxed{2 = 2}$

$L_1 \times L_2 \Rightarrow P = (1, 3, 2)$

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b) (3 marks) Find the angle between the two lines L_1 and L_2.

$\cos\theta = \dfrac{\vec{V}_1 \cdot \vec{V}_2}{|V_1||V_2|} = \dfrac{18 - 7 + 4}{(\sqrt{86})(\sqrt{26})}$

$\theta = \cos^{-1}\left(\dfrac{15}{\sqrt{86 \times 26}}\right)$

$\vec{V}_1 = \langle 6, -7, 1 \rangle$

$\vec{V}_2 = \langle 3, 1, 4 \rangle$

$|\vec{V}_1| = \sqrt{36+49+1} = \sqrt{86}$

$|\vec{V}_2| = \sqrt{9+1+16} = \sqrt{26}$

c) (4 marks) Find the plane containing the two lines L_1 and L_2.

$\vec{N} = \vec{V}_1 \times \vec{V}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & -7 & 1 \\ 3 & 1 & 4 \end{vmatrix} = -29\hat{i} - 21\hat{j} + 27\hat{k}$

$\vec{N} = \langle -29, 21, 27 \rangle,\ P_1(1, 3, 2)$

$P = -29(x-1) + 21(y-3) + 27(z-2) = 0$

$-29x + 29 + 21y - 63 + 27z - 54 = 0$

$29x - 29 - 21y + 63 - 27z + 54 = 0$

$29x - 21y - 27z = 29 - 63 - 54$

$29x - 21y - 27z = -88$

[2] Let $\overline{v} = \langle a, b, a\rangle, \overline{u} = \langle b, a, b\rangle$.

a) (3 marks) Find $\overline{v} \times \overline{u}$

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$$\vec{v} \times \vec{u} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ a & b & a \\ b & a & b \end{vmatrix} = (bb-aa)\hat{i} - (ab-ab)\hat{j} + (aa-bb)\hat{k}$$

$$= |b|^2 - |a|^2\,\hat{i} - 0 + |a|^2 - |b|^2\,\hat{k}$$

$$= |b|^2 - |a|^2\,\hat{i} + |a|^2 - |b|^2\,\hat{k}$$

$$= \langle |b|^2 - |a|^2, 0, |a|^2 - |b|^2 \rangle.$$

b) (3 marks) For what nonzero values of a and b the vectors $\overline{v}$ and $\overline{u}$ are parallel.

Sol: ~~parallel~~ when $\frac{b}{a} = \frac{a}{b} = \frac{b}{a}$ ~~$\Rightarrow b = a = \mathbb{R} - \{0\}$~~

~~parallel~~ $\Rightarrow$ ~~$\frac{a}{b} = \frac{b}{a} = \frac{a}{b}$~~

~~parallel when a and b~~

~~$\vec{v} \cdot \vec{u} = 1$~~

~~$ab + ba + ab = 1$~~

~~$ab = 1$~~

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Look Back

[3] Let L be a line passing through the point $P(3,2,1)$ and perpendicular to the plane π: $2x - y + 2z = -3$.

a) (3 marks) Find the equation of the line L.

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$\vec{V}_L = \vec{N}_P = \langle 2, -1, 2 \rangle$

$P(3,2,1) \Rightarrow L: \frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{2}$

$x = 3 + 2t$

$y = 2 - t$

$z = 1 + 2t.$

b) (3 marks) Find the point of intersection between L and the plane π

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$2(3+2t) - (2-t) + 2(1+2t) = -3$

$6 + 4t - \cancel{2} + t + \cancel{2} + 4t = -3$

$9t = -9$

$\boxed{t = -1}$

$x = 1$

$y = 3$

$z = -1$

$P = (1, 3, -1)$

[4] (3 marks) Name and sketch the surface $z - 3 = \sqrt{1 + x^2 - y^2}$ => Look at Back

$(z-3)^2 = 1 + x^2 - y^2$

~~$z^2 - 6z + 9 = 1 + x^2 - y^2$~~

~~$(z-3)^2$~~

$0 = 1 + x^2 - y^2 - (z-3)^2$

$\boxed{(z-3)^2 = 1 + x^2 - y^2}$

$\therefore\ -1 = \frac{x^2}{1} - y^2 - (z-3)^2$

circular Hyperboloid of Two sheets. along the x-axis.

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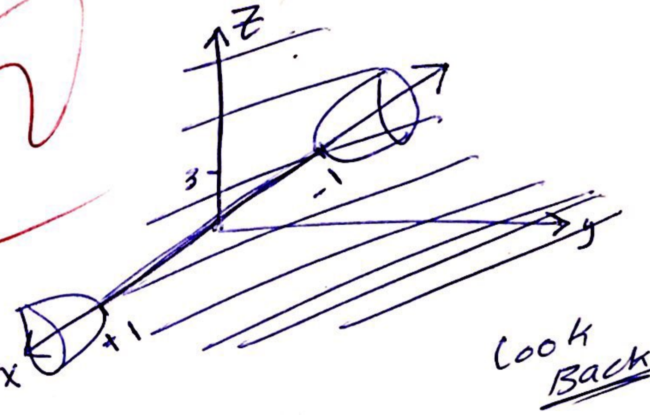

Look Back.

[5] (4 marks) A surface consists of all points $P(x, y, z)$ such that the distance from P to the point $(0, 1, 0)$ is twice the distance from P to the plane $y = 1$. Find an equation for this surface and identify it.

~~x~~ $(0,1,0)$

$y\ (0,1,0)$

$(2|Px|)^2 = (|Py|)^2$

1

~~$2\sqrt{x}$~~

$4(x-0)^2 + (y-1)^2 + (z-0)^2 = (x-0)^2 + (y-1)^2 + (z-0)^2$

~~$4x^2 + 4(y-1)^2 + 4(z)^2$~~

$4x^2 + 4(y-1)^2 + 4z^2 = x^2 + (y-1)^2 + z^2.$

$\Longrightarrow 3x^2 + 3(y-1)^2 + 3z^2 = 0$

$\therefore$ a plane with Normal $\vec{N} = \langle 3,3,3 \rangle$ passes through the point $(0,1,0)$

sketch Q4: