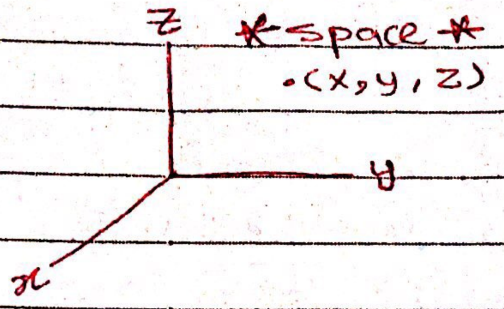
chapter – 12
space.



* space *

$P_1(x_1, y_1, z_1)$

$P_2(x_2, y_2, z_2)$

$$\boxed{|P_1P_2|} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$M_{\overline{P_1P_2}} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

$\boxed{\text{Ex:}}$ $P = (2, 4, -5)$ $Q = (6, -2, 3)$

Find $|PQ|$, $M_{\overline{PQ}}$

$$|PQ| = \sqrt{(4)^2+(-6)^2+(8)^2} = \sqrt{116}$$

$$M_{\overline{PQ}} = (4, 1, -1)$$

* sphere *

equ:

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 = r^2$$

where: $(x_0, y_0, z_0) \to$ center

$r \to$ radius

Ex: Find the equ. center $= (3, -1, 5)$, radius $= 4$.

$$(x-3)^2+(y+1)^2+(z-5)^2 = 16$$

Q: Identify the equ.

$x^2 + y^2 + z^2 + 2x - 4y + 6z + 10 = 0$

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$(x+1)^2 + (y-2)^2 + (z+3)^2 + 10 = 1 + 4 + 9$

$(x+1)^2 + (y-2)^2 + (z+3)^2 = 4$

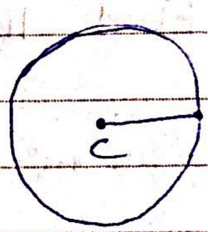
↳ sphere

center $(-1, 2, -3)$

radius $= 2$.

$\frac{15}{797}$ Find equ. of the sphere that passes through the point $P(4, 3, -1)$

center $= C(3, 8, 1)$.



$r = |PC| = \sqrt{1 + 25 + 4} = \sqrt{30}$.

$\boxed{\text{equ:}}$ $(x-3)^2 + (y-8)^2 + (z-1)^2 = 30$.

$\boxed{22}$ 1) Midpoint (center) 2) Distance

$\boxed{23}$ Find equ's center $(2, -3, 6)$ that touch

a) XY-plane b) YZ-plane c) XZ-plane.

↳ $r = 6$ ↳ $r = 2$ ↳ $r = 3$

$\boxed{24}$ Find equ. largest sphere center $(5, 4, 9)$. First octant.

$r = 4$ باخذ أصغر رقم

$(x-5)^2 + (y-4)^2 + (z-9)^2 = 16$

12
796

Q: Find Distance $(3,7,-5)$

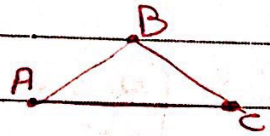
a) xy-plane $\rightarrow D=5$.

b) xz-plane $\rightarrow D=7$

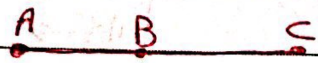
c) x-axis $\rightarrow D=\sqrt{49+25}$

d) y-axis $\rightarrow D=\sqrt{9+25}$

$\boxed{11}$

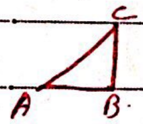


هل هي على استقامة
واحدة ؟!



- If $|AB|+|AC| = |BC| \rightarrow A,B,C$ are liy on the same line "Colinear"

Q: $\boxed{9,10}$

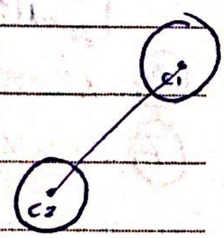


right triangle.

$$(AB)^2+(BC)^2=(AC)^2$$

47
797

Find the Distance between the sphere $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=4x+4y+4z-11$



$C_1=(0,0,0) \quad r_1=2$

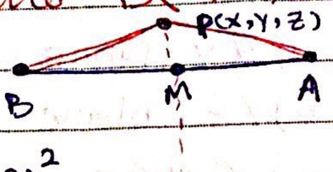
$c_2=(x-2)^2+(y-2)^2+(z-2)^2=1$

$=(2,2,2) \quad r_2=1$

$|C_1C_2|=\sqrt{4+4+4}=\sqrt{12}=2\sqrt{3}$.

* $D=2\sqrt{3}-3$.

$\boxed{45}$ Find equ. of the set of all points equidistant from the point $A(-1, 5, 3)$ and $B(6, 2, -2)$.



$|PA|^2 = |PB|^2$

$(x+1)^2 + (y-5)^2 + (z-3)^2 = (x-6)^2 + (y-2)^2 + (z+2)^2$

$= \cancel{x^2} + 2x + 1 + \cancel{y^2} - 10y + 25 + \cancel{z^2} - 6z + 9 = \cancel{x^2} - 12x + 36 + \cancel{y^2} - 4y + 4$
$+ \cancel{z^2} + 4z + 4$

$= 14x - 6y - 10z = 9 \rightarrow$ plane مستوى

$\mathbb{R} \rightarrow$ خط الأعداد.
$\mathbb{R}^2 \rightarrow$ 2D.
$\mathbb{R} \rightarrow$ 3D.

Q. Describe equ. $\mathbb{R}^3$

① $z = 3$
↳ plane parallel to xy-plane.

② $x = 5$
↳ plane parallel to yz-plane

③ $x^2 + y^2 = 1$
↳ cylinder along z-axis.

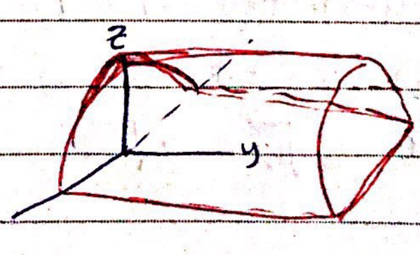
④ $y^2 + z^2 = 5$
↳ cylinder along x-axis.

⑤ $\frac{x^2}{4} + \frac{y^2}{9} = 1$
↳ ellips قطع ناقص (cylinder along z-axis)

⑥ $x^2 - y^2 = 1 \rightarrow$ haiparabola قطع زائد

⑦ $y = x^2 \rightarrow$ parabola قطع مكافئ

Q: sketch $z = 1 - x^2$
parabolic cylinder along y-axis



25–38) Identify in $\mathbb{R}^3$.

29) $1 \le z \le 6$

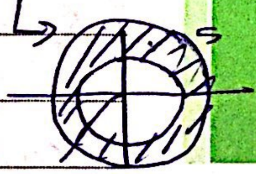
the set of all point on and between the two planes $z=1$ and $z=6$.

34) $x^2+y^2+z^2 \le 4$.

the set of all points on and inside the sphere $x^2+y^2+z^2=4$.

35) $5 \ge x^2+y^2+z^2 \ge 1$

the set of all point on and between the sphere $x^2+y^2+z^2 \geq 1$



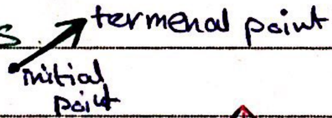
* $x^2+y^2 \le 1$

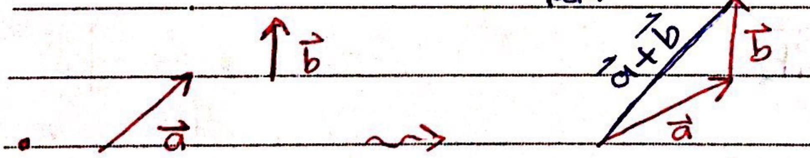
↳ " " " on and inside the cylinder $x^2+y^2 \le 1$

* $4 \ge x^2+y^2 \ge 1$

" " " on and between the cylinder $4 \ge x^2+y^2 \ge 1$

* ch 12–2 Vectors





- $\vec{a} = \langle a_1, a_2, a_3 \rangle$
- $\vec{b} = \langle b_1, b_2, b_3 \rangle$
- $\vec{a}+\vec{b} = \langle a_1+b_1, a_2+b_2, a_3+b_3 \rangle$
- $k\vec{a} = \langle ka_1, ka_2, ka_3 \rangle$.
- $|\vec{a}| = \sqrt{a_1^2+a_2^2+a_3^2}$.
- $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$.

unit vector

Q:

$\vec{a} = \langle 2, 4, -1 \rangle$

Find a vector $\vec{b}$ in the same direction of $\vec{a}$ with length 7

$$\vec{b} = 7 \underbrace{\frac{\vec{a}}{|\vec{a}|}}_{=1} \qquad \vec{b} = \frac{7}{\sqrt{4+16+1}} \langle 2, 4, -1 \rangle$$

$$= \left\langle \frac{14}{\sqrt{21}}, \frac{28}{\sqrt{21}}, \frac{-7}{\sqrt{21}} \right\rangle$$

* اذا كان oppos- بحط بالسالب 7 "

* $\hat{i} = \langle 1, 0, 0 \rangle$
* $\hat{j} = \langle 0, 1, 0 \rangle$
* $\hat{k} = \langle 0, 0, 1 \rangle$

standard basis vectors.

$\langle a, b, c \rangle = ai + bj + ck$.

Q: If $P_1 = (2, 4, 8)$, $P_2(-3, 5, 2)$

Find $\overrightarrow{P_1P_2}$, $\overrightarrow{P_2P_1}$

$\overrightarrow{P_1P_2} = \langle -5, 1, -6 \rangle$

$\overrightarrow{P_2P_1} = \langle 5, -1, 6 \rangle = -\overrightarrow{P_1P_2}$

Q: $\overrightarrow{P_1P_2} = \langle 3, 2, 7 \rangle$. $\overrightarrow{P_1P_3} = \langle 5, -1, 4 \rangle$

Find $\overrightarrow{P_2P_3}$.

$\overrightarrow{P_2P_3} = \overrightarrow{P_2P_1} + \overrightarrow{P_1P_3}$ نهاية الاول بداية الثاني.

$= \langle -3, -2, -7 \rangle + \langle 5, -1, 4 \rangle$

$= \langle 2, -3, -3 \rangle$

12.3 Dot product (scalar product).

$\vec{a} = \langle a_1, a_2, a_3 \rangle$

$\vec{b} = \langle b_1, b_2, b_3 \rangle$

* $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$.
* $\vec{a} \cdot \vec{b} = |a||b| \cos\theta$.
* $\boxed{\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}}$

Ex: let $\vec{v} = \langle 1, 2, 5 \rangle$

$\vec{u} = \langle -2, 4, 0 \rangle$ Find the angle between

$$\cos\theta = \frac{\vec{v} \cdot \vec{u}}{|\vec{v}||\vec{u}|} = \frac{-2+8}{\sqrt{1+4+25}\sqrt{4+16}} = \frac{6}{\sqrt{30 * 20}}.$$

$$\theta = \cos^{-1}\left(\frac{6}{\sqrt{600}}\right).$$

* If $\vec{a} \cdot \vec{b} = 0 \longrightarrow \vec{a} \perp \vec{b}$ *

[1] $\vec{a} \cdot \vec{a} = |a|^2$

[2] $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

[3] $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

[4] $k\vec{a} \cdot \vec{b} = \vec{a} \cdot k\vec{b} = k(\vec{a} \cdot b)$

[5] $|k\vec{a}| = |k||\vec{a}|$.

Ex: $|\vec{a}| = 2$, $|\vec{b}| = 3$

$\vec{a} \perp \vec{b}$ Find:

$* \, |a|^2 = \vec{a} \cdot \vec{a}$

[1] $|a+b| = (a+b) \cdot (a+b)$

$= |a|^2 + \cancel{a \cdot b} + \cancel{b \cdot a} + |b|^2$

$= 4 + 9 = 13$

$|a+b| = \sqrt{13}$.

* [2] $|4a+b| = |4|(\vec{a} \cdot \vec{b}) = 4 \times 0 = 0$.

$\Rightarrow (4a+b) \cdot (4a+b)$

* Direction angles *

$\vec{a} = \langle a_1, a_2, a_3 \rangle$

$\cos\alpha = \dfrac{\vec{a} \cdot \hat{i}}{|\vec{a}||\hat{i}|}$

$\cos\alpha = \dfrac{a_1}{|\vec{a}|}$ x ⅃k

$= \dfrac{a_1 + 0}{|\vec{a}| \cdot |1|}$

$\cos\beta = \dfrac{a_2}{|\vec{a}|}$ y-axis ⅃k

$\cos\gamma = \dfrac{a_3}{|\vec{a}|}$ z-axis

$* \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Ex: let $\vec{b} = \langle 3, -1, 5 \rangle$ Find α, β, γ

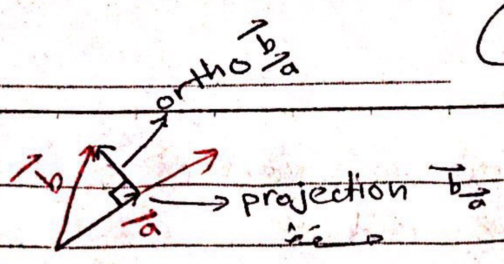
$\cos\alpha = \dfrac{3}{\sqrt{9+1+25}} = \dfrac{3}{\sqrt{35}}$

$\cos\beta = \dfrac{-1}{\sqrt{35}}$ $\cos\gamma = \dfrac{5}{\sqrt{35}}$

$\dfrac{9}{35} + \dfrac{1}{35} + \dfrac{25}{35} = \dfrac{35}{35} = 1$ ✓

Scanned with CamScanner

projection



* Projection $\vec{b}_{\vec{a}} = \dfrac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\,\vec{a} = \dfrac{\vec{b}\cdot\vec{a}}{|a|}\cdot\dfrac{\vec{a}}{|\vec{a}|}$ (vector proj) $\dfrac{\vec{a}}{|\vec{a}|} \to 1$

* ortho $\vec{b}_{\vec{a}} = \vec{b} - $ projection $\vec{b}_{\vec{a}}$

* comp $\vec{b}_{\vec{a}} = \dfrac{\vec{b}\cdot\vec{a}}{|a|}$ (component) (scalar projection)

Q: $\vec{a} = \langle 1, 4, -2\rangle$

$b = \langle 5, 3, -4\rangle$

$$\text{Proj}_{\vec{b}}\,\vec{a} = \frac{a\cdot b}{|b|^2}\,\vec{b}$$

$$= \frac{5+12+8}{25+9+16}\langle 5, 3, -4\rangle = \frac{1}{2}\langle 5, 3, -4\rangle$$

43 / 813 Find scalar and vector projection of b onto $\vec{a}$.

$\vec{a} = \langle 3, -3, 1\rangle$

$\vec{b} = \langle 2, 4, -1\rangle$

- vector projection $= \text{proj}_{\vec{a}}\,\vec{b} = \dfrac{b\cdot a}{|\vec{a}|^2}\,\vec{a} = \dfrac{6-12-7}{9+9+1}\langle 3, -3, 1\rangle$

$= \dfrac{-7}{19}\langle 3, -3, 1\rangle$

- scalar proj $=$ comp $\vec{b}_{\vec{a}} = \dfrac{b\cdot a}{|a|} = \dfrac{-7}{\sqrt{19}}$

$\rightarrow$

[24] Determine wether the given vectors are orthogonal, parallel or neither

↳ = Dot = 0. ↳ $v = ku$.

a) $u = \langle -5, 4, -2 \rangle$

$v = \langle 3, 4, -1 \rangle$ $u \cdot v = -15 + 16 + 2 \neq 0$ ∴ Not orthogonal.

$\frac{3}{-5} \overset{?}{\neq} \frac{4}{4} \overset{?}{\neq} \frac{-1}{-2}$ → Not parallel

∴ neither

b) $u = \langle 9, -6, 3 \rangle$

$v = \langle -6, 4, -2 \rangle$

$u \cdot v \neq 0$ not orthogonal

$\frac{-6}{9} \overset{?}{=} \frac{4}{-6} \overset{?}{=} \frac{-2}{3}$ ✓ ∴ parallel.

↳ $v = \frac{-2}{3} u$.

[25] Determine wether the triangle with vertices $P(1,-3,-2)$, $Q(2,0,-4)$, $R(6,-2,-5)$ is right-angle قائمة

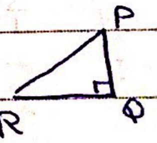
[1] △ فيثاغورس

[2] ⅁

$\overrightarrow{PQ} = \langle 1, 3, -2 \rangle$ $\overrightarrow{PQ} \cdot \overrightarrow{PR} = 5 + 3 + 6 \neq 0$

$\overrightarrow{PR} = \langle 5, 1, -3 \rangle$ $\overrightarrow{PQ} \cdot \overrightarrow{QR} = 4 - 6 + 2 = 0$ ✓

$\overrightarrow{QR} = \langle 4, -2, -1 \rangle$. ∴ right angle

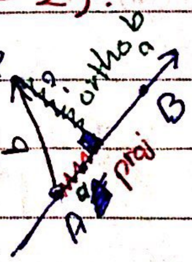


Q: Find the distance $P(2,4,3)$ and the line passes throw $A(1,-2,5)$ and $B(4,3,-2)$.

$\vec{b} = \overrightarrow{AP}$

$\vec{a} = \overrightarrow{AB}$

$\text{Proj}_{\vec{a}} \vec{b} = \frac{b \cdot a}{|a|^2} \vec{a} = \langle \quad \rangle$



$\text{ortho}_{\vec{a}} \vec{b} = \vec{b} - \text{proj}_{\vec{a}} \vec{b}$

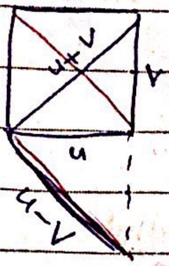
$d = |\text{ortho}_{\vec{a}} \vec{b}|$

Q: $\boxed{\frac{55}{813}}$, $\boxed{\frac{58}{814}}$ $\rightarrow$ أطلع من الكتاب

$\langle a, a, a \rangle$

$\langle a, a, a \rangle$

$\boxed{64}$ Show that $u+v$ and $u-v$ are orthogonal then vectors $\vec{u}$ and $\vec{v}$ must have the same length.



$$(u+v)\cdot(u-v) = 0$$

$$|u|^2 - \cancel{u\cdot v} + \cancel{v\cdot u} - |v|^2 = 0$$

$$|u|^2 = |v|^2$$

$$|u| = \cancel{\pm}|v|.$$

12.4 cross product.

if $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ $\quad |A| = 2\times 3 - \overset{3\times 4}{12} = -2$

if $A = \begin{bmatrix} 1 & 3 & -2 \\ 5 & 4 & 1 \\ 3 & -4 & 2 \end{bmatrix}$ $\quad |A| = ?$

$$|A| = 1\times\begin{vmatrix} 4 & 1 \\ -4 & 2 \end{vmatrix} - 3 * \begin{vmatrix} 5 & 1 \\ 3 & 2 \end{vmatrix} + (-2)\times\begin{vmatrix} 5 & 4 \\ 3 & -4 \end{vmatrix}$$

$$= 12 - 21 + 64 = 55$$

if $\vec{a} = \langle a_1, a_2, a_3 \rangle$ $\quad \vec{b} = \langle b_1, b_2, b_3 \rangle$

$$\vec{a}\times\vec{b} = \begin{array}{c} \begin{matrix} i & j & k \end{matrix} \\ \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \end{array}$$

$$|\vec{a}\times\vec{b}| = |\vec{a}|\,|\vec{b}|\,\sin\theta.$$

Ex:

$\vec{a} = \langle 2, -4, 3 \rangle$

$\vec{b} = \langle 1, 2, 5 \rangle$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i^{+} & j^{-} & k^{+} \\ 2 & -4 & 5 \\ 1 & 2 & 5 \end{vmatrix}$$

$= \langle -26, -7, 8 \rangle$

$\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$

$\vec{b} \cdot (\vec{a} \times \vec{b}) = 0$

$\vec{a} \times \vec{b} = 0$

↳ parallel

$\vec{a} \parallel \vec{b}$

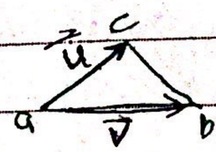
- $\vec{a} \times \vec{a} = 0$
- $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$
- $k\vec{a} \times \vec{b} = \vec{a} \times k\vec{b} = k(\vec{a} \times \vec{b})$
- $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
- $(\vec{b} + \vec{c}) \times \vec{a} = \vec{b} \times \vec{a} + \vec{c} \times \vec{a}$
- $a \times (b \cdot c)$ X خطأ
- $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$ ✓ → scalar triple products
- $a \times (b \times c) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ → vector " "

الترتيب مهم

Q:

$b =$ $a \cdot c =$

$c =$ $a \cdot b =$ Find $a \times b \times c$

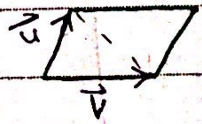
① 

$\vec{u} = ac$

$\vec{v} = ab$

Area $= \frac{1}{2} |\vec{u} \times \vec{v}|$

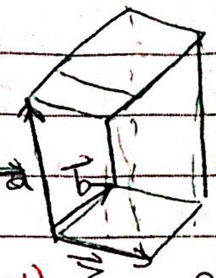
↳ $|u| |v| \sin\theta$

② 

Paddelogram

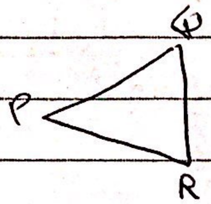
Area $= |\vec{u} \times \vec{v}|$

③ $V = \left| \vec{a} \cdot (\vec{b} \times \vec{c}) \right|$ → magnitude



30/821) Find Area of triangle PQR

$P(-1,3,1)$, $Q(0,5,2)$, $R(4,3,-1)$



$\vec{PQ} = \langle 1,2,1 \rangle$

$\vec{PR} = \langle 5,0,-2 \rangle$

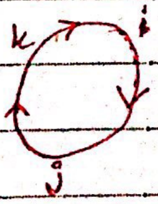
$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 5 & 0 & -2 \end{vmatrix} = \langle -4, 7, -10 \rangle$$

$$|\vec{PQ} \times \vec{PR}| = \sqrt{16+49+100} = \sqrt{165}$$

$$\text{Area} = \frac{1}{2}\sqrt{165}$$

(اذا سألهم على استقامة وحدة او لا
لازم ال cross يساوي صفر)

$i \times j = k$
$j \times k = i$
$k \times i = j$



19/821 Find two unit vectors orthog.

$a \langle 3,2,1 \rangle$ and $b \langle -1,1,0 \rangle$

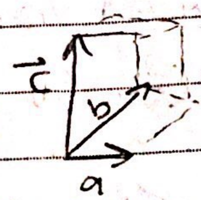
$$c = \pm (a \times b) = \begin{vmatrix} i & j & k \\ 3 & 2 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$

$$= \pm \langle -1, -1, 5 \rangle$$

$$\hat{c} = \pm \frac{\langle -1,-1,5 \rangle}{\sqrt{1+1+25}}$$

$\frac{34}{822}$ Find volume of the paralleledpied by vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$

$\vec{a} = \langle 1, 1, -1 \rangle$, $\vec{b} = \langle 1, -1, 1 \rangle$, $\vec{c} = \langle -1, 1, -1 \rangle$.



$$V = \left| a \cdot (b \times c) \right|$$

قيمة مطلقة

$$= \left| \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & -1 \end{vmatrix} \right| \begin{matrix} \to a \\ \to b \\ \to c \end{matrix} = \left| -2 - 2 + 0 \right| = 4$$

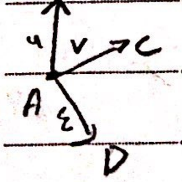
$\boxed{37}$ Determine $u = i + 5j - 2k$, $v = 3i - j$, $w = 5i + 9j - 4k$ are coplanner. (على نفس ال plane)

scalar tripple product..

$$u \cdot (v \times w) = \begin{vmatrix} 1 & 5 & -2 \\ 3 & -1 & 0 \\ 5 & 9 & -4 \end{vmatrix} = 4 + 60 - 64 = 0 \quad \therefore \text{coplanner}$$

$\boxed{38}$ $A \langle 1, 3, 2 \rangle$, $B \langle 3, -1, 6 \rangle$, $C \langle 5, 2, 0 \rangle$ and $D (3, 6, 4)$ lie in the same plane.

نفس الحل



43

$\vec{a} \cdot \vec{b} = \sqrt{3}$ $\quad \vec{a} \times \vec{b} = \langle 1, 2, 2 \rangle$

Find the angle between $\vec{a}$ and $\vec{b}$.

$a \cdot b = |a|\,|b| \cos\theta$

$|a \times b| = |a|\,|b| \sin\theta$.

$$\tan\theta = \frac{|a \times b|}{a \cdot b} = \frac{\sqrt{1+4+4}}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$\theta = \tan^{-1}(\sqrt{3}) = \pi/3$.

47 show that

$|a \times b|^2 = |a|^2 |b|^2 - (a \cdot b)^2$

$|a|^2 |b|^2 - (|a|\,|b| \cos\theta)^2 = |a|^2 |b|^2 (1 - \cos^2\theta)$.

$= |a|^2 |b|^2 \sin^2\theta$.

$= |a \times b|^2$.

48
822

If $\vec{a} + \vec{b} + \vec{c} = 0$

show that $a \times b = b \times c = c \times a$.

$b \times c = b \times -(a+b)$

$= -(b \times (a+b))$

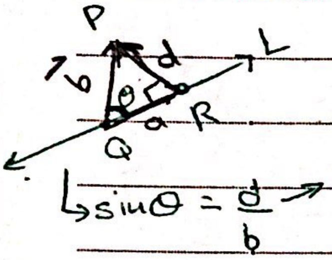
$= -(b \times a + \overset{0}{\cancel{b \times b}})$

$= -(b \times a) = \boxed{a \times b.}$

45 ⓐ let (p) not on the line L passes through (Q) and (R)

show that the Distance from (p) to line (L) is given by

$$d = \frac{|a \times b|}{|a|} \quad \text{where } \vec{a} = \vec{QR} \ , \ \vec{b} = \vec{QP}$$



$$d = \frac{\cancel{|a|}\,|b| \sin\theta}{\cancel{|a|}} = b \sin\theta$$

ⓑ Distance p(1,1,1) , Q(0,6,8) , R(-1,4,7)

$\vec{a} = \vec{QR} = \langle -1, -2, -1 \rangle$

$\vec{b} = \vec{QP} = \langle 1, -5, -7 \rangle$

$$a \times b = \begin{vmatrix} i & j & k \\ -1 & -2 & -1 \\ 1 & -5 & -7 \end{vmatrix} = \langle 9, -8, 7 \rangle$$

$|a \times b| = \sqrt{81 + 64 + 49} = \sqrt{194}$

$|a| = \sqrt{1+4+1} = \sqrt{6}$

$$d = \frac{\sqrt{194}}{\sqrt{6}}$$

"12.5" lines and planes.

- line: The equ. of the line is given by:

$$\left.\begin{aligned} x &= x_0 + at. \\ y &= y_0 + bt. \\ z &= z_0 + ct. \end{aligned}\right\}$$ Parametric of the line.

- (x_0, y_0, z_0) point on the line.
- $\langle a, b, c \rangle$ vector parallel to the line.

" 2D → ميل^{slope}, 3D → vector " بيدي الاتجاه..

$\boxed{Ex:}$ Find the equ. of the line passes through the

ⓐ point $(\underset{x_0}{2}, \underset{y_0}{5}, \underset{z_0}{4})$ and parallel to vector $\langle \underset{a}{2}, \underset{b}{-1}, \underset{c}{3} \rangle$.

- equ. of the line:

$$x = 2 + 2t.$$
$$y = 5 - t$$
$$z = 4 + 3t$$

ⓑ Is the point $(8, 1, 13)$ lies on the line.

$x = 8 \rightarrow 2 + 2t \rightarrow t = 3$

$y = 5 - 3 = 2 \neq 1 \rightarrow \therefore$ not on the line

Ex: Find equ. of line passes through the points $A(\underset{x_0}{1}, 4, -2)$, $B(\underset{x}{5}, \underset{y}{3}, \underset{z}{2})$.

• vector of line

* $V_L = \overrightarrow{AB} = \langle \underset{a}{4}, -1, 4 \rangle$

equ. of line:

$x = 1 + 4t$ ② أو حل

$y = 4 - t$

$z = -2 + 4t$

نعوض نقطة B بالمعادلة

$5 = 1 + 4t$

$\hookrightarrow \boxed{t = 1}$

$\rightarrow y = 4 - 1 = 3$ ✓

$z = -2 + 4(1) = 2$ ✓

حل نهائي ①

$x = 5 + 4t$

$y = 3 - t$

$z = 2 + 4t$

Ex: Find equ. of line passes through the point $P(\underline{\underline{3}}, 4, -1)$ and parallel to the line $x = 3 + \underline{\underline{5}}t$, $y = 7 - \underline{\underline{2}}t$, $z = \underline{\underline{3}}t$.

* $V_L = \langle 5, -2, 3 \rangle$.

equ. of line

$x = 3 + 5t$.

$y = 4 - 2t$.

$z = -1 + 3t$.

* The symmetric for the equ. of line is given by:

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

↳ * vector form:

$$\langle x, y, z \rangle = \langle x_0 + y_0 + z_0 \rangle + t \langle a, b, c \rangle$$

Q: Find equ of line passes through point $(1, 3, 2)$ and parallel to $2x = 3y = 5z$.

S:

$$\frac{x-0}{1/2} = \frac{y-0}{1/3} = \frac{z-0}{1/5}$$

* $V_1 = \langle \underset{a}{\tfrac{1}{2}}, \underset{b}{\tfrac{1}{3}}, \underset{c}{\tfrac{1}{5}} \rangle$

↳ // equ of line:

$x = 1 + 15t$

$y = 3 + 10t$

$z = 2 + 6t$

[بدل أني أكتبها $x = 1 + \frac{1}{2}t$ بضرب جميعها بـ 30] تفسيره

Q: Find symmetric for the equ of the line passes through $(\underset{x_0}{1}, \underset{y_0}{3}, \underset{z_0}{7})$ and parallel to $\langle \underset{a}{2}, \underset{b}{5}, \underset{c}{0} \rangle$.

S:

$$\frac{x-1}{2} = \frac{y-3}{5}, \quad \underline{\underline{z = 7}} \qquad \cancel{\frac{z-7}{0}}$$

متخالفين

" لا موازي ولا متقاطع "

Ex: $\boxed{\frac{19}{831}}$ Determine the two lines parallel, skew, or intersection lines:

- $L_1 \Rightarrow x = 3+2t \ , \ y = 4-t \ , \ z = 1+3t$.
- $L_2 \Rightarrow x = 1+4s \ , \ y = 3-2s \ , \ z = 4+5s$.

S —

اقسم على بعض ...

$\boxed{1}$

- $V_{L_1} = \langle 2, -1, 3 \rangle$.
- $V_{L_2} = \langle 4, -2, 5 \rangle$.

$\frac{2}{4} = \frac{1}{2}$

$\frac{-1}{-2} = \frac{1}{2}$

$\frac{3}{5} \neq \frac{1}{2}$

$V_{L_1} \not\parallel V_{L_2}$

$\rightarrow L_1 \not\parallel L_2$

$\boxed{2}$

$3+2t = 1+4s$ ---- (1)

$4-t = 3-2s$ ---- (2)

$1+3t = 4+5s$ ---- (3)

((اذا متقاطعين فيكون

(($x_1 = x_2$, - - -

لو أخدت (2) + (1) ما بيطلع t , لأنه مش موجودة مع بعض

$3*(2)+(3) \rightarrow 12-\cancel{3t} = 9-6s$

$1+\cancel{3t} = 4+5s$

$\Rightarrow 13 = 13-s \rightarrow \boxed{s=0}$

$\therefore \boxed{t=-1}$ ← عوض s بـ (2)

- sub in (1):

$5 \neq 1 \rightarrow \therefore$ not intersection so "Skew" line

$\frac{22}{831}$ L_1, L_2 are parallel, skew, or intersecting.

$$L_1: \quad x = \frac{y-1}{-1} = \frac{z-2}{3}$$

$$L_2: \quad \frac{x-2}{2} = \frac{y-3}{-2} = \frac{z}{7}$$

s —

$V_{L_1} = \langle 1, -1, 3 \rangle$ $\quad V_{L_1} \not\parallel V_{L_2}$

$V_{L_2} = \langle 2, -2, 7 \rangle$ $\quad L_1 \not\parallel L_2$

$t = 2s + 2$ (1)
$1 - t = 3 - 2s$ (2)
$2 + 3t = 7s$ (3)

$3(2) + (3)$:
$5 = s + 9$
$\boxed{s = -4}$, $\boxed{t = -10}$

sub in (1)

$-10 \neq -6$ $\therefore$ skew line

* planes *

The equ of planes is given by:

↳ $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

↳ where:

(x, y, z) point on the plane.

$\langle a, b, c \rangle$ vector normal to the plane $\perp$

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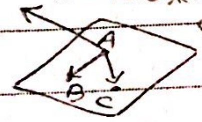
Q: Find the eq. plane passes the point $(2,4,-1)$ and perpendicular to the vector $\langle 5,4,3 \rangle$.

$5(x-2)+4(y-4)+3(z+1)=0$

$5x+4y+3z=23$.

Q: Find equ of the plane passes through $a(1,1,1)$, $b(2,3,-1)$, $c(3,1,5)$.

$n=\overrightarrow{AB}\times\overrightarrow{AC}$ cross



$\overrightarrow{AB}=\langle 1,2,-2 \rangle$

$\overrightarrow{AC}=\langle 2,0,4 \rangle$.

normal $\leftarrow n_p=\overrightarrow{AB}\times\overrightarrow{AC}$

$$\begin{array}{ccc} i & j & k \\ \end{array}$$
$$\begin{vmatrix} 1 & 2 & -2 \\ 2 & 0 & 4 \end{vmatrix} = \langle 8,-8,-4 \rangle$$

equ of plane:

$8(x-1)-8(y-1)-4(z-1)=0$.

Q: Find equ ___ point $(2,1,5)$ and perpendicular to the line $x=1+t$, $y=3-5t$, $z=3t$.

$n_p=V_L=\langle 1,-5,3 \rangle$

equ:

$1(x-2)-5(y-1)+3(z-5)=0$

اذا كان السؤال parallel

$n_{p_1} \parallel n_{p_2}$

العامودي ع الأول بيضل عامودي

عالثاني

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$\frac{45}{832}$ Find intersection of line L with plane P

* L: $x = 2-2t$, $y = 3t$, $z = 1+t$.

plane P: $x + 2y - z = 7$.

s.

① $V_L \perp n_p$ (Dot $\neq 0$) تقاطعها

$V_L = \langle -2, 3, 1 \rangle$

$n_p = \langle 1, 2, -1 \rangle$

$V_L \cdot n_p = -2 + 6 - 1 = 3 \neq 0$

→ the line intersects the plane.

② the intersection:

بنحلها بال t

$n_p = \langle 1, 2, -1 \rangle$

$\hookrightarrow 2-2t + 2(3t) - (1+t) = 7 = 2 - \underline{2t} + \underline{6t} - 1 - \underline{t} = 7$

$= 3t + 1 = 7$

$3t = 6 \rightarrow \boxed{t = 2}$

* بنعوض t بمعادلة ال (L)

→ point of intersection: $(-2, 6, 3)$.

* plane with plane *

$\frac{58}{832}$ Find intersection between $P_1: 3x - 2y + z = 1$,
$P_2: 2x + y - 3z = 3$.

$n_{p_1} = \langle 3, -2, 1 \rangle$ $n_{p_2} = \langle 2, 1, -3 \rangle$ →

$n_{p_1} \not\parallel n_{p_2}$

$\hookrightarrow P_1 \not\parallel P_2$.

* نحتاج معادلة خط التقاطع ← نحتاج نقطتين أو نقطة و vector

← نثبت x ونطلع قيم y , z.

أو نجيب vector موازي للخط.

$\boxed{x = 0}$

$$+\begin{cases} (-2y + z = 1) \times 3 \longrightarrow -6y + 3z = 3 \\ y - 3z = 3 \end{cases}$$

$$\rightarrow -5y = 6$$

$$y = \frac{-6}{5}$$

$$\frac{-6}{5} - 3z = 3$$

$$-3z = 3 + \frac{6}{5} \longrightarrow -3z = \frac{21}{5} \longrightarrow z = \frac{-7}{5}$$

$$\rightarrow \left(0, \frac{-6}{5}, \frac{-7}{5}\right)$$

$$V_L = n_{p_1} \times n_{p_2} = \begin{vmatrix} i & j & k \\ 3 & -2 & 1 \\ 2 & 1 & -3 \end{vmatrix} = \langle 5, 11, 7 \rangle$$

equ of line:

$$x = 5t$$

$$y = \frac{-6}{5} + 11t$$

$$z = \frac{-7}{5} + 7t$$

$\frac{35}{832}$ Find the equ. of the plane passing through $P(3,5,-1)$ and contains the line

$x = 4-t$, $y = 2t-1$, $z = -3t$.

Q is point on the line $(4,-1,0)$

$\overrightarrow{QP} = \langle -1, 6, -1 \rangle$.

$V_L = \langle -1, 2, -3 \rangle$

$$n_p = V_L \times \overrightarrow{QP} = \begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ -1 & 6 & -1 \end{vmatrix} = \langle 16, 2, -4 \rangle$$

equ. of plane:

$$16(x-3) + 2(y-5) - 14(z+1) = 0.$$

65| find equ. of line passes through $p(0,1,2)$ parallel to the plane $P: \underbrace{x+y+z}_{}\overset{n_p}{} = 2$.
and $\perp$ perpendicular to the line L_1:
$x = 1+t$, $y = 1-t$, $z = 2t$.

S

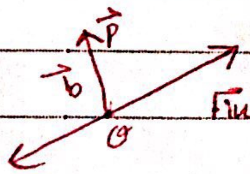
$V_{L_1} = \langle 1, -1, 2 \rangle$

$$V_L = V_{L_1} \times n_p = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 1 & 1 & 1 \end{vmatrix} = \langle -3, 1, 2 \rangle$$

• equ. of line is:

$$x = -3t \ , \ y = 1+t \ , \ z = 2+2t.$$

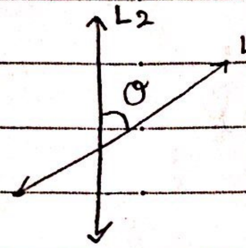
66|



Find: $\text{Proj}_{V_L} b =$

$\text{ortho}_{V_L} =$

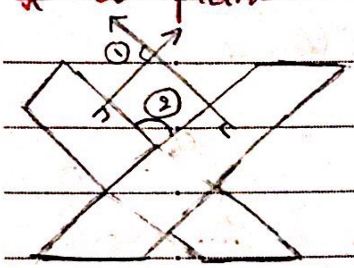
- The largest θ between 2 vectors : 180°
- " " " " 2 lines : 90°



$$\theta_{L_1L_2} = \theta_{V_{L_1}L_2}$$

$$\cos\theta = \left| \frac{V_{L_1} \cdot V_{L_2}}{|V_{L_1}|\ |V_{L_2}|} \right|$$

* 2 planes



$1 + 2 = 90^\circ$

$\theta_{P_1P_2}$:

$180 - 110$

$$\underbrace{\cos\theta = \left| \frac{n_1 \cdot n_2}{|n_1|\ |n_2|} \right|}_{\text{بيعطيني الزاوية الصغيرة}} = |110| \rightarrow 70^\circ$$

$\boxed{57}$ find the angle between the planes:
832

$P_1: x + y + z = 1$

$P_2: x + 2y + 2z = 1$

S

$n_{P_1} = \langle 1, 1, 1 \rangle$

$n_{P_2} = \langle 1, 2, 2 \rangle$

$$\cos\theta = \left| \frac{n_1 \cdot n_2}{|n_1|\ |n_2|} \right| = \left| \frac{1+2+2}{\sqrt{3}\ \sqrt{9}} \right| = \frac{5}{3\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{5}{3\sqrt{3}} \right)$$

* Distance:

1) two points.

2) point and line. « cross »

3) point (x_0, y_0, z_0) and plane $ax+by+cz+d=0$ is given by:

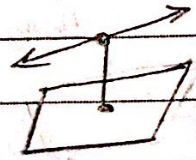
$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2+b^2+c^2}}$$

4) two parallel lines. $\longleftrightarrow L_1$

بجيب نقطة ع L وبطبق 2 $\longleftrightarrow L_2$

5) line parallel to plane:

find point on the line and use 3



6) two parallel planes (P_1, P_2)

find point on P_1 and use 3

7) two skew lines.

Ex:

72/833 Find the distance from point $(-2, 2, 1)$ and the plane $3x - 5y + z = 5$.

$$D = \frac{|3 \times -2 - 5(2) + 1(1) - 5|}{\sqrt{9+25+1}} = \frac{20}{\sqrt{35}}$$

$\frac{73}{833}$ Find the distance between the two planes

$P_1 = 2x - 3y + z = 4$

$P_2 = 4x - 6y + 2z = 3.$

$(2,0,0)$ on P_1

$$D = \frac{|8-3|}{\sqrt{16+36+4}} = \frac{5}{\sqrt{56}}$$

rule:

▷ If $P_1 : ax + by + cz + d_1 = 0$, $P_2 : ax + by + cz + d_2 = 0$

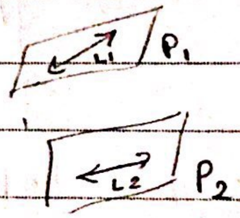
then.

$$D = \frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$$

$\boxed{78}$ Find the distance between skew lines

$L_1: x = 1+t \quad , \quad y = 1+6t \quad , \quad z = 2t$

$L_2: x = 1+2s \quad , \quad y = 5+15s \quad , \quad z = -2+6s.$



$n = V_{L_1} \times V_{L_2}$

$$= \begin{vmatrix} i & j & k \\ 1 & 6 & 2 \\ 2 & 15 & 6 \end{vmatrix} = \langle 6, -2, 3 \rangle.$$

$(P_1 // P_2)$

equ of P_1:

$(1,1,0)$ on P_1

→ equ:

$6(x-1) - 2(y-1) + 3z = 0$

$6x - 2y + 3z - 4 = 0$

$(1,5,-2)$ on P_2.

$$D = \frac{|6-10-6-4|}{\sqrt{36+4+9}} = 2.$$

$\boxed{76}$
833

Find equ. of planes that are parallel to the plane. $x+2y-2z=1$ $d_2=-1$
and two units away from it.

The equ. of plane:

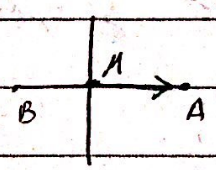
$$x+2y-2z+d_1=0$$

$$2=\frac{|d_1-d_2|}{\sqrt{1+4+4}} \rightarrow 2=\frac{|d_1+1|}{3} \rightarrow 6=|d_1+1|$$

$$d_1+1=\pm 6 \rightarrow d_1=5,-7$$

$$P_1 = x+2y-2z+5=0$$
$$P_2 = x+2y-2z-7=0$$

$\boxed{62}$ Find equ of plane of all points that are equidistant from the points $(2,5,5)$ and $(-6,3,1)$



- M_{AB} on the plane $(-2,4,3)$
- $\overrightarrow{MA}=n_p$

Ex: $x = 1 + t$

$y = 5 - 2t$

$z = 3 - t$

xy-plane $\rightarrow z = 0$

$0 = 3 - t$

$\boxed{t = 3} \rightarrow x = 4 , y = -1$.

Q: $(x-1)^2 + (y-3)^2 + (\underset{\Downarrow\, z=0}{z-2})^2 = 16$

xy-plane

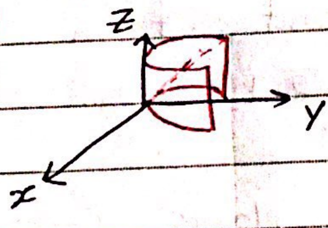
$(x-1)^2 + (y-3)^2 = 12 \Rightarrow$ معادلة دائرة ..

$\boxed{*12.6}$

Quadric surfaces:

cylinder → امتداد

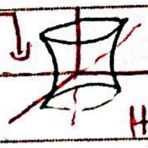
- $x^2 + y^2 + z^2 = r^2 \rightsquigarrow$ sphere.
- $x^2 + y^2 = 1 \rightsquigarrow$ cylinder.
- $y = x^2 \rightarrow$ cylinder (parabola) →



$\boxed{1}$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \rightarrow$ Ellipsoid . شكل بيضوي

الطماطة تخنة

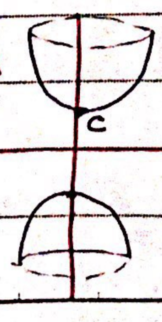
$\boxed{2}$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$



Hyperboloid of one sheet along z-axis.

مسار

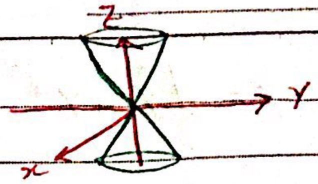
$\boxed{3}$ $\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



Hyperboloid of two sheets along z-axis

موجب

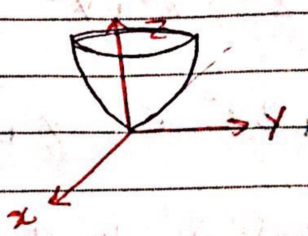
[4] $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$. Cone →

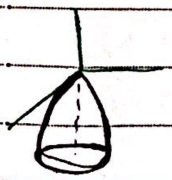


[5] $\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$, $c>0$

↳ paraboloid

along z-axis.



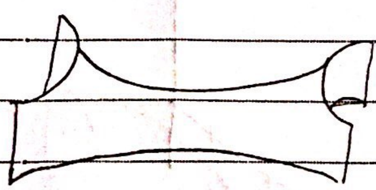


← $c<0$ لو حط

لنفس الاشارة (5)

واحد + وواحد − (6)

[6] $\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ → Hyperbolic paraboloid.



Saddle. زي شكل سرج الحصان

Q: Identify and sketch

[1] $z = 1 - x^2 - y^2$.

s

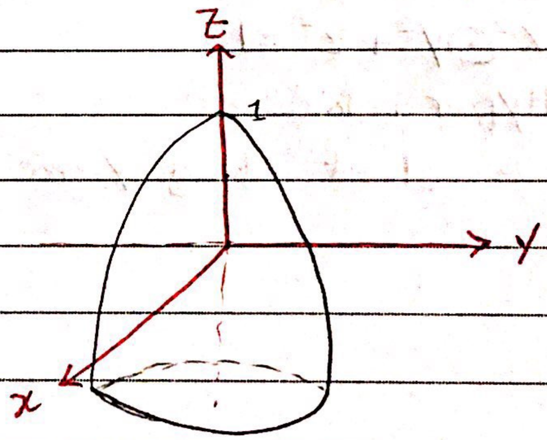
$z = x^2 + y^2$

$z = -(x^2 + y^2)$

$z = 1 - (x^2 + y^2)$

paraboloid along

z-axis

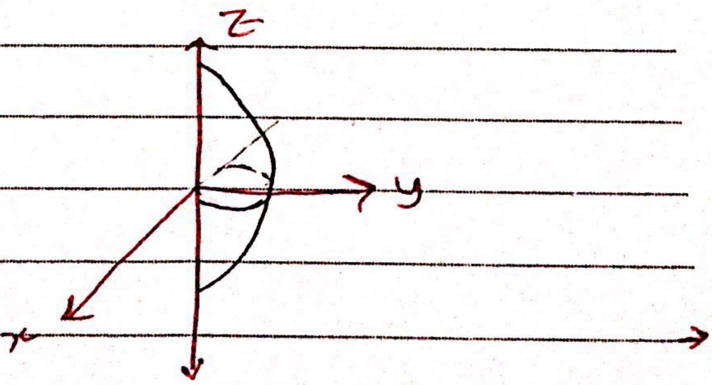


[2] $y = +\sqrt{1 - x^2 - z^2}$.

s $y^2 = 1 - x^2 - z^2$

$x^2 + y^2 + z^2 = 1$

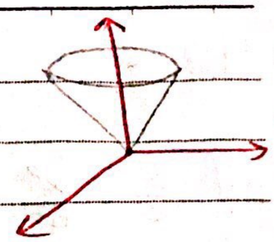
half sphere.



[3] $z = \sqrt{x^2+y^2}$

S

$z^2 = x^2 + y^2 \rightarrow$ cone (z → موجب)



P-837

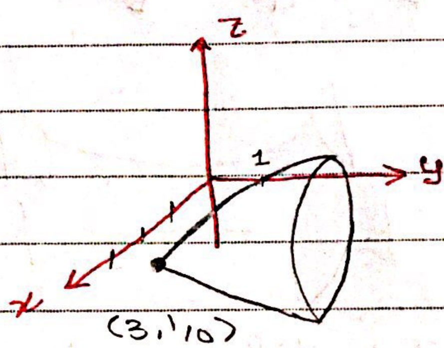
[4] $x^2 + 2z^2 - 6x - y + 10 = 0$

Paraboloid along y-axis

$(x-3)^2 + 2z^2 - 9 - y + 10 = 0$ اكمال مربع لل x

$y = (x-3)^2 + 2z^2 + 1 \rightarrow$ حسب اتجاه y ↑

↳ shift 3 units



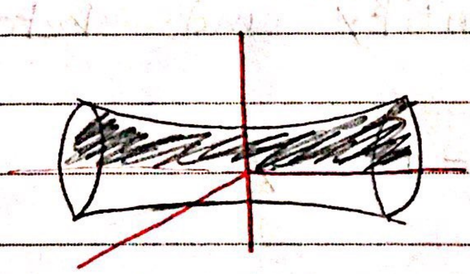
[5] $z = \sqrt{1 - x^2 + y^2}$

$z^2 = 1 - x^2 + y^2$

$x^2 + z^2 - y^2 = 1$

$x^2 - y^2 + z^2 = 1$

Hyperboloid of one sheet along y-axis.



$\boxed{\text{Ch-13}}$

★ Vector valued functions ★

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

Ex: Identify the following $\boxed{\text{curve}}$

$\boxed{1}$ $\vec{r}(t) = \langle 1+2t, 3t, 5-4t \rangle$.

S—

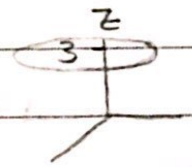
$x = 1+2t$ line parallel to $\langle 2, 3, -4 \rangle$

$y = 3t$ at point $(1, 0, 5)$.

$z = 5-4t$

$\boxed{2}$ $\vec{r}(t) = \langle \sin t, \cos t, 3 \rangle$

S—



$x = \sin t$ $x^2 + y^2 = 1$

$y = \cos t$ circle on the

$z = 3$ plane $z = 3$

$\boxed{3}$ $\vec{r}(t) = \langle \sin t, \cos t, t \rangle$

S—

$x = \sin t$ $\left[\begin{array}{l} x^2 + y^2 = 1 \\ z = t \end{array} \right.$ $z \rightarrow$ "تزداد غير ثابتة"

$y = \cos t$

$z = t$ helix on cylinder $(x^2 + y^2 = 1)$

$\boxed{4}$ $\langle t, t^2, 1 \rangle$

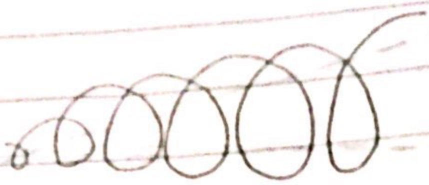
$x = t$ $y = x^2$

$y = t^2$ parabola on $z = 1$

$z = 1$

$\frac{21}{854}$ Identify the following curve.

$x = t\cos t$
$y = t$
$z = t\sin t$



S. " helix on cone along y-axis "

$y^2 = x^2 + z^2$

لو طلع نقطة من الـ t أول ، نعوض t بقيم موجبة .

[لو $y = t^2 \leftarrow$ paraboloid
$x^2 + z^2 = y$]

Rules:

* $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

(1). Domain $\vec{r} = D(x) \cap D(y) \cap D(z)$

(2). $\lim_{t \to t_0} \vec{r}(t) = \langle \lim_{t \to t_0} x(t), \lim_{t \to t_0} y(t), \lim_{t \to t_0} z(t) \rangle$.

(3). $\vec{r}\,'(t) = \langle x'(t), y'(t), z'(t) \rangle$.

(4). $\int \vec{r}(t)\,dt = \langle \int x(t)\,dt, \int y(t)\,dt, \int z(t)\,dt \rangle$.

$\boxed{\frac{1}{853}}$ $\vec{r}(t) = \left\langle \ln(t+1), \frac{t}{\sqrt{9+t^2}}, 2^t \right\rangle$.
Find the domain:

S.

. $D(\ln(t+1)) \rightarrow (-1, \infty)$ ((داخل الـ ln > صفر))

. $D\left(\frac{t}{\sqrt{9+t^2}}\right) \rightarrow (-3, 3)$

. $D(2^t) \rightarrow (-\infty, \infty)$

* $D(\vec{r}(t)) \rightarrow (-1, 3)$ *

[4] Find

$$\lim_{t\to1}\left(\frac{t^2-t}{t-1}\, i + \sqrt{t+8}\, j + \frac{\sin\pi t}{\ln t}\, k\right).$$

S.

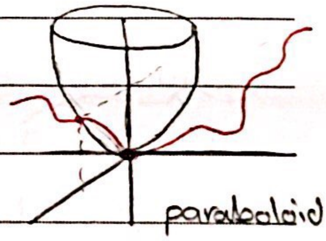
$$\lim_{t\to1}\frac{t^2-t}{t-1} = 1$$

$$\lim_{t\to1}\sqrt{t+8} = 3$$

$$\lim_{t\to1}\frac{\sin\pi t}{\ln t} \xrightarrow{\text{لوبيتال}} \lim_{t\to1}\frac{\pi\cos\pi t}{\frac{1}{t}} = -\pi$$

$$\longrightarrow = \langle 1, 3, -\pi \rangle.$$

[31] (954) at what points does the curve $\vec{r}(t) = \langle \underset{x}{t}, \underset{y}{0}, \underset{z}{2t - t^2} \rangle$ intersect the surface $z = x^2 + y^2$?



S.

$$2t - t^2 = t^2 + 0$$

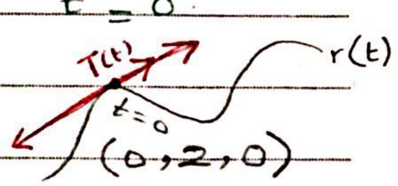
$$2t - 2t^2 = 0$$

$$2t(1-t) = 0$$

$$t = \underline{\underline{0, 1}} \longrightarrow (0,0,0), (1,0,1)$$

عوضها بالمعادلة

[18] (860) Find the unit tangent vector $T(t)$ for $\vec{r}(t) = \langle \tan^{-1} t, 2e^{2t}, 8te^{t} \rangle$ at $t = 0$.



S.

$$\bullet\ T(t) = \frac{\vec{r}\,'(t)}{|r'(t)|}$$

$$\vec{r}\,'(t) = \left\langle \frac{1}{1+t^2}, 4e^{2t}, 8e^{t} + 8te^{t} \right\rangle \longrightarrow$$

$\rightarrow \quad \vec{r}'(0) = \langle 1, 4, 8 \rangle$

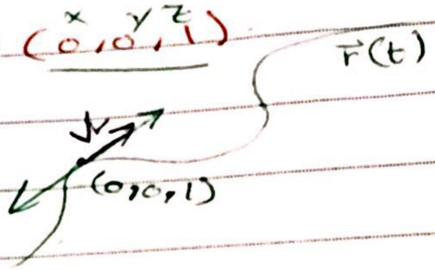
$$T(0) = \frac{r'(0)}{|r'(0)|} = \frac{\langle 1,4,8 \rangle}{\sqrt{1+16+64}} \Rightarrow \frac{1}{9}\langle 1,4,8 \rangle.$$

$\frac{24}{360}$ Find parametric equ. for the tangent line to the curve

$x = \ln(t+1)$ at $p(\overset{x}{0}, \overset{y}{0}, \overset{z}{1})$

$y = t \cos 2t$

$z = 2^t$



S

$V_L = r'$

$$r'(t) = \left\langle \frac{1}{t+1}, \cos 2t - 2\sin 2t, 2^t \ln 2 \right\rangle$$

$\boxed{t=0}$. $\longrightarrow$ $V_L = r'(0) = \langle \overset{a}{1}, \overset{b}{1}, \overset{c}{\ln 2} \rangle$

equ. of line.

$x = 0 + 1t \rightarrow t$

$y = t$

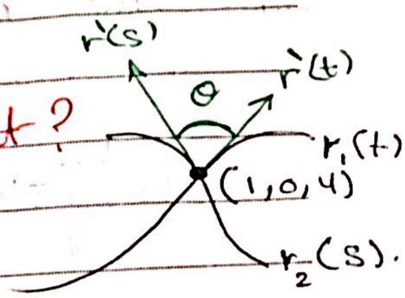
$z = 1 + (\ln 2)t.$

$\frac{34}{861}$ at what point do the curve

$\cdot\ \vec{r}_1(t) = \langle t, 1-t, 3+t^2 \rangle$

and $\vec{r}_2(s) = \langle 3-s, s-2, s^2 \rangle$. Intersect? $\boxed{1}$

Find the $\boxed{2}$ angles of Intersection.



S ___ .

$\boxed{1}$ $\bullet\ t = 3-s$

$\bullet\ 1-t = s-2$

$\bullet\ 3+t^2 = s^2$ → 1+3 نعوض الأولى في المعادلة الثالثة نستبدل ال t

$$3+(3-s)^2 = s^2$$

$$3+9-6s+s^2 = s^2$$

$$12 = 6s \longrightarrow \boxed{s=2}$$

$$\therefore\ t = 3-2 = 1$$

sub in ② عوضنا (t) في $\vec{r}_1(t)$

$0 = 0$ ✓ $\therefore$ Intersect $(1, 0, 4)$

$\boxed{2}$ $\bullet\ r_1'(t) = \langle 1, -1, 2t \rangle$

$r_1'(1) = \langle 1, -1, 2 \rangle$ *

$\bullet\ r_2'(s) = \langle -1, 1, 2s \rangle$

$r_2'(2) = \langle -1, 1, 4 \rangle$ *

$$*\ \cos\theta = \frac{r_1'(1) \cdot r_2'(2)}{|r_1'(1)|\,|r_2'(2)|}\ * = \frac{-1-1+8}{\sqrt{6}\ \sqrt{18}} = \frac{1}{\sqrt{3}}$$

41/861 | Find $\vec{r}(t)$ If $\vec{r}'(t) = 3t^2 i - 8t j + 8t^3 k$. and $\vec{r}(1) = 3i + 7j$.

S.

$$\vec{r}(t) = \int \vec{r}'(t) = \int 3t^2 i - 8tj + 8t^3 k$$

$$= \langle t^3, -4t^2, 2t^4 \rangle + \langle c_1 + c_2 + c_3 \rangle$$

$$\vec{r}(1) = \langle 3, 7, 0 \rangle = \langle 1, -4, 2 \rangle + \langle c_1 + c_2 + c_3 \rangle$$

$3 = 1 + c1 \longrightarrow c_1 = 2$

$7 = -4 + c2 \longrightarrow c_2 = 11$

$0 = 2 + c3 \longrightarrow c_3 = -2$

$$\rightarrow \langle 1, -4, 2 \rangle + \langle 2, 11, -2 \rangle = \underline{\langle 3, 7, 0 \rangle} \checkmark$$

* Properties of der. *

$\boxed{1}$ $(\vec{u}(t) + \vec{v}(t))' \rightarrow (\vec{u}'(t) + \vec{v}'(t))$

$\boxed{2}$ $(k\vec{u}(t))' \rightarrow k\vec{u}'(t)$.

cross $\boxed{3}$ $(\vec{u}(t) \times \vec{V}(t))' = \vec{u}'(t) \times \vec{V}(t) + \vec{V}'(t) \times \vec{u}(t)$

Dot $\boxed{4}$ $(\vec{u}(t) \cdot \vec{V}(t))' = \vec{u}'(t) \cdot \vec{V}(t) + \vec{V}'(t) \cdot \vec{u}(t)$

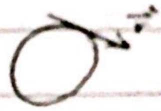
$\boxed{5}$ $(f(t)\,\vec{u}(t))' = f'(t) * \vec{u}(t) + \vec{u}'(t) * f(t)$.

ضرب عادي

$\boxed{6}$ $(\vec{u}(f(t)))' = f'(t) * \vec{u}'(f(t))$.

Q. * If. $|r(t)| = c$, show that $\vec{r} \cdot \vec{r}' = 0$

$|r(t)|^2 = \vec{r} \cdot \vec{r} = c^2$ $\quad c' \to 0$



$(\vec{r} \cdot \vec{r})' = (c^2)'$

$\vec{r}' \cdot \vec{r} + \vec{r}' \cdot \vec{r} = 0$

$2 \, \vec{r} \cdot \vec{r}' = 0$

$\vec{r} \cdot \vec{r}' = 0$

$\boxed{\frac{53}{861}}$ show that $\frac{d}{dt}\left(\vec{r}(t) \times \vec{r}'(t)\right) = \vec{r}(t) \times \vec{r}''(t)$

$\frac{d}{dt}(\vec{r} \times \vec{r}') = \vec{r}' \times \vec{r}' + \vec{r}' \times \vec{r}''$

$= 0 + \vec{r} \times \vec{r}''$.

13.3 Arc length.

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$.

The arc length of $\vec{r}(t)$ from $t=a$ to $t=b$ is given by:

$$L = \int_a^b |\vec{r}'| \, dt$$

$$= \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} \, dt$$

Ex: find the length of the curve

$\vec{r}(t) = \langle \sin t, \cos t, t \rangle$

from $t=0$ to $t=2$.

S

$\vec{r}'(t) = \langle \cos t, -\sin t, 1 \rangle$

$|\vec{r}'(t)| = \sqrt{\cos^2 t + \sin^2 t + 1} = \sqrt{2}$

$L = \int_0^2 \sqrt{2}\, dt \rightarrow 2\sqrt{2}$.

* Arc length function *

$S(t) = \int_{t_0}^{t} |\vec{r}'(u)|\, du$.

14
868

a) Find arc length function for the curve measured from the point $P(0, 1, \sqrt{2})$, b) Find a point of 4 units along the curve from P. in direction of increasing t where $r(t) = \langle e^t \sin t, e^t \cos t, \sqrt{2} e^t \rangle$.

S

$S(t) = \int_0^t |\vec{r}'(u)|\, du$

$(x')^2 = (e^t \sin t + e^t \cos t)^2 = e^{2t}\sin^2 t + e^{2t}\cos^2 t + \cancel{2e^{2t}\sin t \cos t}$

$(y')^2 = (e^t \cos t + (-e^t \sin t))^2 = e^{2t}\cos^2 t + e^{2t}\sin^2 t - \cancel{2e^{2t}\sin t \cos t}$

$(z')^2 = (\sqrt{2} e^t)^2 = 2e^{2t}$

$St = \int_0^t 2e^u\, du = 2(e^t - 1)$.

$\rightarrow |r'|^2 = 4e^{2t}$ / $|r'| = 2e^t$.

* curvature :

$r(t) = \langle x, y, z \rangle$

$$K(t) = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3}$$

Ex: Find $K(t)$ for $\vec{r}(t) = \langle 2\sin t, 2\cos t, 2 \rangle \to$ circle

S

$x^2 + y^2 = 4\sin^2 t + 4\cos^2 t$

$= 4 \to r = 2$

$r'(t) = \langle 2\cos t, -2\sin t, 0 \rangle$

$r''(t) = \langle -2\sin t, -2\cos t, 0 \rangle$

$$\vec{r}'(t) \times \vec{r}''(t) = \begin{array}{c} \begin{matrix} i & j & k \end{matrix} \\ \begin{vmatrix} 2\cos t & -2\sin t & 0 \\ -2\sin t & -2\cos t & 0 \end{vmatrix} \end{array} = \langle 0, 0, -4 \rangle.$$

$$K(t) = \frac{4}{8} = \frac{1}{2} \to$$ مقلوب r

24
865

Find the curvature for

$\vec{r}(t) = \langle e^t \cos t, e^t \sin t, t \rangle$ at point $(1, 0, 0)$

S

$e^t(\cos t - \sin t)$

- $\vec{r}'(t) = \langle e^t \cos t - e^t \sin t, e^t \sin t + e^t \cos t, 1 \rangle$
- $\vec{r}''(t) = \langle e^t(\cos t - \sin t) - e^t(\sin t + \cos t), e^t(\sin t + \cos t) + e^t(\cos t - \sin t), 0 \rangle$
- $\vec{r}'(0) = \langle 1, 1, 1 \rangle$ $\quad r''(0) = \langle 0, 2, 0 \rangle$.
- $$k(0) = \frac{|r'(0) \times r''(0)|}{|r'(0)|^3} = \frac{\sqrt{8}}{3\sqrt{3}}$$
- $$r'(0) \times r''(0) = \begin{array}{c} \begin{matrix} i & j & k \end{matrix} \\ \begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \end{vmatrix} \end{array} = \langle -2, 0, 2 \rangle$$

Q: $y = x^2$ $\quad k(0) = ?$

S —

$x = t$ $\quad \vec{r}(t) = \langle t, t^2, 0 \rangle$

$y = t^2$

Q: If $y = f(x)$, then —

$$K(x) = \frac{|f''(x)|}{(1+(f'(x))^2)^{3/2}}$$

Q: Find $k(0)$ for $f(x) = x^2$

S —

$f'(x) = 2x \quad , f'(0) = 0$

$f''(x) = 2 \quad , f''(0) = 2$

$\rightarrow k(0) = \frac{2}{1} = 2.$

Q: ① Find x such that $k(x) = 5$ $\quad y = f(x)$ معطى

الحل بنشوف قيمة x بالمعادلة بيعطينا جواب 5

② Find max of curvature

بنجيب المشتقة.

42/855 Find vector function that represents the curve of intersection of the two surfaces.

cylinder ← $x^2 + y^2 = 4$ and $z = xy$.

S —

$x = 2\sin t$ $\rightarrow \vec{r}(t) = \langle 2\sin t, 2\cos t, 2\sin 2t \rangle.$

$y = 2\cos t$

$z = 4\cos t \sin t$

$= 2\sin 2t$

Q: $x^2 + y^2 = 4$, $x^2 + z^2 = 9$ Find intersection.

S

$x = 2\sin t$

$y = 2\cos t$

$z^2 = 9 - x^2 \rightarrow 9 - 4\sin^2 t = \pm\sqrt{9 - 4\sin^2 t}$

$\rightarrow \vec{r}(t) = \left\langle 2\sin t \,,\, 2\cos t \,,\, \pm\sqrt{9 - 4\sin^2 t} \right\rangle$.

$\boxed{43}$ Find intersection.

$z = \sqrt{x^2 + y^2}$, $z = 1 + y$.

S

$x^2 + y^2 = (1 + y)^2$

$x^2 + \cancel{y^2} = 1 + 2y + \cancel{y^2}$

$y = \dfrac{x^2 - 1}{2} \xrightarrow{\text{نفرض}} x = t$

$y = \dfrac{t^2 - 1}{2} \rightarrow z = 1 + \dfrac{t^2 - 1}{2} = \dfrac{t^2 + 1}{2}$

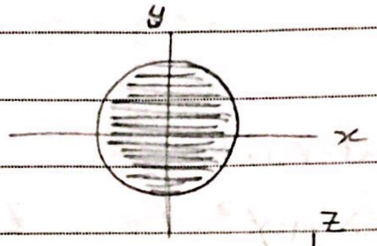
$r(t) = \left\langle t \,,\, \dfrac{t^2 - 1}{2} \,,\, \dfrac{t^2 + 1}{2} \right\rangle$

Ch 14 "Function of several variables"

* $f(x, y) = x^2 + y^2$ « function of two variables »
* $g(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$ " " 3 "

Ex: sketch the Domain.

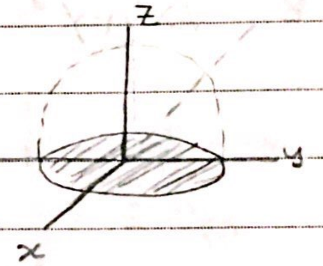
$f(x, y) = \sqrt{1 - x^2 - y^2}$



$\longrightarrow \quad 1 - x^2 - y^2 \geq 0$

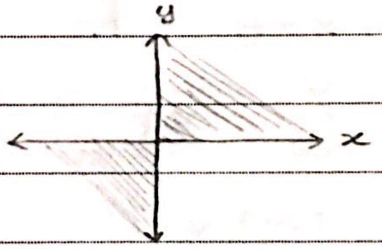
$x^2 + y^2 \leq 1$

$D(f) = \{ (x, y) : x^2 + y^2 \leq 1 \}$



Q: $z = \sqrt{xy}$

$xy \geq 0$



Q: $f(x, y) = \sqrt{1 - x^2} + \sqrt{4 - y^2}$ « Domain 1 $\cap$ Domain 2 »

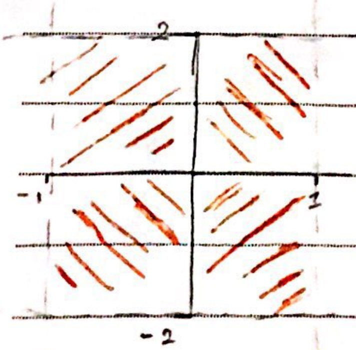
$1 - x^2 \geq 0 \quad \cap \quad 4 - y^2 \geq 0$

$x^2 \leq 1 \qquad y^2 \leq 4$

$-1 \leq x \leq 1 \qquad -2 \leq y \leq 2$

* $D(f) = \{ (x, y) : |x| \leq 1, |y| \leq 2 \}$.

« xy-plane كل »



$\frac{15}{900}$ Find and sketch the domain of:

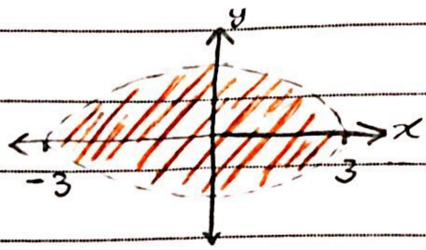
$f(x,y) = \ln(9 - x^2 - 9y^2)$.

$9 - x^2 - 9y^2 > 0$

$x^2 + 9y^2 < 9$

$\frac{x^2}{9} + y^2 < 1$

$D(f) = \{(x,y) : \frac{x^2}{9} + y^2 < 1\}$.

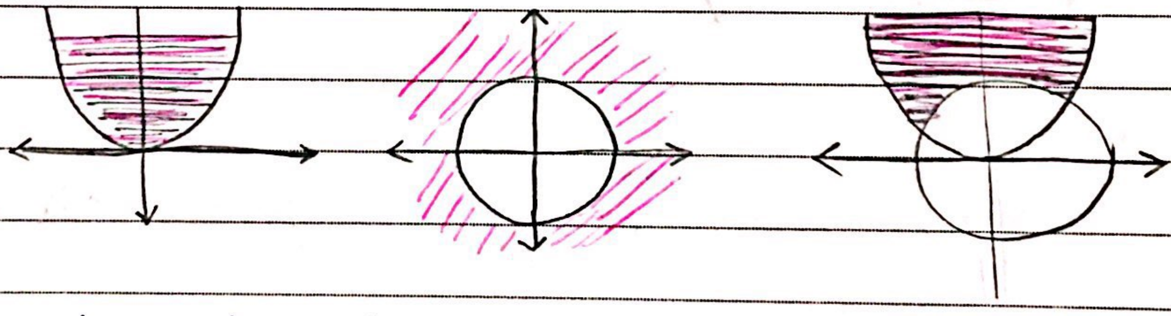


Q: find and sketch the domain

$f(x,y) = \sqrt{y - x^2} + \sqrt{x^2 + y^2 - 4}$

$y - x^2 \geq 0 \longrightarrow y \geq x^2$

$x^2 + y^2 - 4 \geq 0 \longrightarrow x^2 + y^2 \geq 4$



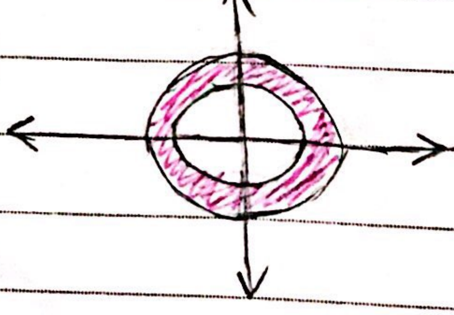
$\boxed{22}$ sketch the domain --

$f(x,y) = \sin^{-1}(x^2 + y^2 - 2)$.

$\hookrightarrow [-1, 1]$

$-1 \leq x^2 + y^2 - 2 \leq 1$

$1 \leq x^2 + y^2 \leq 3$



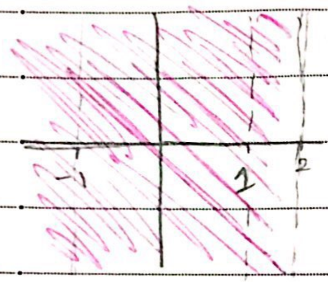
Ex: sketch the domain

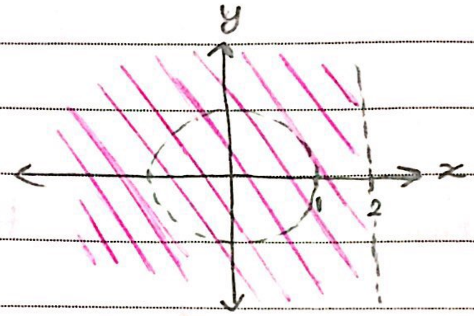
$$g(x) = \frac{\ln(2-x)}{1-x^2-y^2}$$

$* \; 2-x>0$ $\qquad\qquad$ $* \; 1-x^2-y^2>0$

$x<2$

if $\dfrac{\ln(2-x)}{1-x^2}$





Q: find the domain

$$f(x,y,z) = \sqrt{1-x^2-y^2-z^2}$$

$1-x^2-y^2-z^2 \geq 0$

$x^2+y^2+z^2 \leq 1 \rightarrow$ Domain: sphere on & inside.

Ex: $z = x^2 + y^2$ " بنثبت ال range $|z|$ "

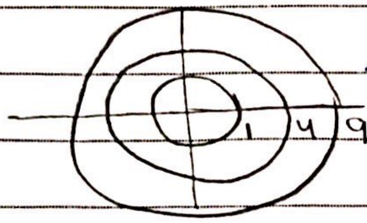
* If $z = 1$

$1 = x^2 + y^2 \rightarrow$ circle.

* $z = 2$

$2 = x^2 + y^2$

" level curves "



" contour map "

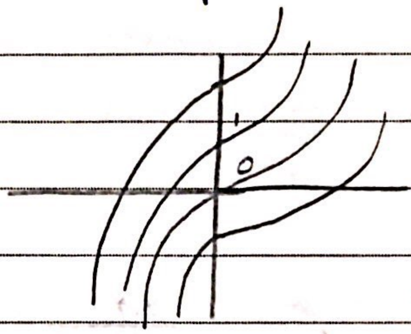
Ex: $z = y - x^3$ sketch contour map.

$z = 0 \rightarrow y = x^3$

$z = 1 \rightarrow y = x^3 + 1$

$z = 2 \rightarrow y = x^3 + 2$

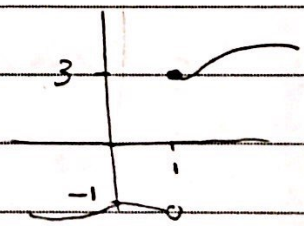
$z = -1 \rightarrow y = x^3 - 1$



$\boxed{14.2}$ limits ..

① $\lim_{x \to 1^+} f(x) = 3$

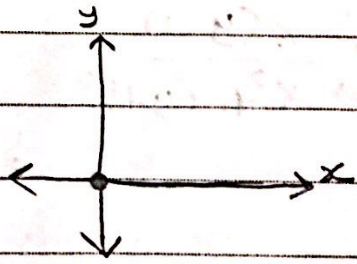
$\lim_{x \to 1^-} f(x) = -1$



$\lim_{x \to 1} f(x)$ not exist

② $z = f(x, y) \rightarrow$ Domain $= \infty$

$\lim f(x, y) \rightarrow (0, 0)$



plane نهاية كلى

Ex: show that the following limit doesn't exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}.$$

S

[1] along x-axis $(y=0)$.

$$\lim_{x\to 0} \frac{x^2}{x^2} = 1$$

[2] along y-axis $(x=0)$

$$\lim_{y\to 0} \frac{-y^2}{y^2} = -1$$

$\therefore \lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ not exist.

Q: Find $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ If exist.

[1] along x-axis $(y=0)$

$$\lim_{x\to 0} \frac{0}{x^2} = 0$$

[2] along $y=x$

$$\lim_{x\to 0} \frac{x^2}{x^2+x^2} = \frac{1}{2}$$

doesn't exist

نفس الحل

$$\lim_{(x,y)\to(0,0)} \frac{x(y-1)}{x^2+(y-1)^2}$$

① lim x-axis $(y=1)$

② $y = x+1$

exist

$\frac{12}{910}$ Find $\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$

$\lim_{(x,y)\to(0,0)} \frac{(x^2-y^2)\cancel{(x^2+y^2)}}{\cancel{x^2+y^2}} = \boxed{0}$ ∴ exist

$\boxed{17}$ Find limit if exist

$\lim_{(x,y)\to(0,0)} \frac{x^2ye^y}{x^4+4y^2}$

$\boxed{1}$ along x-axis $(y=0)$

$\lim_{x\to0} \frac{0}{x^4} = 0$

$\boxed{2}$ along $y=x$

$\lim_{x\to0} \frac{x^3e^x}{x^4+4x^2} \to \lim_{x\to0} \cancel{x^2}\frac{(xe^x)}{x^2+4} = 0$

عامل مشترك

$\boxed{3}$ along $y=x^2$

$\lim_{x\to0} \frac{x^4e^{x^2}}{x^4+4x^4} \to \frac{1}{5} \to$ DNE

Ex: $\lim_{(x,y)\to0} \frac{x^2-y^2}{x^2+y^2}$

$\left[\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}\right.$ ✱

$\lim_{r\to0^+} \frac{\cancel{r^2}(\cos^2\theta-\sin^2\theta)}{\cancel{r^2}} = \cos^2\theta-\sin^2\theta.$

اذا جواب أي سؤال بعتمد على θ بيكون DNE

نعوض باستعمها اذا طلبها أو اذا شكيت بالحل

14/9/10 Find (If exist)

$$\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+xy+y^2}$$

S

$$\lim_{(x,y)\to(0,0)} \frac{(x-y)\cancel{(x^2+xy+y^2)}}{\cancel{x^2+xy+y^2}} = 0$$

$\boxed{18}$

$$\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+2y^2}$$

$\boxed{1}$

$x = r\cos\theta$

$y = r\sin\theta$

$$\lim_{r\to 0^+} \frac{\cancel{r^2}\cos^2\theta\sin^2\theta\, r(\sin\theta)}{\cancel{r^2}(\cos^2\theta + 2\sin^2\theta)} = 0$$

$\boxed{2}$ طريقة ثانية

$$0 \leq \frac{x^2}{x^2+2y^2} \leq 1$$

$x\sin^2 y$

$$0 \leq \frac{x^2\sin^2 y}{x^2+2y^2} \leq \sin^2 y$$

$\lim_{0,0}$

$\downarrow \quad \rightarrow \quad 0 \quad \leftarrow \quad \downarrow$

$0 \qquad\qquad\qquad\qquad 0$

$\boxed{11}$ Find (If exist)

$$\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{3x^2+y^2}$$

① along x-axis $(y=0)$ $= \lim_{x\to 0} \frac{0}{3x^2} = 0$

② along $y=x$ $= \frac{x^2\cos x}{4x^2} = \frac{1}{4} = \frac{1}{4}$

$\neq$

$\therefore$ not exist.

$\left(\frac{\infty}{\infty}, \frac{0}{0}\right) \rightarrow$ لوبيتال

39 911

$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$$

$x = r\cos\theta$

$y = r\sin\theta$

$$\lim_{r\to 0^+} \frac{\cancel{r^3}^{\,r}(\cos^3\theta + \sin^3\theta)}{\cancel{r^2}} = 0$$

40

$$\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$$

$* \left(\ln 0^+ = -\infty\right) *$

$x = r\cos\theta$

$y = r\sin\theta$

$$\lim_{r\to 0^+} r^2 \ln r^2$$

$$\rightarrow \lim_{r\to 0^+} \frac{2\ln r}{\frac{1}{r^2}} \quad \left(\frac{-\infty}{\infty}\right)$$

لوبيتال $\rightarrow 2\lim_{r\to 0^+} \frac{\frac{1}{r}}{\frac{-2}{r^3}} = -\lim_{r\to 0^+} r^2 = 0$

41

$$\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2}$$

$$\lim_{r\to 0^+} \frac{e^{-r^2}-1}{r^2} = \frac{0}{0} \rightarrow \lim_{r\to 0^+} \frac{-2re^{-r^2}}{2r} = -1$$

Q: Find

$$\lim_{(x,y)\to(0,1)} \frac{xy-x}{x^2+y^2-2y+1}$$

(0,1)

$$\lim_{(x,y)\to(0,1)} \frac{x(y-1)}{x^2+(y-1)^2}$$

1) along y-axis $(x=0)$

$$\lim_{y\to 1} \frac{0}{(y-1)^2} = 0$$

2) along $y = x+1$

$$\lim_{x\to 0} \frac{x^2}{x^2+x^2} = \frac{1}{2}$$

$\rightarrow$ not exist.

• If $\lim\limits_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

$\rightarrow$ $f(x,y)$ continuous at (a,b).

$\boxed{32}$ Determine the set of points at which the function is cont.

911

$$H(x,y) = \frac{e^x + e^y}{e^{xy} - 1}$$ أصفار المقام

S

$e^{xy} - 1 = 0$ (ln للطرفين)

$xy = 0$

cont. everywhere but except x-axis & y-axis.

$\boxed{37}$ $f(x,y) = \begin{cases} \dfrac{x^2 y^3}{2x^2+y^2} & (x,y) \neq 0 \\ 1 & (x,y) = (0,0). \end{cases}$

$$\lim \frac{x^2y^3}{2x^2+y^2} = \lim_{r\to0^+} \frac{\cancel{r^5}^{\,r^3} \cos^2\theta \sin^3\theta}{\cancel{r^2}(2\cos^2\theta + \sin^2\theta)} \boxed{=0}$$

∴ not cont. because $\neq 1$

∴ not continuous at $(0,0)$.

find point of discontinuity.

$\boxed{38}$ $f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

(1) along x-axis

$\lim \dfrac{0}{x^2} = 0$ ← ما استفدت

(2) along $y = x$

$\lim \dfrac{x^2}{3x^2} = \dfrac{1}{3}$

not exist ∴ not cont.

14.3 Partial derivative:

Ex: let $f(x,y) = x^2 + y^2 x^3 + y^4$.

Find $f(x)$, $f(y)$.

$f(x) = 2x + 3x^2 y^2 + 0$

$f_y = 0 + 2y x^3 + 4y^3$

$f_{xx} = 2 + 6x y^2$

$f_{xy} = 6x^2 y$.

$\frac{20}{911}$ Find if exist

$$\lim_{(x,y,z)\to(0,0,0)} \frac{yz}{x^2+4y^2+9z^2}$$

1. along x-axis $(y=0, z=0)$

$$\lim \frac{0}{x^2} = 0$$

2. along $y=z$, $x=0$

$$\lim_{y\to 0} \frac{y^2}{13y^2} = \frac{1}{13}$$

→ Doesn't exist.

$f_x \rightarrow$ slope

Q: $z = f(x,y)$

* $f_x(a,b) = g'(a)$ where $g(x) = f(x,b)$.

* $f_y(a,b) = h'(b)$

$h(y) = f(a,y)$.

Q: $f(x,y) = z = x^2 + y^2$ $\quad f(x,0)$

$f_x(a,b)$ ... كمل حل

$\frac{28}{424}$ | $f(x,y) = x^y$. Find f_x, f_y "the 1st partial dir. of $f(x,y)$"

S.

$f_x = y x^{y-1}$

$f_y = x^y \cdot 1 \cdot \ln x$ [نفس X مشتقة القوة X base ln]

$\boxed{35}$ Find 1st P.D of

$u = \underbrace{xy}\ \underbrace{\sin^{-1}(yz)}$.

① $u_x = y \sin^{-1}(yz)$

② $u_y = x \sin^{-1}(yz) + xy \dfrac{z}{\sqrt{1-(yz)^2}}$.

③ $u_z = xy \dfrac{y}{\sqrt{1-(yz)^2}}$

$(\sin^{-1} x)' \longrightarrow \dfrac{x'}{\sqrt{1-x^2}}$

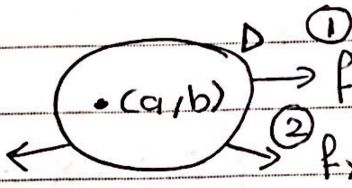
$\boxed{57}$ Find all second p.d. of

$$V = \sin(s^2-t^2).$$

$$\left.\begin{aligned} V_s &= 2s\cos(s^2-t^2) \\ V_t &= -2t\cos(s^2-t^2) \end{aligned}\right] \to 1st$$

2nd
$$\left[\begin{aligned} V_{ss} &= 2\cos(s^2-t^2) - 2s\sin(s^2-t^2)(2s)^2 \\ V_{st} &= +4st\sin(s^2-t^2) \\ V_{tt} &= -2\cos(s^2-t^2) + 2t\sin(s^2-t^2)(-2t) \\ V_{ts} &= 4st\sin(s^2-t^2). \end{aligned}\right.$$

$V_{st} = V_{ts} \to$ متطابقة ..



① f is defined on D.

② f_{xy} and f_{yx} cont. on D.

$f_{xy}(a,b) = f_{yx}(a,b)$

$\boxed{104}$ If $f(x,y) = \sqrt[3]{x^3+y^2}$ Find $f_x(0,0)$

$$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b)-f(a,b)}{h} \to \text{.. قانون التعريف}$$

$$* \left[\, f_y(a,b) = \lim_{h\to 0} \frac{f(a,b+h)-f(a,b)}{h} \right.$$

S —

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{h-0}{h} = \boxed{1}.$$

$$* \left[\begin{array}{l} g(x) = f(x,0) = x \\ f_x(0,0) = g'(0) = 1 \end{array} \right.$$

$\boxed{14.3}$

$$f_{xx} = (f_x)_x = \frac{\partial f^2}{\partial x^2}$$

$$f_{xy} = (f_x)_y = \frac{\partial f^2}{\partial y \, \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

$\boxed{\frac{103}{927}}$ if $f(x,y) = x(x^2+y^2)^{-\frac{3}{2}} e^{\sin(x^2 y)}$

find $f_x(1,0)$

S.

$f_x(a,b) = g'(a)$ where $g(x) = f(x,b)$

$$g(x) = f(x,0) = x^{-2}$$

$$f_x(1,0) = g'(1) = -2x^{-3}\Big|_{x=1} = -2$$

$\boxed{105}$ let $f(x,y) = \begin{cases} \dfrac{x^3y - xy^3}{x^2+y^2} & , (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$

b) Find $f_x(x,y)$, $f_y(x,y)$ when $(x,y) \neq (0,0)$

c) find $f_x(0,0)$, $f_y(0,0)$.

$$f_x(x,y) = \begin{cases} \dfrac{(3x^2y - y^3)(x^2+y^2) - 2x(x^3y - xy^3)}{(x^2+y^2)^2} & ; (x,y) \neq (0,0) \\ 0 & ; (x,y) = (0,0) \end{cases}$$

$$f_x(0,0) = \lim_{h \to 0} \frac{f(h+0,0) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{0}{h^2}}{h} = \lim_{h \to 0} \frac{0}{h^3} = 0.$$

14.4

① Tangent plane
② linear approximation
③ Total differentiation.

* Tangent plane *

equ. of surface $z = f(x, y)$ at point (a, b) is given by:

* " $z - z_0 = f_x(a,b)(x-a) + f_y(a,b)(y-b)$ "

* linear app. *

The LA for $z = f(x,y)$ at (a,b)

$f(x,y) \approx L(x,y)$

$L(x,y)$ = tangent plane ⇒ z = *.

$L(x,y) = z$.

$z = \underbrace{[redacted]}_{z_0} f_x(a,b)(x-a) + f_y(a,b)(y-b)$.

6
434

Find the equ. of tangent plane for.

$z = xe^{xy}$ at $(2, 0, 2)$

S—

$f_x = e^{xy} + xye^{xy}$

$f_x(2,0) = 1$

$f_y = x^2 e^{xy}$

$f_y(2,0) = 4$

→ equ of tangent plane:

$z - 2 = 1(x-2) + 4(y-0)$

$\boxed{z = x + 4y}$

18/934 verify the linear approximation at $(0,0)$.

$$\frac{y-1}{x+1} \approx x+y-1$$

S —

$$f(x,y) = \frac{y-1}{x+1}$$

$$f(0,0) = -1$$

$$f_x = \frac{-(y-1)}{(x+1)^2} \rightarrow f_x(0,0) = 1$$

$$f_y = \frac{1}{x+1} \longrightarrow f_y(0,0) = 1$$

$$L(x,y) = -1 + x + y.$$

11/934 Find the linearization $L(x,y)$ of $f(x,y) = 1 + x\ln(xy-5)$ at $(2,3)$

S —

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$f(2,3) = 1 + 2\ln 1 = 1.$$

$$f_x = \ln(xy-5) + \frac{xy}{xy-5}$$

$$f_x(2,3) = 6$$

$$f_y = \frac{x^2}{xy-5} \qquad f_y(2,3) = \frac{4}{1} = 4$$

$$L(x,y) = 1 + 6(x-2) + 4(y-3)$$

$$f(x,y) \approx L(x,y).$$

[19] Given that f is differentiatable with $f(2,5) = 6$

$= f_x(2,5) = 1 \qquad f_y(2,5) = -1$

use L.A to estimate $f(2.2, 4.9)$.

Sol —

$f(x,y) \approx L(x,y)$

$L(x,y) = 6 + 1(x-2) - 1(y-5)$

$f(2.2, 4.9) \approx L(2.2, 4.9) = 6 + \frac{2}{10} - \left(\frac{-1}{10}\right) = 6.3$

[21] approximate $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2}$

Sol — $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

$(a,b,c) = (3,2,6)$

$f(3,2,6) = 7$

* $f_x = \dfrac{x}{\sqrt{x^2+y^2+z^2}} \rightarrow f_x(3,2,6) = \dfrac{3}{7}$

* $f_z = \dfrac{z}{\sqrt{x^2+y^2+z^2}} \rightarrow f_z(3,2,6) = \dfrac{6}{7}$

* $f_y = \dfrac{y}{\sqrt{x^2+y^2+z^2}} \rightarrow f_y(3,2,6) = \dfrac{2}{7}$

$L(x,y,z) = 7 + \frac{3}{7}(x-3) + \frac{2}{7}(y-2) + \frac{6}{7}(z-6)$

$f(3.02, 1.97, 5.99) \approx L(3.02, 1.97, 5.99)$

$= 7 + \frac{3}{7} \times \frac{2}{100} + \frac{2}{7} \times \frac{-3}{100} + \frac{6}{7} \times \frac{-1}{100}$

$= 7 + \frac{-6}{700}$

$= \dfrac{7 * 700 - 6}{700}$

$\because z = f(x,y)$ The differ. of f is given by :

$dz = f_x\,dx + f_y\,dy.$

26/935 Find the diff. of function $u = \sqrt{x^2 + 3y^2}$

Sol—

$$du = \frac{x}{\sqrt{x^2+3y^2}}\,dx + \frac{3y}{\sqrt{x^2+3y^2}}\,dy$$

$\boxed{31}$ If $z = 5x^2 + y^2$, and (x,y) changes from $(1,2)$ to $(1.05, 2.1)$

compute the values Δz, dz.

Sol—

$$\Delta z = f(1.05, 2.1) - f(1,2)$$

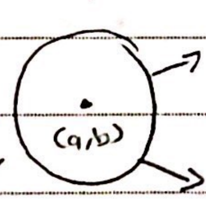
$$dz\Big|_{(1,2)} = f_x(1,2)\,\underbrace{dx}_{\substack{1.05-1 \\ \boxed{=0.05}}} + f_y(1,2)\,\underbrace{dy}_{\substack{2.1-2 \\ =\boxed{0.1}}}$$

$$f_x = 10x \longrightarrow f_x(1,2) = 10$$

$$f_y = 2y \longrightarrow f_y(1,2) = 4$$

$$dz\Big|_{(1,2)} = 10 \times \frac{5}{100} + 4 \times \frac{10}{100} = 0.9$$

$\boxed{46}$



$\left.\begin{matrix} f_x \\ f_y \end{matrix}\right\}$ exist on D

$\left.\begin{matrix} f_x \\ f_y \end{matrix}\right\}$ cont. at (a,b)

f is

diff (a,b)

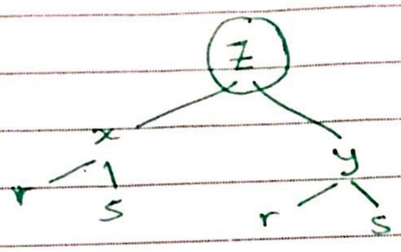
14.5 chain Rule ...

let $z = f(x, y)$

$x = g(r, s)$

$y = h(r, s)$

Find $\dfrac{\partial z}{\partial r}$



$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial r}$$

23/944 | If $w = xy + yz + zx$

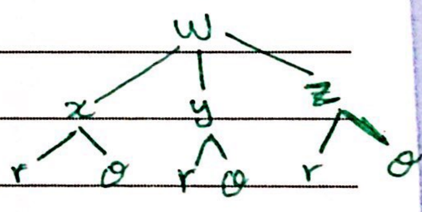
$x = r\cos\theta$

$y = r\sin\theta$

$z = r\theta$

Find $\dfrac{\partial w}{\partial r}$, when $r = 2$, $\theta = \pi/2$.

S —



$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial r} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial r}$$

$$= (y + z)\cos\theta + (x + z)\sin\theta + (y + x)\theta$$

$$= (2 + \pi)0 + \pi \cdot 1 + 2 * \frac{\pi}{2} = 2\pi.$$

$$\left.\frac{\partial w}{\partial r}\right|_{(r,\theta) = (2, \frac{\pi}{2})}$$

$(x, y, z) = (0, 2, \pi)$

13

944

let $p(t) = f(g(t), h(t))$

when f is diff, $g(2) = 4$

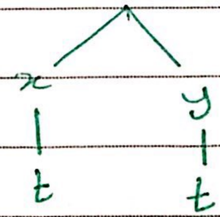
$g'(2) = -3$, $h(2) = 5$, $h'(2) = 6$

$f_x(4,5) = 2$, $f_y(4,5) = 8$.

Find $p'(2)$.

S—.

$g(x) = f(x, y)$



$x = g(t)$

$y = h(t)$

$$p'(t) = \frac{\partial f}{\partial t} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$= f_x(x, y) \cdot g'(t) + f_y(x, y) \cdot h'(t)$$

$$p'(2) = f_x(4,5) \cdot g'(2) + f_y(4,5) \cdot h'(2)$$

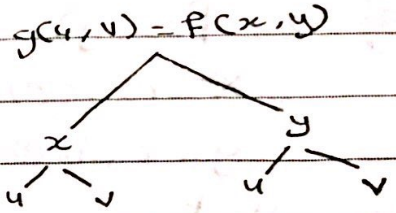
$$= 2 * -3 + 8 * 6$$

$$= 42.$$

15 suppose f is diff fun. of x, y and $g(u, v)$
$= f(e^u + \sin v \;,\; e^u + \cos v)$.
use the table to find $g_u(0,0)$, $g_v(0,0)$

	f	g	fx	fy
(0,0)	3	6	4	8
(1,2)	6	3	2	5

S —



$\boxed{x} = e^u + \sin v$
$\boxed{y} = e^u + \cos v$.

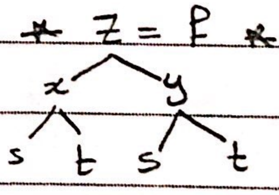
$$g_u = \frac{\partial f}{\partial u} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$g_u(0,0) = f_x(x,y) \cdot e^u + f_y(x,y) \cdot e^u$$
$$g_u(\underset{u}{0}, \underset{v}{0}) = f_x(\underbrace{1,2}) \cdot e^u + f_y(\underbrace{1,2}) \cdot e^0$$
$$= 2 \times 1 + 5 \times 1 = \boxed{7}.$$

عوّضنا في x, y
مفروضه

46/945 If $z = f(x, y)$, $\underbrace{x = s + t}$, $y = s - t$
show that
$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \frac{\partial z}{\partial s} \cdot \frac{\partial z}{\partial t}$$

S —



(1) $$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$$
$$= \frac{\partial z}{\partial x} \cdot 1 + \frac{\partial z}{\partial y} \cdot 1$$
$$= \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$$

(2) $$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}$$

$$\rightarrow \frac{\partial z}{\partial s}\cdot\frac{\partial z}{\partial t} = \left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right)\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

$$= \left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2$$

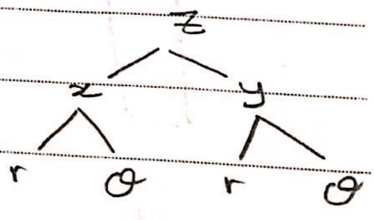
$\boxed{45}$ $z = f(x, y)$

$x = r\cos\theta$

$y = r\sin\theta$.

show that ...

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$



Q: $x^2 y e^{xz} = y z^3 + x^3 z^2$

Find $\dfrac{\partial z}{\partial x}$

S

$$F(x,y,z) = x^2 y e^{xz} - y z^3 - x^3 z^2$$

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}$$

$$F_x = 2xy e^{xz} + x^2 y z e^{xz} - 3x^2 z^2$$

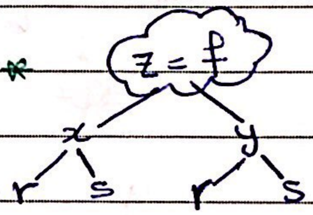
$$F_z = x^3 y e^{xz} - 3yz^2 - 2x^3 z$$

$$* \quad \frac{\partial z}{\partial x} = \frac{-(2xye^{xz} + x^2 y z e^{xz} - 3x^2 z^2)}{x^3 y e^{xz} - 3yz^2 - 2x^3 z}$$

Q: If $z = f(x,y)$

$x = r^2 + s^2$ * Find $\dfrac{\partial^2 f}{\partial r \partial s}$ *

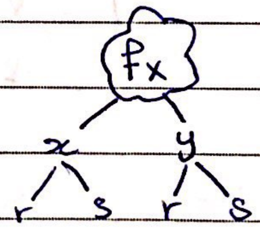
$y = rs$



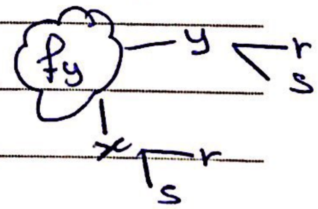
S

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$= 2s f_x + r f_y$$



$$\left(\frac{\partial}{\partial r}\right)\left(\frac{\partial f}{\partial s}\right) = \frac{\partial}{\partial r}(2s f_x + r f_y)$$



$$= \frac{\partial}{\partial r}(2s f_x) + \frac{\partial}{\partial r}(r f_y)$$

$$= 0 \cdot f_x + 2s \frac{\partial}{\partial r}(f_x) + f_y + r \frac{\partial}{\partial r}(f_y)$$

$$= 0 + 2s\left(\frac{\partial f_x}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial f_x}{\partial y} \cdot \frac{\partial y}{\partial r}\right) + f_y + r\left(\frac{\partial f_y}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial f_y}{\partial y} \cdot \frac{\partial y}{\partial r}\right) = 2s(2r f_{xx} + s f_{xy}) + f_y$$

$$+ r(2r f_{yx} + s f_{yy}).$$

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$\frac{33}{944}$ $e^z = xyz$, Find $\frac{\partial z}{\partial y}$.

S —

$F(x,y,z) = e^z - xyz$

$F_y = -xz$

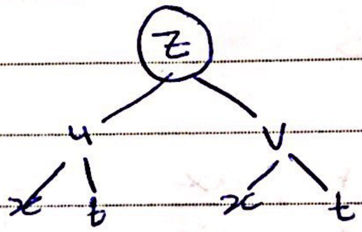
$F_z = e^z - xy$

$$\boxed{\frac{\partial z}{\partial y} = \frac{xz}{e^z - xy}}$$

$\frac{49}{945}$ show that any function of the form:
$z = f(x+at) + g(x-at)$ is the solution of the equ:

$$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial t^2}$$

S —



$u = x + at$

$v = x - at$

$z = f(u) + g(v)$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial t} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial t}$$

$$= a \underbrace{f'(u)}_{\partial f/\partial u} - a \underbrace{g'(v)}_{\partial g/\partial u}$$

$$\frac{\partial z^2}{\partial t^2} = \frac{\partial}{\partial t}\left(a f'(u) - a g'(v) \right) \longrightarrow$$

$$= a \frac{\partial}{\partial t}\left(f'(u)\right) - a \frac{\partial}{\partial t}\left(g'(v)\right)$$

$$= a \frac{\partial f'}{\partial u} \cdot \frac{\partial u}{\partial f} - a \frac{\partial g'(v)}{\partial v} \cdot \frac{\partial v}{\partial t}$$

$$= a^2 f'' + a^2 g''$$

$$\rightarrow \frac{\partial z^2}{\partial x^2} = f'' + g''.$$

* 14.6

[1] Directional Derivation

let $z = f(x,y)$

$u = \langle u_1, u_2 \rangle$ is a unit vector,

$D_{f_u}(a,b) = f_x(a,b)\,u_1 + f_y(a,b)\,u_2$

Ex: Find the directional Derivation for $f(x,y) = x^2 e^{xy}$ at $(2,0)$ in the directional of $u = \langle 3,4 \rangle$.

~~s~~ $\hat{u} = \frac{u}{|u|} = \left\langle \frac{3}{9}, \frac{4}{5} \right\rangle$

$f_x(2,0) = 4$

$f_y = x^3 e^{xy}$

$f_y(2,0) = 8$

$\rightarrow D_{f_u}(2,0) = 4 \cdot \frac{3}{5} + 8 \cdot \frac{4}{5} = \boxed{\frac{44}{5}}$

2| Gradient vector:

- let $z = f(x,y)$, The gradient vector for $f(x,y)$ is given by:

$$\nabla f(x,y) = \langle f_x(x,y),\ f_y(x,y) \rangle$$

- $D_f u(a,b) = \nabla f(a,b) \cdot \hat{u} \longrightarrow$ (Directional Derivation) علاقة (gradient) لـ

- Theorm~~s~~: The maximum rate of change for $z = f(x,y)$ is occur when u is in the same dir~~ection~~ of ∇f & the value of this rate of change is equal $|\nabla f|$.

Ex 8

If $D_f u(2,1,5)$, where $V = \langle 4,1,1 \rangle$

$u = \langle 2,-2,1 \rangle$, $|\nabla f(2,1,5)| = 8$.

Find $D_f u(2,1,5)$ ...

Sol.

$$D_f u(2,1,5) = \nabla f(2,1,5) \cdot \hat{v}$$

$$\nabla f(2,1,5) = 8\hat{u} = 8\left\langle \frac{2}{3}, \frac{-2}{3}, \frac{1}{3} \right\rangle$$

$$= \left\langle \frac{16}{3}, \frac{-16}{3}, \frac{8}{3} \right\rangle$$

$$|V| = \sqrt{18}, \quad \hat{v} = \left\langle \frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}}, \frac{1}{\sqrt{18}} \right\rangle$$

$$D_f v(2,1,5) = \left\langle \frac{16}{3}, \frac{-16}{3}, \frac{8}{3} \right\rangle \cdot \left\langle \frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}}, \frac{1}{\sqrt{18}} \right\rangle$$

Ex: If $D f_u(1,2) = 6$, $u = \langle \frac{3}{5}, \frac{-4}{5} \rangle$

$D f_u(1,2) = 8$

$v = \langle \frac{3}{5}, \frac{4}{5} \rangle$ Find $\nabla f(1,2)$

S. $\nabla f(1,2) = \langle f_x(1,2), f_y(1,2) \rangle$

$$6 = \frac{3}{5} f_x(1,2) - \frac{4}{5} f_y(1,2) \quad \text{①}$$

$$8 = \frac{3}{5} f_x(1,2) + \frac{4}{5} f_y(1,2) \quad \text{②}$$

$$14 = \frac{6}{5} f_x(1,2)$$

$$f_x(1,2) = \frac{35}{3} \longrightarrow \frac{4}{5} f_y(1,2) = 1$$

$$\longrightarrow f_y(1,2) = \frac{5}{4}$$

$\therefore \nabla f(1,2) = \langle \frac{35}{3}, \frac{5}{4} \rangle.$

$\frac{15}{957}$ Find the dir. derivative of $f(x,y,z) = x^2y + y^2z$ at $p(1,2,3)$ in direction of $v = \langle 2,-1,2\rangle$.

S —

$D_v f(1,2,3) = f_x(1,2,3)v_1 + f_y(1,2,3)v_2 + f_z(1,2,3)v_3$

$f_x = 2xy \rightarrow f_x(1,2,3) = 4$

$f_y = x^2 + 2yz \rightarrow f_y(1,2,3) = 13.$

$f_z = y^2 \rightarrow f_z(1,2,3) = 4$

$\hat{v} = \langle \frac{2}{3}, \frac{-1}{3}, \frac{2}{3} \rangle$

$D_v f(1,2,3) = 4 \cdot \frac{2}{3} + 13 \cdot \frac{-1}{3} + 4 \cdot \frac{2}{3} = \frac{3}{3} = 1$

$\boxed{20}$ Find directional derv. of $f(x,y,z) = xy^2z^3$ at $p(2,1,1)$ in the direction of $q\langle 1,-3,5\rangle$.

S —

$u = \overrightarrow{PQ} = \langle -1,-4,4\rangle$ نفس الحل

$D_u f(2,1,1) =$

$\boxed{24}$ Find the max rate of change of $f(x,y,z) = x\ln(yz)$ at $(1,2,\frac{1}{2})$ and find the direction in which it occurs.

S —

$\nabla f = \langle f_x, f_y, f_z\rangle$

$f_x = \ln(yz) \rightarrow f_x(1,2,\frac{1}{2}) = 0$

$f_y = \frac{xz}{yz} \rightarrow f_y(1,2,\frac{1}{2}) = \frac{1}{2}$

$f_z = \frac{xy}{yz} = 2$

$\rightarrow \nabla f(1,2,\frac{1}{2}) = \langle 0, \frac{1}{2}, 2\rangle.$

max rate of change $|\nabla f| = \sqrt{\frac{1}{4}+4} = \frac{\sqrt{17}}{2}.$

$\frac{28}{957}$ Find the directions in which the directional derivative of $f(x,y)=x^2+xy^3$ at the point $(2,1)$ has the value 2.

S —

$D_u f(x_0,y_0) = \nabla f(x_0,y_0) \cdot \hat{u}$

$\hat{u} = \langle u_1, u_2 \rangle$

$\nabla f = \langle 2x+y^3, 3xy^2 \rangle \rightarrow \nabla f(2,1) = \langle 5,6 \rangle$

$2 = 5u_1 + 6u_2$ ——— ①

$u_1^2+u_2^2=1$ ——— ②

$\rightarrow u_2 = \frac{2-5u_1}{6}$

$u_1^2 + \left(\frac{2-5u_1}{6}\right) = 1$

$u_1 =$, $u_2 =$

Ex: Find all points at which the direction of fastest change of the fun. $f(x,y) = x^2+y^2-2x-4y$ is $i+y$.

S —

$\nabla f(x_0,y_0) \parallel i+j$

$\nabla f = \langle 2x-2, 2y-4 \rangle \parallel \langle 1,1 \rangle$

$\rightarrow \langle 2x-2, 2y-4 \rangle = k\langle 1,1 \rangle$

$2x-2 = k$

$2y-4 = k$ " $2x-2 = 2y-4$ "

$2y-2x-2=0$

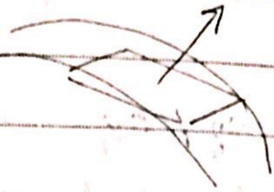
$y-x-1=0$

The set of all points on the line $y=x+1$ $\leftarrow \boxed{y=x+1}$

46 / 958 Find equ. of tangent plane and normal line to the surface:

$$x^4 + y^4 + z^4 = 3x^2y^2z^2 \quad \text{at } (1,1,1)$$

S—.



$F(x,y,z) = x^4 + y^4 + z^4 - 3x^2y^2z^2$

$\nabla F = n_p = V_L$

$\nabla F = \langle 4x^3 - 6xy^2z^2 ,\ 4y^3 - 6x^2yz^2 ,\ 4z^3 - 6x^2y^2z \rangle$

$\nabla f(1,1,1) = \langle -2, -2, -2 \rangle$.

* equ. of tangent plane:

$$-2(x-1) - 2(y-1) - 2(z-1) = 0$$

* equ. of normal line:

$x = 1 - 2t$

$y = 1 - 2t$

$z = 1 - 2t$

If ... $z = f(x,y)$ $\rightarrow$ $F = f(x,y) - z$

$\nabla F = \langle f_x , f_y , -1 \rangle$

$$\underset{\downarrow \atop 0}{z - z_0} = f_x(x - x_0) + f_y(y - y_0) - (z - z_0).$$

$\frac{54}{959}$ At what point on the surface $x^2+y^2+2z^2=1$ is the tangent plane parallel to the plane $x+2y+z=1$?!

S —.

$n_{P_0} // n_{P_1}$

$\nabla F // \langle 1,2,1 \rangle$ when $F = x^2+y^2+2z^2-1$

$\langle 2x, 2y, 4z \rangle // \langle 1,2,1 \rangle$

P_1

$?$ P_0

$\langle 2x, 2y, 4z \rangle = k \langle 1,2,1 \rangle$

$2x = k \rightarrow x = k/2$

$2y = k \rightarrow y = k/2$

$4z = k \rightarrow z = k/4$

$\left(\frac{k}{2}\right)^2 + k^2 + 2\left(\frac{k}{4}\right)^2 = 1$

$\frac{k^2}{4} + k^2 + \frac{k^2}{8} = 1$

$\frac{11}{8} k^2 = 1 \rightarrow k^2 = \frac{8}{11} \rightarrow k = \pm$

Maximum & minimum

14.7

* Def: The function $z = f(x,y)$ has local max. at (a,b) iff $f(a,b) \geq f(x,y)$ for all points (x,y) near (a,b)

* والعكس بالـ min *

* Def: The fun. $z = f(x,y)$ has absolute max at (a,b) iff $f(a,b) \geq f(x,y)$ for all f. Domain

* Def: The fun. has critical point at (a,b) iff one of the following hold.

$\boxed{1}$ $f_x(a,b) = f_y(a,b) = 0$

or

$\boxed{2}$ One of $f_x(a,b)$ or $f_y(a,b)$ not exist.

* Theory "Second derivative test"
suppose the second p.ds of f are cont. on disk with center (a,b) and suppose $\underbrace{f_x(a,b) = f_y(a,b) = 0}_{\text{critical}}$.

Let:

مميز ← $D = D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - \left(f_{xy}(a,b)\right)^2$

a) if $D > 0$ and $f_{xx}(a,b) > 0 \rightarrow f(a,b)$ local min

b) if $D > 0$ and $f_{xx}(a,b) < 0 \rightarrow$ " " max

c) " $D < 0 \longrightarrow f(a,b)$ not Max or Min

→

$\frac{5}{968}$

Find L. Max and L. Min for:

$f(x,y) = 9 - 2x + 4y - x^2 - 4y^2$.

S —.

$f_x = -2 - 2x = 0 \rightarrow x = -1$

$f_y = 4 - 8y = 0 \rightarrow y = \frac{1}{2}$

$\rightarrow (-1, \frac{1}{2})$ c.p.

$$\left.\begin{aligned} f_{xx} &= -2 \\ f_{yy} &= -8 \\ f_{xy} &= 0 \end{aligned}\right] \rightarrow$$ عالقانون $D = 16$

$f_{xx} < 0 \therefore f(-1, \frac{1}{2})$ L. Max

Ex: Find L. Max and L. Min for

$f(x,y) = x^4 + y^4 - 4xy + 1$

S —.

$f_x = 4x^3 - 4y = 0 \rightarrow y = x^3$

$f_y = 4y^3 - 4x = 0$ ----- (2)

sub in (2)

$4x^9 - 4x = 0$

$4x(x^8 - 1) = 0$

$x = 0, 1, -1$

$\rightarrow (0,0), (1,1), (-1,-1)$ c.p.

$f_{xx} = 12x^2$

$f_{yy} = 12y^2$

$f_{xy} = -4$

	f_{xx}	f_{yy}	f_{xy}	D	result
(0,0)	0	0	-4	-16	saddle p.
(1,1)	12	12	-4	+	L. Min
(-1,-1)	12	12	-4	+	L. Min

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Ex: Find L. Max & Min for:

$$f(x,y) = x^3y + 12x^2 - 8y.$$

S —.

$f_x = 3x^2y + 24x = 0$ ----- (1)

$f_y = x^3 - 8 = 0$

$\rightarrow \boxed{x = 2}$

sub in (1)

$12y + 48 = 0 \rightarrow (2,-4)$ c.p.

$\rightarrow \boxed{y = -4}$

$f_{xx} = 6xy + 24$

$f_{yy} = 0$

$f_{xy} = 3x^2 \rightarrow D = -144$

$f(2,-4)$ suddle.p.

Ex: Find L.Min & L.Max for...

$$f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2.$$

S —

$f_x = 6xy - 12x = 0 \rightarrow 6x(y-2) = 0$

$f_y = 3y^2 + 3x^2 - 12y = 0$ $\quad \hookrightarrow (x = 0, y = 2)$

If $\underline{x=0} \rightarrow 3y^2 - 12y = 0$

$3y(y-4) = 0 \rightarrow y = 0, 4$

$\rightarrow (0,0), (0,4)$ c.p, $(2,2), (-2,2)$ c.p.

If $\underline{y=2} \rightarrow 12 + 3x^2 - 24 = 0$

$3x^2 = 12$

$x^2 = 4 \rightarrow x = 2, -2$

cont.

$f_{xx} = 6y - 12$
$f_{yy} = 6y - 12$
$f_{xy} = 6x$

	f_{xx}	f_{yy}	f_{xy}	D	Result
(0,0)	-12	-12	0	144	L. Max
(0,4)	12	12	0	144	L. Min
(2,2)	0	0	12	-144	saddle
(-2,2)	0	0	-12	-144	saddle

$\boxed{5}$ Find C.P. for $f(x,y) = e^x \cos y$

S

$f_x = e^x \cos y = 0 \longrightarrow \cos y = 0 \longrightarrow y = \frac{\pi}{2} + n\pi$

$f_y = -e^x \sin y = 0 \longrightarrow \sin y = 0 \longrightarrow y = n\pi$

"No critical points."

مافيه قيم لـ x أو y بتعوض المعادلة [e^x مـ أعطيت صفر لأ

Q: ① If $f(x,y) = x \cos y$; Find c.p.

S

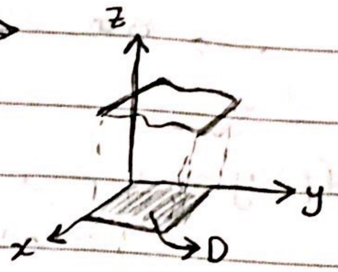
$f_x = \cos y = 0$

$f_y = -x \sin y = 0$ $\qquad$ c.p $= (0, \frac{\pi}{2} + n\pi)$

② classify → L.max, min, saddle → عالقاعدة

Q: If $z=f(x,y)$ is cont. on closed region $\underline{\underline{D}}$, then $\underline{\underline{f}}$ must have abs. max and abs. min.

$\hookrightarrow$

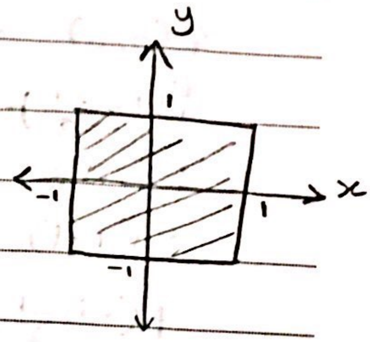


$\boxed{33}$ Find abs. Mas & abs. Min for

$f(x,y) = x^2+y^2+x^2y+4$

on the region D, when:

$D = \{(x,y) : |x| \leq 1, |y| \leq 1\}$



S—.

$f_x = 2x+2xy = 0 \rightarrow 2x(1+y) = 0$

$f_y = 2y + x^2 = 0$ $\qquad (x=0, y=-1)$

- If $\boxed{x=0} \rightarrow 2y=0 \rightarrow \boxed{y=0}$
- If $\boxed{y=-1} \rightarrow x^2 = 2 \rightarrow x = \pm\sqrt{2} \rightarrow \times$ أكبر من الفترة

$\therefore$ C.P $(0,0)$ abs. Max.

* boundary:

$\boxed{1}$ $x=1 \rightarrow f(1,y) = y^2+y+5$

$f'(1,y) = 2y+1 = 0 \rightarrow y = \frac{-1}{2}$ C.N

$\rightarrow (1,-\frac{1}{2}),\ (1,-1),\ (1,1)$

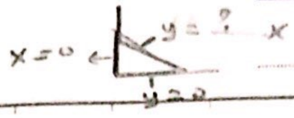
$\boxed{2}$ $x=-1 \rightarrow f(-1,y) = y^2+y+5$ $\qquad -1 \leq y \leq 1$

$f'(-1,y) = 2y+1 \rightarrow y = \frac{-1}{2}$ C.N

$\rightarrow (-1, \frac{-1}{2}),\ (-1,-1),\ (-1,1)$

حل من الكتاب ---

$$\frac{31/32}{968}$$



[3] $y=1 \longrightarrow f(x,1) = 2x^2+5 \qquad -1 \le x \le 1$

$f'(x,1) = 4x = 0 \longrightarrow x=0$

$\longrightarrow (0,1)$

[4] $y=-1 \longrightarrow f(x,-1) = 5 \qquad -1 \le x \le 1$

$f'(x,-1) = 0$

$f(0,0) = 4 \rightsquigarrow$ Abs. min

$f(1,-\frac{1}{2}) = 4.75$

$f(1,-1) = 5$

$f(1,1) = 7 \rightsquigarrow$ Abs. max

$f(-1,-\frac{1}{2}) = 4.75$

$f(-1,-1) = 5$

$f(-1,1) = 7$

$f(0,1) = 5$

$\longleftarrow \boxed{f(x,y)} = x^2+y^2+x^2y+4$

تعويض

"14.8" Lagrange's multiplies.

To Find the max & min values for $f(x,y,z)$ subject to constrain $g(x,y,z)=k$.

a) Find all values of x,y,z such that:

$$\nabla f(x,y,z) = \lambda \nabla g(x,y,z), \quad g(x,y,z)=k.$$

b) f at the points in $\underline{a}$, the largest value is max, and the smallest value is min.

Ex: Find extreme values of $f(x,y)=x^2+2y$ on the circle $x^2+y^2=1$,

$g(x,y)=x^2+y^2-1$

S.

$$\langle 2x, 4y\rangle = \lambda \langle 2x, 2y\rangle$$

$$x^2+y^2=1.$$

$2x = 2\lambda x$ ------ (1) from (1)

$4y = 2\lambda y$ ------ (2) $x=0$, $\lambda=1$

$x^2+y^2=1$ ------ (3)

* If $\lambda=1 \rightarrow y=0 \rightarrow x=\pm 1$

$\rightarrow (1,0), (-1,0)$

* If $x=0 \rightarrow \lambda=2 \rightarrow y=\pm 1.$

$\rightarrow (0,1), (0,-1)$

- $f(1,0)=1 \rightarrow$ min

- $f(-1,0)=1$

- $f(0,1)=2 \rightarrow$ max.

- $f(0,-1)=2.$

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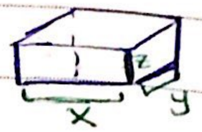
Ex: A rectangular box without a lid , made from $12\ m^2$ card board. Find the max volume of each box.

S.



* Volume $= f(x,y,z) = xyz$

$xy + 2xz + 2yz = 12$

$\rightarrow g(x,y,z) = xy + 2xz + 2yz$

- $yz = \lambda(y + 2z)$ $\cancel{*x}$ ---- 1
- $xz = \lambda(x+2z)$ $\cancel{*y}$ ---- 2
- $xy = \lambda(2x+2y)$ $\cancel{*z}$ ---- 3
- $xy + 2xz + 2yz = 12$ ---- 4

1 + 2 $\rightarrow 2xz = 2yz \longrightarrow \boxed{x=y}$

1 + 3 $\rightarrow y\cancel{x} + 2xz = 2xz + 2yz \longrightarrow \boxed{z = \frac{x}{2}}$

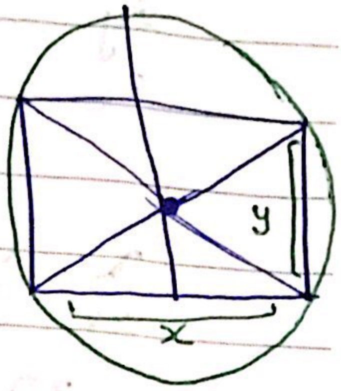
- Sub in 4

$x^2 + x^2 + x^2 = 12$

$x = 2 \longrightarrow y=2,\ z=1.$

* max Volume = 4. *

Q: Find largest Area for rectangular inside circle $x^2+y^2=4$.



S—

$$\text{Area} = f(x,y) = xy$$

$$x^2+y^2=16$$

$$g(x,y) = x^2+y^2$$

$$\begin{cases} y = 2\lambda x \\ x = 2\lambda y \\ x^2+y^2 = 16 \\ x, y, \lambda \neq 0 \end{cases}$$

لنشوف أبعاد.

$y \neq x$.

$x = \sqrt{8}$

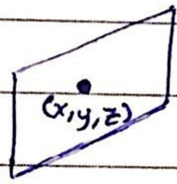
$y = \sqrt{8}$.

$\frac{42}{969}$ Find the point on the plane $x-2y+3z=6$, that is close to the point $(0,1,1)$

S—

$$f(x,y,z) = d^2 = x^2+(y-1)^2+(z-1)^2$$

$$g(x,y,z) = x-2y+3z$$



$\frac{7}{977}$ Find max and min for

$$f(x,y,z) = x^2+y^2+z^2$$

$$x+y+z=12 \rightarrow g(x,y,z)$$

S—

$$\Delta f = \lambda \Delta g. \rightarrow g = k$$

$$\langle 2x, 2y, 2z \rangle = \lambda \langle 1,1,1 \rangle.$$

$$x+y+z=12$$

Cont.

[14.7 → Q 41 – 52
حل من الكتاب]

→ $2x = \lambda \rightsquigarrow x = \lambda/2$

$2y = \lambda \qquad y = \lambda/2$

$2z = \lambda$

$x + y + z = 12$

$\frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{2} = 12$

$3\lambda = 24 \longrightarrow \boxed{\lambda = 8}$

$x = 4 \;,\; y = 4 \;,\; z = 4$

$(4,4,4) \rightarrow f(4,4,4) = \underline{48}$ = min.

45
969

Find 3 positive numbers whose sum is 12 and the sum of whose squares is as small as possible.

. بنا نعمل minimize لمجموع مربعاتهم ..

$f(x,y,z) = x^2 + y^2 + z^2$

$\underbrace{x + y + z = 12}_{g(x,y,z)}$

نفس حل السؤال اللي قبله .

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Ch-15 " Double Integrals "

Ex:
$$\iint_D f(x,y)\, \underline{dA} \longrightarrow \begin{matrix} dx\,dy \\ dy\,dx \end{matrix}$$

Ex: Find
$$\underbrace{\int_0^2 \underbrace{\int_0^1 (x^3y+y^2)\, dy}_{①}\, dx}_{②}$$

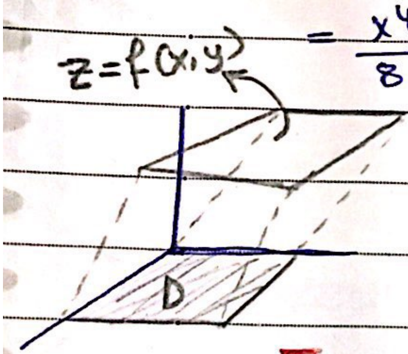
الداخل مع الداخل و الخارج مع الخارج ...

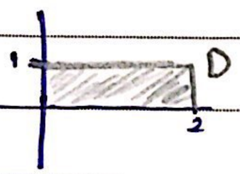
S —.
$$\int_0^2 \left. \frac{x^3y^2}{2} + \frac{y^3}{3} \right|_0^1 dx$$

$$= \int_0^2 \left(\frac{x^3}{2} + \frac{1}{3}\right) dx$$

$$= \left.\frac{x^4}{8} + \frac{1}{3}x \right]_0^2 = 2 + \frac{2}{3} = \frac{8}{3}$$

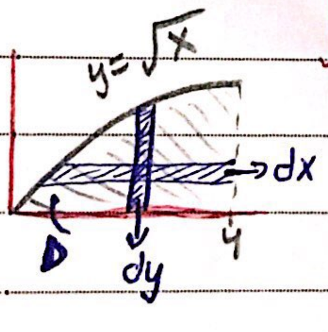
الجواب حجم مجسم





Ex: Find
$$\iint_D xy^2\, dA$$

where D is the shaded region

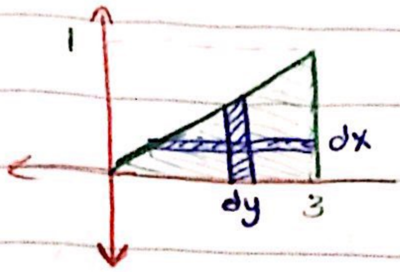


S —.
$$\iint_D xy^2\, dA = \int_0^4 \int_0^{\sqrt{x}} xy^2\, dy\, dx$$

$$= \int_0^4 \frac{x^{3/2}}{3}\, dx = \left.\frac{2x}{21}\right]_0^4 = \frac{2}{21}(2)^7 = \frac{2^8}{21}.$$

$\frac{51}{1009}$ Find

$$\int_0^1 \int_{3y}^{3} e^{x^2}\, dx\, dy$$



S — $$\int_0^3 \int_0^{\frac{x}{3}} e^{x^2}\, dy\, dx$$

$$= \frac{1}{3} \int_0^3 x e^{x^2}\, dx$$

$z = x^2$

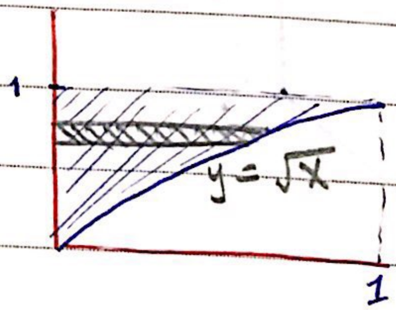
$dz = 2x\, dx$

$$= \frac{1}{6} \int_0^9 e^{z}\, dz$$

$$= \frac{1}{6} e^{z} \Big|_0^9 = \frac{1}{6}(e^9 - 1).$$

$\frac{53}{1009}$ Find:

$$\int_0^1 \int_{\sqrt{x}}^{1} \sqrt{y^3+1}\, dy\, dx$$



S — $$= \int_0^1 \int_0^{y^2} \sqrt{y^3+1}\, dx\, dy$$

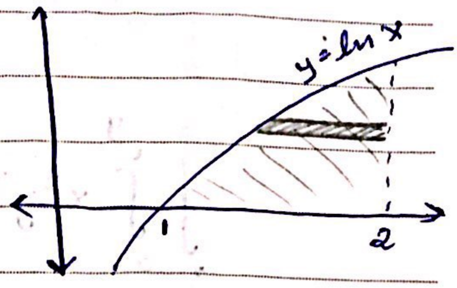
$$= \int_0^1 y^2 \underbrace{\sqrt{y^3+1}}\, dy$$

sub in $z = y^3 + 1$

49
1009

Sketch the region of integration & change the order of integration

$$\int_1^2 \int_0^{\ln x} f(x,y)\, dy\, dx$$



S —

أكبر قيمة
لـ y

$$= \int_0^{\ln 2} \int_{e^y}^{2} f(x,y)\, dx\, dy$$

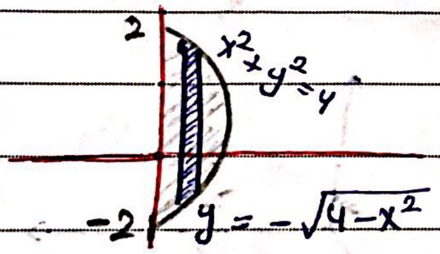
من المعروض في الرسم

أصغر قيمة لـ y

48

$$\int_{-2}^{2} \int_0^{\sqrt{4-y^2}} f(x,y)\, dx\, dy$$

sketch and change the order of integ.



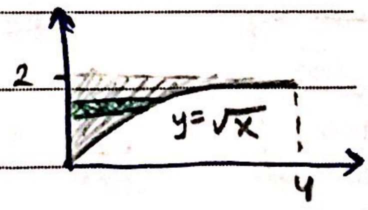
S — $x=\sqrt{4-y^2}$

$x^2+y^2=4$

$$= \int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y)\, dy\, dx$$

55 Evaluate.

$$\int_0^4 \int_{\sqrt{x}}^{2} \frac{1}{y^3+1}\, dy\, dx$$

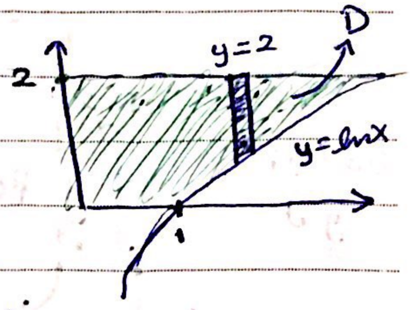


$$= \int_0^2 \int_0^{y^2} \frac{1}{y^3+1}\, dx\, dy \qquad = \int_0^2 \frac{y^2}{y^3+1}\, dy$$

Ex: Find

$$\iint_D x^2y^3\,dy\,dx$$

$$= \int_0^2 \int_0^{e^y} x^2y^3\,dx\,dy$$



S —

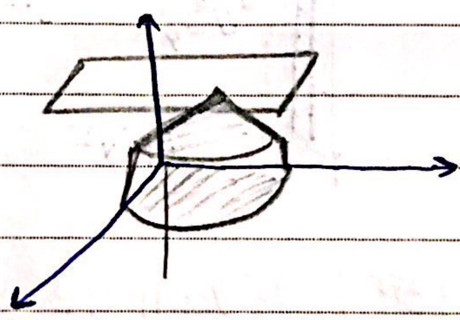
$$\int_0^1 \int_0^2 x^2y^3\,dy\,dx + \int_1^{e^2} \int_{\ln x}^2 x^2y^3\,dy\,dx$$

23/1008 Find the volume of the solid under the plane $3x+2y-z=0$ and above the region bounded by $y=x^2$ and $x=y^2$.

S —

$$V = \iint_D f(x,y)\,dA$$

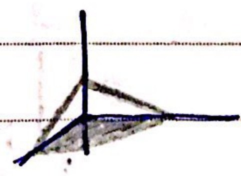
$$= \int_0^1 \int_{x^2}^{\sqrt{x}} (3x+2y)\,dy\,dx$$

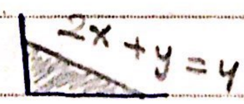


27/1008 Find the volume tetrahedron enclosed by the coordinates planes and the plane $2x+y+z=4$

S —

$$V = \int_0^4 \int_0^{\frac{4-y}{2}} (4-y-2x)\,dx\,dy$$





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"15.3" * Double integral in polar Coordinates *

$$* \iint_D f(x,y)\, dA = \int_{\theta_1}^{\theta_2}\int_{\alpha_1}^{\alpha_2} f(r\cos\theta, r\sin\theta)\, \underbrace{r\,dr\,d\theta}_{\text{الترتيب ثابت}}$$

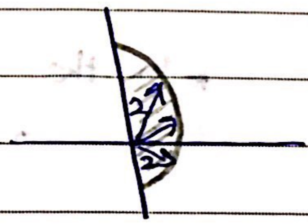
Ex: evaluate:

$$\iint e^{-x^2-y^2}\, dA$$ where D is the region bounded by y-axis and $x = \sqrt{4-y^2}$

S ___

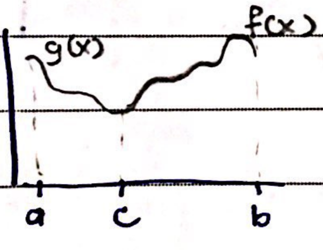
$x = r\cos\theta$

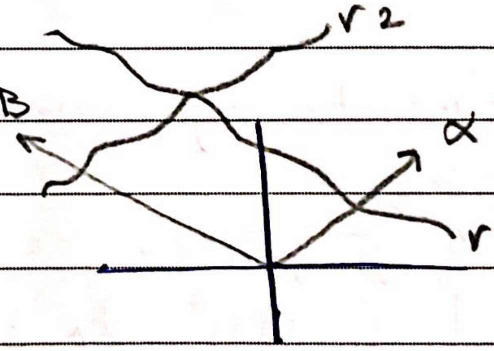
$y = r\sin\theta$



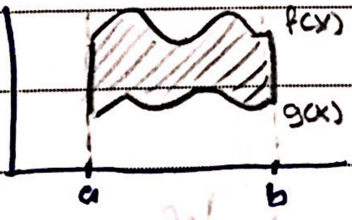
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{0 \to \text{origin}}^{2 \to r} e^{-r^2}\, r\,dr\,d\theta$$ ___ sub.

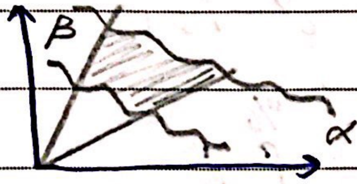
تجزيء *





طرح *





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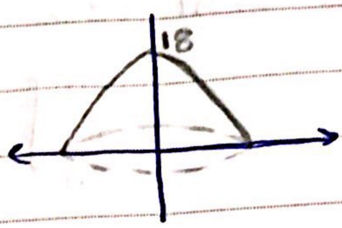
Ex: Find the volume of solid below $z = 18 - x^2 - y^2$ & above xy-plane.

S —

$$V = \iint_D 18 - x^2 - y^2 \, dA$$



$$= \int_0^{2\pi} \int_0^{\sqrt{18}} (18 - r^2) \, r \, dr \, d\theta$$ — sub.

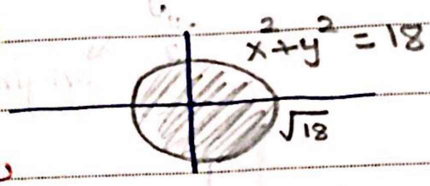
* In the first octan *

$0 \to \pi/2$

+ − | + +
− − | − −

$\left.\begin{matrix} x<0 \\ y>0 \end{matrix}\right] \to \left(\frac{\pi}{2} \to \frac{3\pi}{2}\right)$

Bounded below by f(x)



Ex: Evaluate by converting to polar

$$\int_0^{1/2} \int_{\sqrt{3}y}^{\sqrt{1-y^2}} x y^2 \, dx \, dy$$

$y = r\sin\theta$

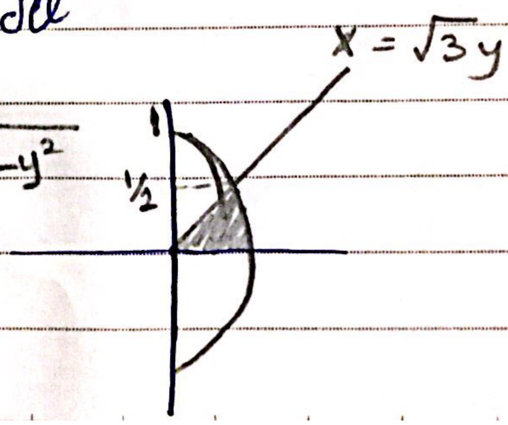
$\frac{1}{2} = 1 \sin\theta$

$\theta = \frac{\pi}{6}$

$$\int_0^{\pi/6} \int_0^1 r^4 \cos\theta \sin^2\theta \, dr \, d\theta$$

$\sqrt{3}\, y = \sqrt{1-y^2}$

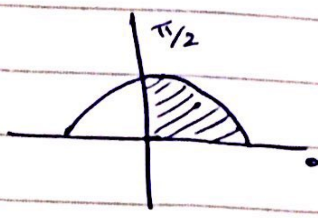
$y = \pm \frac{1}{2}$



Ex: Evaluate by converting to polar

$$\int_0^{\pi/2} \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2} \, dy \, dx$$

S — $\displaystyle\int_0^{\pi/2} \int_0^{2} e^{-r^2} \, r \, dr \, d\theta$ —— sub



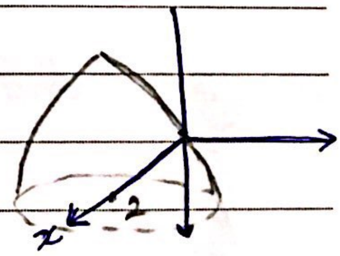
Ex: Find the volume of the solid under $z = 4 - (x-2)^2 - y^2$ above xy-plane

$$V = \iint 4 - (x-2)^2 - y^2 \, dA$$

$$= \iint \cancel{4} - x^2 - y^2 + 4x - \cancel{4} \, dA$$

$$= \int_0^{\pi} \int_0^{4\cos\theta} (4r\cos\theta - r^2) \, r \, dr \, d\theta$$

اذا كان ال center على المحاور دائماً من 0 لـ π)

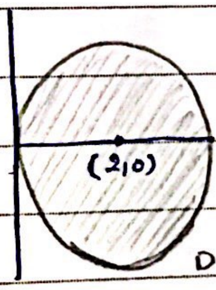


$$(x-2)^2 + y^2 = 4$$

$$x^2 - 4x + \cancel{4} + y^2 - \cancel{4} = 0$$

$$r^2 = 4r\cos\theta$$

$$r = 4\cos\theta.$$

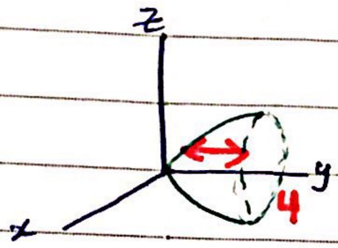


* Triple integrals *

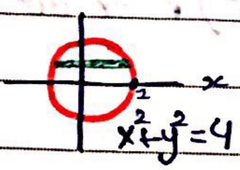
* $\iiint f(x,y,z)\, \underbrace{dv}_{dz\,dy\,dx}$

Ex:

① $\iiint \sqrt{x^2+z^2}\; dv$. when E is the region bounded by $y = x^2 + z^2$ and $y = 4$.



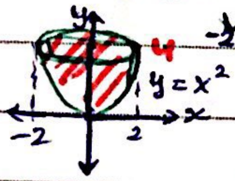
S ... $\int_{-2}^{2}\int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}}\int_{x^2+z^2}^{4} \sqrt{x^2+z^2}\; dy\,dx\,dz$

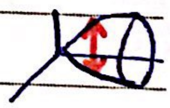


$$= \int_{-2}^{2}\int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}} (4-x^2-z^2)\sqrt{x^2+z^2}\; dx\,dz$$

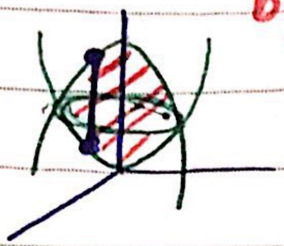
$$= \int_{0}^{2\pi}\int_{0}^{2} (4-r^2)\, r^2\; dr\,d\theta .$$

② $\int_{-2}^{2}\int_{x^2}^{4}\int_{-\sqrt{y-x^2}}^{\sqrt{y-x^2}} \sqrt{x^2+z^2}\; dz\,dy\,dx$

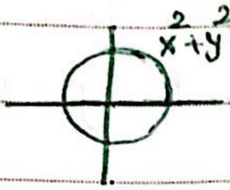




Q: Find the Volume of the solid enclosed by $z = x^2+y^2$ and $z = 8-x^2-y^2$.



$$V = \iiint_E 1 \, dV.$$



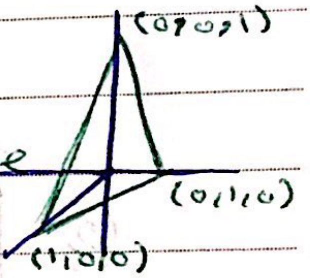
$x^2+y^2=4$

$$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{8-x^2-y^2} dz\,dy\,dx$$

$$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left(8-2(x^2+y^2)\right) dy\,dx = \int_{0}^{2\pi}\int_{0}^{2} (8-2r^2)\, r\, dr\, d\theta.$$

Q: Find volume of the solid enclosed by the coordinam planes and the planes passes through.

(0,0,1), (1,0,0), (0,1,0).



S ——

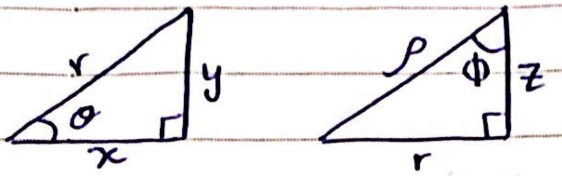
equ. of plane

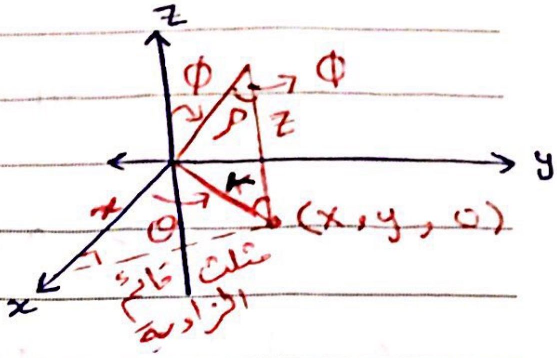
$$\int_0^1\int_0\int_0 1\, dz\, dy dx.$$

(15.7) Triple Integrals
cylindrical coordinates.

(15.8) Triple Integrals
spherical coordinates

$(x, y, z) = (r, \theta, z) = (\rho, \theta, \phi)$.





① rec → cy
- $r = \sqrt{x^2+y^2}$
- $\tan\theta = \frac{y}{x}$
- $z = z$

② cy → rec
- $x = r\cos\theta$
- $y = r\sin\theta$
- $z = z$

③ cy → sph
- $\rho = \sqrt{r^2+z^2}$
- $\tan\theta = \frac{r}{z}$
- $\theta = \theta$

4) sph → cy
- $r = \rho\sin\phi$
- $\theta = \theta$
- $z = \rho\cos\phi$

5) rec → sph
- $\rho = \sqrt{x^2+y^2+z^2}$
- $\tan\theta = \frac{y}{x}$
- $\tan\phi = \frac{\sqrt{x^2+y^2}}{z}$

6) sph → rec
- $x = \rho\sin\phi\cos\theta$
- $y = \rho\sin\phi\sin\theta$
- $z = \rho\cos\phi$

Ex: change from rec to cy
$(3, -3, 2)$

"حسب النقطة بالربع الرابع أم الثاني"

S. — • $r = \sqrt{9+9} = 3\sqrt{2}$

• $\tan\theta = -1 \longrightarrow \theta = \frac{-\pi}{4}$

• $z = 2$

$\rightarrow \left(3\sqrt{2}, \frac{-\pi}{4}, 2\right)$

* Describe in $\mathbb{R}^3$

• $r = 2 \longrightarrow x^2 + y^2 = 4$ " cylinder "

• $\theta = \frac{\pi}{6} \longrightarrow \tan\theta = \tan\frac{\pi}{6}$

$\frac{y}{x} = \frac{1}{\sqrt{3}} \rightarrow y = \frac{1}{\sqrt{3}} x$ " plane "

• $r^2 + z^2 = 4$

$x^2 + y^2 + z^2 = 4$ " sphere "

• $(r = 2\sin\theta) * r$

$r^2 = 2r\sin\theta$

$x^2 + y^2 = 2y$

$x^2 + (y-1)^2 = 1$ " Cylinder with center $(0, 1)$. "

Q: change from sph to rec $(6, \frac{\pi}{3}, \frac{\pi}{6})$.

s—.

$$x = 6 \sin \frac{\pi}{6} \cos \frac{\pi}{3} = 6 \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{2}.$$

$$y = 6 \sin \frac{\pi}{6} \sin \frac{\pi}{3} = \overset{3}{\cancel{6}} \times \frac{1}{\cancel{2}} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}.$$

$$z = 6 \cos \frac{\pi}{6} = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}.$$

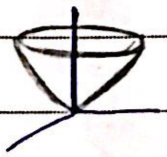
$$\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}, 3\sqrt{3}\right).$$

* Describe in $\mathbb{R}^3$

- $\phi = \frac{\pi}{3} \longrightarrow \tan\phi = \tan\frac{\pi}{3} \longrightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\frac{\sqrt{x^2+y^2}}{z}$

$z = \sqrt{3x^2 + 3y^2} \longleftarrow$



" cone "

- $\rho \cos\phi = 1$

$z = 1 \longrightarrow$ plane.

- $\rho = \cos\phi$

$x^2 + y^2 + z^2 = z \longrightarrow$ sphere

- $\rho^2 - 3\rho + 2 = 0$

$(\rho - 2)(\rho - 1) = 0$

$\rho = 2, 1 \longrightarrow$ sphere

$\boxed{\text{rec}}$ $\iiint f(x, y, z)\ dV$

$\boxed{\text{cy}}$ $\iiint f(r\cos\theta, r\sin\theta, z)\ r\ dz\,dr\,d\theta.$

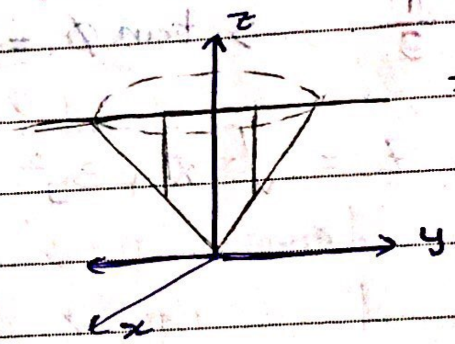
$\boxed{\text{sph}}$ $\iiint f(\_\_\_\_, \_\_\_\_, \_\_\_\_)\ \rho^2 \sin\phi\ d\rho\ d\theta\ d\phi.$

Ex: Find the volume of the solid

$z = 9$ and $z = \sqrt{x^2+y^2}$

s—

$$V = \iiint_E 1\ dV$$



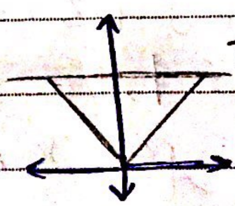
$\boxed{\text{cy}}$

$$V = \int_0^{1v} \int_0^{9} \int_r^{9} r\ dz\ dr\ d\theta$$

$x^2 + y^2 = 81$

$\boxed{\text{sph}}$

$$V = \int_0^{\pi/4} \int_0^{2\pi} \int_0^{9\sec\phi} \rho^2 \sin\phi\ d\rho\ d\theta\ d\phi$$



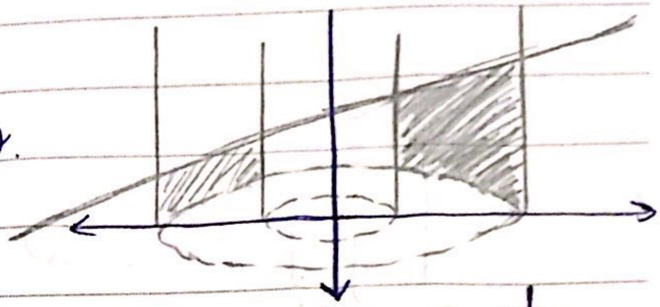
$\rho\cos\phi = 9$

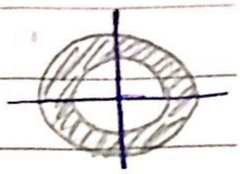
22 Evaluate $\iiint_E x \, dv$

↳ when E is the region enclosed by $z=0$, $z = x+y+5$ and the cylinders $x^2+y^2=4$, $x^2+y^2=9$

S—.

$$\int_0^{2\pi}\int_2^3\int_0^{r(\cos\theta+\sin\theta)+5} r^2\cos\theta \, dz\,dr\,d\theta.$$



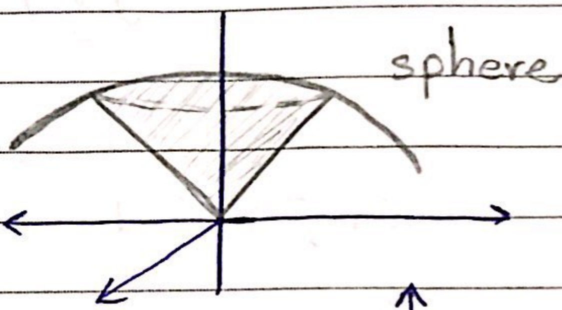


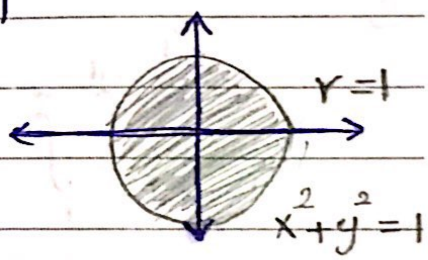
23 Find the volume of solid enclosed by $z=\sqrt{x^2+y^2}$ and $x^2+y^2+z^2=2$

S—.

cy

$$\int_0^{2\pi}\int_0^{1}\int_r^{\sqrt{2-r^2}} r \, dz\,dr\,d\theta$$





sph

$$V = \int_0^{\pi/4}\int_0^{2\pi}\int_0^{\sqrt{2}} \rho^2 \sin\phi \, d\rho\,d\theta\,d\phi$$

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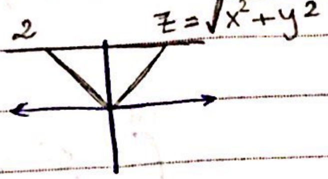
Ex → P-1048

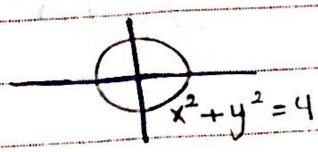
$\boxed{29}$ Evaluate by converting to cy

$$\hookrightarrow \int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\underset{\text{cone}}{\sqrt{x^2+y^2}}}^{2} xz\,dz\,dx\,dy$$



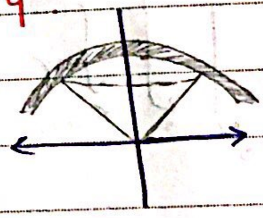
S —

$$\int_0^{2\pi}\int_0^{2}\int_r^{2} 2r^2\cos\theta\,dz\,dr\,d\theta$$



$\boxed{26}$ Evaluate $\iiint_E \sqrt{x^2+y^2+z^2}\,dv$

when E above $z=\sqrt{x^2+y^2}$ and between $x^2+y^2+z^2=1$, and $\underbrace{x^2+y^2+z^2=4}_{\rho=2}$

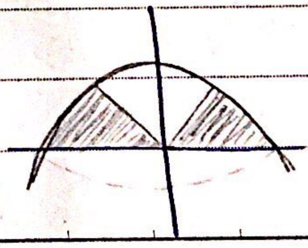


S —

$$\int_0^{\pi/4}\int_0^{2\pi}\int_1^{2}\rho^3\sin\phi\,d\rho\,d\theta\,d\phi$$

$\boxed{30}$ Find the volume of the solid within the sphere $x^2+y^2+z^2=4$, above xy-plane and below $z=\sqrt{x^2+y^2}$

$$V=\int_{\pi/4}^{\pi/2}\int_0^{2\pi}\int_0^{2}\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$$



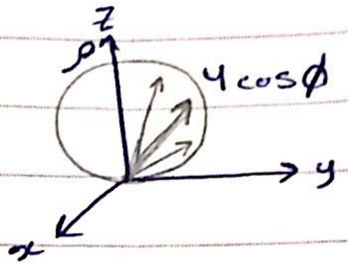
لا نطلبش نخط ال cone م

43 Evaluate by converting to sph.

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)^{3/2}\, dz\, dy\, dx$$

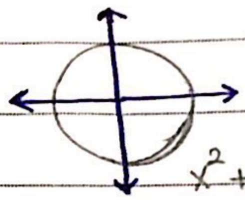
S

$$z = 2 - \sqrt{4-x^2-y^2}$$



$$(z-2)^2 = 4 - x^2 - y^2$$

$$x^2+y^2+(z-2)^2 = 4$$



$$= \int_0^{\pi/2}\int_0^{2\pi}\int_0^{4\cos\phi} \rho^5 \sin\phi \, d\rho\, d\theta\, d\phi$$

$$x^2+y^2+(z-2)^2 = 4$$

$$x^2+y^2+z^2-4z+4 = 4$$

$$4z = x^2+y^2+z^2$$

$$4\rho\cos\phi = \rho^2$$

$$\rho = 4\cos\phi$$