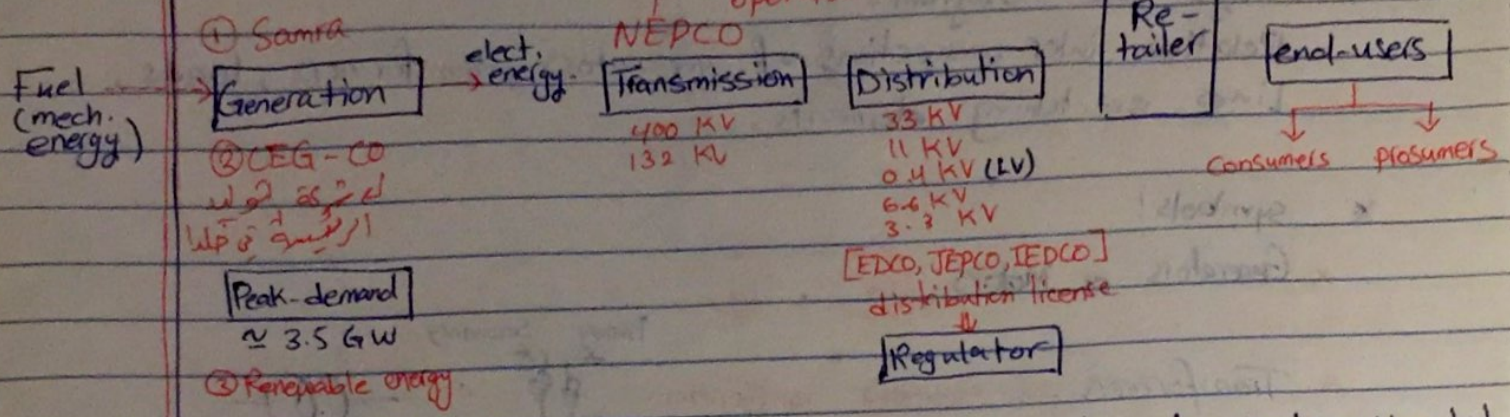
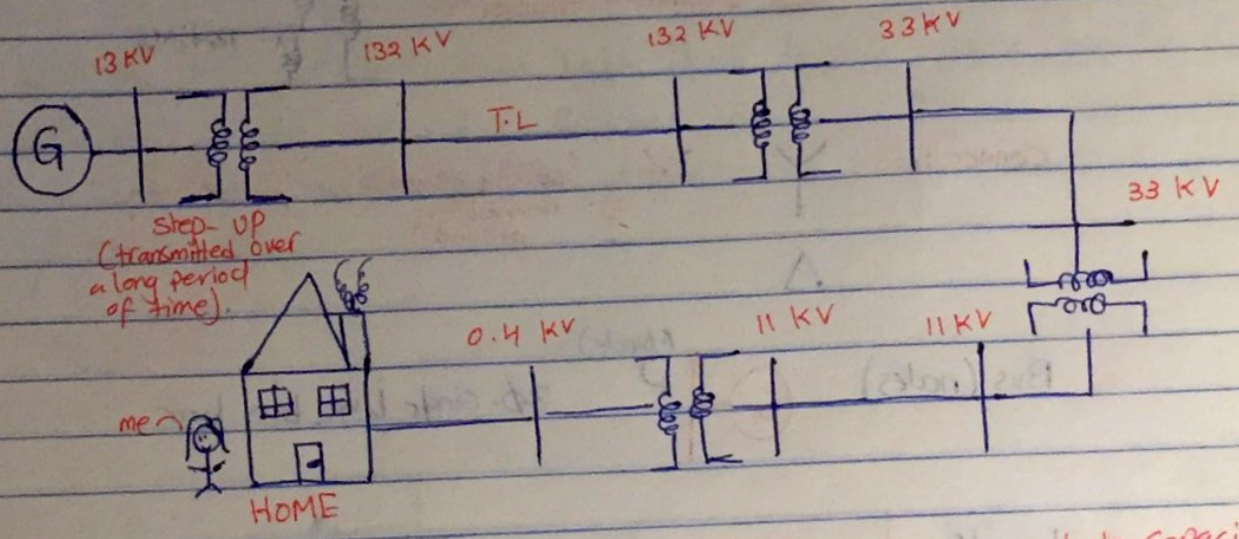


Thursday / 13-9-2018

* Power systems analysis:



* Transmission & distribution blocks are discriminated by their voltage lvl.



- * Models
- Generator
 - Transmission lines (T.L.)
 - cables
 - over-head lines (OHL)
 - Loads
 - Transformers

* every T.L. has a certain capacity, SC → I ↑ so you need a cable that's able to handle this current.

- * Analysis
- load-flow studies
 - short-circuit studies

Tuesday / 18-9-2018.

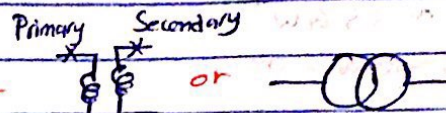
* Single line Diagram:-

Relative inter connections of generators, transformers, loads, lines, switching elements.

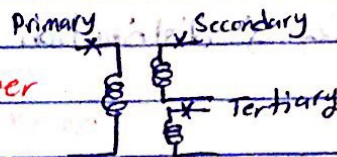
* Symbols!

* Generators or Motors (G), (M)

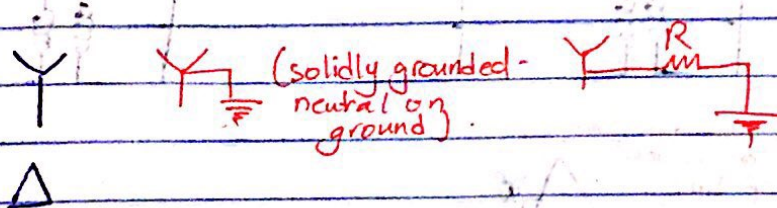
* Transformers → 2 winding transformer



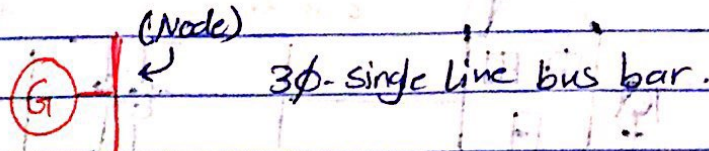
→ 3 winding transformer



Connections:



* Bus (nodes)



* Loads



* Switching elements — / — : circuit breaker

Isolator

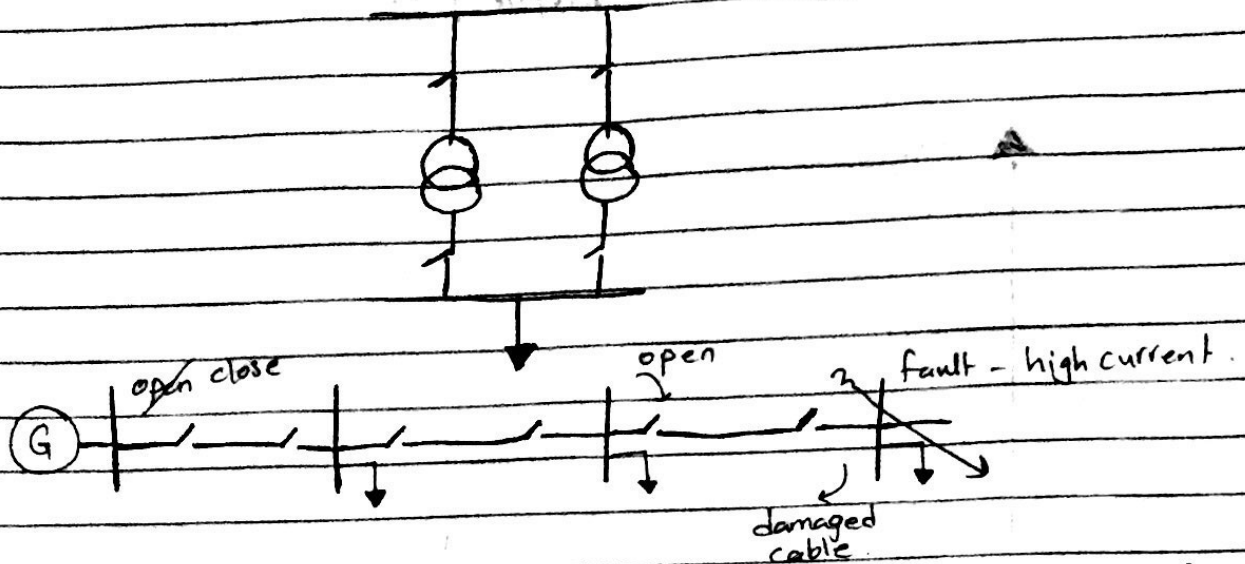
load-break switch

Re-closer

Sectionalizers

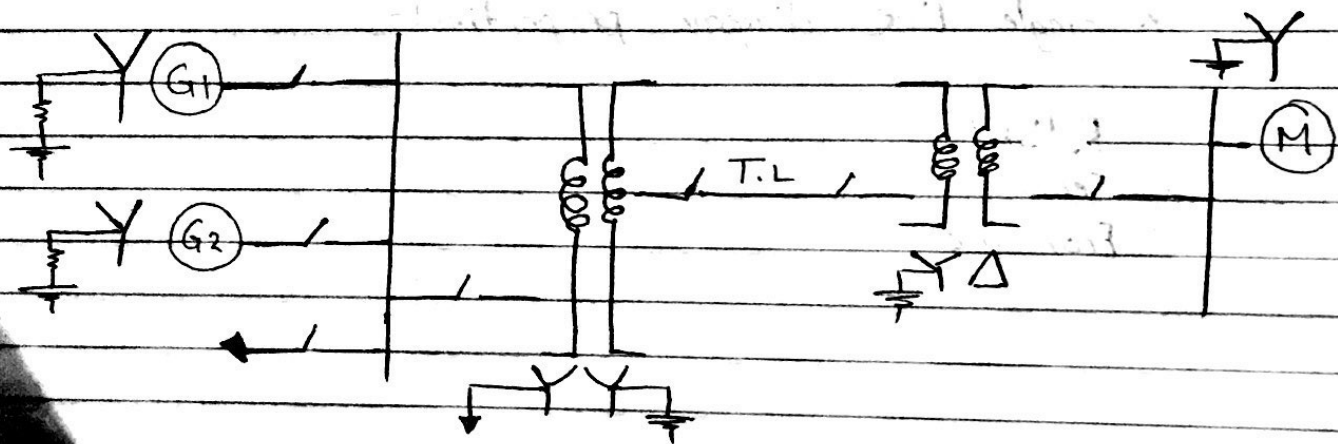
fuse

* delta filters the third harmonic.

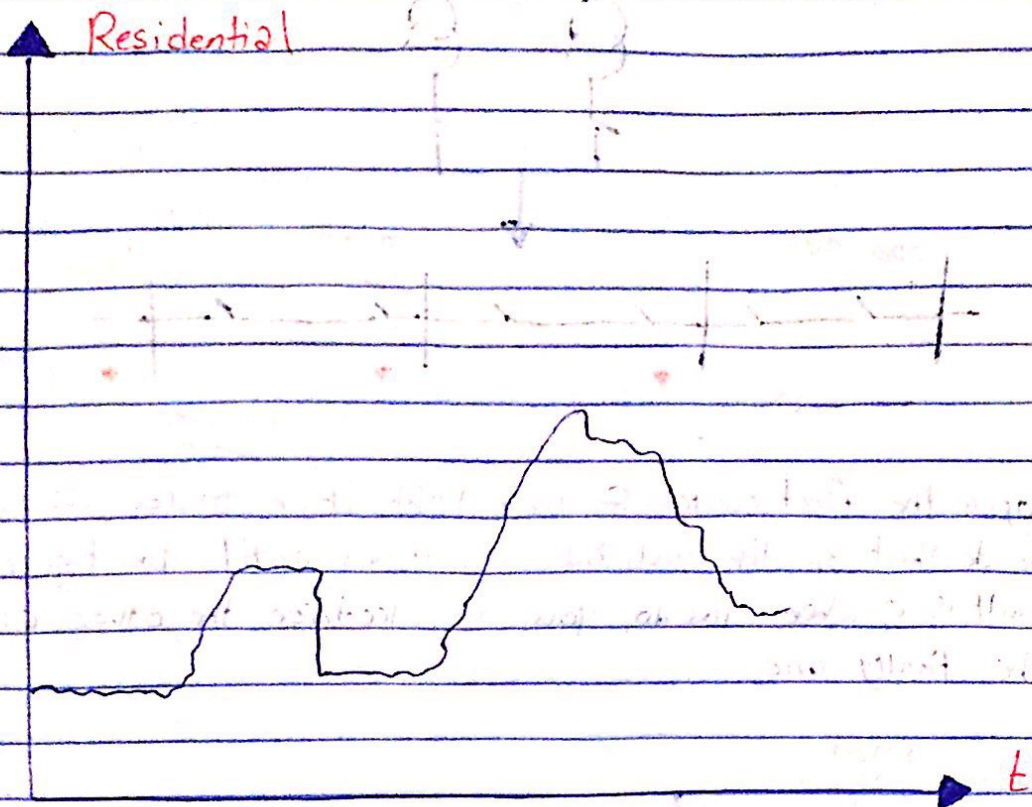


open the first switch & then light it on to see where the fault is.
 Re-do that on the 2nd, 3rd --- switches until you figure where the
 fault is & when you do, you re-electrify the other cables & fix
 the faulty one.

x Ratings $\begin{cases} \rightarrow \text{MVA} \\ \rightarrow \text{MW} \\ \rightarrow \text{KV} \end{cases}$



* Load profile [demand profile] 80

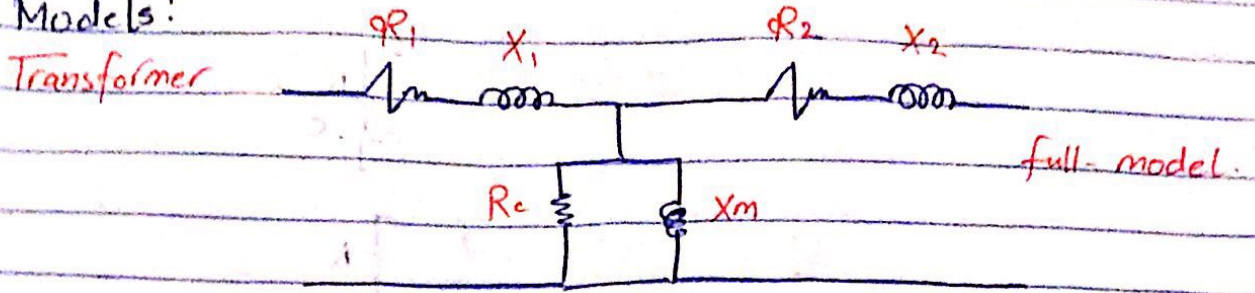


* single line diagram properties:-

- ① Reliable
- ② Secure
- ③ Economic

* to solve the previous system on the page before the graph:-

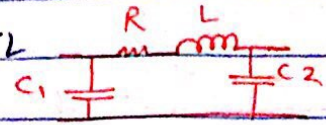
* Models:



* Lines: short T.L

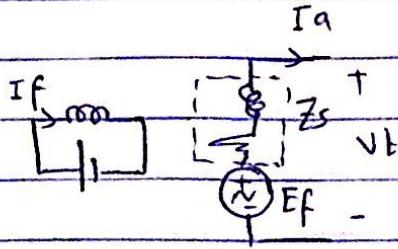


→ medium long T.L



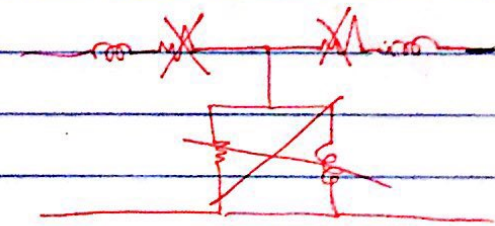
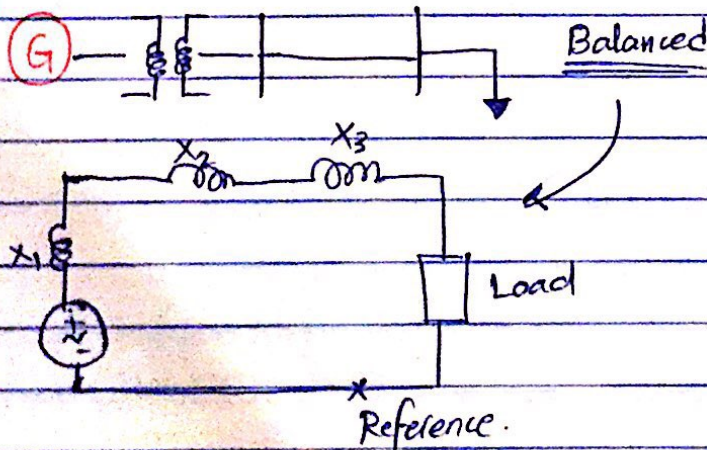
→ long T.L

* synchronous generator:



* Reactance Diagram: $\frac{X}{R} > 30$, 132 kV, 275 kV, 400 kV

simplification:

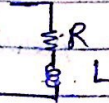


* in HV there's only X, so the R is gone at HV.

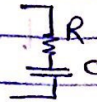
Per phase.

[Neutral-ground are the same because it's balanced]

* load full. specif : [1] Inductive - load



[2] Capacitive - load



[3] unity PF - load



Second specification:

[1] constant power $|S| = |V| |I|$

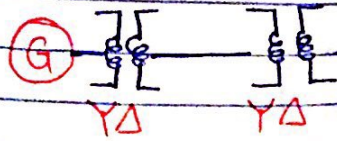
[2] constant Impedance $Z = |V| \uparrow \downarrow$ [to keep the impedance constant]
 $|I| \uparrow \downarrow$

[3] constant current

[4] mix

* The per-unit system

Thursday - 20/9/2018



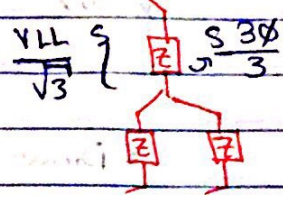
$$\text{per unit} = \frac{\text{Actual value}}{\text{Base value}}$$

things we want to analyze:

VLL, S 3φ, I, Z

* Base selection: S 3φ base, VLL base

$$* Z_{\text{base}} = \frac{(V_{\text{LL base}})^2}{S_{3\phi \text{ base}}}, \quad * I_{\text{base}} = \frac{S_{3\phi \text{ base}}}{\sqrt{3} V_{\text{L base}}}$$



Ex: Vbase 3φ = 0.4 kV, T2 → 33 kV / 0.4 kV

$$V_{\text{base 2}} = 0.4 * \left(\frac{33 \text{ kV}}{0.4} \right) = 33 \text{ kV}$$

Ex: S 3φ base = 100 MVA

$$P_{\text{load}} = 8 \text{ MW}$$

$$P_{\text{load}} \Big|_{\text{pu}} = \frac{8}{100}$$

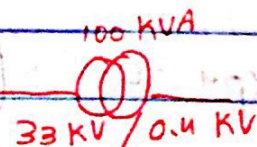
$$S_{3\phi \text{ base}} = P_{3\phi \text{ base}} = \phi_{3\phi \text{ base}}$$

$$R_{\text{base}} = X_{\text{base}} = Z_{\text{base}}$$

Transformer → Z = 5% (per-unit) for the Rating of the transformer

Ex: Z = 5% (100 KVA, 33/0.4 kV)

Z_{pu} (100 KVA, 33 kV) ??

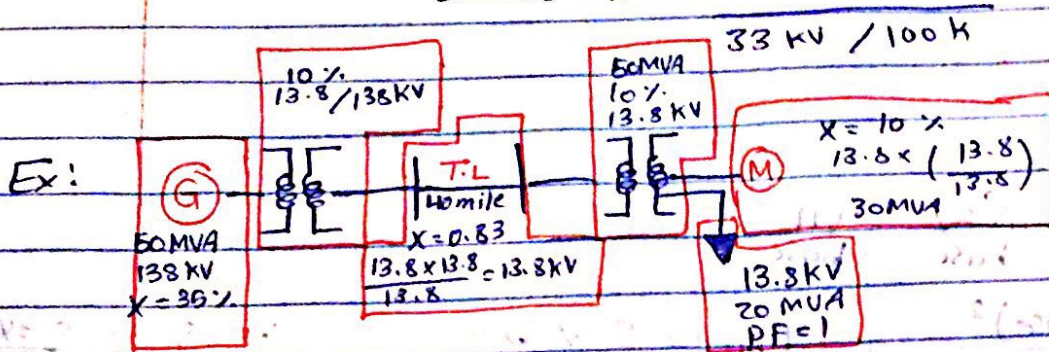


$$Z_{\text{per-phase Actual}} (\Omega) = \text{pu} * \text{Base}$$

$$= 0.05 * \frac{(0.4)^2}{S_{100k}} = \text{actual value (LV)}$$

$$Z_{actual} (\Omega) \Big|_{HV} = 0.05 \times \frac{(33)^2}{100k}$$

$$Z_{pu}(new) = \frac{Z_{actual}}{Z_{base}(new)} = 0.05 * \left(\frac{33^2}{100k} \right)$$

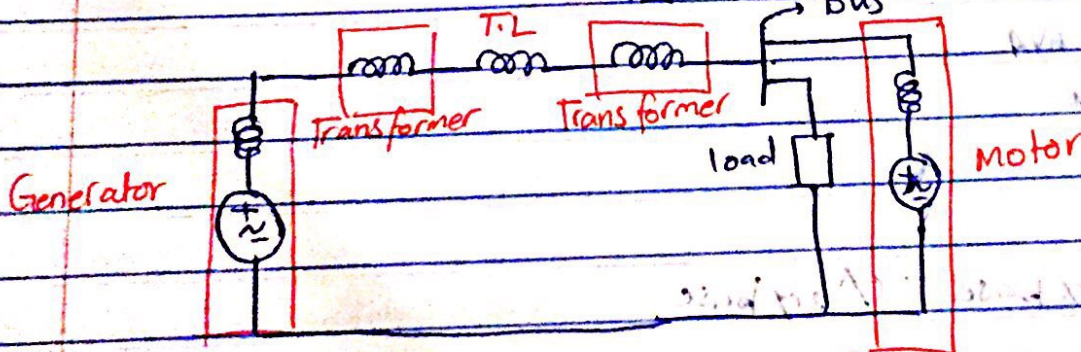


* Draw Reluctance Diagram given that you should represent them in per units (values)

base 13.8 kV
50 MVA

1] find all 'Regions' bases :-

2] Draw Bus



$$Z_{pu}(new) = Z_{pu}(old) * \left(\frac{S_{New}}{S_{Old}} \right) * \left(\frac{V_{old}}{V_{new}} \right)^2$$

$$T \rightarrow X_{pu} = 0.1 * \left(\frac{50}{50} \right) * \left(\frac{13.8}{13.8} \right)^2 = 0.1$$

$$G \rightarrow X_{pu} = 0.35 * \left(\frac{50}{50} \right) * \left(\frac{13.8}{13.8} \right)^2 = 0.35$$

$$T.L. = \frac{34 \Omega}{(138)^2 / 50}$$

$$T_2 \rightarrow X_{pu} = 0.1 \times \left(\frac{50}{50}\right)^2 \times \left(\frac{13.8}{13.8}\right)^2 = 0.1$$

$$X_{Mobr pu} = 0.1 \times \left(\frac{50}{30}\right) \times \left(\frac{13.8}{13.8}\right)^2 = 0.167 pu$$

$$\text{unity} \rightarrow Z_{load} \Big|_{pu} = \frac{(13.8)^2 / 20}{(13.8)^2 / 50} = \frac{50}{20} = 2.5 \angle 0^\circ$$

if it was lagging : $2.5 \angle +\cos^{-1}(PF)$

if it was leading : $2.5 \angle -\cos^{-1}(PF)$

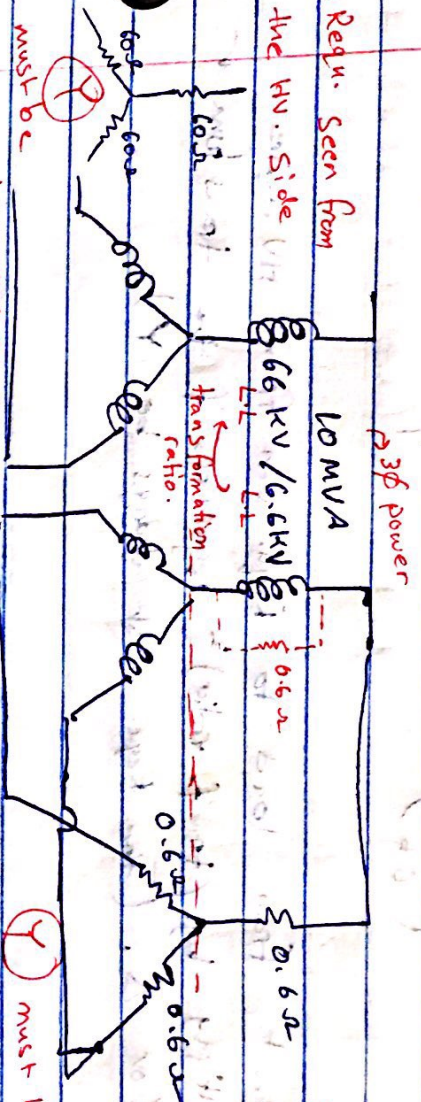
If it was leading: $2.5 \times \cos(\text{pt})$

Sunday 20/1/2018

* Three-phase transformer

① Requ. seen from

the HV side



3φ power

Assuming each branch is a single φ.

turns ratio $\leftarrow \frac{66/\sqrt{3}}{6.6/\sqrt{3}}$ (Y connection)

• $R_{HV} = 0.6 \times \left(\frac{66/\sqrt{3}}{6.6/\sqrt{3}} \right)^2$

$$= 0.6 \times \left(\frac{66}{6.6} \right)^2 = 60 \Omega$$

you get $\left(\frac{66}{6.6} \right)^2$ (Y)

* $\left(\frac{66}{6.6} \right)^2$ imp.

• $R_{LV} = 0.6$

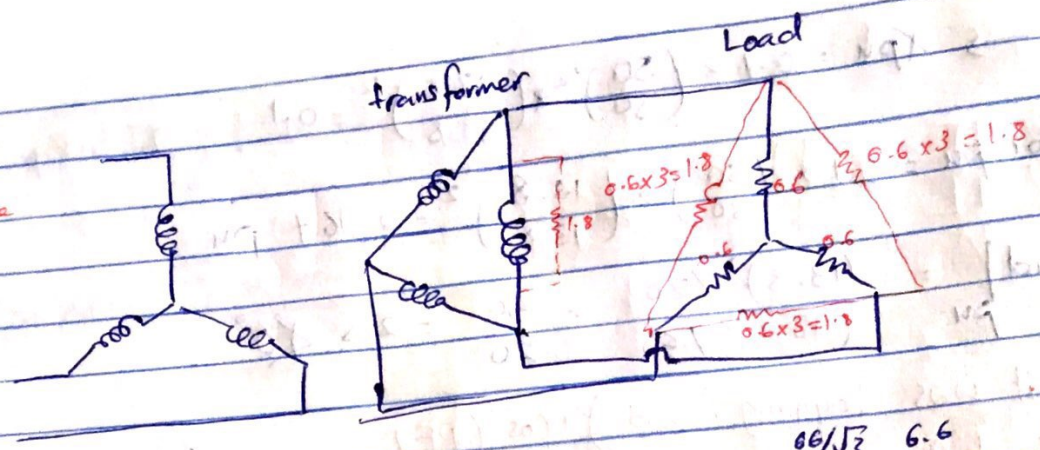
$$R_{LV} = \frac{60}{(6.6)^2/10}$$

$$= 0.6 \times \left(\frac{66^2}{6.6^2} \right)$$

$$= 0.6 \times \frac{(6.6)^2/10}{6.6^2/10}$$

(per unit LV, HV doesn't differ)
Per unit must be Y

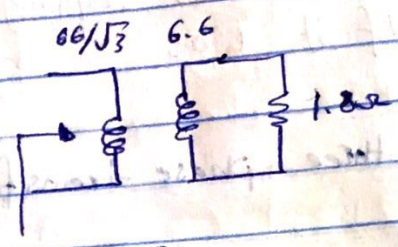
Req. seen from the HV side



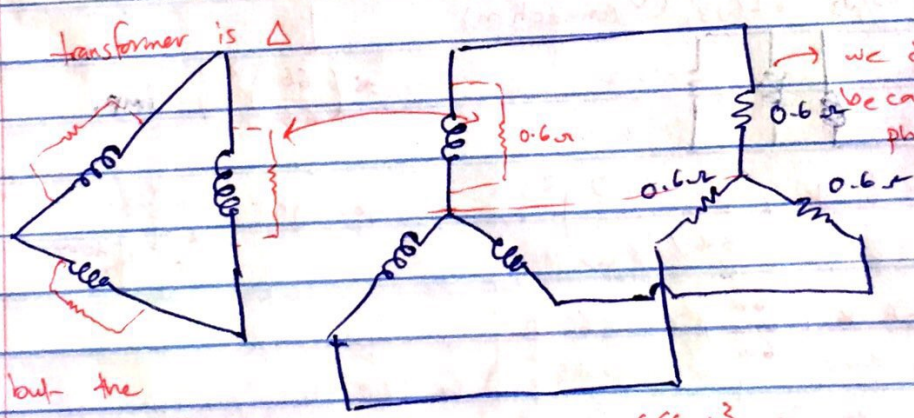
(must always be Y)

$$R_{HV} = 0.6 \times 3 \times \left(\frac{66/\sqrt{3}}{6.6} \right)^2$$

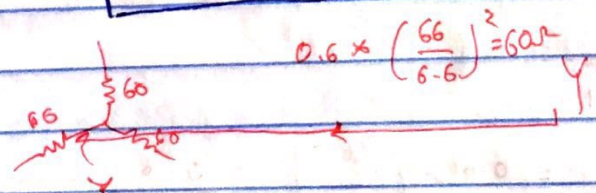
$$= 0.6 \times \left(\frac{66}{6.6} \right)^2$$



To convert from load to Req. seen from HV side we multiply by (transf. ratio)² regardless if the transformer was Y or Δ. (the Req. must always be Y)



but the answer will be Y



$$R(\omega)_{Hv} = 0.6 \times \frac{(66^2)}{\left(\frac{6.6}{\sqrt{3}} \right)^2} \times \frac{1}{3}$$

$$R_{Yeqn.} = 0.6 \times \left(\frac{66}{6.6} \right)^2$$

$$= 60 \Omega$$

Very important (check configuration in the book).

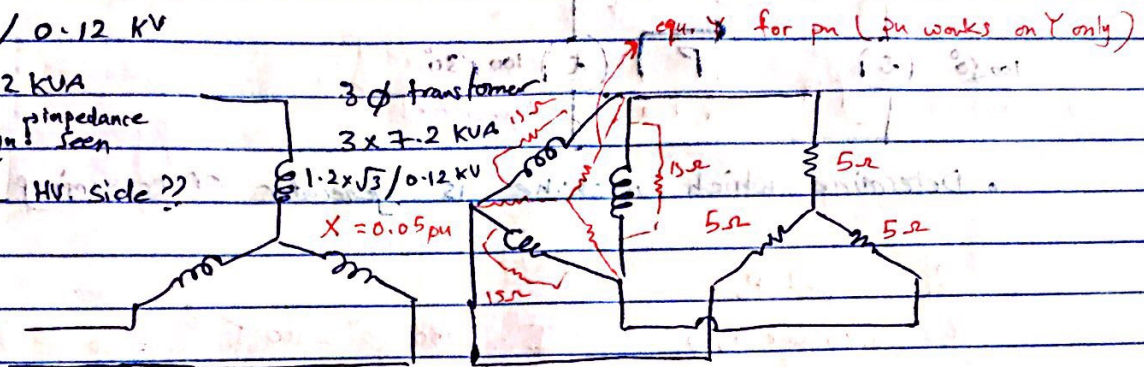
Example: 3 single phase transformer

1.2 / 0.12 kV

7.2 kVA

Find Z_{eqn} impedance seen

from the HV side??



Solution: $5 \times \left(\frac{1.2 \times \sqrt{3}}{0.12} \right)^2 = R_{eqn} |_{HV}$

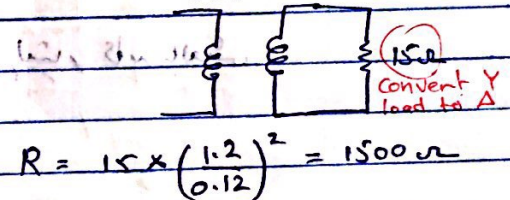
$R_{eqn} |_{HV} = 1500 \Omega$

$X_{transf} |_{HV} = 0.05 \times \left(\frac{(1.2 \times \sqrt{3})^2}{3 \times 7.2 \text{ kVA}} \right) = 10 \Omega$

$Z_{eqn} = 1500 + j10 \Omega$

* solution for explanation:

1.2 / 0.12

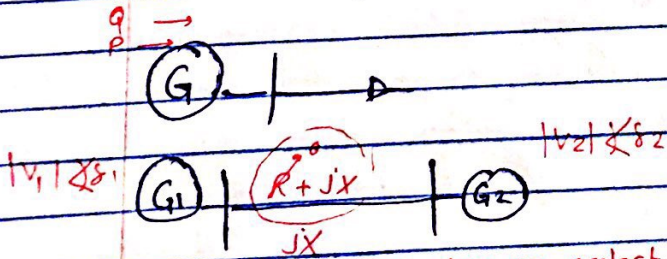


$R = 15 \times \left(\frac{1.2}{0.12} \right)^2 = 1500 \Omega$

$X = 0.05 \times \left(\frac{1.2}{0.12} \right)^2 = 10 \Omega$

$Z = 1500 + j10$

* Direction of power flows:-



• $P \propto \delta$

• $Q \propto |V|$

leads \rightarrow gives power

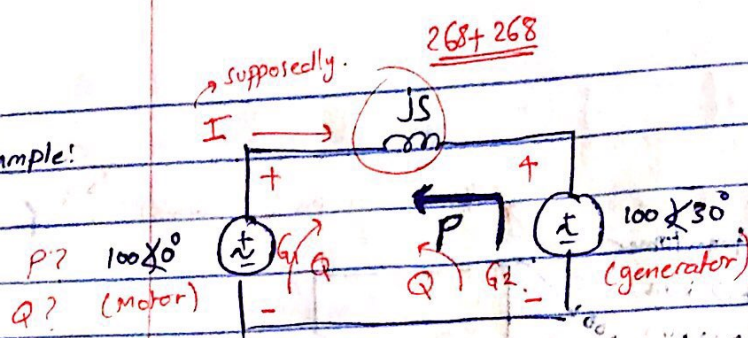
$Q \rightarrow$ depending on magnitude
 $Q >$ supplies.
 $Q <$ absorbs.

Real power decides whether its motor or generator.

motor $\leftarrow P \uparrow$

generator $\leftarrow P \downarrow$

Example!



Determine which machine is generator or consuming P/Q?

G₁ → inject P, Q
G₂ → consume P, Q

$$I = \frac{100 \angle 0^\circ - 100 \angle 30^\circ}{j5} = 10.35 \angle -35^\circ$$

$$S_1 = V I^* = 100 \angle 0^\circ \times 10.35 \angle 35^\circ = -1000 + j268 \text{ VA}$$

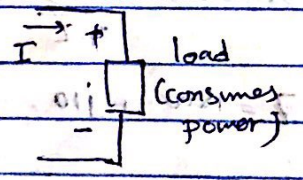
(consume P) inject Q

$$S_2 = V I^* = -1000 - j268 \text{ VA}$$

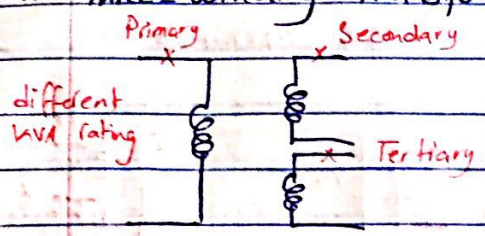
both give Q so it goes to JS (268 + 268)

لأنه لا يتغير مقدار Q
هنا لا يتغير Q

* Passive sign convention:

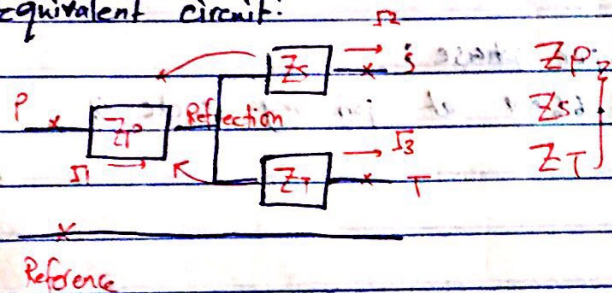


* Three winding transformer:-



* 2-winding transformer:
 kVA rating = kVA rating for secondary.

* Equivalent circuit:

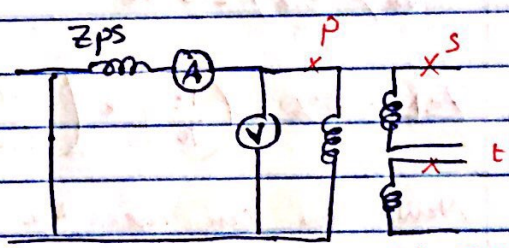


Zs , Zt } Reflected to primary side.

* 3-short circuit tests needed

Z_{ps} ; t:o:c

Impedance seen from primary
 Secondary short-circuited



$Zs' = Zs + Zt''$

* $Z_{ps} = Z_p + Z_s$ (1) Reflection primary.

$Z_{pt} = Z_p + Z_t$ (2) Reflected to primary

$Z_{st} = Z_s + Z_t$ (3) Reflected to secondary.

check reflected to primary:

* $Z_p = \frac{1}{2} (Z_{ps} + Z_{pt} - Z_{st})$

Memorize.

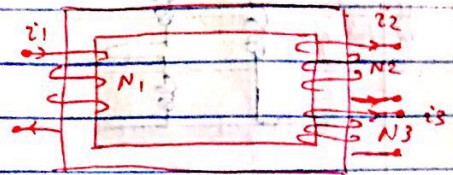
* $Z_s = \frac{1}{2} (Z_{ps} + Z_{st} - Z_{pt})$

* $Z_t = \frac{1}{2} (Z_{pt} + Z_{st} - Z_{ps})$

Example: 3 winding transformer;
 Primary Y-connected, 66 kV, 15 MVA
 Secondary Y-connected, 13.2 kV, 10 MVA
 tertiary Δ-connected, 2.3 kV, 5 MVA

* Rated: load (pu)

$$N_1^2 = N_2^2 + N_3^2$$



they must have common base to use the 1/2 equations

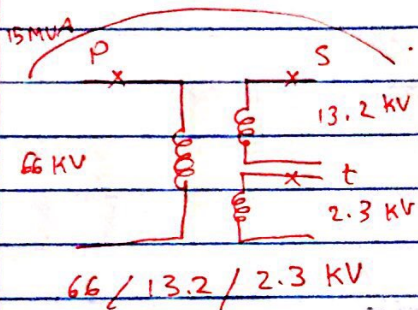
$$Z_{ps} = 7\% \text{ (15 MVA, 66 kV)}$$

$$Z_{pt} = 9\% \text{ (15 MVA, 66 kV)}$$

$$Z_{st} = 8\% \text{ (10 MVA, 13.2 kV)}$$

Final per unit impedances per phase:
 considering base of 15 MVA, 66 kV at primary side??

Solution:



correction $\rightarrow Z_{st} = 8\% \times \left(\frac{15}{10}\right) \times \left(\frac{13.2 \text{ kV}}{13.2 \text{ kV}}\right)^2 = 12\%$ using $Z_{pu(\text{new})} = Z_{pu(\text{old})} \times \frac{S_{\text{new}}}{S_{\text{old}}} \left(\frac{V_{\text{old}}}{V_{\text{new}}}\right)^2$

Now you can use the 1/2 equations:-

$$Z_p = \frac{1}{2} (Z_{ps} + Z_{pt} - Z_{st})$$

$$Z_s = \frac{1}{2} (Z_{ps} + Z_{st} - Z_{pt})$$

$$Z_t = \frac{1}{2} (Z_{pt} + Z_{st} - Z_{ps})$$

another solution using (Ω) without correction:

$$Z_{ps}(\Omega) = 0.07 \times \left(\frac{66^2}{15}\right)$$

$$Z_{pt}(\Omega) = 0.09 \times \left(\frac{66^2}{15}\right)$$

$$Z_{st}(\Omega) = 0.08 \times \left(\frac{13.2^2}{10}\right)$$

Primary.

reflect from secondary to primary

$$Z_{st}^{\text{ref}}(\Omega) = 0.08 \times \left(\frac{13.2^2}{10}\right) \times \left(\frac{66}{13.2}\right)^2$$

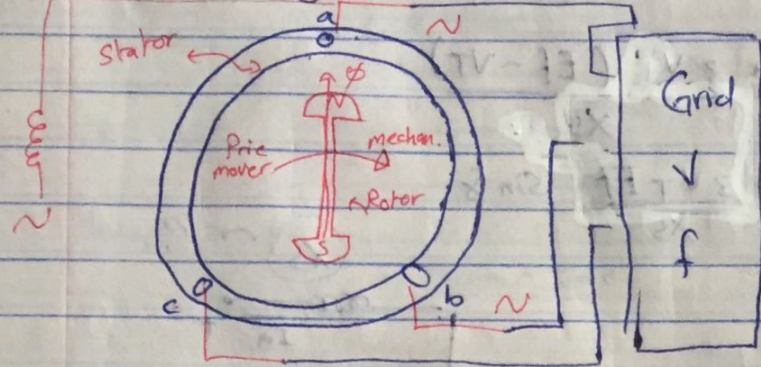
Actual at secondary Reflection to primary

$$Z_{st}^{\text{ref}}(\text{pu}) = 0.08 \times \left(\frac{13.2^2}{10}\right) \times \left(\frac{66}{13.2}\right)^2 \times \frac{15}{66^2}$$

$$Z_{st}^{\text{ref}}(\text{pu}) = 8\% \times \frac{15}{10} = 12\%$$

which is the same answer

* Synchronous generator:-

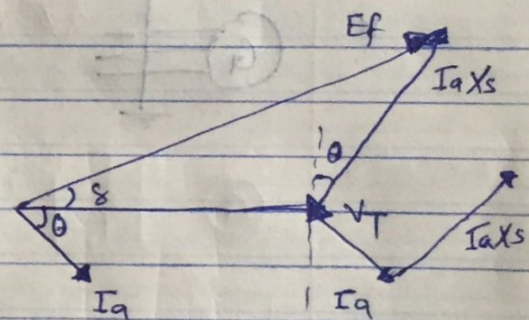
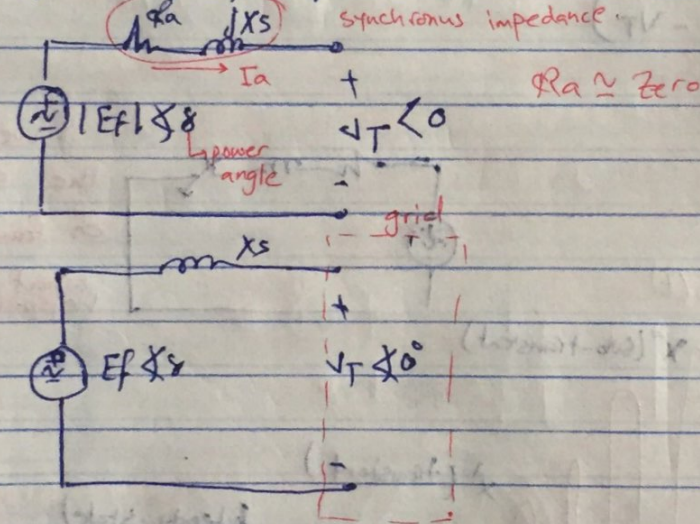


$$e(t) = N \frac{d\phi}{dt} \quad \phi(t) \downarrow e(t)$$

$$N_s = \frac{120f}{P}$$

↳ synchronous speed
 $f = \frac{P \times N_s}{120}$

equi circuit per phase:



$$P = 3 V_T I_a \cos \theta$$

$$P = \frac{3 V_T E_f \sin \delta}{X_s}$$

$Q \propto (E_f - V_T)$
 $P \propto \delta$

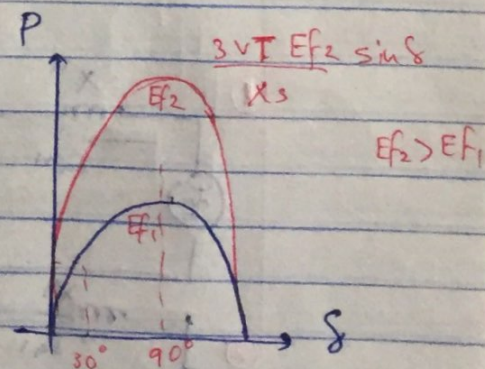
$$I_a X_s \cos \theta = E_f \sin \delta$$

$$P = \frac{3 V_T E_f \sin \delta}{X_s}$$

$$Q = 3 V_T I_a \sin \theta$$

$$I_a X_s \sin \theta = E_f \cos \delta - V_T$$

$$I_a \sin \theta = \frac{E_f \cos \delta - V_T}{X_s}$$



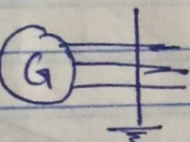
$$Q = \frac{3V_T(E_f \cos \delta - V_T)}{X_s}$$

δ small so $\rightarrow Q \approx \frac{3V_T(E_f - V_T)}{X_s}$

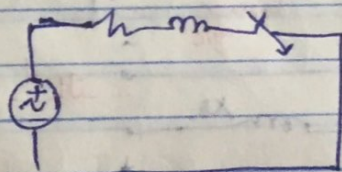
$$P = \frac{3V_T E_f}{X_s} \sin \delta$$

$$P = \frac{3V_T E_f \sin \delta}{X_s}$$

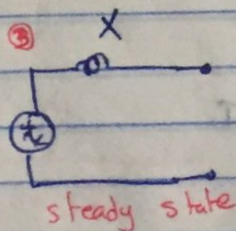
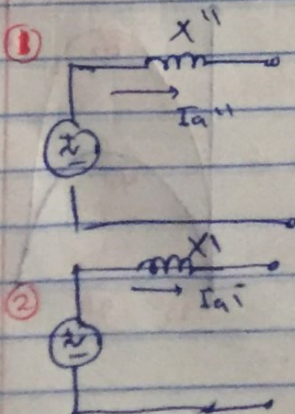
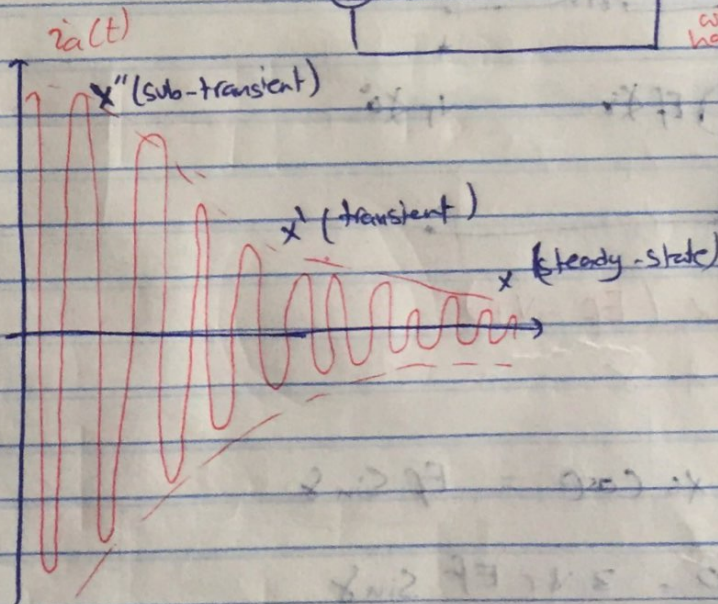
$$Q = \frac{3V_T(E_f \cos \delta - V_T)}{X_s}$$



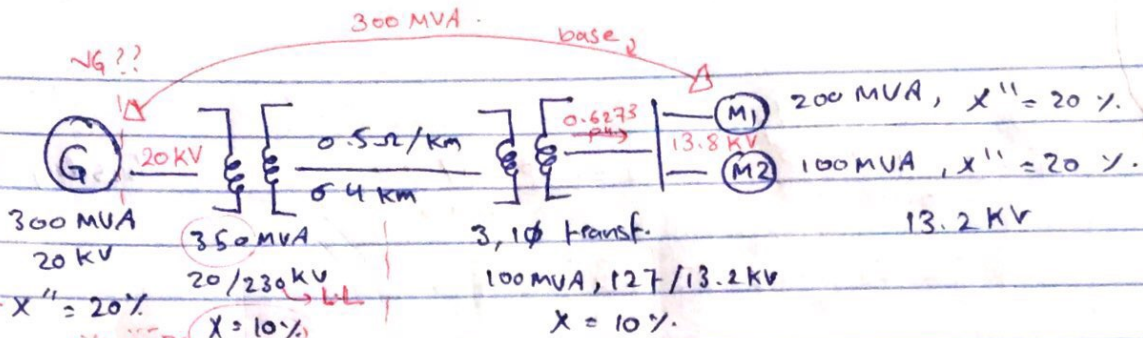
3 ϕ fault.



if we closed the switch on fault, what happens??



Example:



$X'' = 20\%$
 800 MVA system
 350 MVA base
 300 system

base 300 MVA

13.8 kV at motor side.

find: ① Draw pu reactance.

② Motor inputs are: $M_1 \rightarrow 120 \text{ MW}$
 at (13.2 kV/unity PF) $M_2 \rightarrow 60 \text{ MW}$

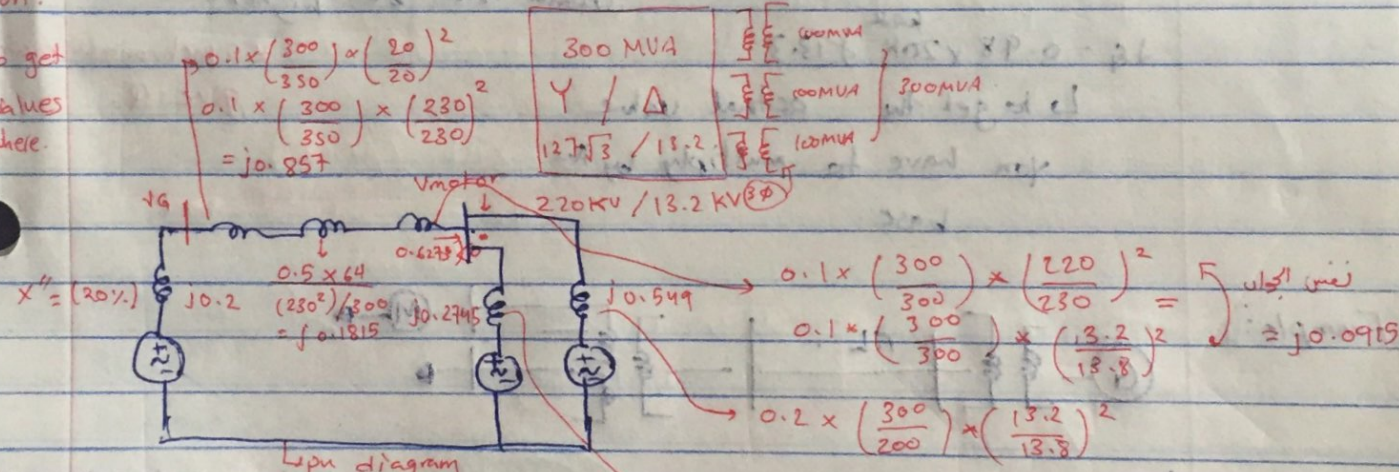
find voltage @ generator bus??

$$\frac{13.8 \times 220}{15.2} = 230 \text{ kV}$$

L-L-V transf. ratio

Solution:

red → to get base values everywhere.



$$\frac{13.2}{13.8} = 0.96 \text{ pu} = V_{\text{motor}}$$

$$|S_{3\phi}| = \sqrt{3} V_L I_L$$

$$|S_{3\phi \text{ pu}}| = \frac{\sqrt{3} V_L I_L}{\sqrt{3} V_{L \text{ base}} I_{L \text{ base}}}$$

$$|S_{3\phi \text{ pu}}| = V_L \text{ pu} \cdot I_p \text{ pu}$$

$$S_p \text{ pu} = V_p \text{ pu} \cdot I_p \text{ pu}$$

$$\frac{120}{300} \text{ at unity PF} = 0.96 I_p \text{ pu}$$

$$I_1 \text{ pu} = \dots$$

leading $\cos^{-1} \text{ pf}$
 lagging $\cos^{-1} \text{ pf}$

* $V_{\text{motor}} = \sqrt{j0.549}$
 only if the current was no load.

Tuesday 9/10/2018

$$I_1 = \frac{(120/1)}{0.96} \rightarrow I_2 = \frac{(60/1)}{0.96}$$

$$I = I_1 + I_2 = 0.6273 \text{ kA}$$

$$V_G = 0.96 \angle 0^\circ + 0.6273 \angle 0^\circ (j0.0857 + j0.1815 + j0.0915)$$

$V_G = 0.98 \angle 13.3^\circ$ pu. $> V_{\text{motor}}$ so that Q can flow which makes sense.

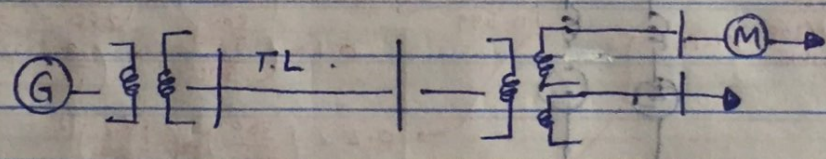
V_G gives the power & the angle should also be higher.

$$V_G = 0.98 \times 20 \text{ kV} \angle 13.3^\circ$$

* synchronous motors give Q.

↳ to get the actual value you have to multiply by the base.

Example:



- V_{motor} is given,
- motor load ($P_{\text{motor}}, P.F.$)
- load given ($\frac{P}{PFL}$) constant impedance

load 100 MW, 0.8 PF lag $\rightarrow Z(+)$ inductive

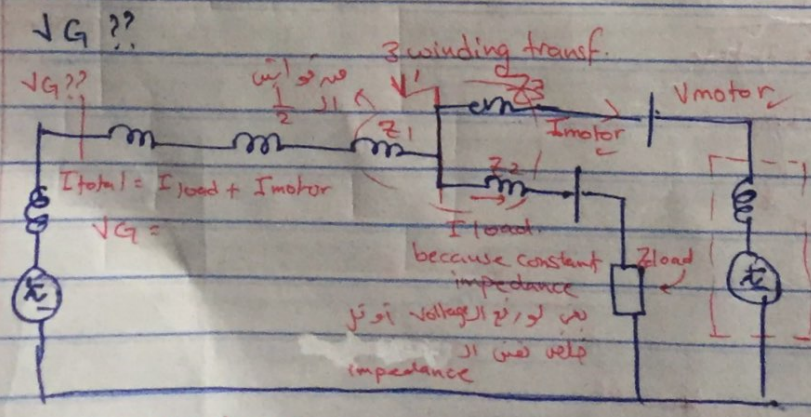
base 300 MVA, 13.8 kV
 Z_L pu ??

$$Z_L \text{ pu} = \frac{(13.8)^2}{(100/0.8)}$$

$$= \frac{(13.8)^2}{(300)}$$

$$= \dots \angle \cos^{-1}(PF)$$

Solution:



reactance diagram.

You have voltage & power
 so find $I_{\text{motor pu}}$.

$$* |S_{\text{pu}}| = |V_{\text{pu}}| |I_{\text{pu}}| \rightarrow I_{\text{pu}} = \frac{|S_{\text{pu}}|}{|V_{\text{pu}}|} \angle -\cos^{-1}(\text{PF})$$

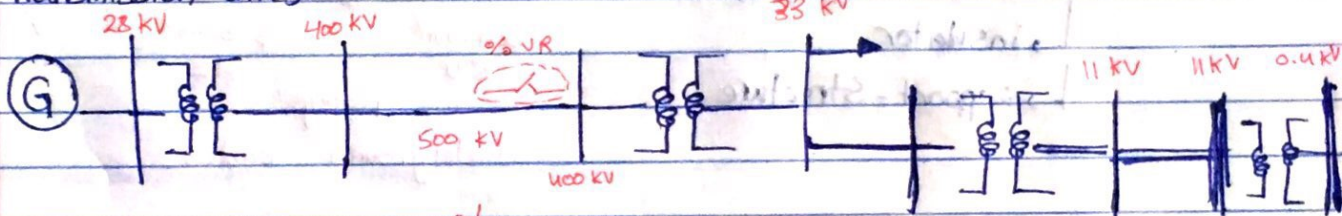
Motor

$$* V' = V_{\text{Motor}} + I_{\text{Motor}} \times Z$$

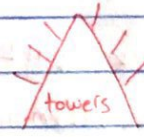
$$I_{\text{load}} = \frac{V'}{Z_{\text{t}} + Z_{\text{load}}}$$

Thursday 11/10/2018

* Transmission Lines:-



- * OHL
- * cables (underground)



* Modeling for Transmission Lines:

- Short length < 80 km
 - medium $80 \text{ km} < \text{Length} < 240$ km
 - long ≥ 240 km
- Approximate model.

"full model"

- * Performance under steady state analysis.
- * voltage drop
- * power losses.
- * voltage regulation.

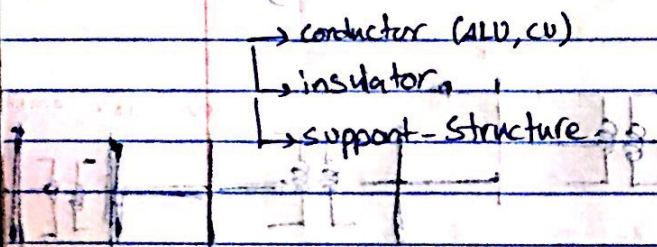
- * voltage regulation
 - no-load voltage rise
 - full-load voltage drop.

* Line Tractability:

- ↳ thermal limit.
- ↳ voltage drop limit.
- ↳ stability limit.

* reactive power compensation.

↳ Line compensation techniques.



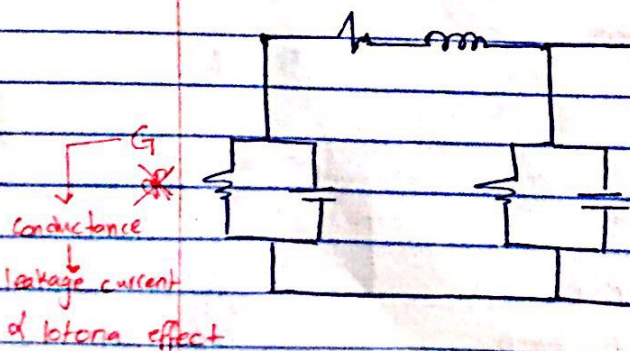
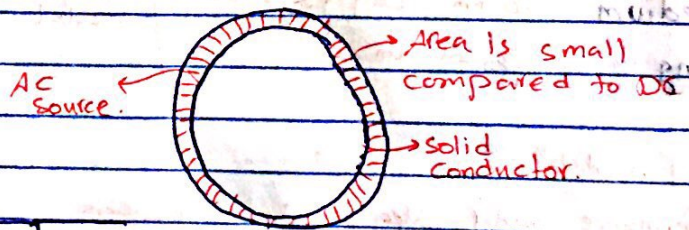
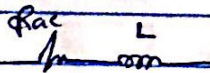
33 kV Δ 33 kV
 24 mm² 500 mm²
 CU "Ratings" AL
 underground cables.

* OHL (ACSR)

ALuminium conductor steel reflected.

* Transmission lines:

- ↳ electrical design
- ↳ mathematical design



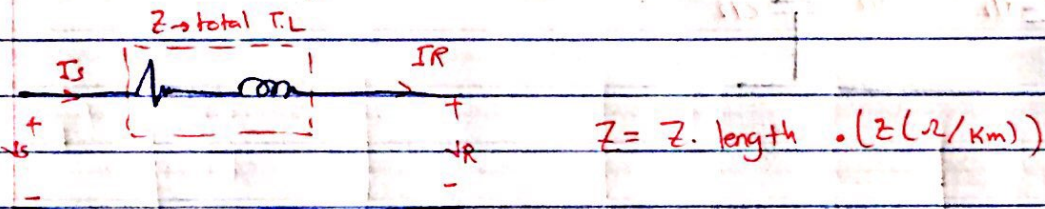
data sheet voltage: تصنيف الجهد الكهربائي

33 KV, U.G cables

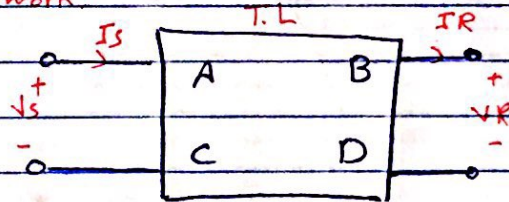
cross section

	R _{dc}	L	C	(A) Continuous ratings:
150 mm ²				300 A
24 mm ²				

* Short T.L (length ≤ 80 km)



2 port-network:



$$V_s = AV_R + BI_R$$

$$I_s = CI_R + DI_R$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$A = \frac{V_s}{V_R} \Big|_{I_R=0 \text{ (No-load)}}$$

$$B = \frac{V_s}{I_R} \Big|_{V_R=0}$$

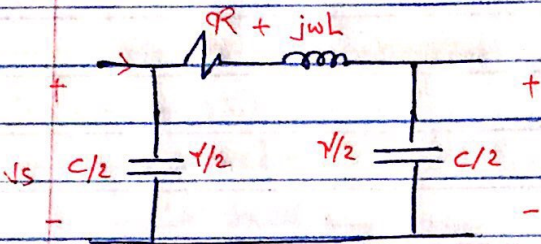
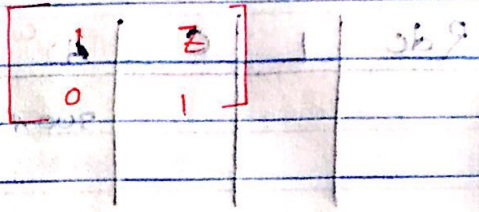
$C \sim S$ (Siemens)

$D \sim$ Dimensionless.

* $AD - BC = 1$

$V_s = V_R + I_R(Z)$
large

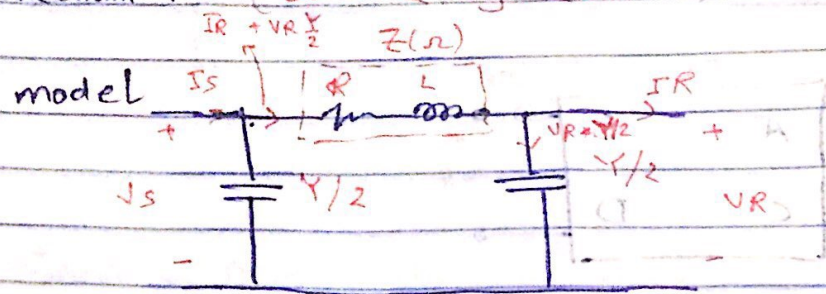
$I_s = I_R$



Find A, B, C, D ??

* medium T.L (80 km ≤ length ≤ 240 km)

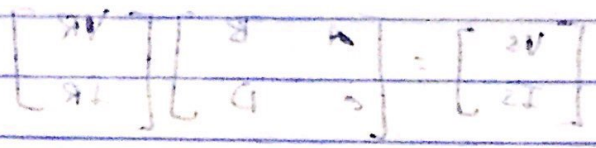
Sunday 14/10/2018



currents which go through $Y/2$ (capacitor) are called charging currents.

$y = j\omega C \rightarrow \mu F/\text{unit length}$

$Y = j\omega C * \text{length}$



ending $\leftarrow V_s = A V_R + B I_R$

$I_s = C V_R + D I_R$

to find A, B we use V_s' :

$V_s = V_R + (I_R + V_R \frac{Y}{2}) Z$

$V_s = (1 + \frac{Y Z}{2}) V_R + Z I_R$

* $A = 1 + \frac{Y Z}{2}, B = Z$

$A = \frac{V_s}{V_R} \Big|_{I_R=0} = 1 + \frac{Y Z}{2}$
 $B = \frac{V_s}{I_R} \Big|_{V_R=0} = Z$

(unitless) $Z \rightarrow \Omega$

to find C, D we use Is:

$$I_s = I_R + V_R \frac{Y}{2} + V_s \frac{Y}{2}$$

subs the Vs you found earlier in this equation:

$$I_s = Y \left(1 + \frac{YZ}{4} \right) V_R + \left(1 + \frac{YZ}{2} \right) I_R$$

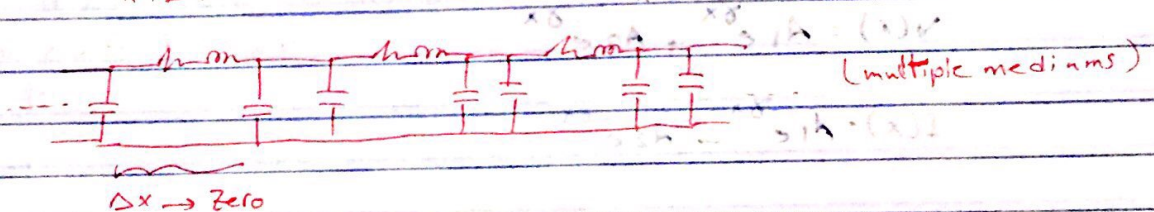
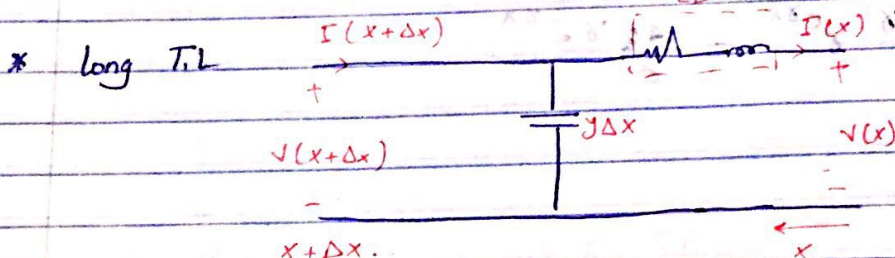
$$\therefore A = D = 1 + \frac{YZ}{2} \quad (\text{unitless})$$

$$B = Z(\Omega)$$

$$C = Y \left(1 + \frac{YZ}{4} \right) \quad (\text{siemens})$$

same conclusion in short T.L.

$$AD - BC = 1$$



long T.L is the full model. (Medium/short are approximations).

$$V(x+\Delta x) = V(x) + Z \Delta x I(x)$$

$$V(x+\Delta x) - V(x) = Z \Delta x I(x)$$

$$\Delta x \rightarrow$$

$$\frac{dV(x)}{dx} = Z I(x) \quad \text{1st order DE.}$$

$$I(x+\Delta x) = I(x) - Y \Delta x V(x+\Delta x)$$

$$I(x+\Delta x) - I(x) = -Y \Delta x V(x+\Delta x)$$

$$\Delta x \rightarrow$$

$$\Delta x \rightarrow 0$$

$$\frac{dI(x)}{dx} = -Y V(x)$$

$$\frac{d^2 V(x)}{dx^2} = Z_y V(x)$$

$$\frac{d^2 I(x)}{dx^2} = Z_y I(x)$$

$\gamma \frac{dV}{dx} = Z I(x)$

γ^2
 γ : propagation constant

$$\gamma = \alpha + j\beta$$

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

$$I(x) = \frac{1}{Z} \frac{dV(x)}{dx} = \frac{A_1 \gamma}{Z} e^{\gamma x} - \frac{A_2 \gamma}{Z} e^{-\gamma x}$$

$$I(x) = \frac{A_1 \gamma}{Z} e^{\gamma x} - \frac{A_2 \gamma}{Z} e^{-\gamma x}$$

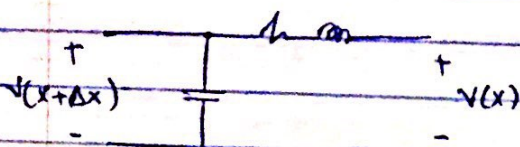
$$\gamma^2 = Z_y \rightarrow \gamma = \sqrt{Z_y}$$

$$\frac{\gamma}{Z} = \frac{\sqrt{Z_y}}{Z} = \sqrt{\frac{Y}{Z}}$$

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

$\sqrt{\frac{Z}{Y}} \rightarrow Z_c \triangleq$ c/s impedance of a line.

$$I(x) = A_1 e^{\gamma x} - A_2 e^{-\gamma x}$$



$$x = Z_c t_0, \quad V(x = Z_c t_0) = V_R$$

$$I(x = Z_c t_0) = I_R$$

$$x = Z_c t_0$$

$$V_R = A_1 + A_2$$

$$I_R = \frac{A_1 - A_2}{Z_c}$$

$$A_1 = (V_R + Z_c I_R) / 2$$

$$A_2 = (V_R - Z_c I_R) / 2$$

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x}$$

$$V(x) = V_R \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2} \right) + I_R \left(Z_c \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2} \right) \right)$$

$$V(x) = V_R \cosh \gamma x + Z_c \sinh \gamma x I_R$$

$$I(x) = \frac{V_R \sinh \gamma x}{Z_c} + \cosh \gamma x I_R$$

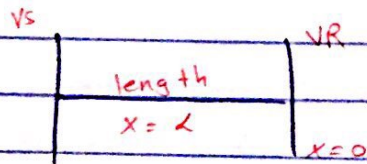
$$\gamma = \sqrt{ZY} = \alpha + j\beta$$

$$Z_c = \sqrt{\frac{Z}{Y}}$$

• $A = \cosh \gamma L$ → length of the line.

where did L come from?

$$V(x) = \cosh \gamma x V_R + Z_c \sinh \gamma x I_R$$



$$V_S = \cosh \gamma L V_R + Z_c \sinh \gamma L I_R$$

$$I_S = \frac{\sinh \gamma L V_R}{Z_c} + \cosh \gamma L I_R$$

important conclusions.

$$\gamma = \alpha + j\beta$$

$$\cosh \gamma L = \frac{1}{2} (e^{\gamma L} + e^{-\gamma L})$$

$$= \frac{1}{2} (e^{\alpha L} e^{j\beta L} + e^{-\alpha L} e^{-j\beta L})$$

$$\cosh \gamma L = \frac{1}{2} (e^{\alpha L} \angle \beta L + e^{-\alpha L} \angle -\beta L)$$

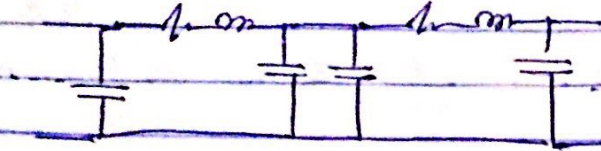
$$\sinh \gamma L = \frac{1}{2} (e^{\alpha L} \angle \beta L - e^{-\alpha L} \angle -\beta L)$$

formulas to find \cosh , \sinh because you can't find $\cosh(\text{complex})$, $\sinh(\text{complex})$ directly on the calculator.

* Π equivalent of a long T.L: (ABCD)

Thursday 18/10/2018

Mod-L of long:

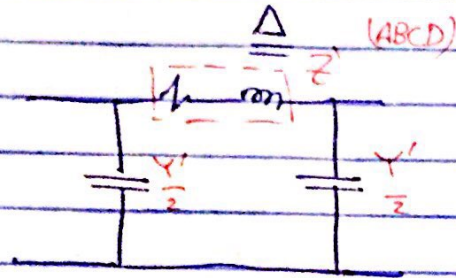


$$D = A = \cosh \gamma l$$

$$B = Z_c \sinh \gamma l$$

$$C = \frac{\sinh \gamma l}{Z_c}$$

Π model of long:



Z' ?

$\frac{Y'}{2}$?

$$D = A = 1 + \frac{Y' Z'}{2}$$

$$B = Z'$$

$$C = 1 + \frac{Y' Z'}{4}$$

$$Z' = Z_c \sinh \gamma l$$

multiply by:

$$Z' = \sqrt{\frac{Z}{Y}} \sinh \gamma l \cdot \begin{bmatrix} Z l \\ Z c \end{bmatrix}$$

$$= Z l \cdot \sqrt{\frac{Z}{Z^2 l^2 Y}} \sinh \gamma l$$

$$Z' = Z l \cdot \sqrt{\frac{1}{Z Y l^2}} \sinh \gamma l$$

$$Z' = Z l \cdot \frac{\sinh \gamma l}{\gamma l} = Z \cdot \frac{\sinh \gamma l}{\gamma l} \quad * \text{ memorize.}$$

factor ≈ 1 b. β
2nd order approx

Approximation:
 $Z' = Z$ Medium?

App. of 1st order dev of 1st order b. β
Valid.

$\sqrt{\frac{Z'}{Y}}$

$$Z' = Z_c \sinh \gamma l = \underbrace{Z_c}_{\text{charact.}} \cdot \underbrace{\frac{\sinh \gamma l}{\gamma l}}_{\text{length.}} \cdot \gamma l$$

$\rightarrow Z l$

$$\cosh \gamma l = 1 + \frac{Y' Z'}{2}$$

$$\frac{Y'}{2} = \frac{\cosh \gamma l - 1}{Z'}$$

$$\frac{Y'}{2} = \frac{\cosh \gamma l - 1}{Z' \sinh \gamma l} \quad \tanh \gamma l / 2$$

$$\frac{Y'}{2} = \frac{\tanh \gamma l / 2}{Z' \sqrt{\frac{Z}{Y}}} \cdot \frac{Y l / 2}{Y l / 2}$$

$$\frac{Y'}{2} = \frac{Y l}{2} \cdot \frac{\tanh \gamma l / 2}{\sqrt{Z Y l^2 / 4}}$$

$$\frac{Y'}{2} = \frac{Y l}{2} \tanh \gamma l / 2$$

$$\frac{Y'}{2} = \frac{Y}{2} \cdot \frac{\tanh \gamma l / 2}{\gamma l / 2} \quad \text{factor} \quad * \text{ memorize}$$

* lossless Transmission Lines (T.L): $R = \text{Zero}$

hand calculations (initial design):

$$Z = r + j\omega L = j\omega L$$

$$Y = j\omega C$$

$$\gamma = \sqrt{ZY} = j\omega \sqrt{LC} = \alpha + j\beta \quad \text{Zero } \alpha$$

$$\beta = \omega \sqrt{LC} \quad \text{rad/sec}$$

$$V(x) = A V_R + B I_R$$

* ABCD constant.

$$A = \cosh \gamma l$$

$$= \frac{1}{2} (e^{\alpha l} \angle \beta l + e^{-\alpha l} \angle -\beta l)$$

$\alpha = \text{Zero}$ [lossless]

$$A = \frac{1}{2} (e^{j\beta l} + e^{-j\beta l})$$

$$A = \cos(\beta l) \quad \text{at no load } V_R > V_S \quad A < 1$$

$$V_S = AV_R + BIR$$

$$V_R = \frac{V_S}{A} = \frac{V_S}{\cos(\beta l)}$$

B constant:

$$B = Z_c \sinh \gamma l$$

$$= Z_c \left[\frac{1}{2} \left\{ e^{\gamma l} - e^{-\gamma l} \right\} \right]$$

$$B = Z_c \left[\frac{1}{2} (1 - 1) \right]$$

$$= Z_c - j \sin \beta l$$

$$= \sqrt{\frac{Z}{Y}} j \sin \beta l$$

$$= \sqrt{\frac{j\omega L}{j\omega C}} j \sin \beta l$$

$$B = j \sqrt{\frac{L}{C}} \sin \beta l$$

* wave length (λ) meter

is the distance required to change the phase of V & I by 360° .

λ ??

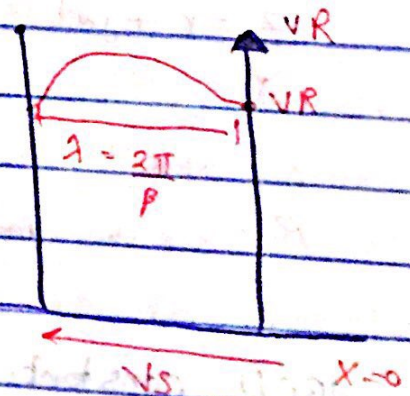
$$V = AV_R + BIR$$

$$V = \cos \beta l V_R + j Z_c \sin \beta l IR$$

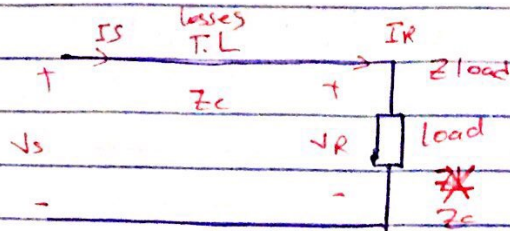
$$\beta l = 2\pi$$

$$l = \frac{2\pi}{\beta}$$

$$\lambda = l = \frac{2\pi}{\beta}$$



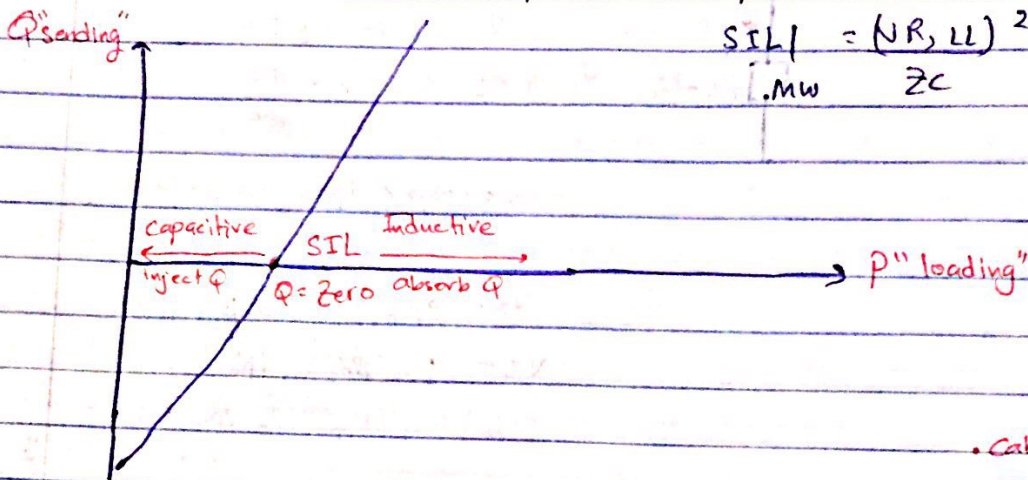
* Surge Impedance loading (SIL) MW.



when $Z_{load} = Z_c = \sqrt{\frac{L}{C}}$ $L \rightarrow \text{red.}$

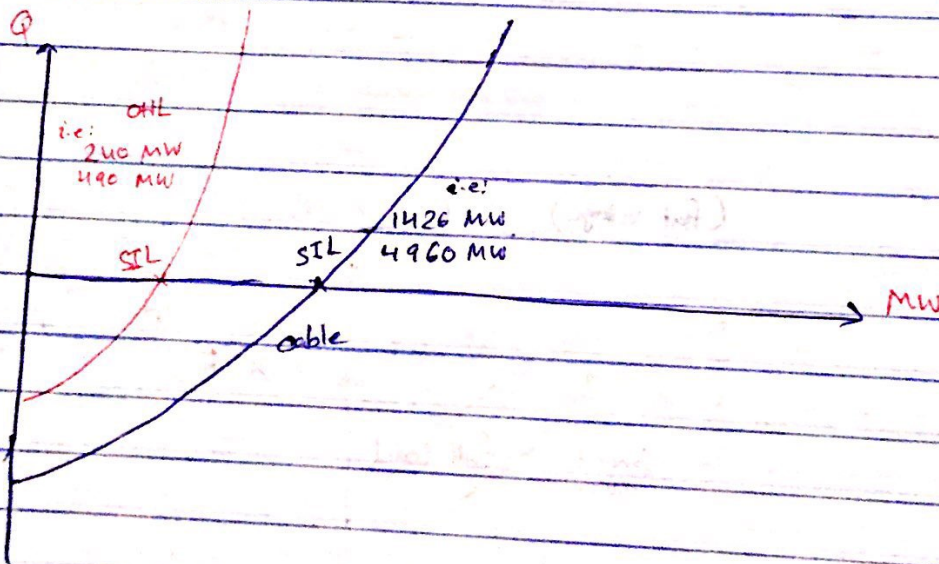
SIL loading $\text{MW} = \frac{3 \times (NR, LN)^2}{Z_c}$ *line-neutral*

SIL $\text{MW} = \frac{(NR, LL)^2}{Z_c}$ \otimes



Q
indec.

• cable, $C \gg L$
 $Z_c \text{ min for cable.}$



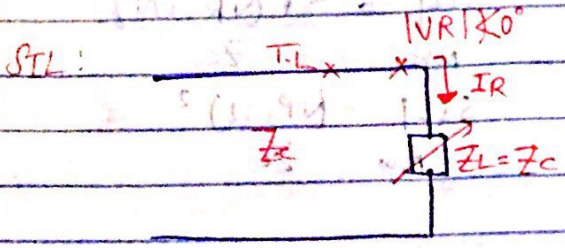
Sunday 21/10/2018

Lossless T.L:

$A = \cos \beta l$

$B = j Z_c \sin \beta l$

$Z_c = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$



$V_S = AV_R + BI_R$

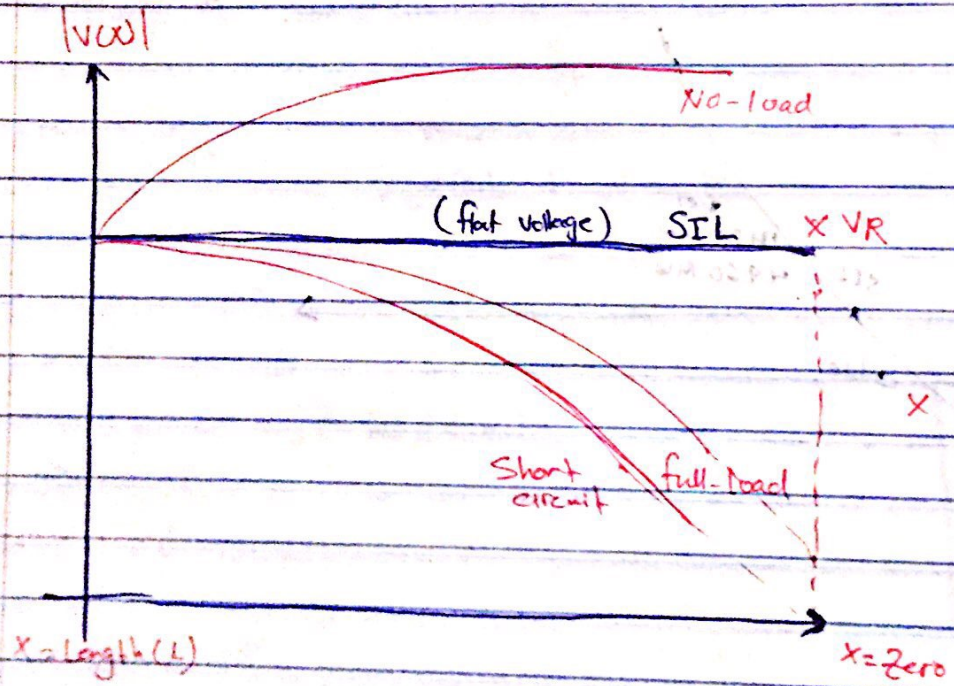
@ SIL $\rightarrow I_R = \frac{V_R}{Z_c}$

$V_S = \cos \beta l V_R + j Z_c \sin \beta l \frac{V_R}{Z_c}$

$V_S = (\cos \beta l + j \sin \beta l) V_R$

$V_S = 1 \angle \beta l V_R$

$|V_S| = |V_R|$ constant voltage but varying phase.



$f = 3 \times 10^8$

$\lambda = \frac{3 \times 10^8}{3 \times 10^8} = 1 \text{ m}$

Length = λ

$$V_s = A V_R + B I_R$$

$$I_s = C V_R + D I_R$$

at no load $\Rightarrow I_R = \text{zero} \Rightarrow I_s \neq \text{zero}$

at short ckt $\Rightarrow V_R = \text{zero}$

* complex power flow through T.L :

objective $\Rightarrow P_R$
 $Q_R = f(A, B, C, D)$

$$S_R = V_R I_R^*$$

(S_R can be 1ϕ or 3ϕ)

• if for 3ϕ then multiply by 3

$$V_s = A V_R + B I_R$$

$$I_R = \frac{V_s - A V_R}{B}$$

$$A = |A| \angle \theta_A$$

$$B = |B| \angle \theta_B$$

$$V_s = |V_s| \angle \delta^\circ$$

$$V_R = |V_R| \angle 0^\circ$$

$$I_R = \frac{V_s}{|B|} \angle \delta - \theta_B - \frac{|V_R| |A|}{|B|} \angle \theta_A - \theta_B$$

$$\therefore S_R = \frac{|V_R| |V_s|}{|B|} \angle \theta_B - \delta - |A| |V_R|^2 \angle \theta_B - \theta_A$$

$V_R, V_s \in \text{LN}$ Line-Neutral

$$S_{R,3\phi} = 3 S_{R,1\phi} = 3 * \frac{|V_{R,LL}|}{\sqrt{3}} \cdot \frac{|V_{s,LL}|}{\sqrt{3}} \angle \theta_B - \delta - 3 |A| \left(\frac{|V_{R,LL}|}{\sqrt{3}} \right)^2 \angle \theta_B - \theta_A$$

LL 3ϕ

$$S_{R,3\phi} = \frac{|V_R| |V_s|}{|B|} \angle \theta_B - \delta - |A| |V_R|^2 \angle \theta_B - \theta_A$$

$$P_R = \frac{|V_R| |V_S|}{|B|} \cos(\theta_B - \delta) - \frac{|A| |V_R|^2}{|B|} \cos(\theta_B - \theta_A)$$

$$Q_R = \frac{|V_R| |V_S|}{|B|} \sin(\theta_B - \delta) - \frac{|A| |V_R|^2}{|B|} \sin(\theta_B - \theta_A)$$

* Lossless short T.L:



$$\begin{aligned} V_S &= V_R + I_R jX \\ I_S &= I_R \end{aligned} \quad \left| \begin{aligned} A &= 1 \\ B &= jX \end{aligned} \right.$$

$$P_R = \frac{|V_R| |V_S|}{X} \cos(90^\circ - \delta) - \frac{|V_R|^2}{X} \cos(90^\circ)$$

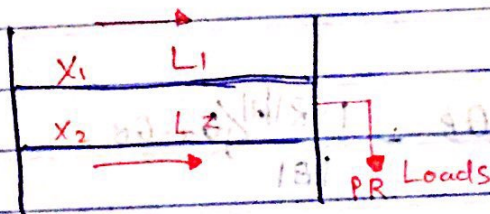
$$P_R = \frac{|V_R| |V_S|}{X} \sin \delta$$

power angle

1] P \propto δ

2] P \propto V²

3] P \propto $\frac{1}{X}$



• Lossless short T.L

$$Q_R = \frac{|V_R| |V_S|}{|B| X} \sin(90^\circ - \delta) - \frac{|V_R|^2}{X} \sin(90^\circ)$$

$$Q_R = \frac{|V_R| |V_S|}{|X|} \cos \delta - \frac{|V_R|^2}{X}$$

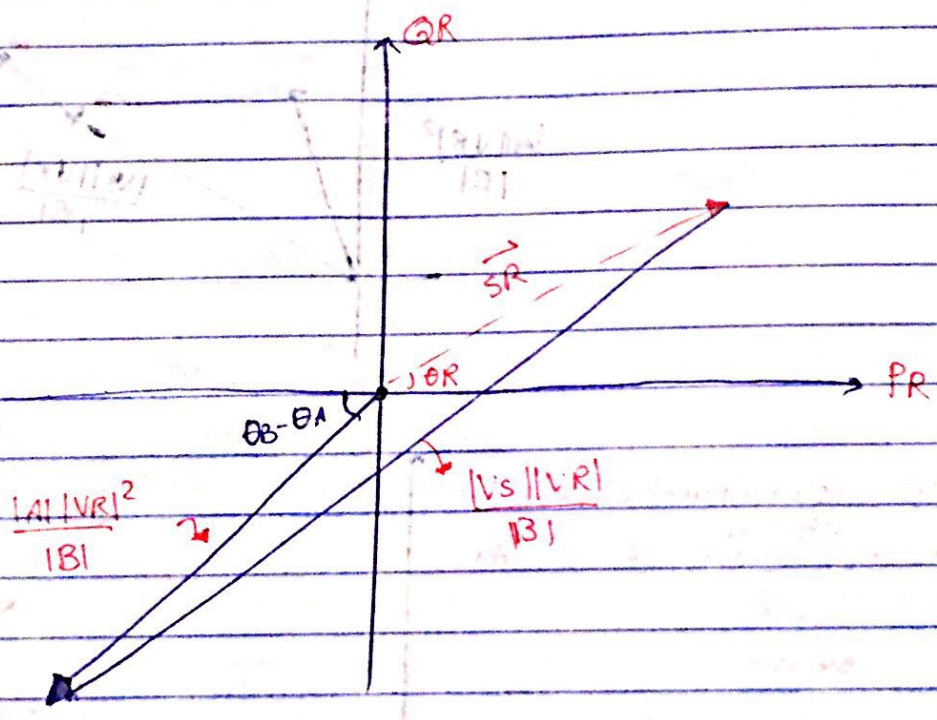
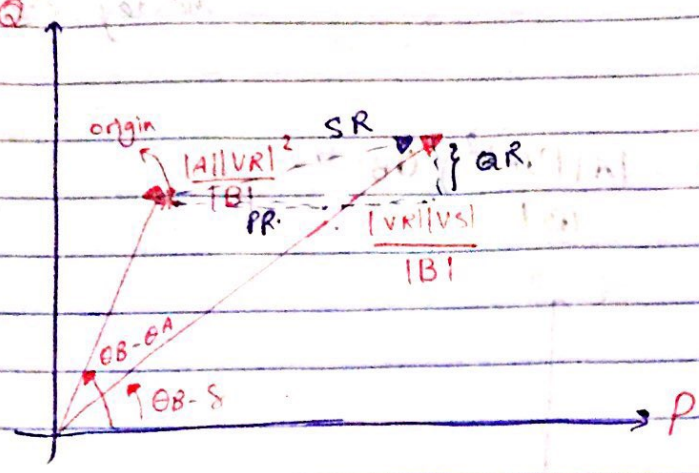
if δ is small:

$$Q_R \approx \frac{|V_R|}{X} (|V_S| - |V_R|)$$

$\therefore Q \propto |V|$

- $\theta_A \approx 0^\circ$
- $\theta_B \approx 90^\circ$

* Receiving end circle diagram:



Tuesday 23/10/2018

* Circle diagram:

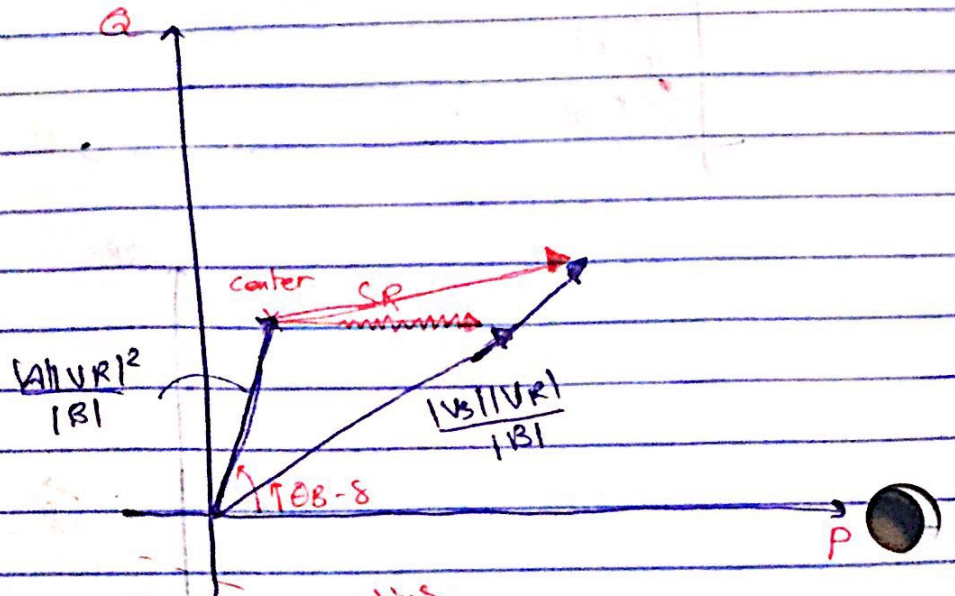
$$S_{R3\phi} = \frac{|V_s||V_R|}{|B|} \angle \theta_B - \delta - \frac{|A||V_R|^2}{|B|} \angle \theta_B - \theta_A$$

$$V_s = |V_s| \angle \delta$$

$$B = |B| \angle \theta_B$$

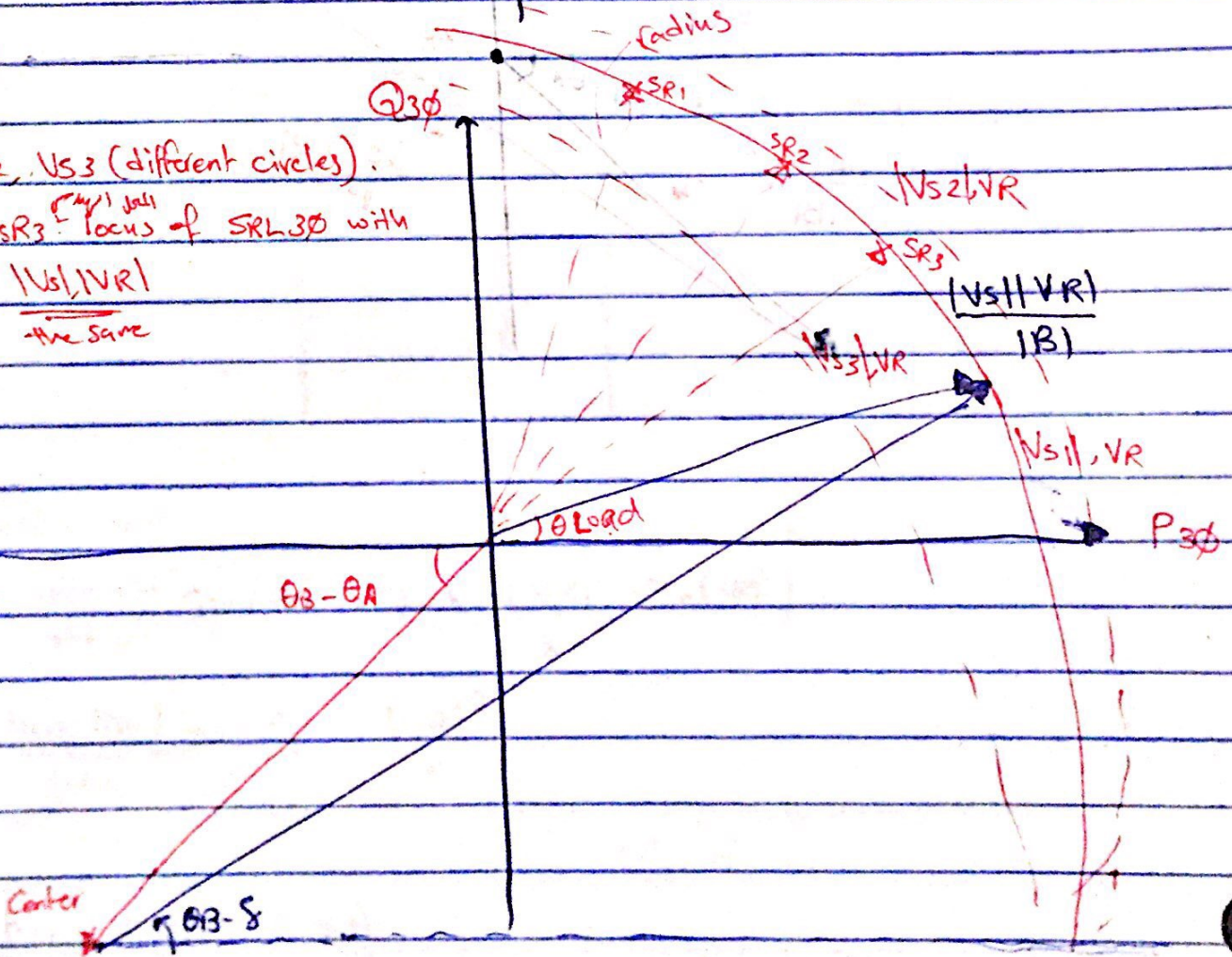
$$A = |A| \angle \theta_A$$

$$V_R = |V_R| \angle 0^\circ$$



V_{s1}, V_{s2}, V_{s3} (different circles).

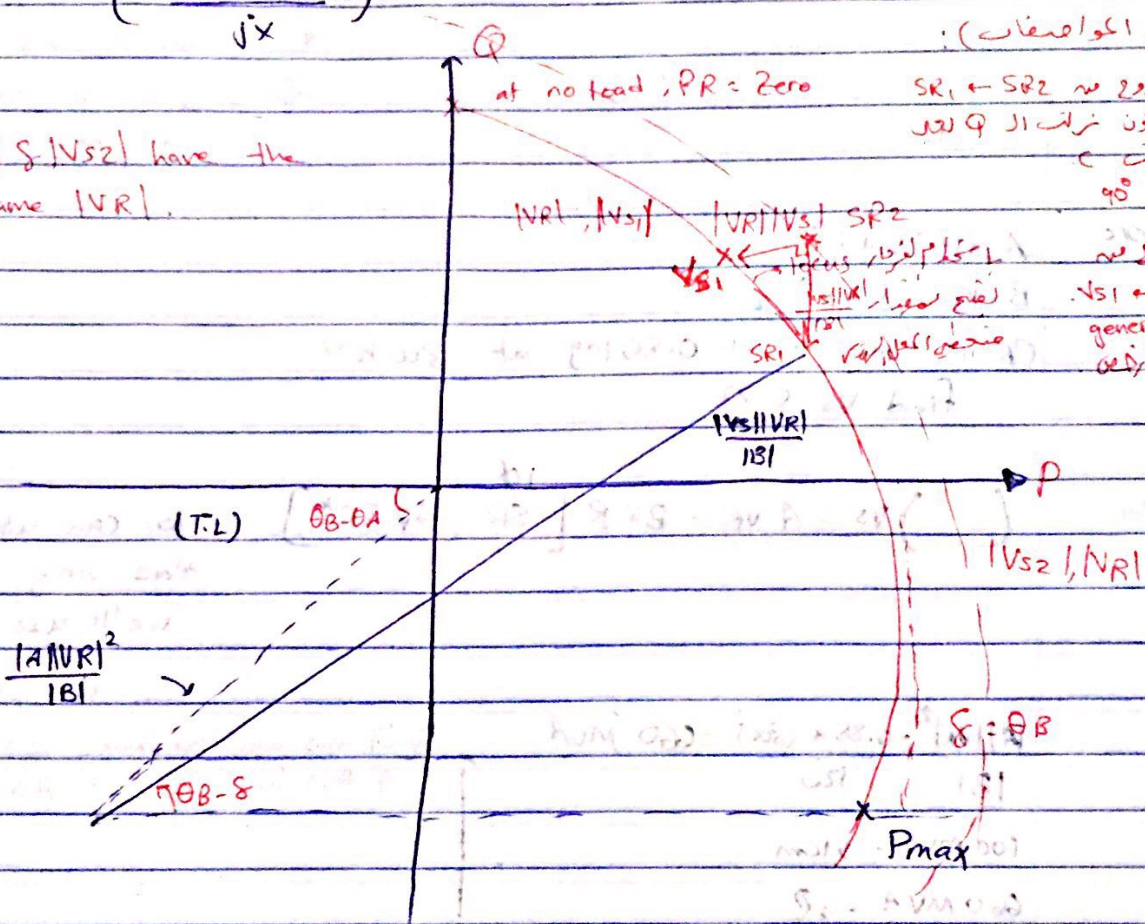
S_{R1}, S_{R2}, S_{R3} locus of $S_{R3\phi}$ with constant $|V_s|, |V_R|$ the same



Example: $|V_s| \ll \delta$? $V_R \ll 0$ Find SR ?? (Receiving power).

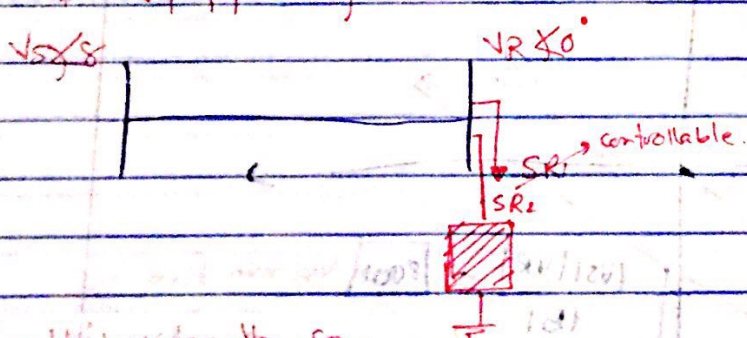
Solution: $SR = V_R I^*$
 $SR = V_R \left(\frac{|V_s| \angle \delta - V_R}{jX} \right)^*$

* $|V_s| \ll \delta$ & $|V_s|$ have the same $|V_R|$.



(المختبر الكواصفات):
 * $SR_1 \leftarrow SR_2$ مع δ زياد
 يكون زاوية ال Q زياد
 زاوية 90°
 مع δ زياد
 $V_s \leftarrow SR_2$
 generation
 ال Q زياد

* $\uparrow P \uparrow \text{cost} \uparrow$



I can't increase V_s so
 I control SR_1 (by adding C, Q)

Example: $V_s \angle \delta$? I_L $V_R \angle 0^\circ$ find SR, δ ?



reactor, ωL , C etc. $\cos \phi$ is $\cos \delta$
 in ω $\cos \phi$ no V_s $\cos \phi$ is $\cos \delta$
 Inductor $\cos \phi$ $\cos \delta$ is $\cos \delta$

* if the PF was less than limit $\cos \phi = 0.88$ then it absorbs more Q & the voltage will decrease.

Example: $A = 0.88 \angle 2^\circ$
 $B = 120 \angle 77^\circ \Omega$
 PR is 170 MW at 0.86 lag at 300 kV
 find V_s, δ ?

Solution: $V_s = A V_R + B I_R$ [$SR = V_R I_R^*$]

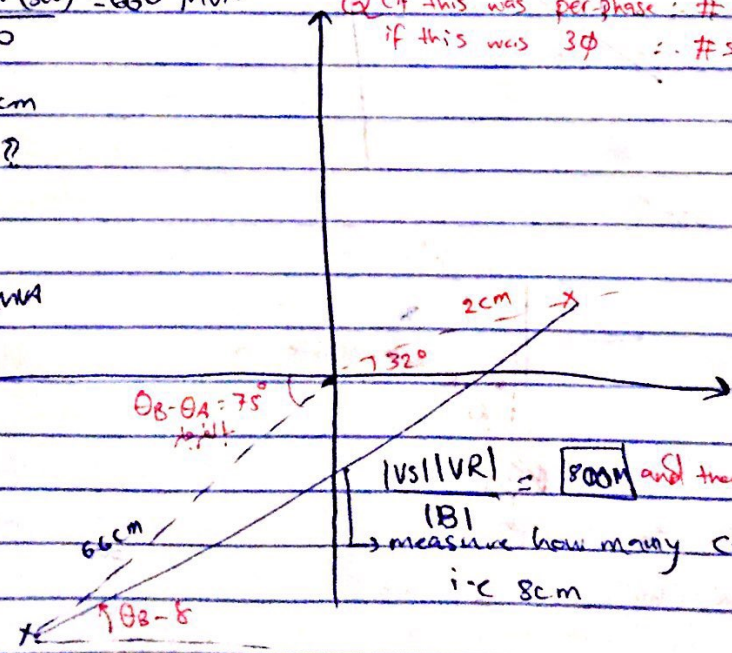
you can use this way but we'll use the circle diagram.

$|A| |V_R|^2 = 0.88 \times (300)^2 = 660 \text{ MVA}$
 $|B| = 120$
 $100 \text{ MVA} \rightarrow 1 \text{ cm}$
 $660 \text{ MVA} \rightarrow ?$
 $\times 6.6 \text{ cm}$

Q If this was per-phase: #s are L-N
 if this was 3 ϕ \therefore #s are L-L } Question

$\times SR = \frac{170}{0.86} = 200 \text{ MVA}$

$\times \cos^{-1}(0.86) = \theta_R = 32^\circ$

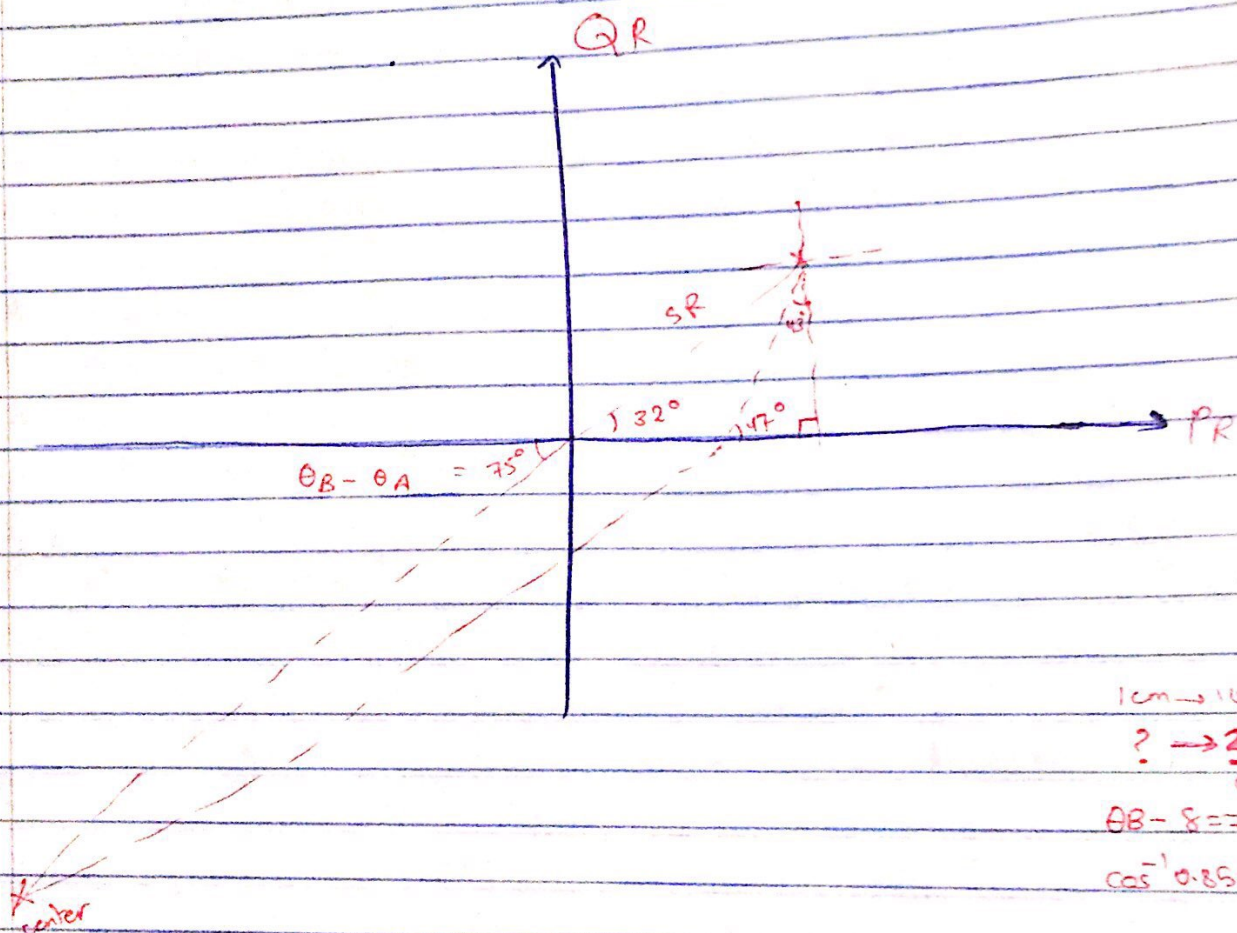


$|V_s| |V_R| = 800$ and then find V_s
 $|B|$ measure how many cms this is i.e. 8 cm

end of first exam material.

Example *: find V_S, V_R if power is increased to 255 MW at the same power factor, $\delta = 30^\circ$, $A = 0.88 \angle 2^\circ$, $B = 120 \angle 77^\circ$ (0.85 lag)

Solution:



$$1 \text{ cm} \rightarrow 100 \text{ MW}$$

$$? \rightarrow \frac{255}{0.85}$$

$$\theta_B - \delta = 77 - 30 = 47^\circ$$

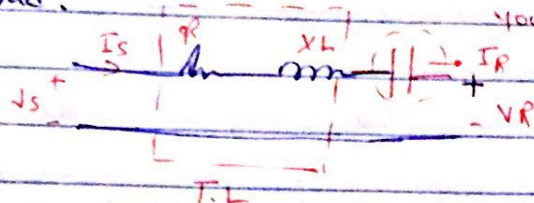
$$\cos^{-1} 0.85 = 32^\circ$$

* Reactive Power compensation:

• heavy load \rightarrow Voltage drop
Problem

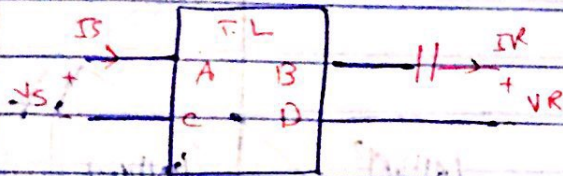
• Light Load \rightarrow Voltage rise
Problem

* heavy load:



you add the capacitor bank

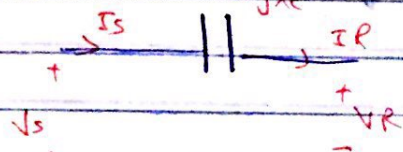
compensation factor = $\frac{|X_L|}{|X_C|}$



But find!

ABCD constant for series capacitor

A	B	1	$-jX_C$
C	D	0	1



$$V_S = V_R - jX_C I_R$$

$$I_S = I_R$$

A _{eq.}	B _{eq.}	A	$A(-jX_C) + B$
C _{eq.}	D _{eq.}	C	$C(-jX_C) + D$

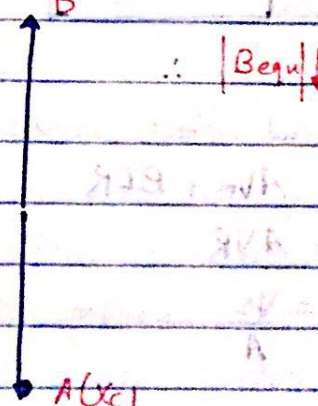
1	$-jX_C$
0	1

ABCD constant for series Capacitor.

equivalent matrix for T.L with series capacitor.

$$B_{eq.} = A(-jX_C) + B$$

\downarrow phase 30° \rightarrow phase $\approx 90^\circ$



$$V_s = AV_R + BIR$$

V_R fixed, I_R fixed

$$V_{s1} = AV_R + BIR \text{ without capacitor}$$

$$V_{s2} = AV_R + B_{eqn} I_R \text{ with capacitor}$$

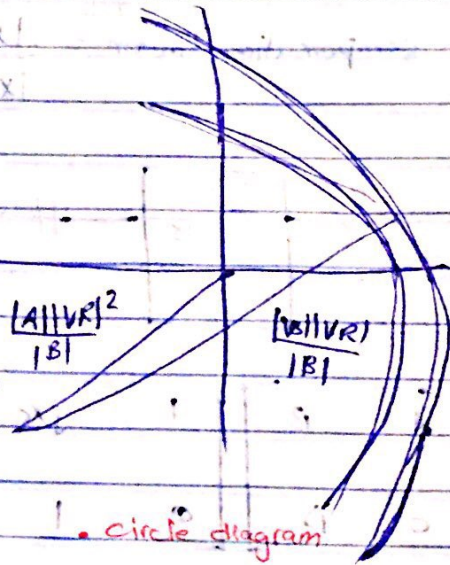
$$|V_{s1}| = |V_R|$$

$|V_{s2}| = |V_R| \rightarrow$ voltage drop \downarrow when you add series capacitor.

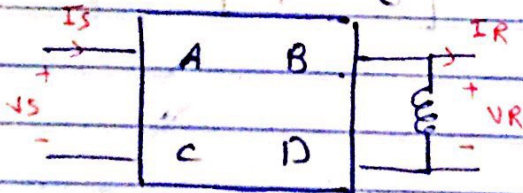
- maximum power transfer when $\phi = \theta_B$

$$P_{max} \approx \frac{|V_{s1}| |V_R| \sin \phi}{|B|}$$

$$|B_{eqn}| \Rightarrow P_{max} \uparrow$$



- * light load (shunt inductor):



No load ($I_R = \text{zero}$)

$$V_s = AV_R + BIR$$

$$V_s = AV_R$$

$$\frac{V_R}{N_L} = \frac{V_s}{A} \rightarrow \text{voltage rise}$$

objective is to enlarge A $[A \uparrow]$

* Load Flow / power flows

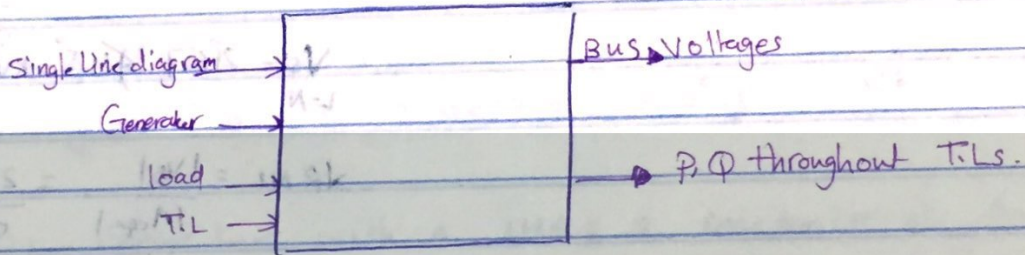
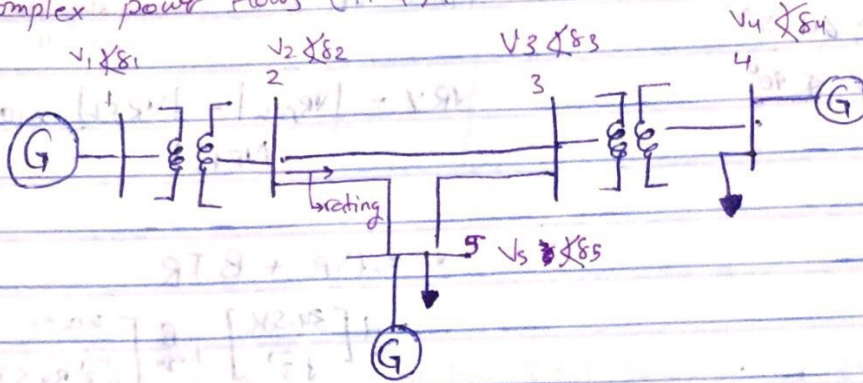
steady state operating point of an electric power system.

"steady-state analysis tool"

* voltage & flows must be acceptable all the time.

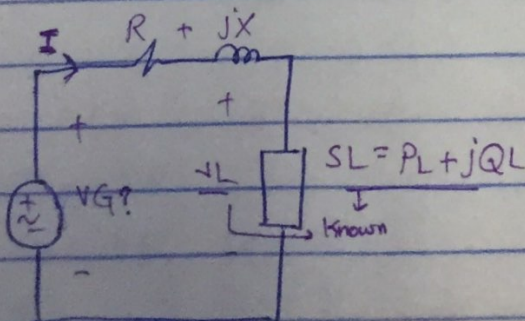
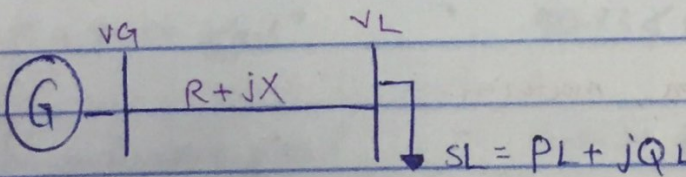
- Bus voltages

- complex power flows (P, Q)



* Power flow analysis needs non-linear analysis technique.

* loads are modeled as constant power.



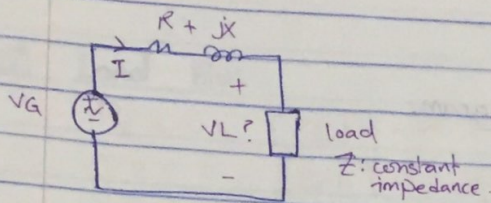
$$S = V I^*$$

$$P + jQ_L = V_L I^*$$

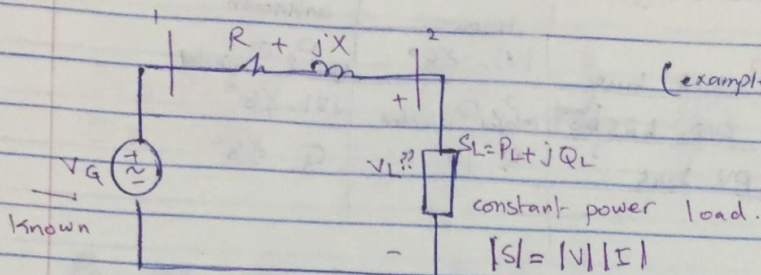
$$I = \frac{P_L - jQ_L}{V_L^*}$$

$$V_G = V_L + I(R + jX)$$

$$V_G = V_L + \frac{P_L - jQ_L}{V_L^*} (R + jX)$$



(examples in ch-2 & machines).



(examples in power)

$$S_L = V_L I^*$$

$$P_L + jQ_L = V_L I^*$$

$$I = \frac{P_L - jQ_L}{V_L^*}$$

$$V_L = V_G - I(R + jX)$$

$$V_L = V_G - \frac{(R + jX)(P_L - jQ_L)}{V_L^*} \quad [\text{numerical techniques}]$$

using iteration:

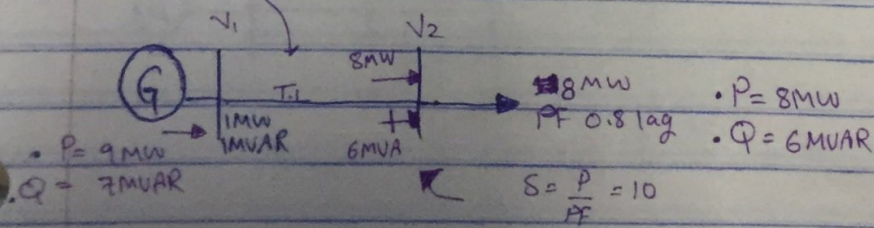
Iteration	k	$V_L(k)$
0		$1 \angle 0^\circ$ pu (initial case) $V_L^{(1)}$
1		$V_L^{(1)}$ $V_L^{(2)}$
2		$V_L^{(2)}$ $V_L^{(3)}$
3		$V_L^{(3)}$ $V_L^{(4)}$

Convergence

(تقریباً ہر iteration کے لیے)

losses (MW
MVAR)

▶ for power
+ ▶ for Q



KCL (balanced equation).

* Power flow problem:

* starting point \Rightarrow Single Line diagram.

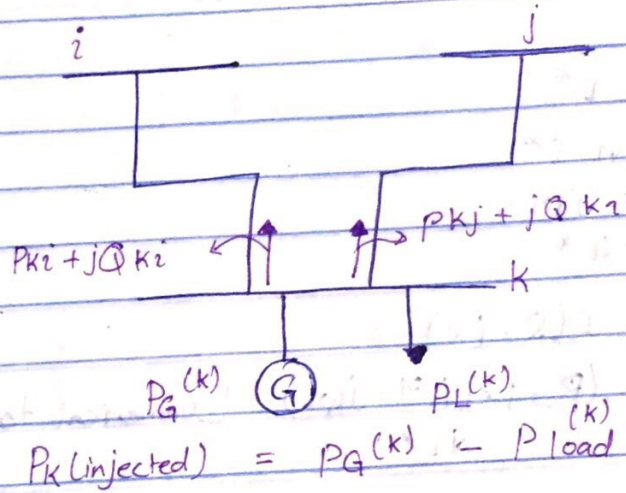
Data.

Bus types.

• Bus types:

$|V|$
 $\angle \delta^\circ$
 P
 Q

reference	known	unknown
slack bus	$ V , \angle \delta^\circ$	$P_{\text{injected}}, Q_{\text{injected}}$
PQ bus (load bus)	$P_{\text{injected}}, Q_{\text{injected}}$	$ V , \angle \delta^\circ$
PV bus	$P, V $	$Q, \angle \delta^\circ$



$$P_2 \triangleq P_2^{\text{injected}} = P_{2,G} - P_{2,\text{load}}$$

$$* P_k^{\text{injected}} = P_G^{(k)} - P_{\text{load}}^{(k)} = \sum_{i=1}^N P_{ki}$$

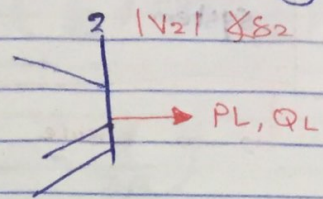
* inject $P, Q \rightarrow +ve$
output $P, Q \rightarrow -ve$

Sunday 4/11/2018

1 Load Bus

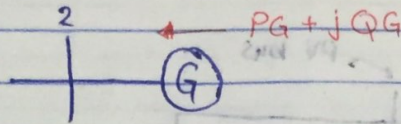
$$P_2 = 0 - [P_L] = -P_L$$

$$Q_2 = 0 - [Q_L] = -Q_L$$



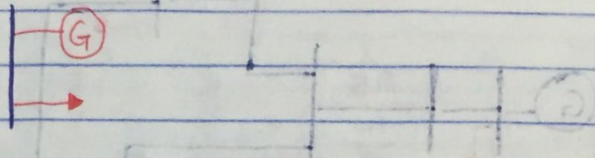
$$P_2 = P_{G2}$$

$$Q_2 = Q_{G2}$$



$$P_2 = P_G - P_L$$

$$Q_2 = Q_G - Q_L$$



2 PV Bus

$|V|$ Known, δ unknown

$$P_L + jQ_L$$

$$P_G + jQ_G$$

capacitor

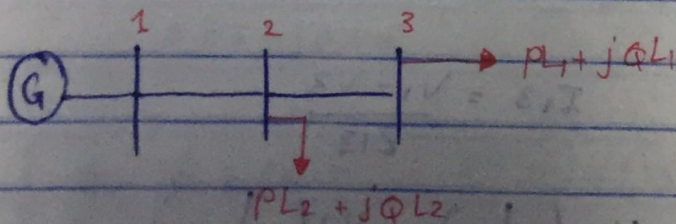
$$Q??$$

$$P_2 = P_G - P_L$$

$$|V|$$

$$Q? \quad \delta?$$

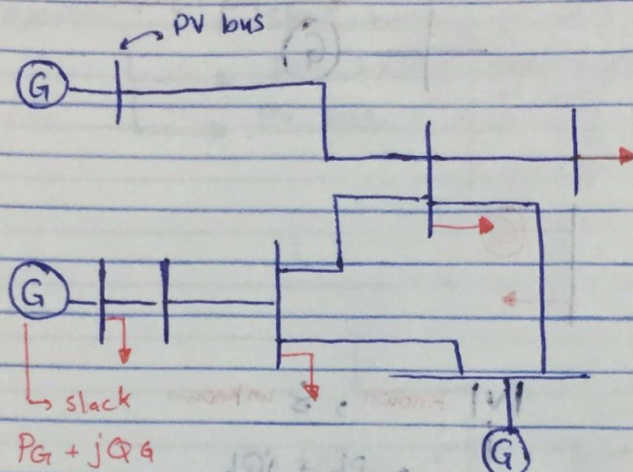
3 Slack Bus



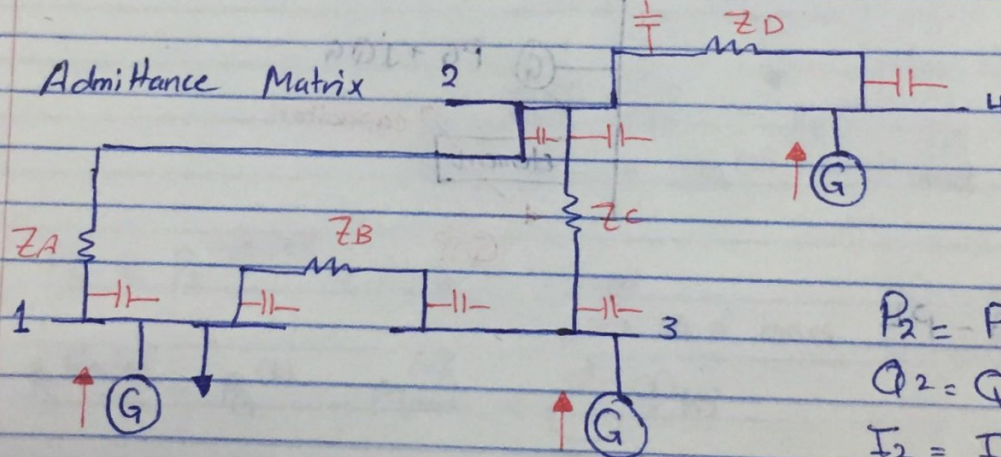
3105/11/10

$$P_{\text{Generation System}} = P_{\text{system Load}} + P_{\text{system losses}}$$

↳ to ensure balance for the system.



* Admittance Matrix



$$P_2 = P_2 \text{ injected}$$

$$Q_2 = Q_2 \text{ injected}$$

$$I_2 = I_2 \text{ injected}$$

KCL:

$$I_1 = I_{G1} - I_{L1} = I_{12} + I_{13}$$

$$I_1 = I_{12} + I_{13}$$

$$I_{12} = \frac{V_1 - V_2}{Z_A}$$

$$I_{13} = \frac{V_1 - V_3}{Z_{13}}$$

$$* I_1 = V_1 \left(\frac{1}{Z_A} + \frac{1}{Z_{13}} \right) - V_2 \left(\frac{1}{Z_A} \right) - V_3 \left(\frac{1}{Z_{13}} \right)$$

$$* I_2 = V_2 \left(\frac{1}{Z_A} + \frac{1}{Z_D} + \frac{1}{Z_C} \right) - V_1 \left(\frac{1}{Z_A} \right) - V_3 \left(\frac{1}{Z_C} \right) - V_4 \left(\frac{1}{Z_D} \right)$$

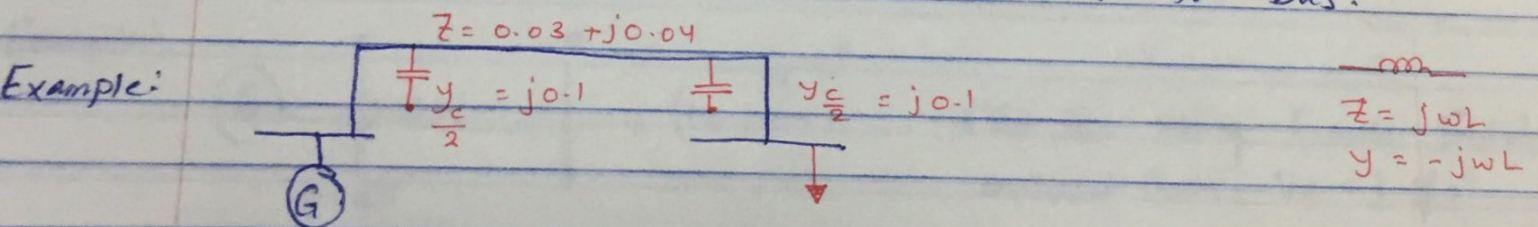
$$* I_3 = -V_1 \left(\frac{1}{Z_B} \right) - V_2 \left(\frac{1}{Z_C} \right) + V_3 \left(\frac{1}{Z_C} + \frac{1}{Z_B} \right)$$

$$* I_4 = -\frac{V_2}{Z_D} + \frac{V_4}{Z_D}$$

$$* \boxed{I = YV}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{Z_A} + \frac{1}{Z_B} \right) & -\frac{1}{Z_A} & -\frac{1}{Z_B} & 0 \\ -\frac{1}{Z_A} & \frac{1}{Z_A} + \frac{1}{Z_C} + \frac{1}{Z_D} & -\frac{1}{Z_C} & -\frac{1}{Z_D} \\ -\frac{1}{Z_B} & -\frac{1}{Z_C} & \frac{1}{Z_C} + \frac{1}{Z_B} & 0 \\ 0 & -\frac{1}{Z_D} & 0 & \frac{1}{Z_D} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

* Diagonal are self-admittance terms equal the sum of admittances of all devices connected to the bus.



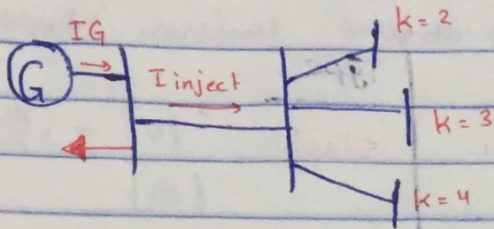
Find the Admittance matrix?

Ans: $y_{TL} = \frac{1}{0.03 + j0.04} = 12 - j16$

$|y_c| = 0.1$
 $y_c = j0.1$
 $|y_{T-L}| = 16$
 $y_{T-L} = -j16$

$$\begin{bmatrix} 12 - j16 + j0.1 & -(12 - j16) \\ -j(12 - j16) & 12 - j16 + j0.1 \end{bmatrix}$$

Tuesday 6/11/2018



$$I_{\text{injected}} = \frac{S_{\text{injected}}}{V_{\text{injected}}}$$

$$S = VI^*$$

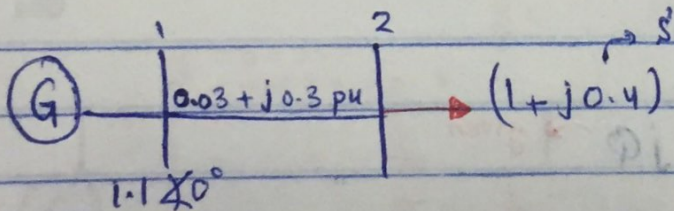
$$I = YV$$

$$\frac{S_i^*}{V_i^*} = \sum_{k=1}^N y_{ik} V_k$$

$$\frac{S_i^*}{V_i^*} = y_{ii} V_i + \sum_{\substack{k=1 \\ k \neq i}}^N y_{ik} V_k$$

$$V_i = \frac{1}{y_{ii}} \left[\frac{S_i^*}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^N y_{ik} V_k \right]$$

example:



Find V_2 using Gauss Sidel method Given $V_2 = 1 \angle 0^\circ$

$$y_{T.L} = \frac{1}{0.03 + j0.3} = 0.33 + j3.3 \text{ pu.}$$

$$Y = \begin{bmatrix} 0.33 - j3.33 & -0.33 + j3.33 \\ -0.33 + j3.33 & 0.33 - j3.33 \end{bmatrix}$$

Classification:

	Known	unknown	Type
Bus 1	$1.1 \angle 0^\circ$	P_1, Q_1	slack
Bus 2	$P_2 = -1$ $Q_2 = +0.4$ $S_2 = -1 - j0.4$	$V_2 ?$ $S ?$	

$$* V_i = \frac{1}{y_{ii}} \left[\frac{S_i^*}{V_i^*} - \sum_{j=1}^N y_{ij} V_j \right]$$

$$[V]Y = [I]$$

$$V_2 = \frac{1}{y_{22}} \left[\frac{S_2^*}{V_2^*} - \sum_{j=1}^N y_{2j} V_j \right]$$

$$V_2^{(1)} = \frac{1}{0.33 - j3.3} \left[\frac{(-1 - j0.4)^*}{1 \angle 0^\circ} - (0.33 + j3.33 \times 1.1 \angle 0^\circ)^* \right]$$

$$V_2^{(0)} = 1 \angle 0^\circ, \quad V_2^{(1)} = 0.99 \angle -16^\circ, \quad V_2^{(2)} = 0.9 \angle -15^\circ$$

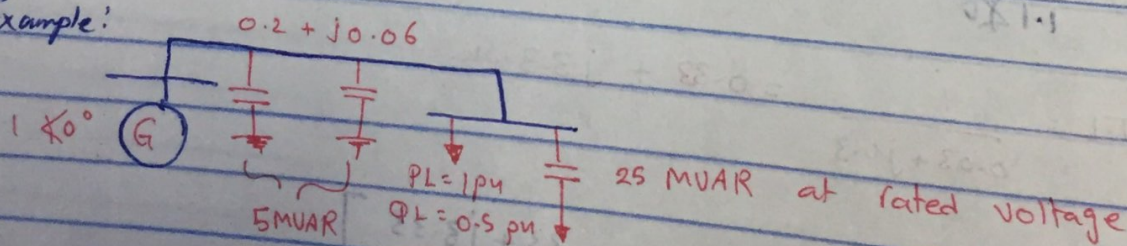
find it but substitute $V_2^{(1)}$ in it.

$$I_{12} = V_1 - V_2$$

$$S_2 = I_{12} \times V_2 \quad \text{must equal}$$

$P + jQ$ given

* Example:



Line charging 5 MVAR on each end,
Find V_2 using Gauss Sidel?

Model constant impedance.

$$Q_c = \frac{|V|^2}{|X_c|}$$

Constant impedance $\rightarrow X_c$ constant

$$|V| = 1 \text{ pu} \rightarrow 25 \text{ MVAR}$$

Base = 100 MVA

Note: if $V = 0.9 \text{ pu}$
 $Q_c = (0.9)^2 \times 25 \text{ MVAR}$

$$Q = |V|^2 y_c$$

$$Q_c \text{ pu} = \frac{25 \text{ MVAR}}{100} = 0.25 \text{ pu}$$

$$|y_c| = \frac{0.25}{1 \text{ pu}} = 0.25 \quad y_c = j0.25$$

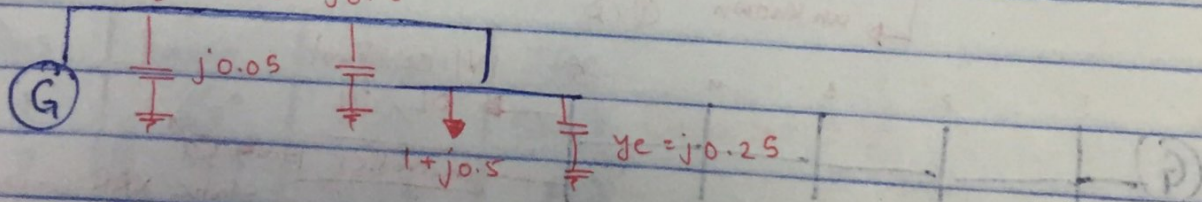
charging end.

$$Q = V^2 y_c$$

$$\frac{S}{100} = 1 \text{ pu} \cdot |y_c| \quad |y_c| = 0.05$$

$$y = 5 - j15$$

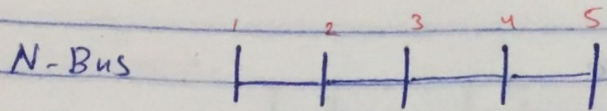
$$z = 0.02 + j0.6$$



$$Y = \begin{bmatrix} 5 - j15 + j0.05 & -5 + j15 \\ -5 + j15 & 5 - j15 + j0.05 + j0.25 \end{bmatrix}$$

Gauss sidel:

$$V_i = \frac{1}{y_{ii}} \left[\frac{S_i^*}{V_i^*} - \sum_{j=1, j \neq i}^N y_{ij} V_j \right]$$



Thursday 8/11/2018

- ① slack bus
- ②, ③, ④, ⑤ PQ buses

$$V_2^{(1)} = \frac{1}{y_{22}} \left[\frac{S_2^*}{V_2^{(0)*}} - y_{21} V_1 - y_{23} V_3^{(0)} \right]$$

$$V_3^{(1)} = \frac{1}{y_{33}} \left[\frac{S_3^*}{V_3^{(0)*}} - y_{32} V_2^{(1)} - y_{34} V_4^{(0)} \right]$$

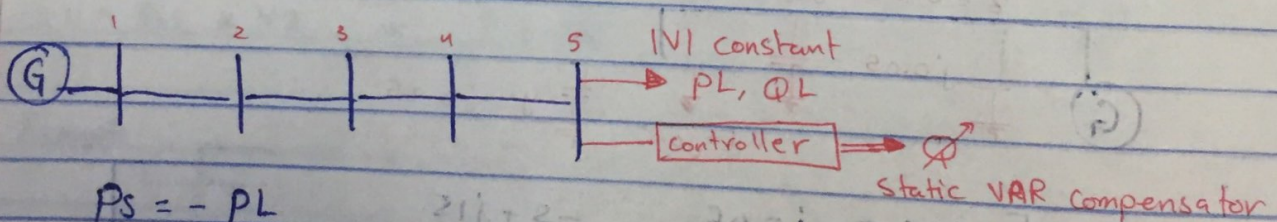
$$V_4^{(1)} = \frac{1}{y_{44}} \left[\frac{S_4^*}{V_4^{(0)*}} - y_{43} V_3^{(1)} - y_{45} V_5^{(0)} \right]$$

$$V_5^{(1)} = \frac{1}{y_{55}} \left[\frac{S_5^*}{V_5^{(0)*}} - y_{54} V_4^{(1)} \right]$$

$$V_1 = |V| \angle 0^\circ$$

Slack bus * PV Bus

- known $P, |V|$
- unknown Q, δ



$$V_5 = \frac{1}{y_{55}} \left(\frac{S_5^*}{V_5^{(0)*}} - y_{54} V_4^{(1)} \right)$$

$$S_5 = P - jQ_S$$

$$S_i = V_i I_i^*$$

$$P_i + jQ_i = V_i \left[\sum_{j=1}^n y_{ij} V_j \right]^*$$

$$P_i - jQ_i = V_i^* \left[\sum_{j=1}^n y_{ij} V_j \right]$$

$$Q_i = -\text{Im} \left[V_i^* \sum_{j=1}^n y_{ij} V_j \right]$$

Estimation for Q

$$P_i = P_i - jQ_i$$

$$V_i = \frac{1}{y_{ii}} \left[\frac{S_i^*}{V_i} - \sum_{j=1, j \neq i}^n y_{ij} V_j \right]$$

Let $V_i^{(0)} = V_5 \angle 85$

↳ correction [only |PVI| changed]

$V_i^{(1)}$ corrected = |V5 desired| $\angle 85$
 Logiven in Question.

x Controller:

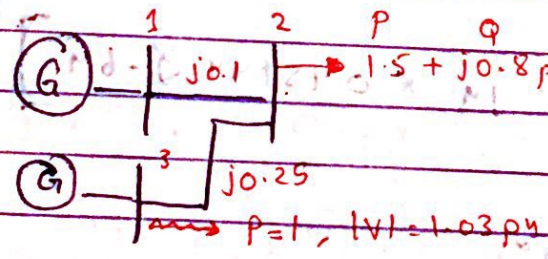
$$Q_{min} \leq Q \leq Q_{max}$$

$$Q_i \gg Q_{max}$$

(We can't fix the voltage on this bus)

PV \rightarrow PQ bus, $Q = Q_{max}$ [known]

x Example:



x Y-matrix

* perform 2 iterations of the [Gauss/Sidel], [load flow].

Solution: □

Bus	known	unknown	Type
1	$1 \angle 0^\circ$	P, Q	Slack (balanced P, Q)
2	$P_2 = -1.5$ $Q_2 = -0.8$	V, S_2	PQ
3	$P = 1 \text{ pu}$ $V = 1.03 \text{ pu}$	Q, S_3	PV

$$S_2 = -1.5 - j0.8$$

2 Y-matrix

$j0.1 \rightarrow -j10$
 $j0.25 \rightarrow -j4$

$$Y = \begin{bmatrix} -j10 & j10 & 0 \\ j10 & -j14 & j4 \\ 0 & j4 & -j4 \end{bmatrix}$$

$$V_2^{(1)} = \frac{1}{y_{22}} \left[\frac{-1.5 + j0.8}{1 \angle 0^\circ} - j10 \times 1 \angle 0^\circ + j4 \times 1.02 \angle 0^\circ \right]$$

$$V_2^{(1)} = 0.95744 \angle -6.4^\circ$$

$$Q_3 = -\text{Im} \left[V_3 \times \sum_{j=1}^3 y_{3j} V_j \right]$$

$V_1 = 1 \angle 0^\circ$
 $V_2^{(0)} = 1 \angle 0^\circ$ (supposedly)
 $V_3^{(0)} = 1.02 \angle 0^\circ$

$$Q_3 = -\text{Im} \left\{ 1.03 \angle 0^\circ \left[j4(0.95744 \angle -6.4^\circ) + -j4 \times 1.03 \angle 0^\circ \right] \right\}$$

$$Q_3 = 0.32 \text{ pu}$$

$Q_3 > Q_{\text{max}} ??$

$$V_3^{(1)} = \frac{1}{y_{33}} \left[\frac{S_3^*}{V_3^A} - y_{32} V_2^{(1)} \right]$$

$$V_3^{(1)} = \frac{1}{-j4} \left[\frac{1 - j0.32}{1.03 \angle 0^\circ} - j4 \times 0.95744 \angle -6.4^\circ \right]$$

$$V_3^{(1)} = 1.0395 \angle 7.2^\circ$$

corrected

$$V_3^{(1)} = 1.03 \angle 7.2^\circ$$

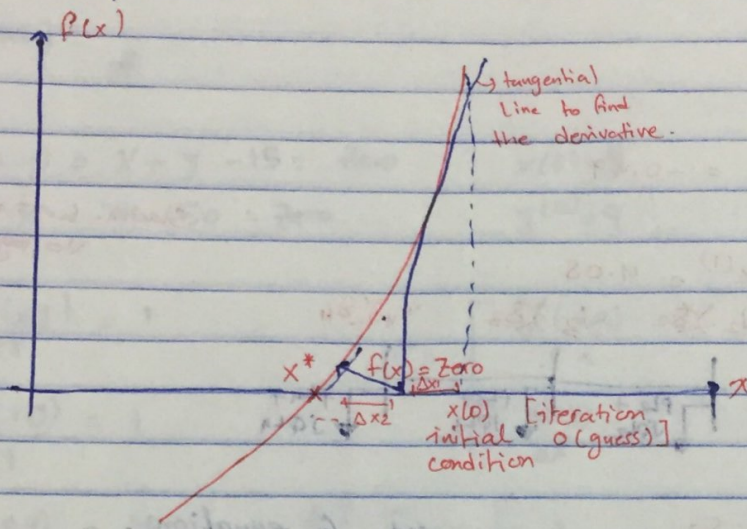
$$V_2^{(1)} = 0.95 \angle -6.4^\circ$$

$$V_1 = 1 \angle 0^\circ$$

Bus	Known	Unknown	Type
1	$V = 1 \angle 0^\circ$		slack
2	$P = -1.2$ $Q = -0.8$	$V = 1.25$	PQ
3	$P = 1.02$ $Q = 0.32$	$V = 1.23$	PV

* Newton Raphson.

Tuesday 13/11/2018



You start taking tangential lines to find the derivative to reach the point.

- low voltage $f \gg x$ unbalanced
- 100kV
- $\frac{x}{R} \gg 1 \approx 30$

* Taylor Series:

$$f(x^*) = f(x^{(0)}) + f'(x^{(0)})\Delta x^{(0)} + \frac{1}{2} f''(x^{(0)})(\Delta x^{(0)})^2 + \frac{1}{3!} f'''(x^{(0)})(\Delta x^{(0)})^3 + \dots$$

Approximation \approx Zero

$$f(x^*) = f(x^{(0)}) + f'(x^{(0)})\Delta x^{(0)} = \text{Zero}$$

unknown

$$f(x^*) = f(x^{(0)}) + f'(x^{(0)})(x - x^{(0)}) = \text{Zero}$$

$$\Delta x^{(0)} = - \frac{f(x^{(0)})}{f'(x^{(0)})} = - \left[f'(x^{(0)}) \right]^{-1} f(x^{(0)})$$

$f(x) = x^2 - 5x + 4, f(x^*) = \text{Zero}, x^*, x^{(0)} = 6$

1st iteration:
 $f(x) = x^2 - 5x + 4$
 $f'(x) = 2x - 5$

$x^{(0)} = 6$
 $f(x^{(0)}) = 10$
 $f'(x^{(0)}) = 7$

$\Delta x^{(0)} = - \frac{10}{7} = x^{(1)} - x^{(0)}$

$x^{(1)} = 6 - \frac{10}{7} = 4.7$

2nd iteration:

$$x^{(1)} = 4.57$$

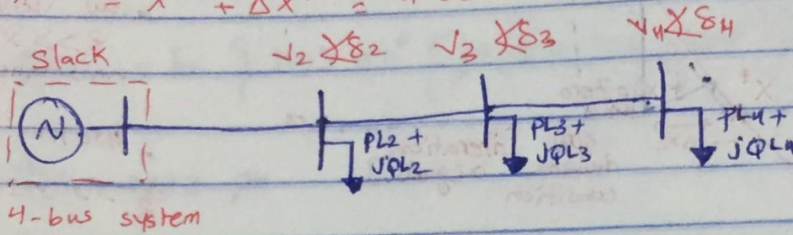
$$f(4.57) = 2.04$$

$$f'(4.57) = 4.14$$

$$\Delta x^{(1)} = \frac{-2.04}{4.14} = -0.49$$

$$x^{(2)} = x^{(1)} + \Delta x^{(1)} = 4.08$$

Slack: Controllable voltage, I -



unknowns: $|V_2|, \delta_2$
 $|V_3|, \delta_3$
 $|V_4|, \delta_4$

we need 6 equations.

balanced equations $\begin{cases} P \\ Q \end{cases}$

injected $P_2 = -PL_2 \Rightarrow F_1(V, \delta) = -PL_2$

$$P_2(\text{injected})(V, \delta) = -PL_2$$

$$P_2^{\text{injected}}(V, \delta) + PL_2 = \text{Zero}$$

$$F_1(x) = P_2^{\text{injected}}(V, \delta) + PL_2$$

$$x^{(k+1)} = x^{(k)} - [f'(x^{(k)})]^{-1} f(x^{(k)})$$

* Jacobian Matrix

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}, \quad F(x) = \begin{bmatrix} F_1(x) \\ F_2(x) \\ F_3(x) \\ \vdots \\ F_n(x) \end{bmatrix}, \quad F'(x) = \begin{bmatrix} \frac{\partial F_1(x)}{\partial x_1} & \frac{\partial F_1(x)}{\partial x_2} & \dots & \frac{\partial F_1(x)}{\partial x_n} \\ \frac{\partial F_2(x)}{\partial x_1} & \frac{\partial F_2(x)}{\partial x_2} & \dots & \frac{\partial F_2(x)}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_n(x)}{\partial x_1} & \dots & \dots & \frac{\partial F_n(x)}{\partial x_n} \end{bmatrix}$$

load flow \Rightarrow Multi-variables.

$$\Delta x^{(k)} = -J^{-1}(x^{(k)}) F(x^{(k)})$$

→ constant.

$$x^{(k+1)} \neq$$

Example: ① $f(x,y) = x+y-15 = \text{zero}$ $x^{(0)} = 4$

$$g(x,y) = xy - 50 = \text{zero}$$

sol:

$$\frac{\partial f(x,y)}{\partial x} = 1$$

$$\frac{\partial f(x,y)}{\partial y} = 1$$

$$\frac{\partial g(x,y)}{\partial x} = y$$

$$\frac{\partial g(x,y)}{\partial y} = x$$

$$J = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} & \frac{\partial f(x,y)}{\partial y} \\ \frac{\partial g(x,y)}{\partial x} & \frac{\partial g(x,y)}{\partial y} \end{bmatrix}$$

$$\textcircled{2} J = \begin{bmatrix} 1 & 1 \\ y & x \end{bmatrix}$$

$$\textcircled{3} J^{-1} = \frac{1}{x-y} \begin{bmatrix} x & -1 \\ -y & 1 \end{bmatrix}$$

to find inverse:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\Delta} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\textcircled{4} \Delta x^{(0)} = -J^{-1}(x^{(0)}) F(x^{(0)})$$

$$* J(x^{(0)} = 4, y^{(0)} = 9) = \begin{bmatrix} 1 & 1 \\ 9 & 4 \end{bmatrix}$$

$$* J^{-1} = \begin{bmatrix} -0.8 & 0.2 \\ 1.8 & -0.2 \end{bmatrix}$$

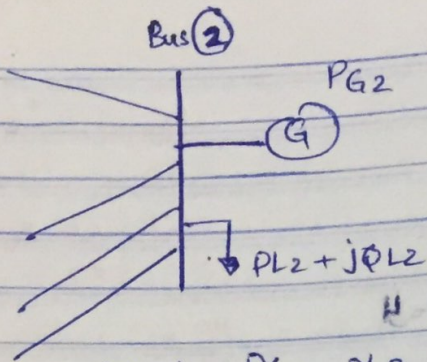
$$F(x^{(0)} = 4, y^{(0)} = 9) = \begin{bmatrix} -2 \\ -14 \end{bmatrix}$$

$$\Delta x^{(0)} = J^{-1}(x^{(0)}) [F(x^{(0)})] = \begin{bmatrix} 1.2 \\ 0.8 \end{bmatrix}$$

$$x^{(1)} = x^{(0)} + \Delta x^{(0)}$$

$$x^{(1)} = \begin{bmatrix} 4 \\ 9 \end{bmatrix} + \begin{bmatrix} 1.2 \\ 0.8 \end{bmatrix}$$

$$\begin{bmatrix} x^{(1)} \\ y^{(1)} \end{bmatrix} = \begin{bmatrix} 5.2 \\ 9.8 \end{bmatrix}$$



$$P_2(x) = PG_2 - PL_2$$

$$F_1(x) = P_2(x) - PG_2 + PL_2$$

$$F_2(x) = Q_2(x) - Q_{G2} + Q_{L2}$$

$$S = VI^*$$

$$S_2 = V_2 I_2^*$$

$$I_2 = V_2 [YV]$$

how to write these 2 equations

using this!

$(10) x$ $(11) x$ $(12) x$ $(13) x$ $(14) x$ $(15) x$

$(16) x$ $(17) x$ $(18) x$ $(19) x$ $(20) x$

$(21) x$ $(22) x$ $(23) x$ $(24) x$ $(25) x$

$(26) x$ $(27) x$ $(28) x$ $(29) x$ $(30) x$

$(31) x$ $(32) x$ $(33) x$ $(34) x$ $(35) x$

$(36) x$ $(37) x$ $(38) x$ $(39) x$ $(40) x$

$(41) x$ $(42) x$ $(43) x$ $(44) x$ $(45) x$

$(46) x$ $(47) x$ $(48) x$ $(49) x$ $(50) x$

$(51) x$ $(52) x$ $(53) x$ $(54) x$ $(55) x$

$(56) x$ $(57) x$ $(58) x$ $(59) x$ $(60) x$

$(61) x$ $(62) x$ $(63) x$ $(64) x$ $(65) x$

$(66) x$ $(67) x$ $(68) x$ $(69) x$ $(70) x$

$(71) x$ $(72) x$ $(73) x$ $(74) x$ $(75) x$

$(76) x$ $(77) x$ $(78) x$ $(79) x$ $(80) x$

$(81) x$ $(82) x$ $(83) x$ $(84) x$ $(85) x$

$(86) x$ $(87) x$ $(88) x$ $(89) x$ $(90) x$

$(91) x$ $(92) x$ $(93) x$ $(94) x$ $(95) x$

$(96) x$ $(97) x$ $(98) x$ $(99) x$ $(100) x$

Thursday 15/11/2018

Load flow:

Newton Raphson:

$F_1(x) = \text{Zero}$

$F_2(x) = \text{Zero}$

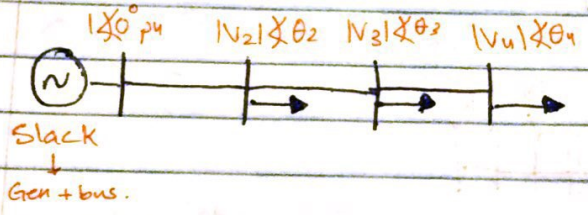
$F_n(x) = \text{Zero}$

$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ unknown

$x^{(k+1)} = x^{(k)} - J^{-1}(x^{(k)}) F(x^{(k)})$

$J = \begin{bmatrix} \frac{\partial F_1(x)}{\partial x_1} & \frac{\partial F_1(x)}{\partial x_2} & \dots & \frac{\partial F_1(x)}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_n(x)}{\partial x_1} & \dots & \dots & \frac{\partial F_n(x)}{\partial x_n} \end{bmatrix}$

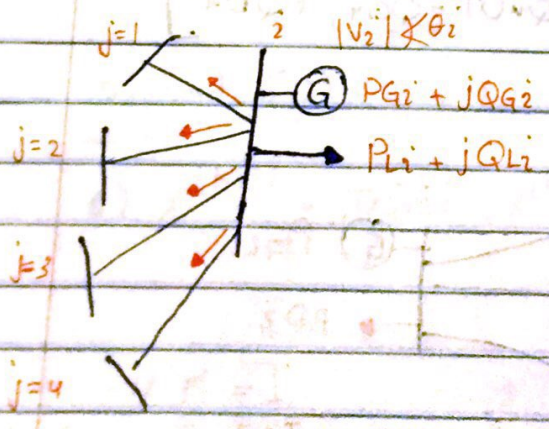
what's the relation between this & the load flow??



$x = \begin{bmatrix} |V_2| \\ |V_3| \\ |V_n| \\ \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_n \end{bmatrix} = \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ |V_2| \\ |V_3| \\ |V_n| \end{bmatrix}$

↳ most common

$F(x)$



$P_i^{\text{injected}} = P_i(x) = P_{Gi} - P_{Di}$ demand (load) --- (1)

$F_1(x) = \text{Zero} \rightarrow P_i(x) - P_{Gi} + P_{Di} = \text{Zero}$

$F_1(x) = P_i(x) - P_{Gi} + P_{Di}$

$Q_i^{\text{injected}} = Q_i(x) = Q_{Gi} - Q_{Di}$

$F_2(x) = Q_i(x) - Q_{Gi} + Q_{Di} = \text{Zero}$

of $F(x)$

4 Buses \rightarrow 1 slack
 \rightarrow 3 PQ Bus

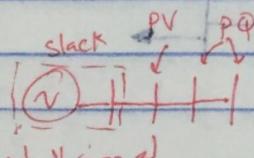
6 $F(x)$?

$$F(x) = \begin{bmatrix} P_2(x) - P_{G2} + P_{D2} \\ P_3(x) - P_{G3} + P_{D3} \\ P_4(x) - P_{G4} + P_{D4} \\ Q_2(x) - Q_{G2} + Q_{D2} \\ Q_3(x) - Q_{G3} + Q_{D3} \\ Q_4(x) - Q_{G4} + Q_{D4} \end{bmatrix}$$

we don't add the slack because it's known.

that's why we write P first.

4 Buses \rightarrow 1 slack
 \rightarrow 1 PV Bus
 \rightarrow 2 PQ bus



(unknown) V, θ

$$X = \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ V_3 \\ V_4 \end{bmatrix}$$

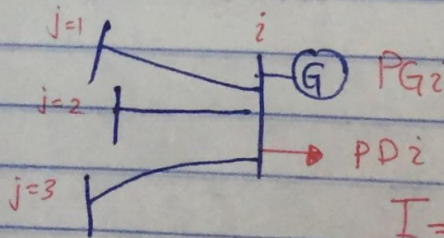
(known) P, Q

$$F(x) = \begin{bmatrix} P_2(x) - P_{G2} + P_{D2} \\ P_3(x) - P_{G3} + P_{D3} \\ P_4(x) - P_{G4} + P_{D4} \\ Q_3(x) - Q_{G3} + Q_{D3} \\ Q_4(x) - Q_{G4} + Q_{D4} \end{bmatrix}$$

finding Y-Matrix :

$$S_i = V_i (I_i)^*$$

$$S_i = V_i \left(\sum_{j=1}^N y_{ij} V_j \right)^*$$



$$I = YV$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_N \end{bmatrix}$$

$$V_i = |V_i| \angle \theta_i$$

$$Y_{ij} = g_{ij} + jb_{ij}$$

$$S_i = \frac{V_i}{V_i \angle \theta_i} \left[\sum_{j=1}^N (g_{ij} + jb_{ij}) |V_j| \angle \theta_j \right]^* \quad * \theta_{ij} = \theta_i - \theta_j$$

$$S_i = \sum_{j=1}^N |V_i| |V_j| (g_{ij} - jb_{ij}) (1 \angle \theta_i - \theta_j)$$

$\cos \theta_{ij} + j \sin \theta_{ij}$

$P_i + jQ_i$

$$P_i(x) = \sum_{j=1}^N |V_i| |V_j| (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij})$$

$$Q_i(x) = \sum_{j=1}^N |V_i| |V_j| (g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij})$$

steps:

1) Bus classifications

2) Y-matrix

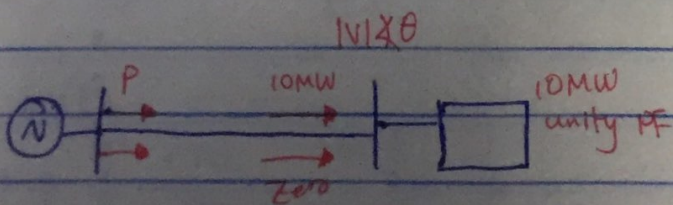
3) find $x = \begin{bmatrix} \theta \\ |V| \end{bmatrix}$

4) find $F(x) = \begin{bmatrix} P(x) - \dots \\ Q(x) - \dots \end{bmatrix}$

5) find $J(x)$

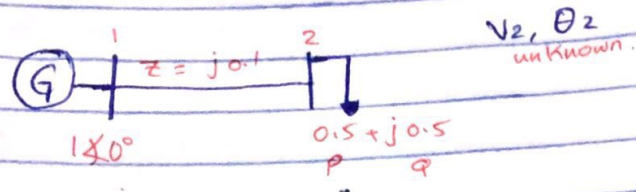
iteration#

6) $x^{(k+1)} = J_{(x^{(k)})}^{-1} F(x^{(k)})$



balanced equation $\begin{bmatrix} P \\ Q \end{bmatrix}$

Example:



Solution:

Bus ① → $1 \angle 0^\circ$, P_1, Q_1 (slack)

Bus ② → $P_2 = -0.5$, $Q_2 = -0.5$ (PQ Bus)

$$z = j0.1 \rightarrow y = -j10$$

$$Y = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$$

$$X = \begin{bmatrix} \theta_2 \\ |V_2| \end{bmatrix}, \quad F(x) = \begin{bmatrix} P_2(x) = -0.5 \\ * P_2(x) + 0.5 = 0 \\ Q_2(x) = -0.5 \\ * Q_2(x) + 0.5 = 0 \end{bmatrix}$$

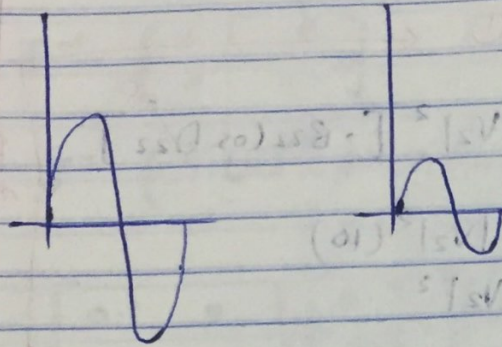
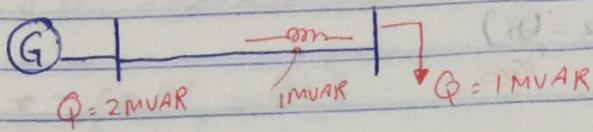
$$P_2(x) = \sum_{j=1}^2 |V_2| |V_j| (g_{2j} \cos \theta_{2j} + b_{2j} \sin \theta_{2j})$$

Zero (lossless system)

$$P_2(x) = |V_2| |V_1| (g_{21} \cos \theta_{21} + b_{21} \sin \theta_{21}) + |V_2|^2 b_{22} \sin \theta_{22}$$

Zero

$$P_2(x) = |V_2| (10) \sin \theta_2 = 10 |V_2| \sin \theta_2$$



reactive power losses

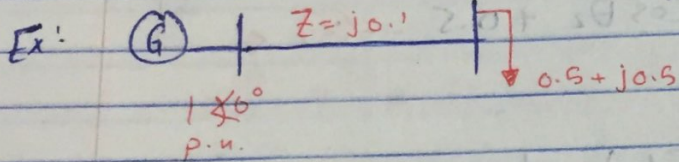
Solve for V_2 by using

Newton-Raphson?

initial given conditions:

* $V_2^{(0)} = 1 \text{ pu}$

* $\theta_2^{(0)} = 0$



bus 1 → slack

bus 2 → PQ [$P_2 = -0.5, Q_2 = -0.5$]

$Z \rightarrow Y = -j10$

$$Y = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$$

① $X = \begin{bmatrix} \theta_2 \\ V_2 \end{bmatrix}$

② $P_2(x) = -0.5 \rightarrow F_1(x) = P_2(x) + 0.5$

$Q_2(x) = -0.5 \rightarrow F_2(x) = Q_2(x) + 0.5$

$$P_k = \sum_i |V_k| |V_i| [G_{ki} \cos \theta_{ki} + B_{ki} \sin \theta_{ki}]$$

$$Q_k = \sum_i |V_k| |V_i| [G_{ki} \sin \theta_{ki} - B_{ki} \cos \theta_{ki}]$$

$G + jB$

* $0 + j \begin{bmatrix} -10 & 10 \\ 10 & -10 \end{bmatrix}$

$$P_2 = |V_2| |V_1| B_{21} \sin \theta_{21} + |V_2|^2 \sin \theta_{22} B_{22}$$

$$P_2 = |V_2| |V_1| (10) \sin(\theta_2 - \theta_1)$$

zero

$$P_2 = 10 |V_2| \sin \theta_2$$

$$F_1(x) = 10 |V_2| \sin \theta_2 + 0.5 \quad \text{--- (1)}$$

$$Q_2 = |V_2| |V_1| [-B_{21} \cos \theta_{21}] + |V_2|^2 [-B_{22} \cos \theta_{22}]$$

$$Q_2 = |V_2| |V_1| [-10 \cos \theta_{21}] + |V_2|^2 (10)$$

$$Q_2 = -10 |V_2| \cos \theta_2 + 10 |V_2|^2$$

$$F_2(x) = 10 |V_2|^2 - 10 |V_2| \cos \theta_2 + 0.5 \quad \text{--- (2)}$$

Power mismatches.

$$J = \begin{bmatrix} \frac{\partial F_1(x)}{\partial \theta_2} & \frac{\partial F_1(x)}{\partial V_2} \\ \frac{\partial F_2(x)}{\partial \theta_2} & \frac{\partial F_2(x)}{\partial V_2} \end{bmatrix}$$

$$J = \begin{bmatrix} 10 V_2 \cos \theta_2 & 10 \sin \theta_2 \\ 10 V_2 \sin \theta_2 & 20 V_2 - 10 \cos \theta_2 \end{bmatrix}$$

$$X^{(k+1)} = X^{(k)} - J^{-1}(X^{(k)}) F(X^{(k)})$$

$$* J^{-1} = \frac{1}{\Delta} \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix}$$

$$J(\theta_2^{(0)} = \text{Zero}, V_2^{(0)} = \text{Zero}) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$J^{-1}(\theta_2^{(0)}, V_2^{(0)})$$

$$J^{-1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$-J^{-1}(F(x)^{(0)}) = \Delta x^{(0)}$$

$$-\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} -0.05 \\ -0.05 \end{bmatrix} = \Delta x^{(0)}$$

$$x^{(1)} = x^{(0)} + \Delta x^{(0)}$$

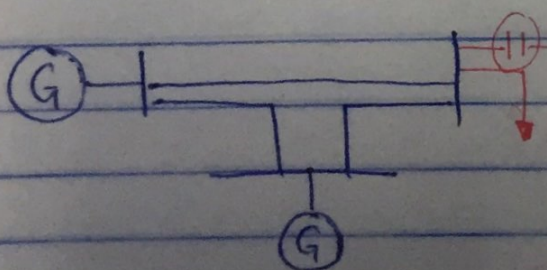
$$\begin{bmatrix} 0 \text{ rad} \\ 1 \end{bmatrix} + \begin{bmatrix} -0.05 \\ -0.05 \end{bmatrix}$$

$$V_2^{(1)} = \begin{bmatrix} 0.95 \text{ rad} \\ 0.95 \end{bmatrix}$$

$$V_2^{(1)} = 0.95 \text{ degree}$$

after 3 iterations:

$$\begin{bmatrix} -0.05288 \\ 0.94575 \end{bmatrix}$$



Admittance (capacitance) matrix for the capacitor.

$$PV \text{ bus } (V_3) = 1 \text{ p.u.}$$

$$P_2(x) = -0.5 \rightarrow F_1(x) = P_2(x) + 0.5$$

$$Q_2(x) = -0.5 \rightarrow F_2(x) = Q_2(x) + 0.5$$

$$P_3(x) = 0.5 \rightarrow F_3(x) = P_3(x) - 0.5$$

$$\begin{bmatrix} F_1(x) \\ F_2(x) \\ F_3(x) \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{\partial F_1(x)}{\partial \theta_2} & \frac{\partial F_1(x)}{\partial \theta_3} & \frac{\partial F_1(x)}{\partial v_2} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$* J = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial v} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial v} \end{bmatrix}$$

lossless line \rightarrow Transmission $X \gg R$

* Fast decoupled load flow:

methods of approximation

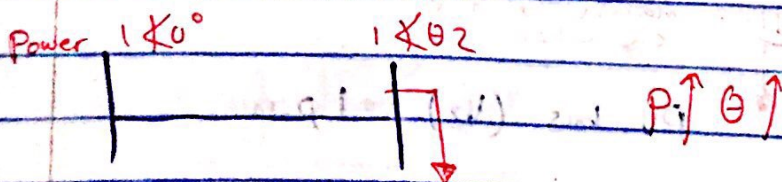
$$J = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial v} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial v} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\partial P}{\partial \theta} \\ \frac{\partial Q}{\partial v} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \end{bmatrix}$$

Approximation

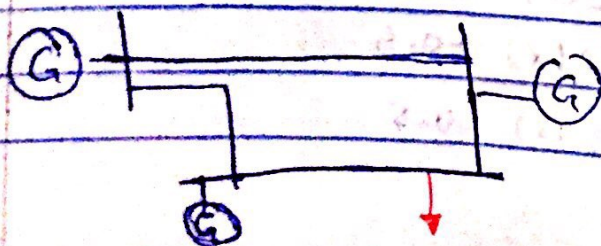
* DC - Load flow:

\rightarrow economics
system reacts with power only.

lossless system
 $|V_k| = 1 \text{ p.u.}$
 $|\theta_k|$

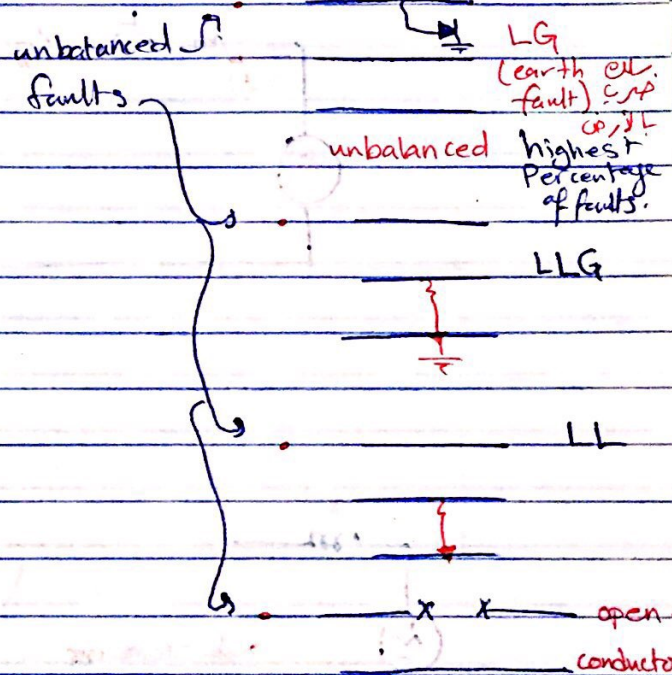
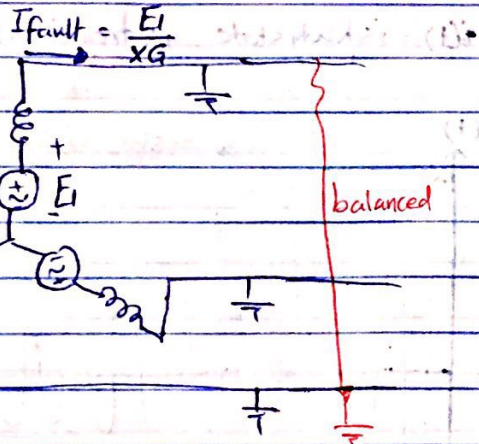
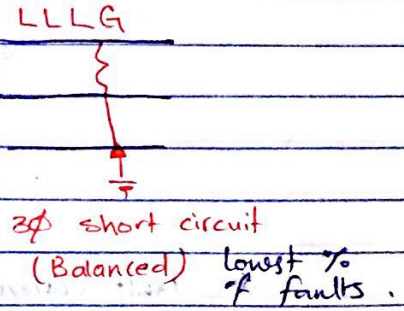
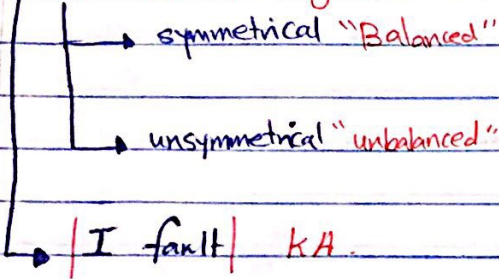


$P \propto (\theta_2 - \theta_1)$ Linear relation.



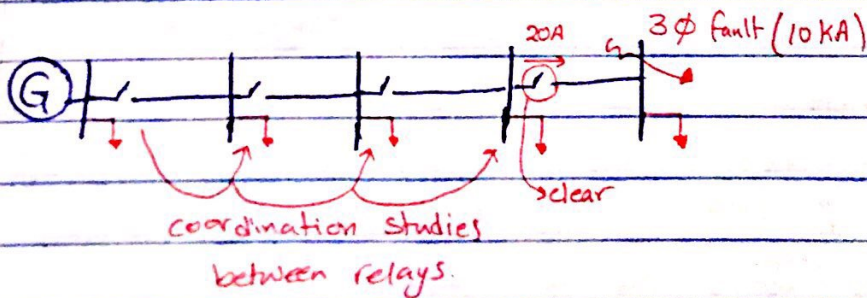
cost / load flow
 \rightarrow 2 complicated cases.

* Fault analysis:
(short-circuit analysis):



- * Design elements.
- x Design switching elements.

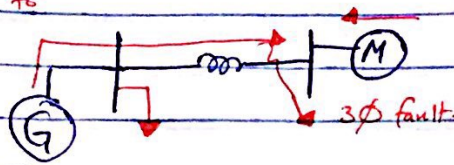
- x protection Design.
 - detect
 - clear



* Contributions to fault:

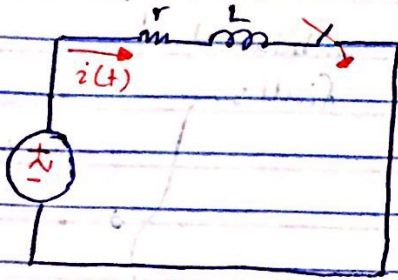
* Generators.

* Motors: converts to generator in some faults' cases.



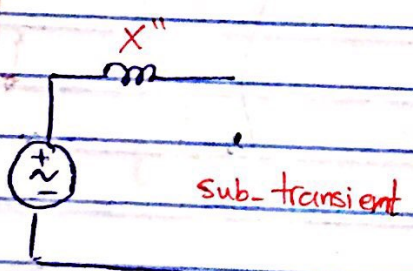
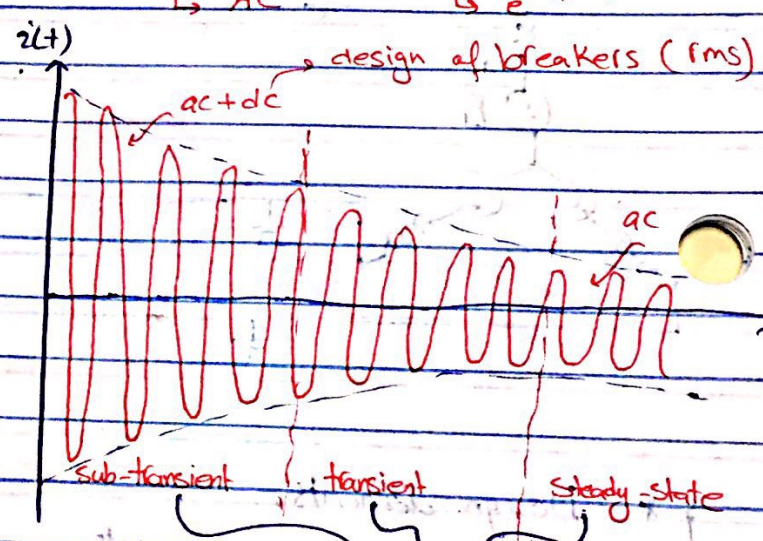
* Fault current.

* Fault flow.



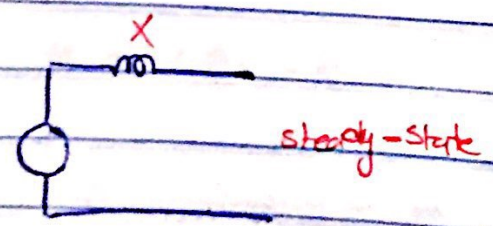
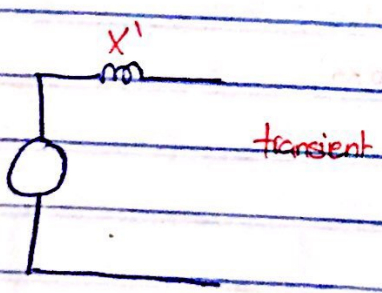
$i(t) = i_{\text{steady state}} + i_{\text{transient}}$

↳ AC ↳ $e^{-t/\tau}$



they differ in the reactances.

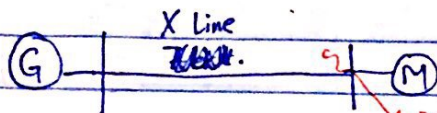
$X'' < X' < X$



Sunday 25/11/2018

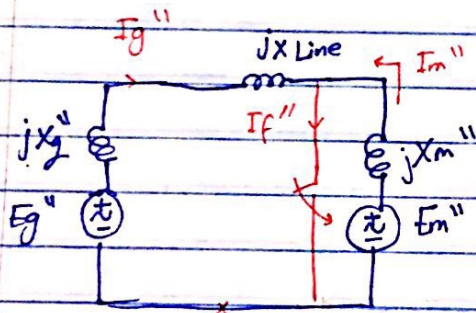
* 3 ϕ Fault analysis: \rightarrow Fault current contributions: from G, M.

- internal voltage method.
- thevenin.
- super position.
- Z-bus.



- 1) I_{fault}''
- 2) I_G''
- 3) I_M''

3 ϕ fault (balanced)



neutral-ground (same in balanced)

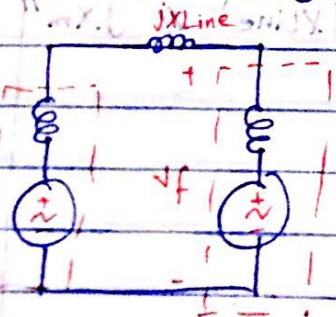
$$I_f'' = I_g'' + I_m''$$

$$I_f'' = \frac{E_g''}{j(X_g'' + X_{line})} + \frac{E_m''}{j(X_m'')} =$$

internal voltage meth.

I have to E_g'' , E_m'' pre-fault, what was the system? know these:

* Pre-fault analysis:



V_f : pre-fault voltage

* Neglect impact of load current

$$I_L = \text{zero}$$

$$E_g'' = V_f = E_m''$$

* impact of load current $I_L =$

$$E_g'' = V_f + I_L (j(X_g'' + X_{line}))$$

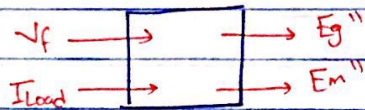
$$E_m'' = V_f - I_L (jX_m'')$$

$$I_f'' = \frac{V_f + I_L (j(X_g'' + X_{line}))}{j(X_g'' + X_{line})} + \frac{V_f - I_L (jX_m'')}{jX_m''}$$

$$I_f'' = \frac{V_f}{j(X_g'' + X_{line})} + \frac{I_L}{j(X_g'' + X_{line})} + \frac{V_f}{jX_m''} - \frac{I_L}{jX_m''}$$

$$I_g'' = \frac{V_f}{j(X_g'' + X_{line})} + I_{Load}$$

$$I_m'' = \frac{V_f}{jX_m''} - I_{Load}$$

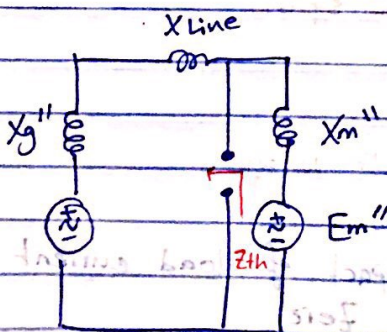


I_{Fault}''
 I_g''
 I_m'' } fault flow

$$I_f'' = V_f \left(\frac{1}{j(X_g'' + X_{line})} + \frac{1}{jX_m''} \right)$$

$$= \frac{V_f}{Z_{th}}$$

seen from fault location



Thevenin Method:

$$Z_{th} = (jX_g'' + jX_{line}) \parallel jX_m''$$

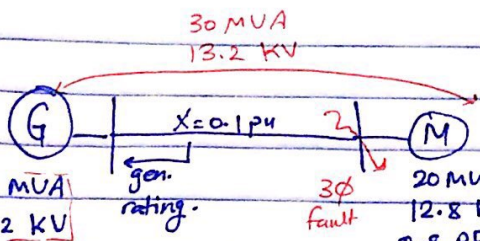
$$I_f = \frac{V_f}{Z_{th}}$$

$$I_g'' = I_f'' + I_L$$

$$I_m'' = I_f'' - I_L$$

~~X wrong~~

Ex:



Choose it to be base

30 MVA
13.2 KV

gen. rating

3 ϕ fault

20 MW
12.8 KV
0.8 PF lead

$\frac{12.8}{13.2}$ to get pu.

$X'' = 0.2 \text{ pu}$

$X'' = 0.2 \text{ pu}$

Solve it using ① internal ② thevini n

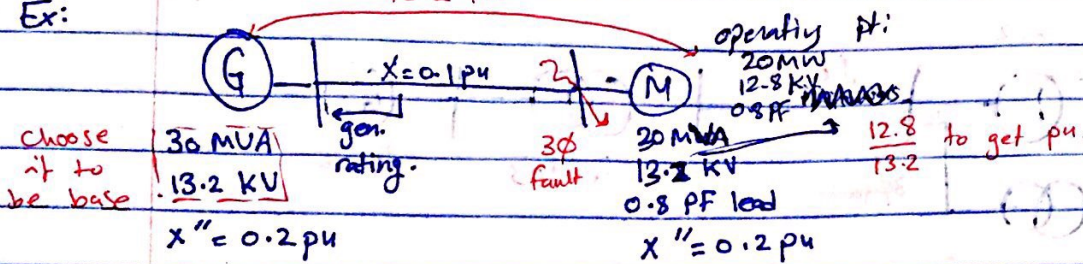
3 ϕ fault:

$V_f = 12.8 \text{ KV}$

Find: ① I_f'' ② I_g'' ③ I_m''

- $\frac{1}{Z}$ konst. impedance \rightarrow Y matrix
- $\frac{1}{Z}$ constant Power $\phi=?$ S_r injected

Ex:



Solve it using
① internal
② thevenin

3 ϕ fault:

$V_f = 12.8$ kV

Find: ① I_f'' ② I_g'' ③ I_m''

1. E_g'' / E_m'' (internal EMF)

① pre-fault.

$S = \sqrt{3} V_L I_L$, $S_{pu} = V_{pu} I_{pu}$

$\frac{20}{0.8} = \sqrt{3} (12.8) (I)$ actual $\frac{20/0.8}{\text{base}} = 0.97$ pu.

$I_{pu} = 0.86 \angle +\cos^{-1} \text{PF}$

$I_L = 1128 \angle 36.9^\circ$

$I_L / \text{pu} = \frac{I_L (\text{actual})}{I_L (\text{base})} = \frac{1128 \angle 36.9^\circ}{30 \text{ MVA} / (\sqrt{3} \times 13.2)}$

$E_g'' = 0.97 + I_L (j0.1 + j0.2) = 0.814 + j0.207$ pu.

$E_m'' = 0.97 - I_L (j0.2) = 1.074 - j0.138$ pu.

$I_g'' = \frac{E_g''}{j0.2 + j0.1} = 0.814 + j0.207$ pu.

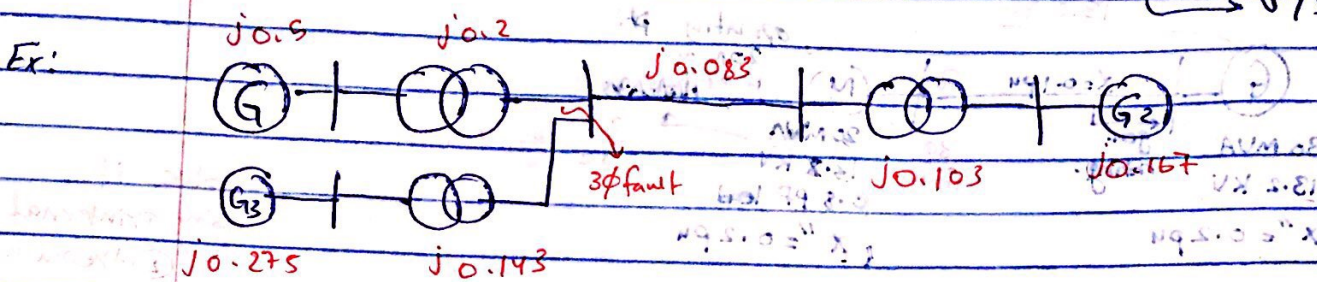
$I_m'' = \frac{E_m''}{j0.2} = 0.69 - j5.37$ pu.

$I_f'' = I_g'' + I_m'' = \dots$ pu.

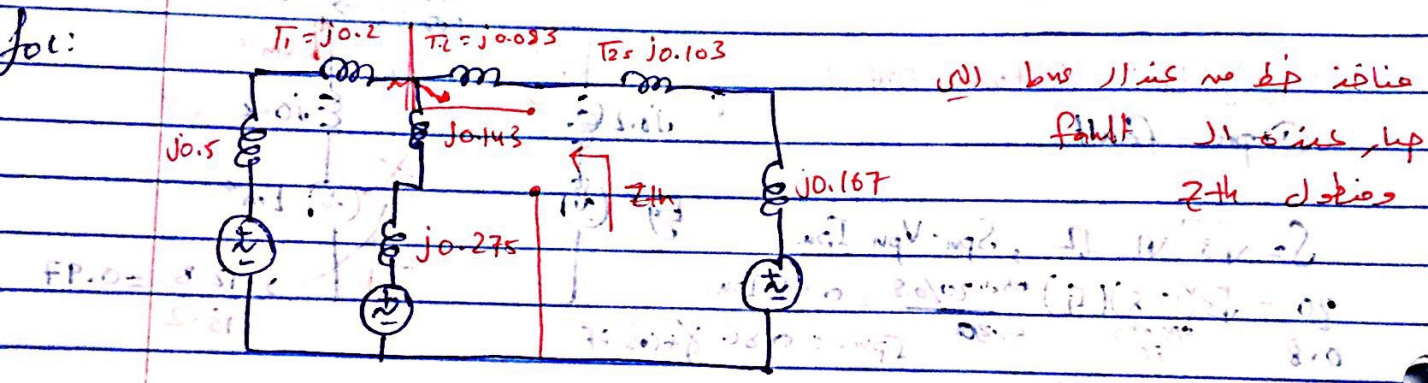
$I_f'' (\text{KA}) = I_f'' \text{ pu.} \left(\frac{\text{base}}{\sqrt{3} \cdot 13.2} \right)$

$$X''_{new} = X''_{old} \times \left(\frac{V_{old}}{V_{new}} \right)^2 \cdot \left(\frac{S_{new}}{S_{old}} \right)$$

$$X_{pu} = \frac{X_{actual}(\Omega)}{Z_{base}} \quad \leftarrow \frac{V^2}{S}$$



find I_{3φ} fault using Thevenin?? (neglect load currents, V_f = 1 pu).

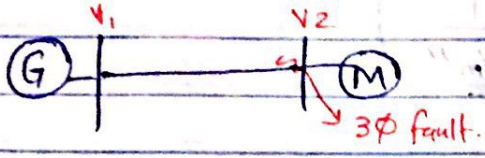


$$Z_{th} = (j0.083 + j0.103 + j0.167) \parallel (j0.143 + j0.275) \parallel (j0.2 + j0.5)$$

$$I_{f''} = \frac{V_{th}}{Z_{th}} = \frac{1}{Z_{th}}$$

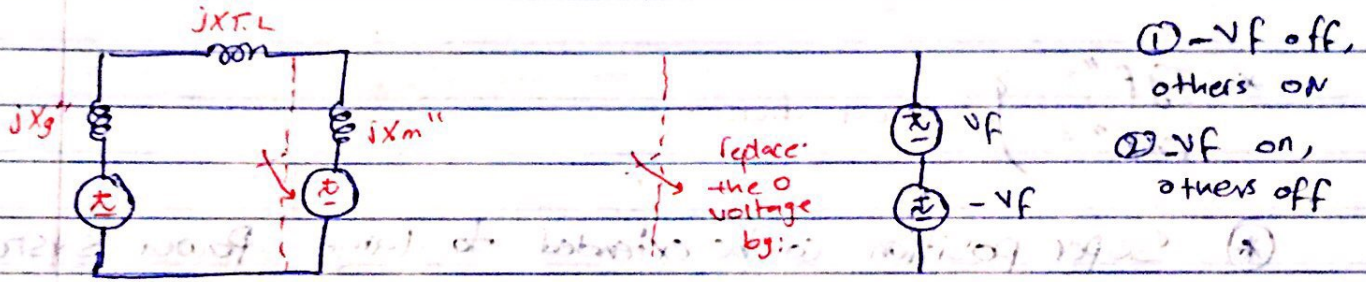
* Fault analysis:

super position method:



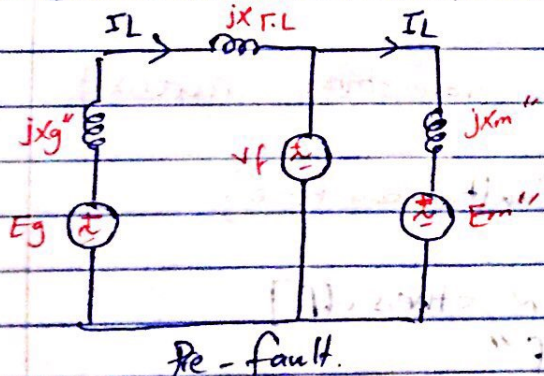
$V_f = V_{pre-fault}$ voltage.

- $V_2 = V_f - V_f = \text{Zero}$ (neglecting I_L)
- $V_1 = V_f + \Delta V_1$



- ① - V_f off, others ON
- ② - V_f on, others off

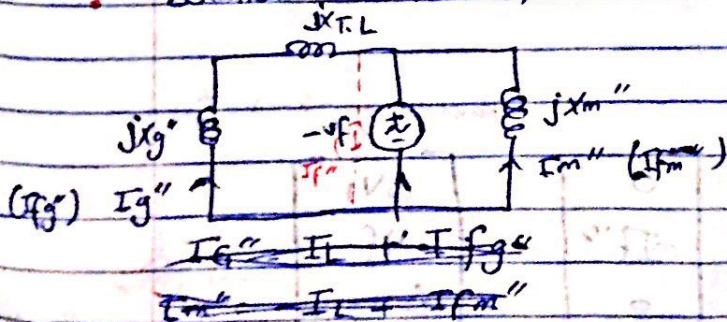
Solution 1: - V_f off, others ON:



$I_g'' = I_L + I_{fg}''$
 $I_m'' = -I_L + I_{fm}''$

pre-fault.

Solution 2: - V_f on, others off:



current division:

$I_{fg}'' = I_f'' \times \frac{jX_m''}{jX_m'' + jX_g'' + jX_{TL}}$
 $I_{fm}'' = I_f'' \times \frac{jX_g'' + jX_{TL}}{jX_{line} + jX_g'' + jX_m''}$

for solution 2:

$$I_f'' = \frac{+V_f}{j(x_g'' + x_{line})} + \frac{V_f}{jX_m''}$$

$$I_f'' = V_f \left(\frac{1}{j(x_g'' + x_{line})} + \frac{1}{jX_m''} \right)$$

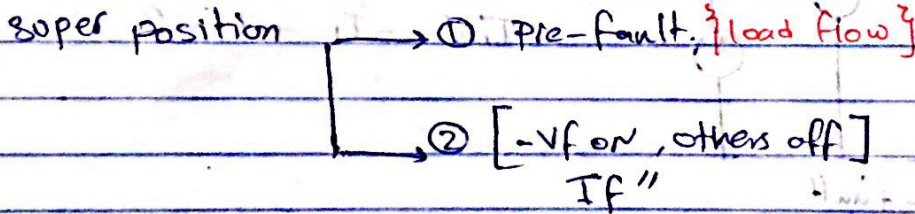
$$I_f'' = \frac{V_f}{Z_{th \text{ seen from the fault}}}$$

$$\left. \begin{aligned} E_{gf}'' &= \\ I_{mf}'' &= \end{aligned} \right\} \text{current division.}$$

* Super position can be extended to large power system.

$$I_f'', V_1, V_2, \dots, V_n,$$

$$YV = I_{\text{injected}} \quad (\text{Admittance matrix})$$



$$V = Z_{\text{bus}} I \quad (\text{Impedance matrix})$$

Solution

②:

4 Bus

$$Y^{-1} \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} 0 \\ -I_f'' \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \Delta V_1 \\ -V_f \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix}$$

fault location column

-Vf on, others off
I_f'', ΔV₁, ΔV₃, ΔV₄

thevenin impedance seen from each bus.

Solution
 ①: -Vf off, others ON
 Neglect load current
 $V_1 = V_2 = V_3 = V_4 = V_f$

Final step (Sol. ① + Sol. ②):

- $V_1 = V_f + \Delta V_1 \rightarrow V_1 = V_f - Z_{12} \left(\frac{V_f}{Z_{22}} \right)$
- $V_2 = V_f - V_f = \text{Zero} \rightarrow V_2 = \text{Zero}$
- $V_3 = V_f + \Delta V_3 \rightarrow V_3 = V_f - Z_{32} \left(\frac{V_f}{Z_{22}} \right)$
- $V_4 = V_f + \Delta V_4 \rightarrow V_4 = V_f - Z_{42} \left(\frac{V_f}{Z_{22}} \right)$

$I_f'' = \frac{-V_f}{Z_{22}}$ \rightarrow Z_{22} seen from the fault.

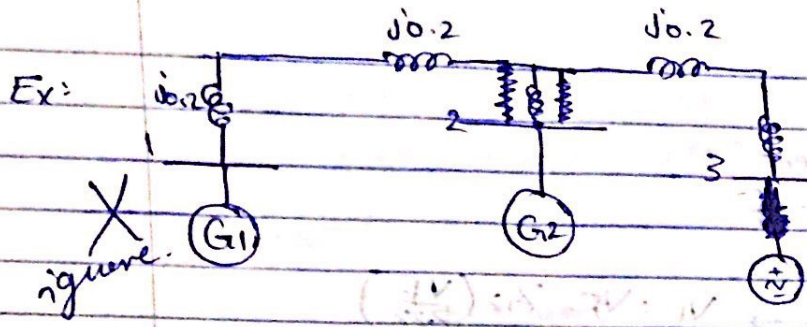
$\Delta V_1 = Z_{12} (-I_f'')$

$\Delta V_1 = -Z_{12} \left(\frac{V_f}{Z_{22}} \right)$

$\Delta V_3 = -Z_{32} \left(\frac{V_f}{Z_{22}} \right)$

$\Delta V_4 = -Z_{42} \left(\frac{V_f}{Z_{22}} \right)$

Shunt 2,3 → 23, 32, 22, 33



Tuesday 4/12/2018

Hint: $|X_L| = 0.5$
 $Z_L = j0.5$
 $Y_C = -j2$
 $Z_C = -j0.5$
 $Y_C = \frac{1}{-j0.5} = j2$

Ex: $Y = j \begin{bmatrix} -j15 & j10 & 0 \\ j10 & -j20 & j5 \\ 0 & j5 & -j9 \end{bmatrix}$ * find I_f at bus 1. assuming $V_f = 1.05$.
 * neglect TL.
 * find V_1, V_2, V_3

Solution: $Z_{bus} = Y^{-1}$
 $= -j \begin{bmatrix} -0.108 & -0.063 & -0.035 \\ -0.063 & -0.094 & -0.052 \\ -0.035 & -0.052 & -0.1409 \end{bmatrix}$
 $Z_{bus} = j \begin{bmatrix} 0.108 & 0.063 & 0.035 \\ 0.063 & 0.094 & 0.052 \\ 0.035 & 0.052 & 0.1409 \end{bmatrix}$
 because fault at bus 1

$$I_f = \frac{V_f}{Z_{11}} = \frac{1.05}{j0.108} = -j9.6 \text{ pu.}$$

$$V_2 = V_2(\text{pre-fault}) + \Delta V_2 \quad \text{superposition.}$$

$$\begin{bmatrix} Z_{bus} \end{bmatrix} \begin{bmatrix} -I_f \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} V_f \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix}$$

to find ΔV_2 :

$$-Z_{21} I_f = \Delta V_2$$

$$\Delta V_2 = -j0.063 \times \frac{1.05}{j0.108} = -0.6048$$

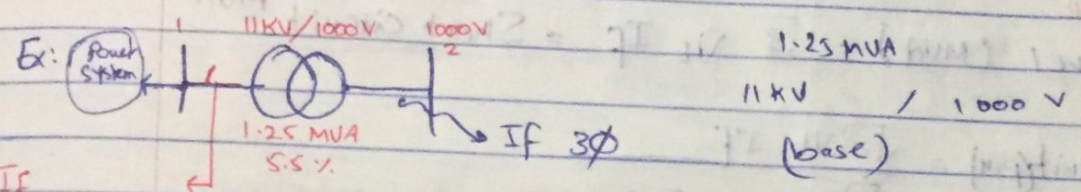
$$V_2 = 1.05 - 0.6048 = 0.4452$$

$$V_3 = 1.05 - \frac{2.313}{2.12} V_f$$

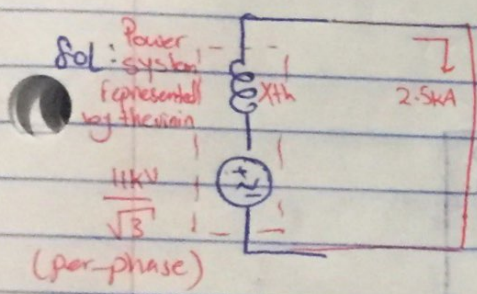
$$V_3 = 0.714 \text{ pu.}$$

$$V_1 = V_f - V_f = 0$$

before → after.



IF
switch is opened
3φ fault at bus 2
is 2.5 KA.

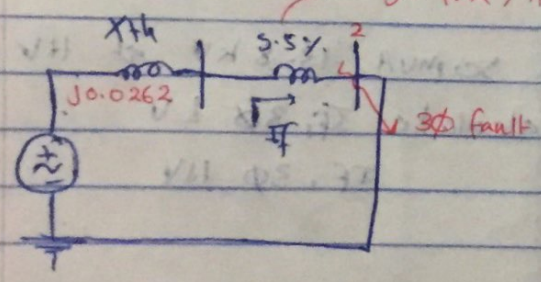


$$X_{th} = \frac{11 \text{ kV} / \sqrt{3}}{2.5 \text{ KA}}$$

$$X_{th}(\text{pu}) = \frac{(11 \text{ kV} / \sqrt{3}) / 2.5 \text{ k}}{(11 \text{ kV})^2 / 1.25 \text{ M}}$$

$$= 0.0262 \text{ pu.}$$

leakage flux, if we had R → copper losses.



$$I_f \text{ pu.} = \frac{1}{j0.0262 + j0.055} = 12.315 \text{ pu.}$$

$$I_f \text{ (KA)}_{\text{actual}} = 12.315 \times \left(\frac{1.25 \text{ MVA}}{\sqrt{3} \times 1000} \right) = 8.9 \text{ KA}$$

worst case:

$$X_{th} = \text{zero}$$

$$I_f = \frac{1}{0.055} = 18.2 \text{ pu.}$$

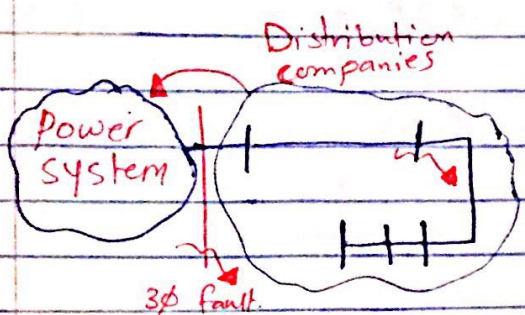
(if we neglect capability)
Xth = 0 oil & fault LL

$$I_f \text{ (KA)} = 18.2 \times \left(\frac{1.25 \text{ MVA}}{\sqrt{3} \times 1000} \right)$$

(worst scenario)

* learn inverse on calculator.
* solve Ybus, Zbus questions.

Thursday 6/12/2018



* Fault Level (MVA) = $\sqrt{3} V_L I_f$ = Short circuit MVA

Short Circuit (p.u.) = $\frac{\sqrt{3} V_L I_f}{\sqrt{3} V_{L,base} I_{L,base}}$

If $V_L = V_{L, system base}$

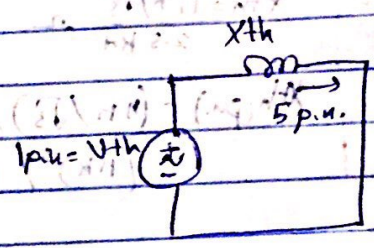
Sc. pu. = I_f p.u.

if $Sc MVA = 500 MVA$

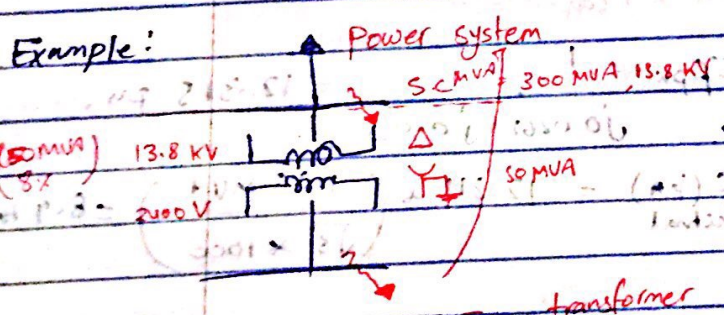
MVA base = 100 MVA

$\therefore I_f p.u. = 5 p.u.$

$X_{th} p.u. = \frac{1}{5} = 0.2 p.u.$



Example:

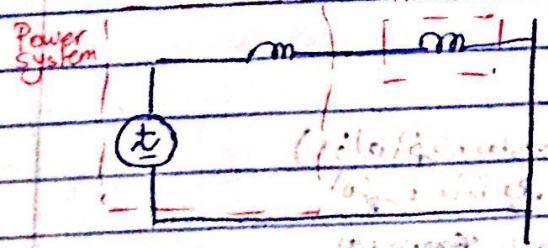


Use 500 MVA, 13.8 kV at HV

to calculate I_f 3 ϕ LV

I_f , 3 ϕ HV

Solution:



500 MVA, 13.8 kV

$\therefore 500 MVA = \sqrt{3} V_L I_f$

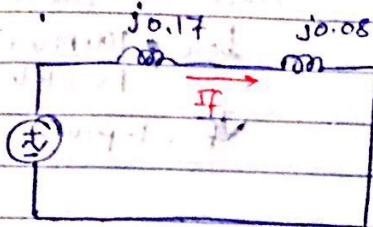
$500 M = \sqrt{3} (13.8 kV) I_f$

$I_f = 0.300 M$

$X_{th} = \frac{13.8 / \sqrt{3}}{300 MVA} = \frac{(13.8 kV)^2}{300 MVA} = \frac{V^2}{S}$

$$X_{th} (\text{p.u.}) = \frac{(13.8 \text{ k})^2 / 300^{\text{actual}}}{(13.8 \text{ k})^2 / 50^{\text{base}}} = \frac{50}{300} = 0.17 \text{ p.u.}$$

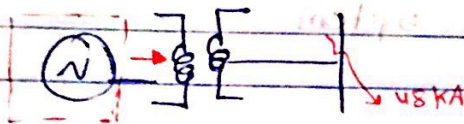
at bus 2000 V.



$$I_f^{\text{p.u.}} = \frac{1}{j0.17 + j0.08} = 4 \text{ p.u.}$$

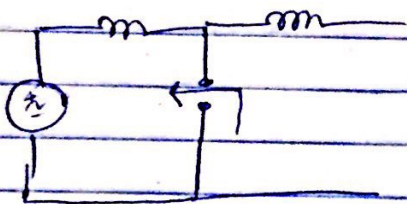
$$I_f^{\text{actual}} = 4 \times \left(\frac{50 \text{ MVA}}{\sqrt{3} \cdot 2000} \right) = 48 \text{ kA.}$$

Simplifying the circuit:



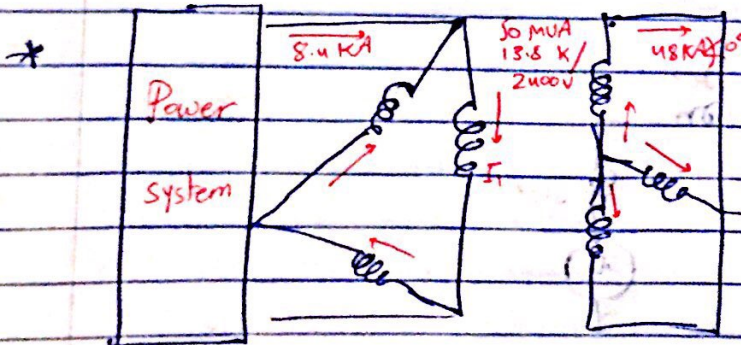
power system.

$$I_f^{\text{actual}} = 4 \times \left(\frac{50 \text{ MVA}}{\sqrt{3} \cdot 13.8 \text{ k}} \right) = 8.4 \text{ kA}$$



$$Z_{th} = j0.17$$

$$I_f = \frac{1}{j0.17} = 6 \text{ p.u.}$$



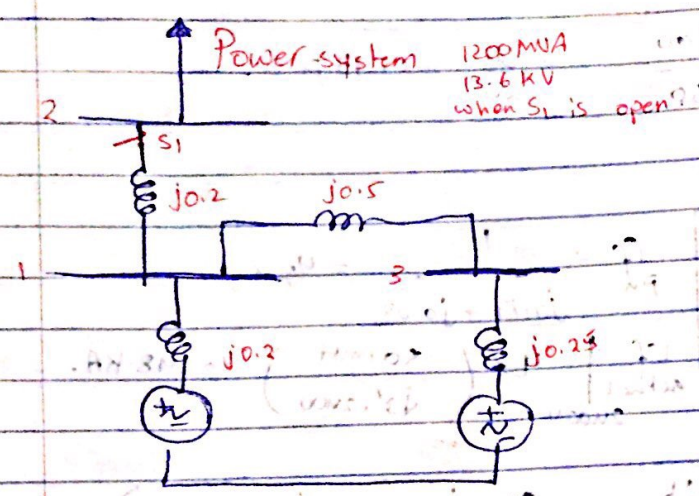
fault 2000 V.

(3φ that represents the single line diagram above).

Natural phase shift between the line currents = 30°.

$$I_f = 48 \text{ k} \times \left[\frac{2000 / \sqrt{3}}{13.8 \text{ k}} \right]$$

Example:



Find 3 ϕ fault at bus 1?

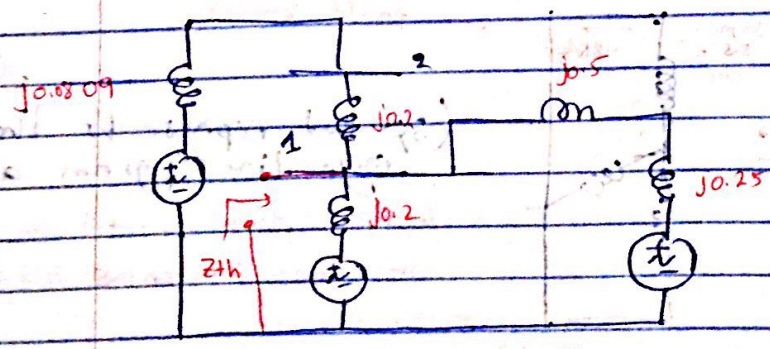
base = 100 MVA
13.8 kV

$V_f = 1.0 \text{ p.u.}$

Solution: ① Firstly find X_{th} of the power system:

$$X_{th} = \frac{(13.6 \text{ kV})^2}{1200 \text{ MVA}}$$

$$X_{th} \text{ p.u.} = \frac{(13.6 \text{ kV})^2}{1200} \cdot \frac{100}{(13.8 \text{ kV})^2}$$



to find Z_{th} (kill all sources)

Z_{th} ✓

$$I_f = \frac{1}{Z_{th}} \text{ p.u. (must convert to actual)}$$

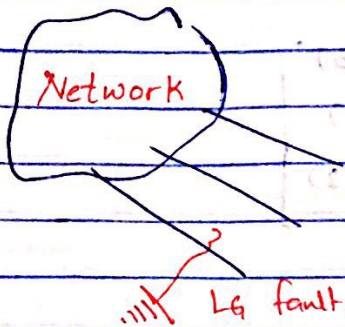
I_f (KA)

* if we had a delta system \rightarrow convert to Y

* Unbalanced Systems $\begin{matrix} \rightarrow LG \\ \rightarrow LL \\ \rightarrow LLG \end{matrix}$

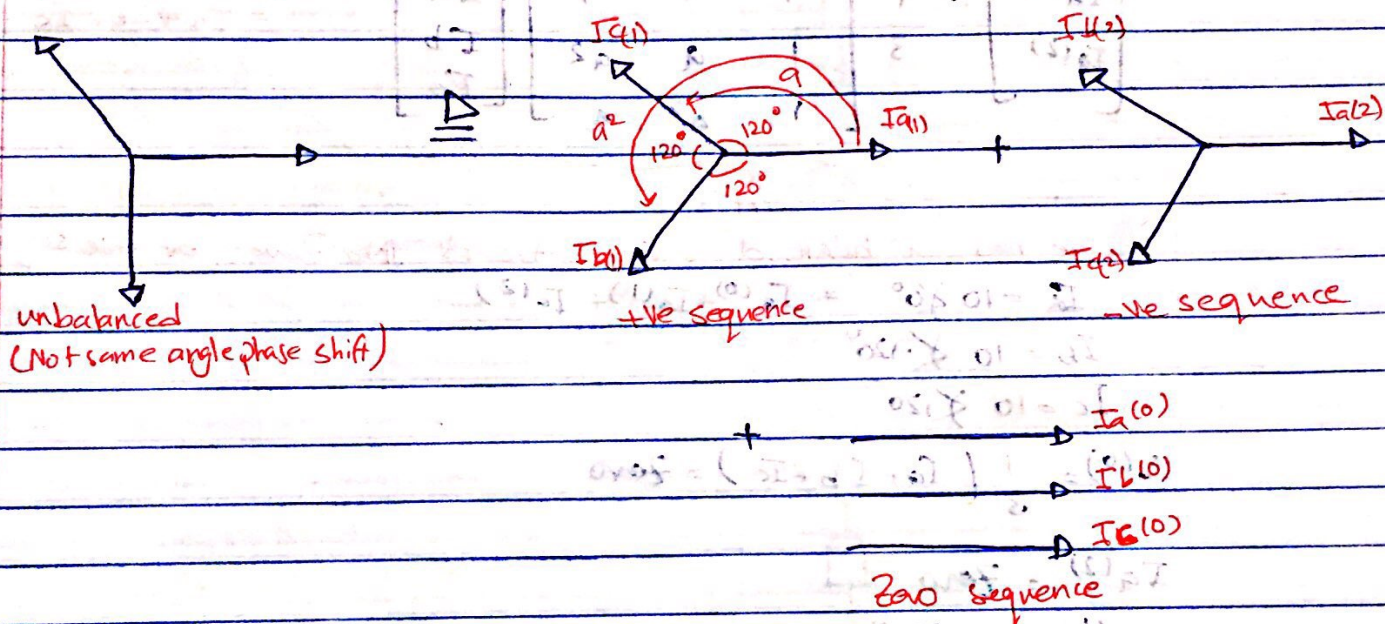
Sunday 9/12/2018

* Symmetrical components & Asymmetrical faults



(we need to know what happens in each phase)

symmetrical components:



$$I_a = I_a^{(1)} + I_a^{(2)} + I_a^{(0)}$$

$$I_b = I_b^{(1)} + I_b^{(2)} + I_b^{(0)}$$

$$I_c = I_c^{(1)} + I_c^{(2)} + I_c^{(0)}$$

$$\begin{matrix} I_a^{(0)} & I_a^{(1)} & I_a^{(2)} \\ I_b^{(0)} & I_b^{(1)} & I_b^{(2)} \\ I_c^{(0)} & I_c^{(1)} & I_c^{(2)} \end{matrix}$$

• 3 ϕ

I_a, I_b, I_c

$$j = 1 \angle 90^\circ$$

$$a = 1 \angle 120^\circ$$

$$a^2 = 1 \angle 240^\circ$$

$$a^3 = 1 \angle 0^\circ$$

$$1 + a + a^2 = \text{zero}$$

• $I_a^{(0)} = I_b^{(0)} = I_c^{(0)}$ zero seq.

• $I_b^{(1)} = a^2 I_a^{(1)}$ } +ve seq.

• $I_c^{(1)} = a I_a^{(1)}$

• $I_b^{(2)} = a I_a^{(2)}$ } -ve seq.

• $I_c^{(2)} = a^2 I_a^{(2)}$

$$* I_a = I_a^{(0)} + I_a^{(1)} + I_a^{(2)}$$

$$* I_b = I_a^{(0)} + a^2 I_a^{(1)} + a I_a^{(2)}$$

$$* I_c = I_a^{(0)} + a I_a^{(1)} + a^2 I_a^{(2)}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

A

$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_a^{(0)} = \frac{1}{3} (I_a + I_b + I_c) \\ = I_a + I_b + I_c = 3 I_a^{(0)}$$

A⁻¹

if we had a balanced system, there's no zero or -ve seq.

$$I_a = 10 \angle 0^\circ = I_a^{(0)} + I_a^{(1)} + I_a^{(2)}$$

$$I_b = 10 \angle -120^\circ$$

$$I_c = 10 \angle 120^\circ$$

$$I_a^{(0)} = \frac{1}{3} (I_a + I_b + I_c) = \text{zero}$$

$$I_a^{(2)} = \text{zero}$$

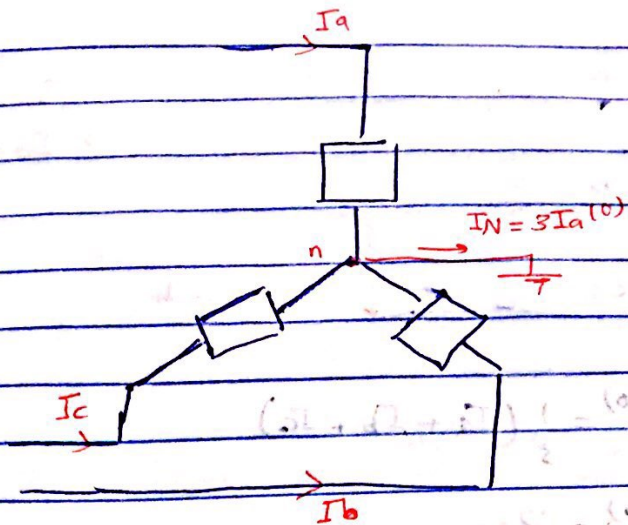
$$I_a^{(1)} = 10 \angle 0^\circ$$

$$I_b^{(1)} = 10 \angle -120^\circ$$

$$I_c^{(1)} = 10 \angle 120^\circ$$

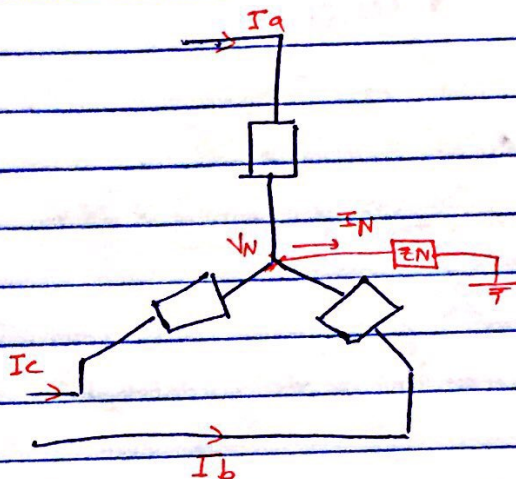
$$I_N = I_a + I_b + I_c$$

$$I_N = 3I_a^{(0)}$$



$$V_N = -I_N Z_N$$

$$V_N = -3I_a^{(0)} \cdot Z_N$$

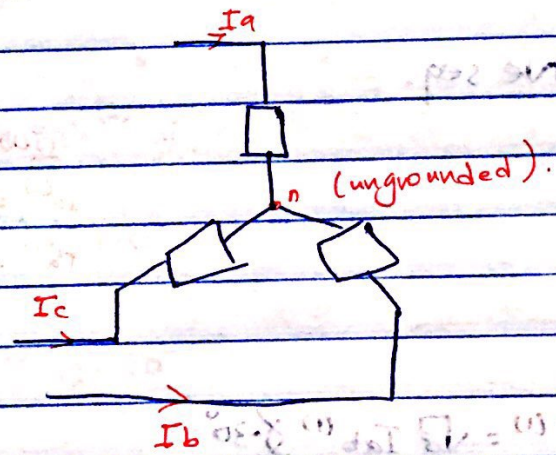


$$I_N = I_a + I_b + I_c$$

$$I_N = \text{Zero}$$

$$I_a^{(0)} = \text{Zero}$$

Zero sequence is only available when grounded.



a $I_a = 0$ (Assumption)

$$I_a = 0$$

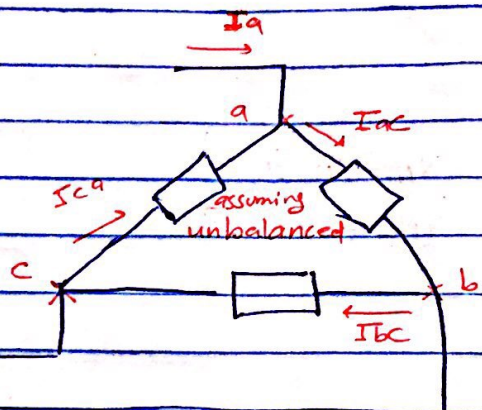
$$I_b = -I_c$$

b I_b

c I_c LL fault

$$I_a^{(0)} = \frac{1}{3} (I_a + I_b + I_c)$$

$$I_a^{(0)} = 0$$



$$I_a = \sqrt{3} I_{ab} \angle -30^\circ$$

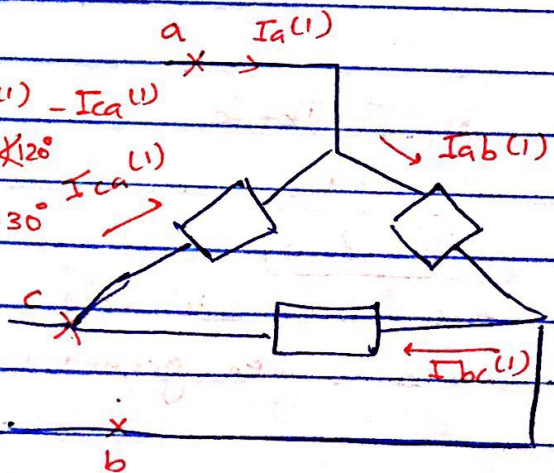
cannot be used in unbalanced.

the seq.

$$I_a^{(1)} = I_{ab}^{(1)} - I_{ca}^{(1)}$$

$$I_a^{(1)} = 1 \angle 0^\circ - 1 \angle 120^\circ$$

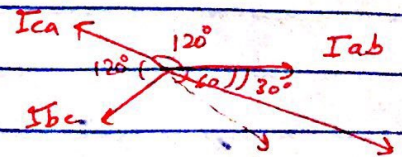
$$I_a^{(1)} = \sqrt{3} \angle -30^\circ$$



$$I_a^{(1)} = \sqrt{3} I_{ab}^{(1)} \angle -30^\circ$$

$$V_{ab}^{(0)} = \frac{1}{3} (V_{ab} + V_{bc} + V_{ca})$$

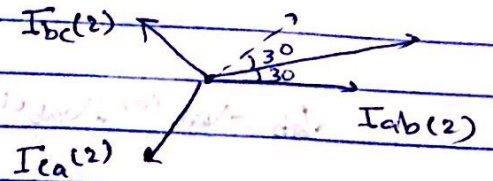
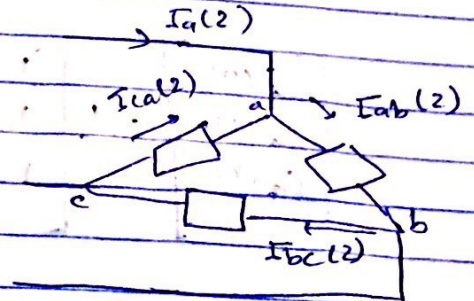
$$V_{ab}^{(0)} = 0$$



→ Ne seq.

$$I_a^{(2)} = I_{ab}^{(2)} - I_{ca}^{(2)}$$

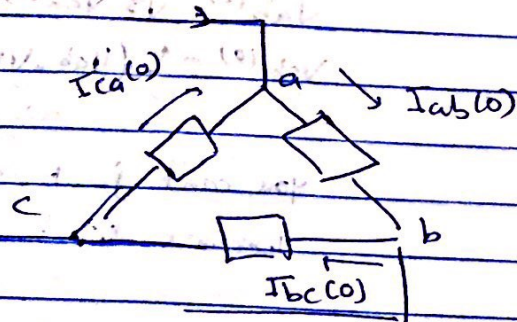
$$I_a^{(2)} = \sqrt{3} I_{ab}^{(2)} \angle 30^\circ$$



Zero seq.

$$I_{ab}^{(0)} = I_{ca}^{(0)}$$

$$I_a^{(0)} = \text{Zero}$$



* Line current of delta has a zero, zero seq. always.

$$* \therefore I_a = \sqrt{3} I_{ab}^{(1)} \angle -30^\circ + \sqrt{3} I_{ab}^{(2)} \angle 30^\circ$$

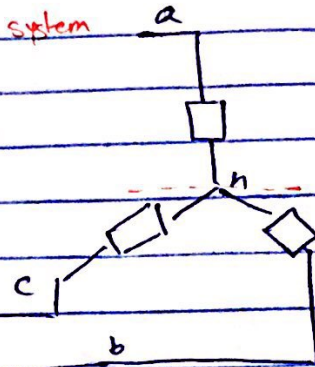
$$* I_b = I_a^{(1)} \angle -120^\circ + I_a^{(2)} \angle 120^\circ$$

$$* I_c = I_a^{(1)} \angle 120^\circ + I_a^{(2)} \angle -120^\circ$$

Tuesday 11/12/2018

$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

unbalanced system



$$V_{ab} = V_{ab}^{(0)} + V_{ab}^{(1)} + V_{ab}^{(2)}$$

$$V_{ab}^{(1)} = \sqrt{3} V_{an}^{(1)} \angle +30^\circ$$

$$V_{ab}^{(2)} = \sqrt{3} V_{an}^{(2)} \angle -30^\circ$$

$$V_{ab}^{(0)} = \frac{1}{3} (V_{ab} + V_{bc} + V_{ca}) = \text{Zero}$$

$KVL = 0$

V_{an}
 V_{bn}
 V_{cn}

$V_{ab} = V_{an} - V_{bn}$
 $V_{bc} = V_{bn} - V_{cn}$
 $V_{ca} = V_{cn} - V_{an}$

You can't just say $V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$

because this is only valid for balanced systems.

* 3 ϕ complex power:

$$S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^*$$

in terms of symmetrical components.

$$S_{3\phi} = 3V_a^{(0)} I_a^{(0)*} + 3V_a^{(1)} I_a^{(1)*} + 3V_a^{(2)} I_a^{(2)*}$$

* Sequence Networks:

ground earth

$$V_a \triangleq V_{ag} \triangleq V_{ce}$$

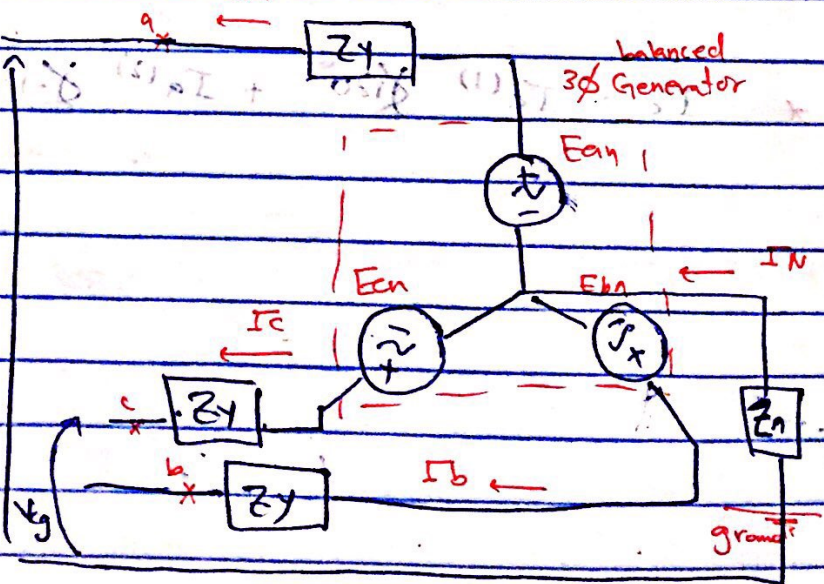
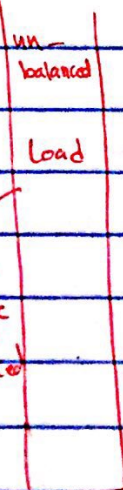
$$V_b \triangleq V_{bg} \triangleq V_{ce}$$

$$V_c \triangleq V_{cg} \triangleq V_{ce}$$

$$V_{an} \neq V_{ag}$$

(unbalanced system)

affects I_a, I_b, I_c
unbalanced



$$-V_{ag} - I_a Z_y + E_{an} - I_N Z_N = 0$$

$$V_{ag} = E_{an} - I_a Z_y - I_N Z_N \rightarrow I_a + I_b + I_c$$

* balanced only
have +ve seq.
not zero or -

$$V_{ag} = E_{an} - I_a (Z_y + Z_N) - I_b Z_N - I_c Z_N$$

coupling

$$V_{bg} = E_{bn} - I_b (Z_y + Z_N) - I_c Z_N - I_a Z_N$$

$$V_{cg} = E_{cn} - I_c (Z_y + Z_N) - I_a Z_N - I_b Z_N$$

coupling between phases

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} E_{an} \\ E_{bn} \\ E_{cn} \end{bmatrix} - \begin{bmatrix} (Z_y + Z_N) & Z_N & Z_N \\ Z_N & (Z_y + Z_N) & Z_N \\ Z_N & Z_N & (Z_y + Z_N) \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$V_{\phi} = E_{\phi} - Z_{\phi} I_{\phi}$$

$$V_{\phi} = A V_s, I_{\phi} = A I_s$$

$$A V_s = E_{\phi} - Z_{\phi} A I_s$$

$$V_s = A^{-1} E_{\phi} - (A^{-1} Z_{\phi} A) I_s$$

$$A^{-1} Z_{\phi} A = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_y + Z_N & Z_N & Z_N \\ Z_N & Z_y + Z_N & Z_N \\ Z_N & Z_N & Z_y + Z_N \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_y + 3Z_N & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix}$$

$$A^{-1} E_{\phi}$$

balanced

$$\frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \begin{bmatrix} 0 \\ E_{a(1)} \\ 0 \end{bmatrix}$$

only +ve because balanced.

$$V_s = A^{-1} E_{\phi} - (A^{-1} Z_0 A) I_s$$

$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a^{(1)} \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} Z_y + 3Z_N & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix} \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

$$V_a^{(0)} = -(Z_y + 3Z_N) I_a^{(0)}$$

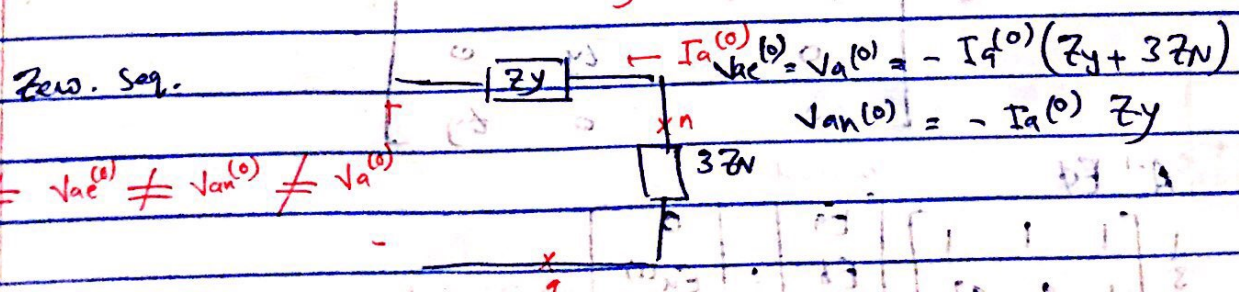
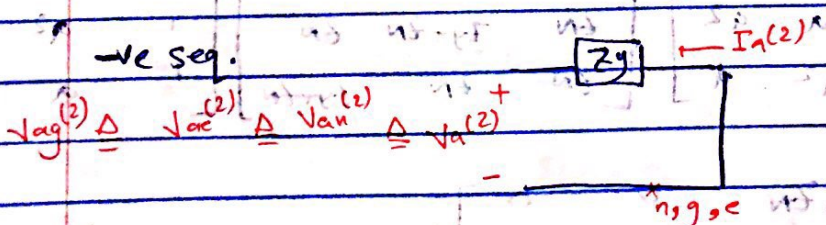
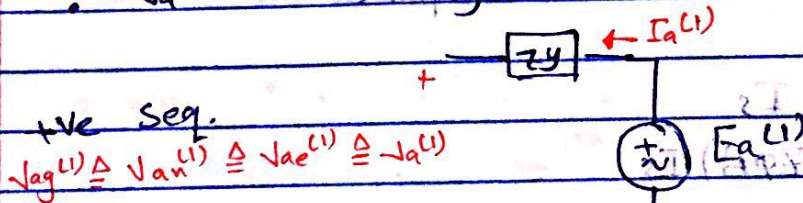
$$V_a^{(1)} = E_a^{(1)} - Z_y I_a^{(1)}$$

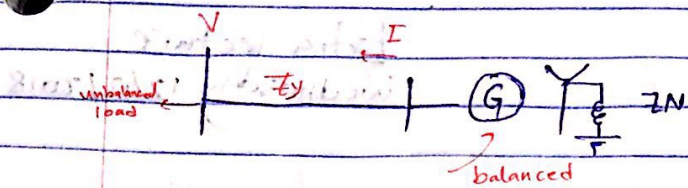
$$V_a^{(2)} = 0 - Z_y I_a^{(2)}$$

unbalanced system \rightarrow coupling between phases.

symmetrical
(uncoupled)

$$\left. \begin{aligned} V_a^{(0)} &= -(3Z_N + Z_y) I_a^{(0)} \\ V_a^{(1)} &= E_a^{(1)} - Z_y I_a^{(1)} \\ V_a^{(2)} &= 0 - Z_y I_a^{(2)} \end{aligned} \right\} \begin{aligned} &\xrightarrow{\text{find}} \begin{matrix} I_a^{(1)} \\ I_a^{(2)} \\ I_a^{(0)} \end{matrix} \Rightarrow \begin{matrix} I_a \\ I_b \\ I_c \end{matrix} \end{aligned}$$





Symmetrical components / Sequence Networks (+ve, -ve, zero)
 Seq. Networks, $V_a^{(1)}$, $V_a^{(2)}$, $V_a^{(0)}$
 $I_a^{(1)}$, $I_a^{(2)}$, $I_a^{(0)}$

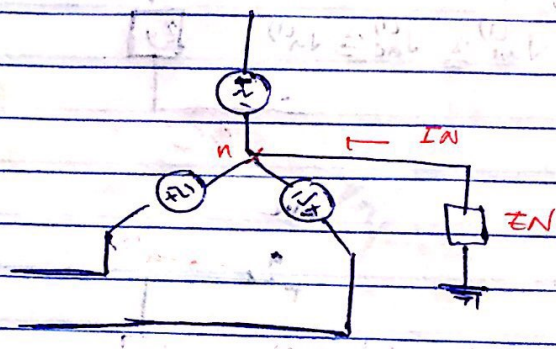
I_a , V_a
 I_b , V_b
 I_c , V_c

$$V_n = Z_n I_n$$

$$I_n = I_a + I_b + I_c$$

$$= 3 I_a^{(0)}$$

$$\therefore V_n = -3 Z_n I_a^{(0)}$$

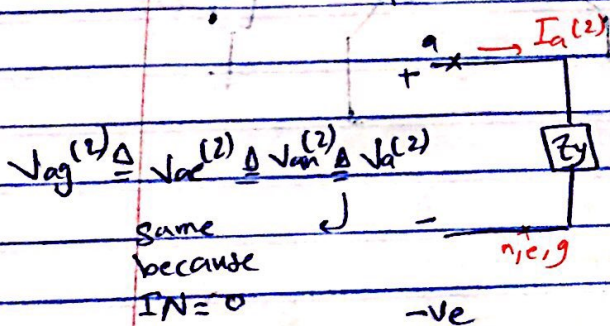
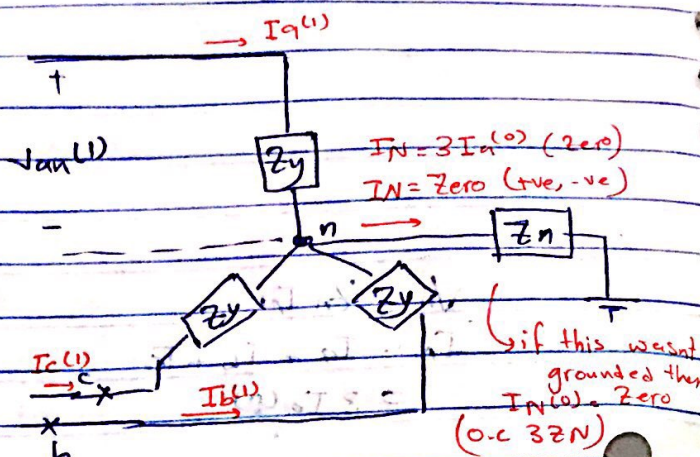
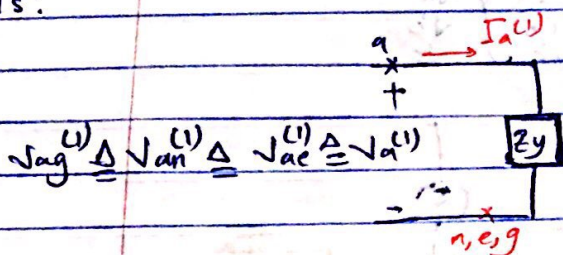


Extra lecture
Wednesday 12/12/2018

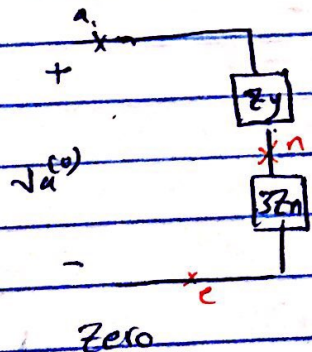
* Sequence Networks:-

- Loads +ve -ve Zero
- Generators
- T.L.S.
- Transformers

loads:

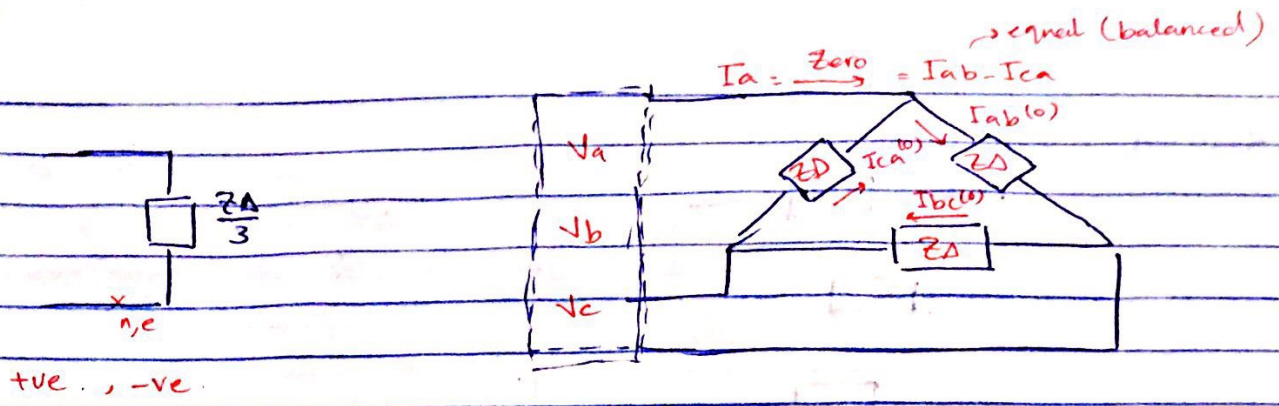


+veⁿ sequence
(balanced always)

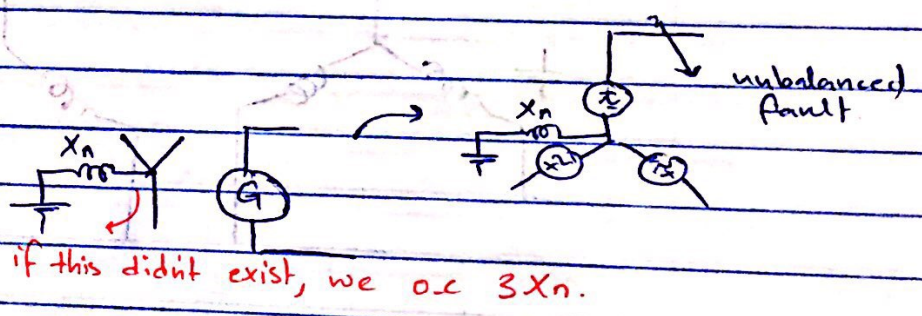
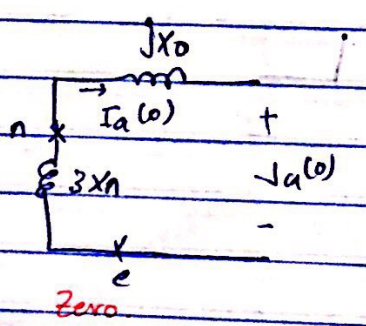
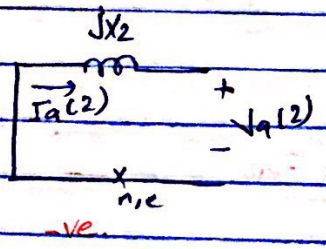
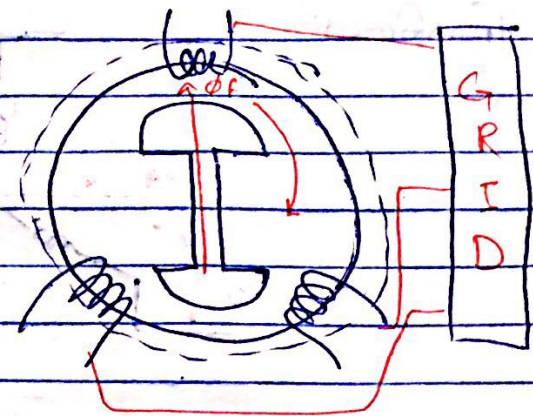
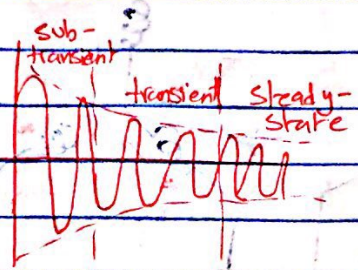
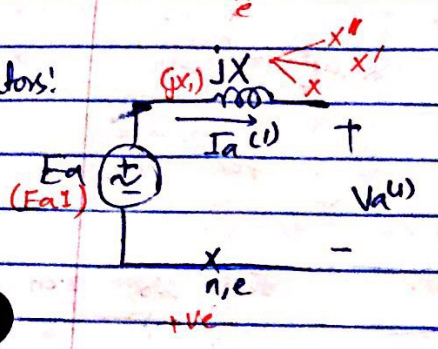


$$V_N = V_N^{(0)} + V_N^{(1)} + V_N^{(2)} \quad \bullet \quad V_N \triangleq V_{NE}$$

$$V_{NE} = 3I_a^{(0)} Z_N$$



generators!



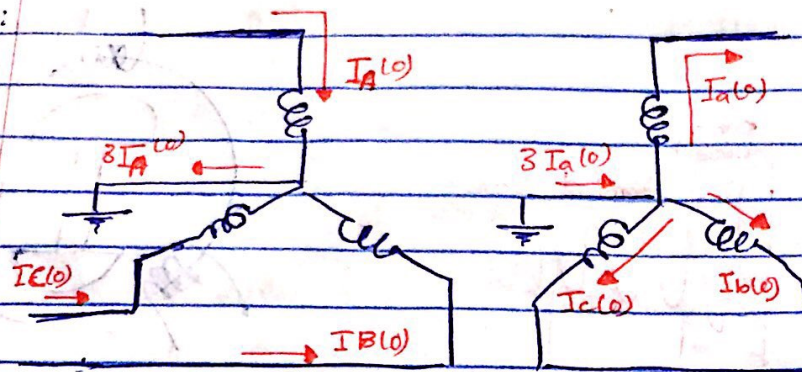
Transmission lines:

Symmetrical TL.
 $Z_a = Z_b = Z_c$



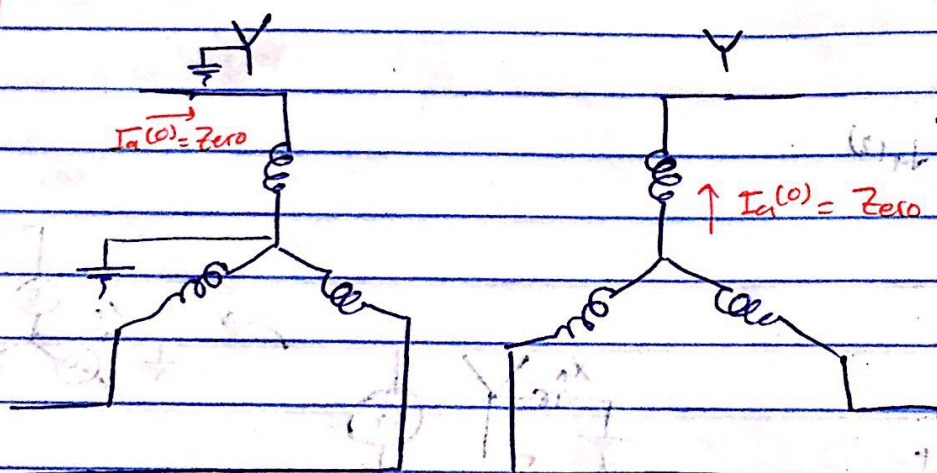
(3 times larger)
 $X_0 \gg X_2$

Transformers:



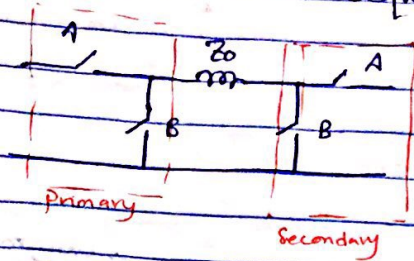
$X_1 = X_2$
 $N_1 I_A(t) = N_2 I_D(t)$
 (+ve, -ve (per-unit))

$I_A(t) X_0 = 3Z_n$
 $+ V_A(t)$

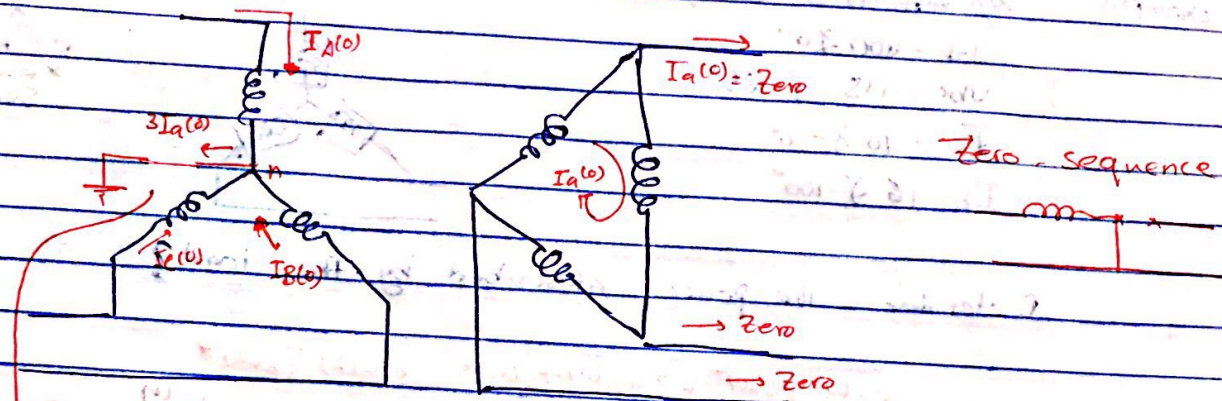


Z_0

* How to find Zero sequence of transformer:

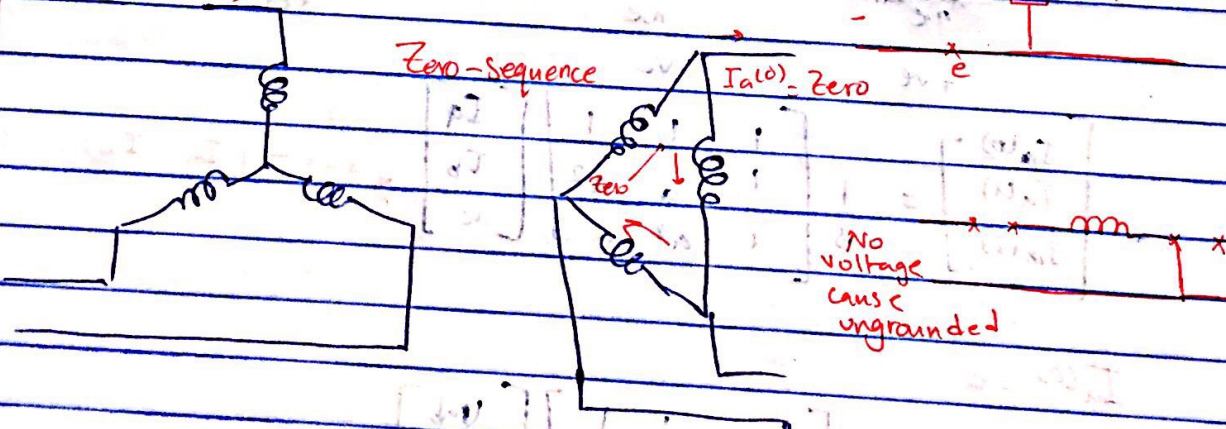


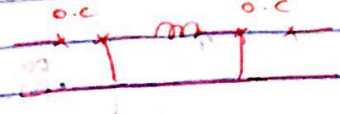
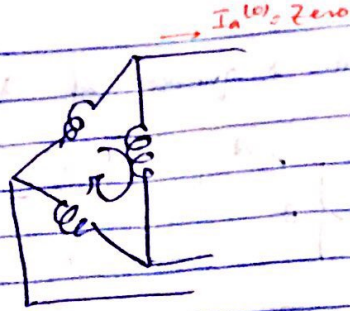
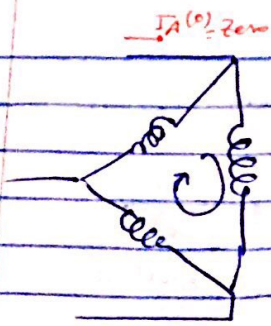
	A	B
	closed	opened
	opened	closed
	opened	opened
	there's no ground for Δ .	



if we add $-[Z_0]$ then \Rightarrow Zero Sequence!

(ungrounded) $\rightarrow I_A(0) = \text{Zero}$





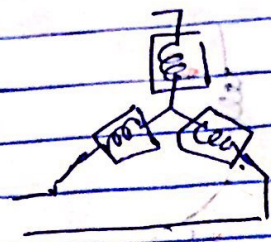
Example: 3 ϕ unbalanced voltages are applied to a star/ (ungrounded) balanced load.

$$V_{ab} = 400 \angle 0^\circ$$

$$V_{bc} = 415 \angle -135^\circ$$

$$I_a = 10 \angle -30^\circ$$

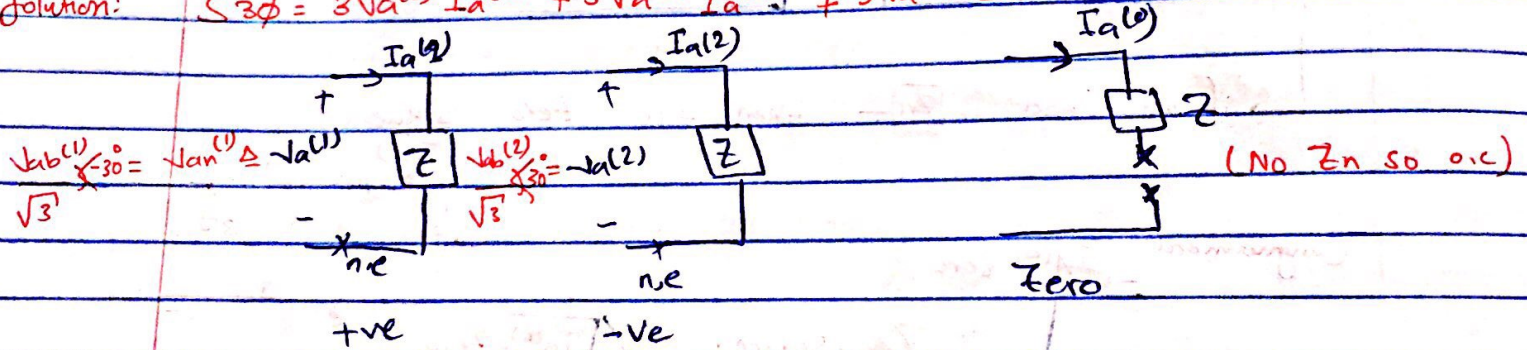
$$I_b = 16 \angle -100^\circ$$



$$\sum I = 0$$

Determine the power absorbed by the load?

Solution: $S_{3\phi} = 3V_a^{(0)} I_a^{(0)*} + 3V_a^{(1)} I_a^{(1)*} + 3V_a^{(2)} I_a^{(2)*}$



$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_c = -(I_a + I_b)$$

$$\sum I = 0$$

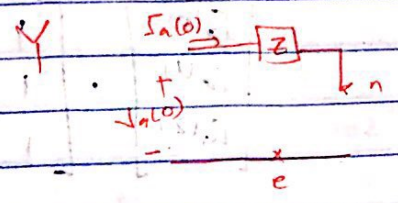
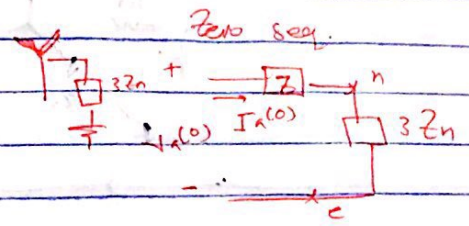
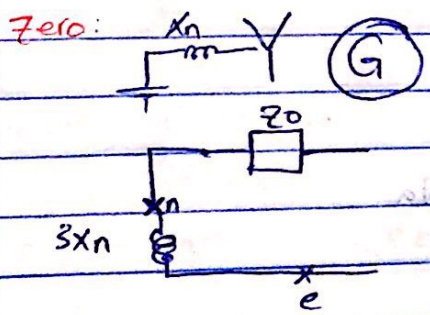
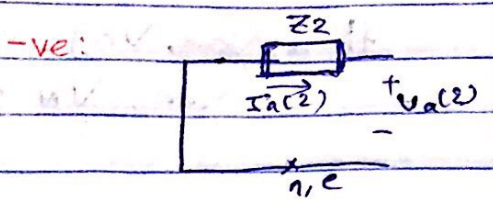
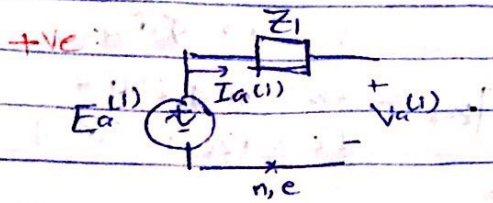
$$I_a, I_b, I_c$$

$$I_a^{(0)} = 0$$

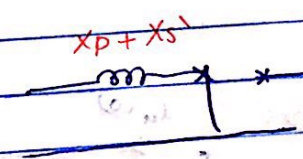
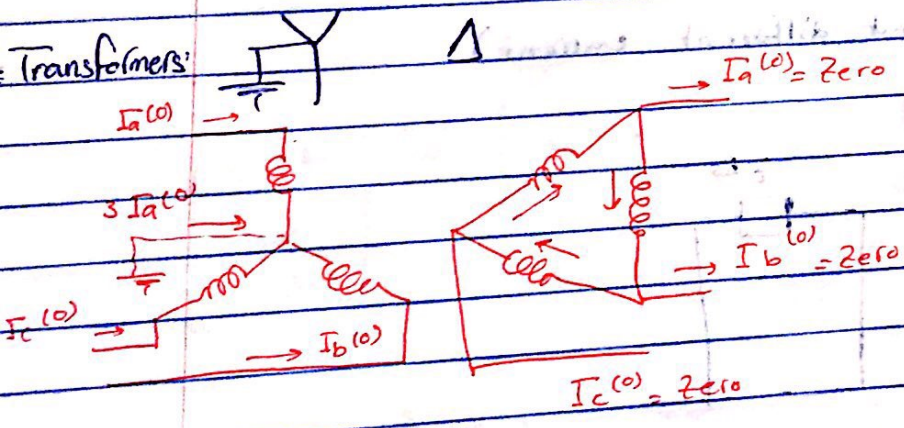
$$\begin{bmatrix} V_{ab}^{(0)} \\ V_{ab}^{(1)} \\ V_{ab}^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}$$

$$V_{ab}^{(0)} = \text{zero}$$

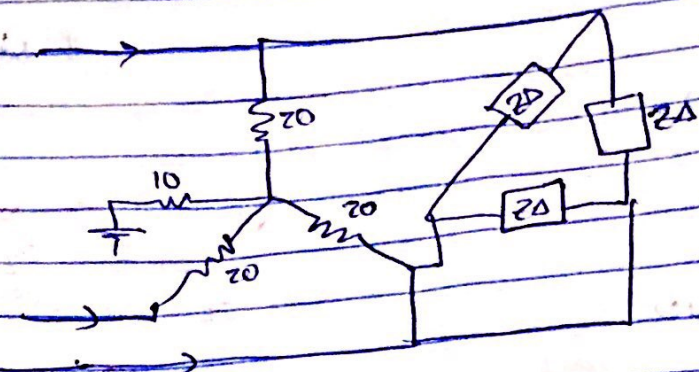
Thursday 13/12/2018



Transformers:



Ex: 11.11



$$I_a = 100 \angle 0^\circ \text{ A}$$

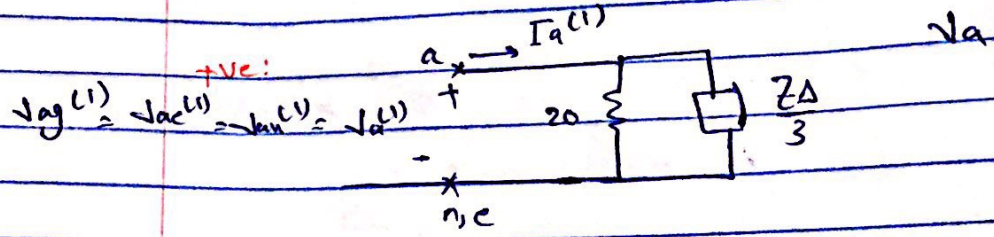
$$I_b = 80 \angle 240^\circ \text{ A}$$

$$I_c = 120 \angle 130^\circ \text{ A}$$

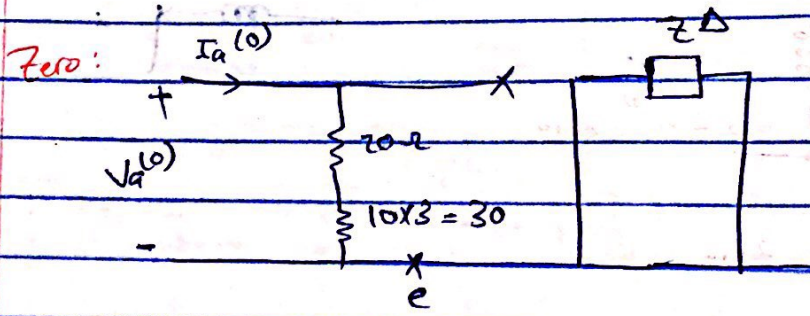
Find $V_{an}, V_{ag}, V_{ab}, V_a$
 V_{ac}, V_N ??

Solution:

$$\begin{bmatrix} I_a^{(1)} \\ I_a^{(2)} \\ I_a^{(3)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$



-ve: (same as +ve but different current)



$$V_{an} = V_{an}^{(1)} + V_{an}^{(2)} + V_{an}^{(3)}$$

$$V_{an} = I_a^{(1)} \left(20 \parallel \frac{20}{3} \right) + I_a^{(2)} \left(20 \parallel \frac{20}{3} \right) + I_a^{(3)} (20)$$

$$V_{ac} = V_{an} + V_N$$

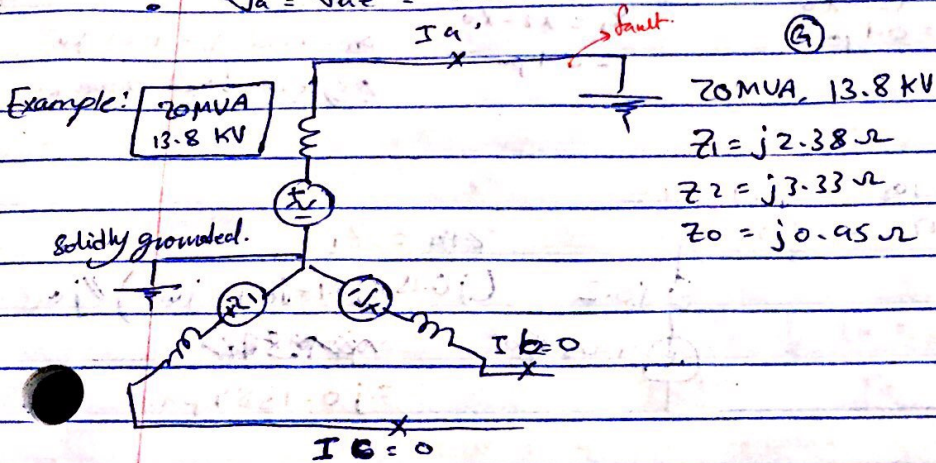
$$V_{ac} = V + 3I_a^{(3)} (10) \rightarrow 2N.$$

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{an(0)} \\ V_{an(1)} \\ V_{an(2)} \end{bmatrix}$$

$$V_{ab} = V_{an} - V_{bn}$$

$$V_{bc} = V_{bn} - V_{cn}$$

$$V_a = V_{an} = V_{an} + V_n$$



$$Z_1 = j2.38 \Omega$$

$$Z_2 = j3.33 \Omega$$

$$Z_0 = j0.95 \Omega$$

find V_{ag} ?

$$V_{bg} = 8.071 \angle 102^\circ \text{ kV}$$

$$V_{cg} = 8.071 \angle 102^\circ \text{ kV}$$

Solve in pu?

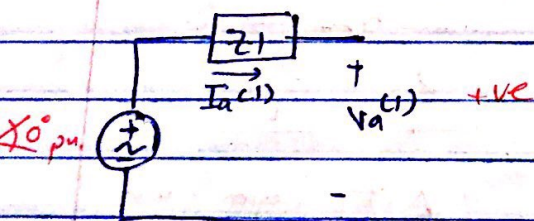
V_{ab}, V_{bc}, V_{ca} ?

(system unloaded

$V_f = 1 \angle 0^\circ \text{ pu}$ (pre fault voltage).

Sol: $V_{ag} = \text{zero}$

$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_c \end{bmatrix}$$

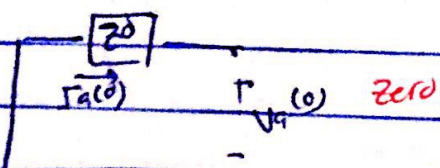
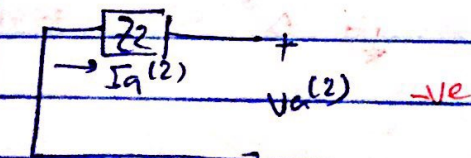


$$V_{ab} = V_{ag} - V_{bg}$$

$$I_a^{(1)} = 1 \angle 0^\circ = \frac{V_a^{(1)}}{Z_1}$$

$$I_a^{(2)} = -\frac{V_a^{(2)}}{Z_2}$$

$$I_a^{(0)} = -\frac{V_a^{(0)}}{Z_0}$$



$$I_a = I_a^{(0)} + I_a^{(1)} + I_a^{(2)}$$

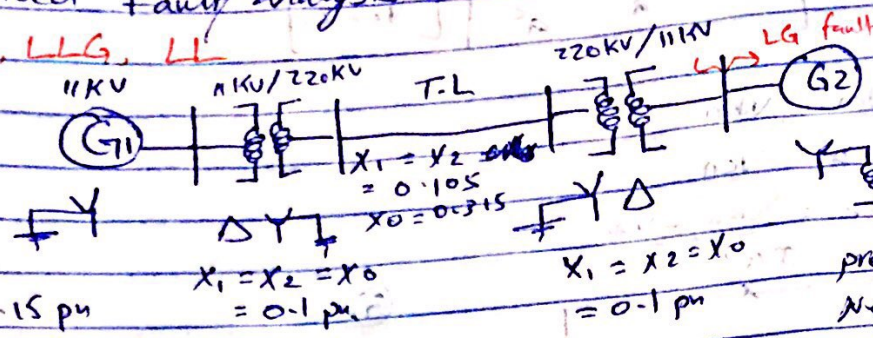
fault current

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

Sunday 16/12/2018

* Unbalanced fault analysis:
 LG, LLG, LL

Example:
 base:
 100 MVA



$X^{(1)} = 0.2 \text{ pu}$
 $X^{(2)} = 0.21 \text{ pu}$
 $X_0 = 0.1 \text{ pu}$

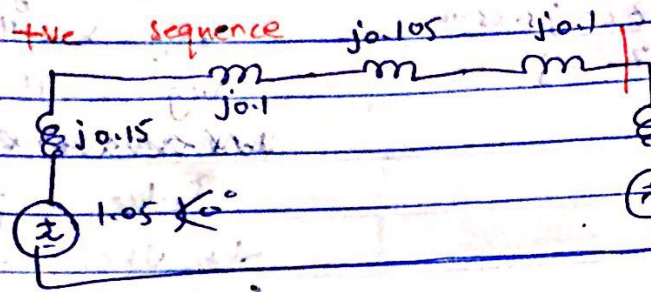
$X^{(1)} = 0.15 \text{ pu}$
 $X^{(2)} = 0.17 \text{ pu}$
 $X_0 = 0.05 \text{ pu}$

$X_1 = X_2 = X_0 = 0.1 \text{ pu}$

$X_1 = X_2 = X_0 = 0.1 \text{ pu}$

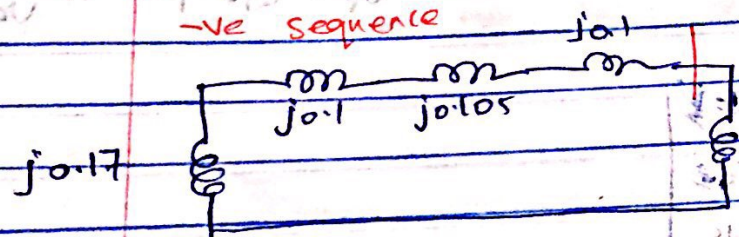
pre-fault $V = 1.05 \text{ kV}$
 Neglect Load current.

Solution:



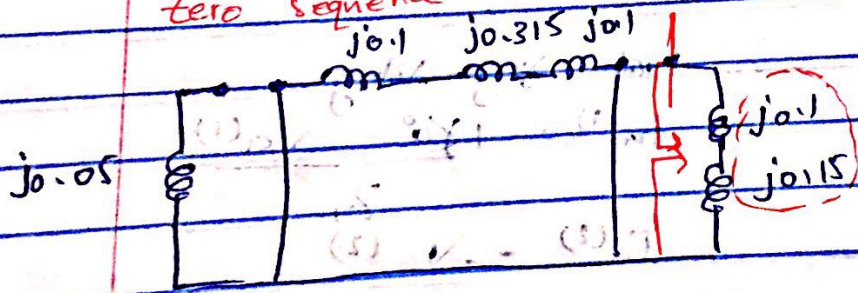
$Z_{th} = Z_1 = (j0.15 + j0.1 + j0.1 + j0.1) \parallel j0.2$
 $= j0.1389 \text{ pu}$

-ve sequence



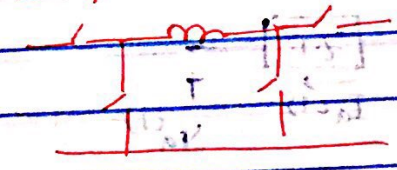
$Z_{th} = Z_2 = j0.1456 \text{ pu}$

zero sequence



Thevenin: $Z_{th} = Z_0 = j0.25$

$\Delta \rightarrow$ open



$\Delta \rightarrow$ open close
 $\rightarrow \Delta$ close open

$$E^{(1)} = 1.05 \angle 0^\circ \text{ pu} = V_F$$

$$Z_1 = j0.1389 \text{ pu}$$

$$Z_2 = j0.1456 \text{ pu}$$

$$Z_0 = j0.25 \text{ pu}$$

* unloaded

LG

$$\begin{matrix} a \rightarrow I_f \\ b \rightarrow \text{zero} \\ c \rightarrow \text{zero} \end{matrix} \quad \left. \begin{matrix} \\ \\ \\ \end{matrix} \right\} V_a = 0$$

$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \rightarrow \text{zero} \\ I_c \rightarrow \text{zero} \end{bmatrix}$$

$V_a = 0 \rightarrow I_f Z_f$ (0.5 pu)
 $V_a^{(0)} + V_a^{(1)} + V_a^{(2)} = 0$ (stand by) (0.5 pu)

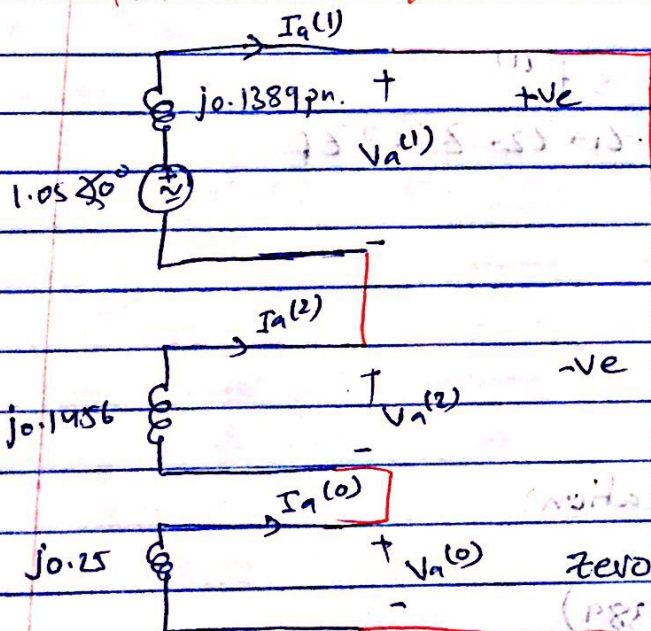
$$I_a^{(0)} = \frac{1}{3} I_a = \frac{1}{3} I_f$$

$$I_a^{(0)} = I_a^{(1)} = I_a^{(2)}$$

$$I_a^{(1)} = \frac{1}{3} I_a = \frac{1}{3} I_f$$

$$I_a^{(2)} = \frac{1}{3} I_a$$

+ve, -ve, zero eqn. ckts are coupled.



$$I_a^{(1)} = I_a^{(2)} = I_a^{(0)}$$

$$I_a^{(0)} = \frac{1.05 \angle 0^\circ}{Z_1 + Z_2 + Z_0}$$

$$I_f = I_a^{(1)} + I_a^{(2)} + I_a^{(0)} = 3 I_a^{(0)}$$

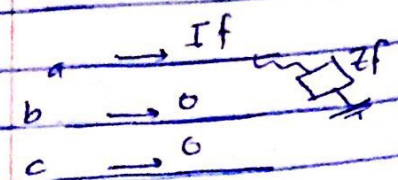
$$|I_f| = 3 \times 1.05 \angle 0^\circ$$

(inside of LG) $Z_1 + Z_2 + Z_0$

$$I_a^{(0)} = -j1.964 \text{ pu}$$

$$I_f = 3 \times (-j1.964) = -j5.892 \text{ pu}$$

if we had Zf load



$$V_a = I_f Z_f$$

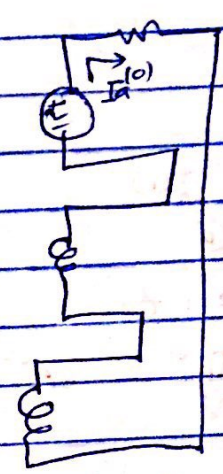
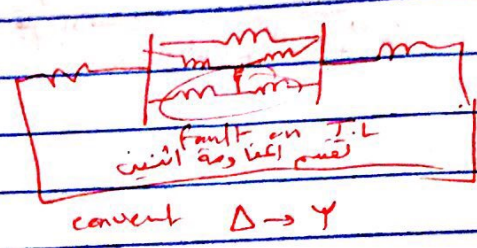
$$V_a^{(0)} + V_a^{(1)} + V_a^{(2)} = (I_a^{(0)} + I_a^{(1)} + I_a^{(2)}) Z_f$$

$$V_a^{(0)} + V_a^{(1)} + V_a^{(2)} = 3 I_a^{(0)} Z_f$$

$$I_a^{(0)} = \frac{1.05 \angle 0^\circ}{Z_1 + Z_2 + Z_0 + 3Z_f}$$

steps:

- ① +ve seq. \rightarrow thevinin
- ② -ve seq. \rightarrow thevinin
- ③ zero seq. \rightarrow thevinin



$$I_f = 3I_a^{(0)} = \frac{3 E^{(1)}}{Z_1 + Z_2 + Z_0' + 3Z_f}$$

* going back to previous page question (no Zf)

V_a, V_b, V_c at fault location:

$$V_a^{(1)} = 1.05 \angle 0^\circ - I_a^{(1)} (j0.1389)$$

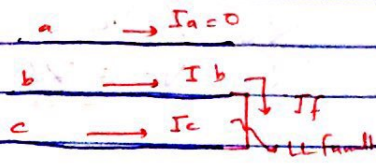
$$V_a^{(2)} = - I_a^{(2)} (j0.1456)$$

$$V_a^{(0)} = - I_a^{(0)} (j0.25)$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix}$$

- $V_a = 0$
- $V_b = 1.166 - j0.178 \text{ pu}$
- $V_c = 1.166 + j0.178 \text{ pu}$

LL fault :



Current analysis

$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_a^{(0)} = \frac{1}{3} (I_a + I_b + I_c) = 0$$

$$I_a^{(1)} = \frac{1}{3} (a I_b - a^2 I_c)$$

$$I_a^{(2)} = \frac{1}{3} (a^2 I_b - a I_c)$$

- $I_a^{(0)} = 0$
- $I_a^{(1)} = -I_a^{(2)}$

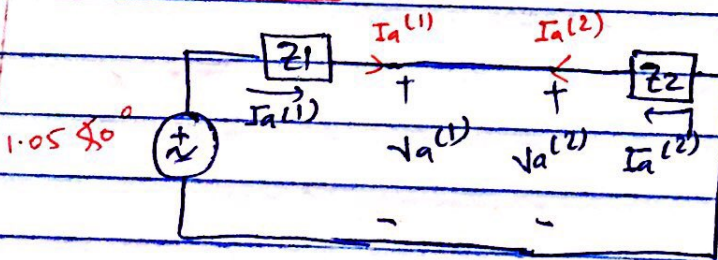
Voltage analysis

$$V_b = V_c$$

$$V_a^{(0)} + a^2 V_a^{(1)} + a V_a^{(2)} = V_a^{(0)} + a V_a^{(1)} + a^2 V_a^{(2)}$$

$$V_a^{(1)} (a^2 - a) = V_a^{(2)} (a^2 - a)$$

$$V_a^{(1)} = V_a^{(2)}$$



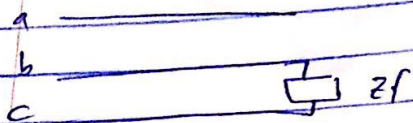
$$I_a^{(1)} = \frac{1.05}{Z_1 + Z_2} = 3.691 \angle -90^\circ$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 3.691 \angle -90^\circ \\ -3.691 \angle -90^\circ \end{bmatrix}$$

$$\begin{bmatrix} I_a = 0 \\ I_b = -6.39 \\ I_c = 6.39 \end{bmatrix}$$

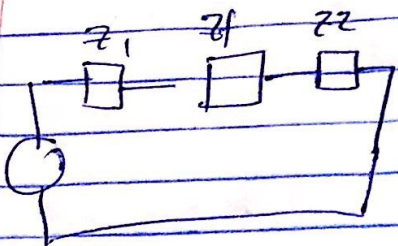
$$I_b = I_f$$

if we had Z_f



$$V_b \neq V_c$$

$$V_b - V_c = Z_f I_f$$



$$I_a^{(1)} = \frac{1.05}{Z_1 + Z_2 + Z_f}$$

$$I_f = I_b = I_a^{(0)} + a^2 I_a^{(1)} + a I_a^{(2)}$$

$$= a^2 I_a^{(1)} - a I_a^{(1)}$$

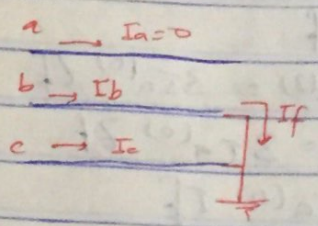
LL $\rightarrow I_f = \sqrt{3} \frac{E^{(1)}}{Z_1 + Z_2 + Z_f}$

$$I_f = (\underbrace{a^2 - a}_{\sqrt{3}}) I_a^{(1)}$$

$$(1 \angle 120^\circ - 1 \angle 240^\circ) = \sqrt{3}$$

LG $\rightarrow I_f = \frac{3 \times E^{(1)}}{Z_1 + Z_2 + Z_0 + 3Z_f}$

* Double Line to ground: (LLG)



• $I_a = 0$ (unloaded)
 $I_a^{(0)} = I_a^{(1)} = I_a^{(2)} = 0$ --- (1)

• $V_b = V_c$
 $V_a^{(0)} + a^2 V_a^{(1)} + a V_a^{(2)} = V_a^{(0)} + a V_a^{(1)} + a^2 V_a^{(2)}$
 $V_a^{(1)} (a^2 - a) = V_a^{(2)} (a^2 - a)$

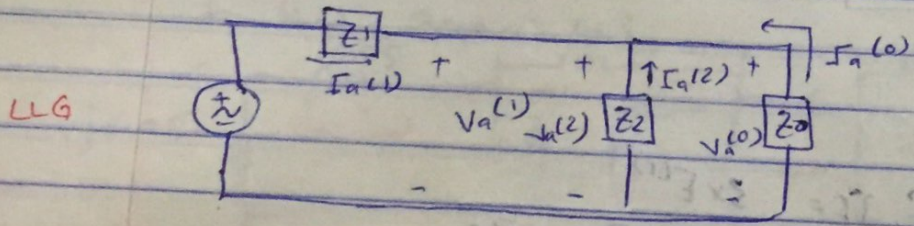
$V_a^{(1)} = V_a^{(2)}$ --- (2)

• $V_b = 0$
 $V_a^{(0)} + a^2 V_a^{(1)} + a V_a^{(2)} = 0$
 $V_a^{(0)} + a^2 V_a^{(1)} + a V_a^{(1)} = 0$
 $V_a^{(0)} + V_a^{(1)} (a^2 + a) = 0$

$V_a^{(0)} = -V_a^{(1)}$ --- (3)

$V_a = V_a^{(0)} + V_a^{(1)} + V_a^{(2)}$
 find V_b, V_c from matrix.

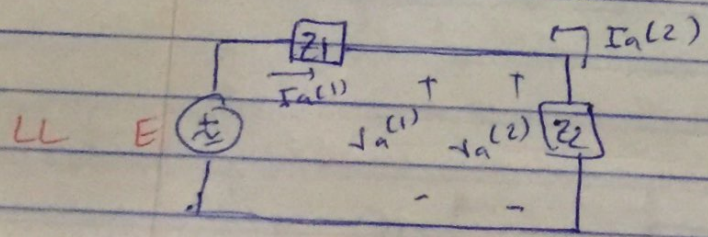
$I_f = I_b + I_c$
 $(I_a + I_b + I_c) \frac{1}{3} = I_a^{(0)}$
 $I_f = 3 I_a^{(0)}$



equ. ckt of LLG.

$I_f = 3 I_a^{(0)}$

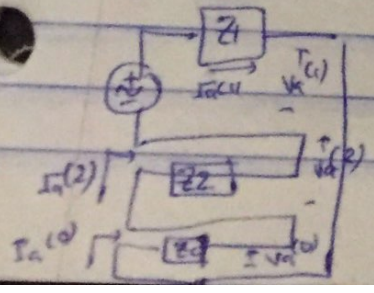
→ you have to find $I_a^{(0)}$ from the equ. ckt.

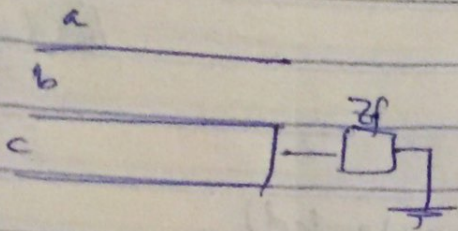


$I_f = I_b$
 $I_b = I_a^{(0)} + a^2 I_a^{(1)} + a I_a^{(2)}$

$I_b = I_a^{(1)} (a^2 - a)$

$|I_f| = \frac{\sqrt{3} E}{Z_1 + Z_2}$





$$V_a^{(1)} = V_a^{(2)}$$

$$I_a^{(0)} + I_a^{(1)} + I_a^{(2)} = 0$$

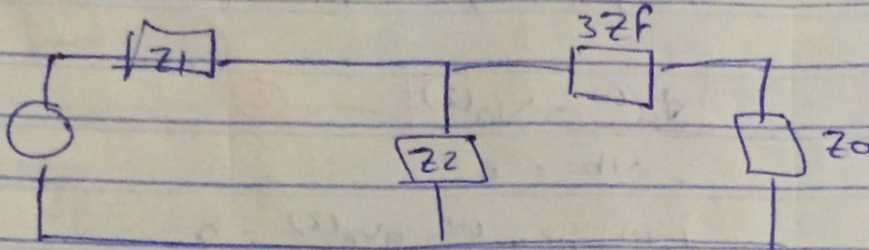
$$V_b = I_f Z_f$$

$$V_a^{(0)} + a^2 V_a^{(1)} + a V_a^{(2)} = 3 I_a^{(0)} Z_f$$

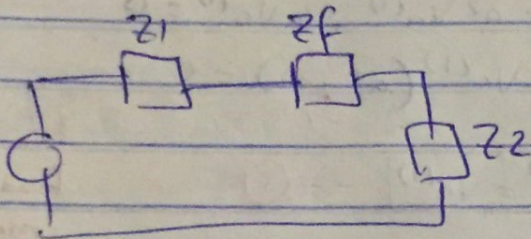
$$V_a^{(0)} + V_a^{(1)} (a^2 + a) = 3 I_a^{(0)} Z_f$$

$$V_a^{(0)} - V_a^{(1)} = 3 I_a^{(0)} Z_f$$

LLG

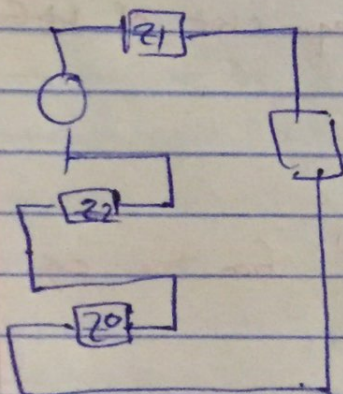


LL



$$I_f = \frac{\sqrt{3} E}{Z_1 + Z_2 + Z_f}$$

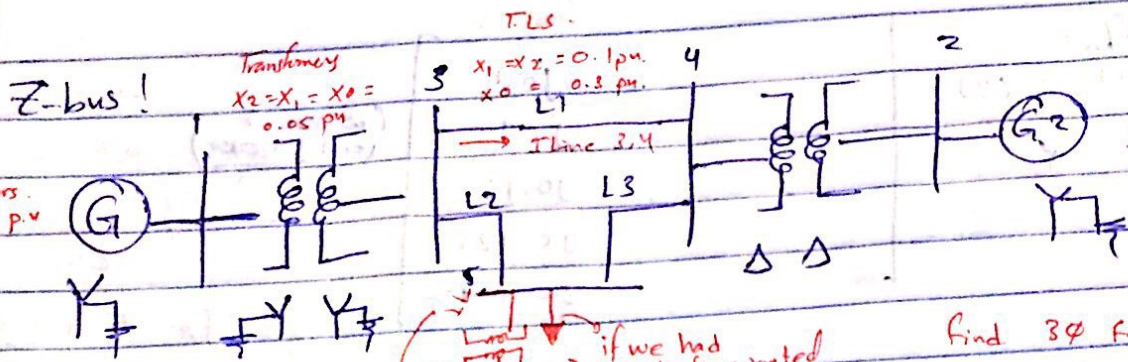
LG



$$I_f = \frac{3 \times E^{(1)}}{Z_1 + Z_2 + Z_0 + 3Z_f}$$

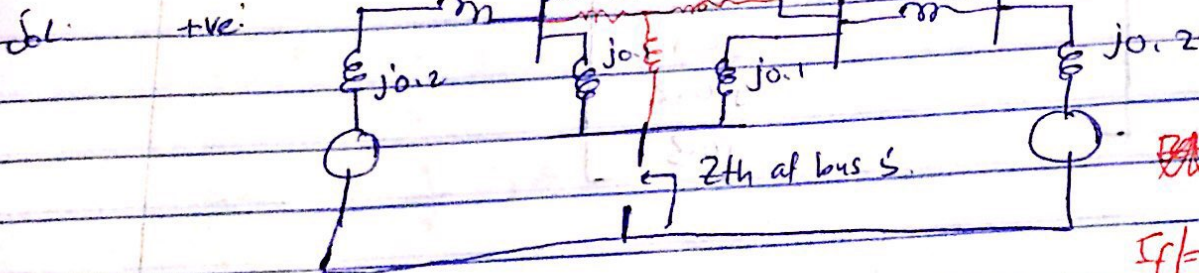
Example:

Generators:
 $X_1 = X_2 = 0.2 \text{ pu}$
 $X_0 = 0.05 \text{ pu}$



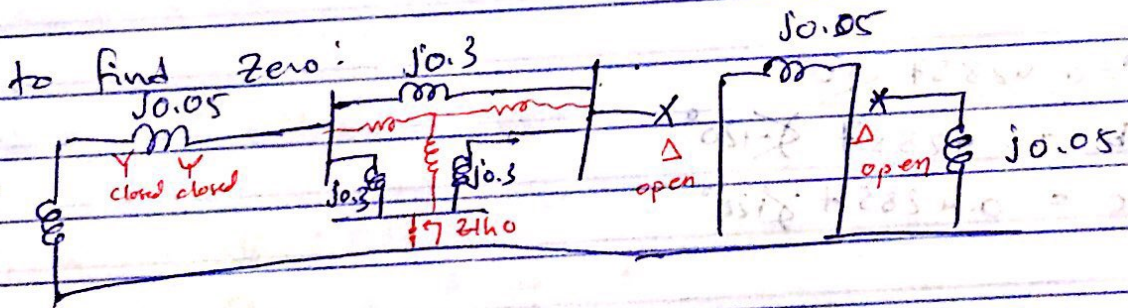
if we had a load & wanted to solve using internal voltage.

find 3 ϕ fault at bus
 find LG = ... =
 (unloaded system
 $V_f = 1 \angle 0^\circ \text{ pu}$)



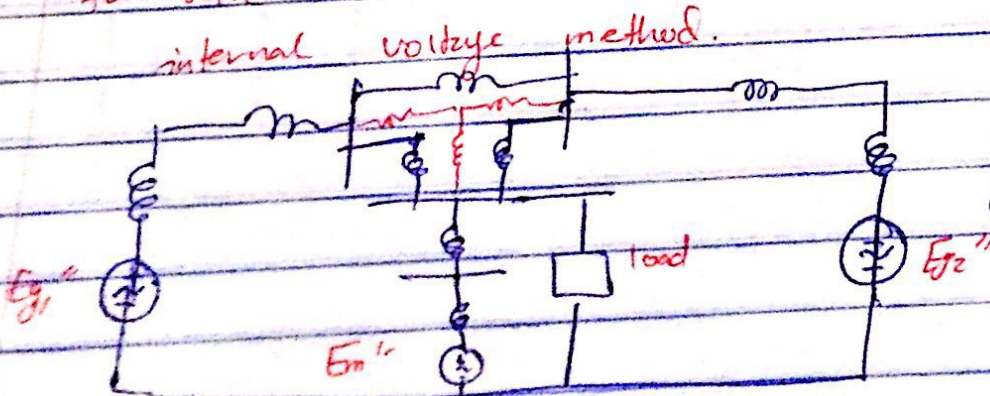
$I_f = \frac{1}{Z_{th}} \text{ p.u.}$
 3 ϕ

$Z_{th2} = Z_{th1}$ cause $X_1 = X_2$.



$I_{LG} = \frac{3 \times 1 \angle 0^\circ}{Z_1 + Z_2 + Z_0} \text{ p.u.}$

to solve with load & motor: find 3 ϕ fault at bus 3 using internal voltage method.



$S_3 = V_s I_s^*$
 $G_1 \rightarrow V_s I_{line} G_1^* = S_{line}$
 $G_2 \rightarrow V_s I_{line} G_2^* = S_{line}$

using Zbus:

$$Z_{bus}$$

$$Z^{(1)} = Z^{(2)} =$$

$$Z = Y^{-1}$$

$$\begin{bmatrix} \text{bus 5} \\ j0.1 \\ j0.1 \\ j0.125 \\ j0.125 \\ j0.175 \end{bmatrix}$$

(without loop) find all fault at bus 5
 $I_f = V_f / Z_{SS} = 1 \angle 0^\circ$
 $Z_{SS} = j0.175$
 $I_{f,a} = 5.714 \angle -90^\circ$
 $I_{f,b} = 5.714 \angle -210^\circ$
 $I_{f,c} = 5.714 \angle 30^\circ$

$$Z^{(0)}$$

$$Z = Y^{-1}$$

$$\begin{bmatrix} \text{bus 5} \\ j0.05 \\ 0 \\ j0.1 \\ j0.2 \\ j0.3 \end{bmatrix}$$

V?

$$V_1 = V_f + \Delta V_1$$

$$V_1 = V_f - Z_{15} I_f$$

$$V_{1,a} = 0.42857 \angle 0^\circ$$

$$V_{1,b} = 0.42857 \angle -120^\circ$$

$$V_{1,c} = 0.42857 \angle 120^\circ$$

$$I_{\text{line } 3,4} = \frac{V_3 - V_4}{j0.1}$$

$$V_3 = V_f - Z_{35} I_f$$

$$V_4 = V_f - Z_{45} I_f$$

* Z_{bus} (Fault at bus 5)

$$I_{LG} = \frac{3 \times V_f}{Z_{SS}^{(1)} + Z_{SS}^{(0)} + Z_{SS}^{(2)}}$$

$$I_{fa}^{(0)} = \frac{1}{3} I_{LG}$$

if we asked for LL you remove this

$$I_{fa}^{(1)} = \frac{1}{3} I_{LG}$$

$$I_{fa}^{(2)} = \frac{1}{3} I_{LG}$$

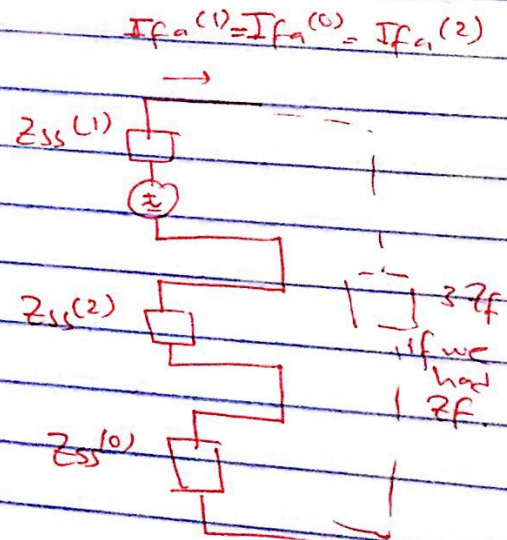
find V_i ?? $\left\{ \begin{array}{l} V_{i,a} \\ V_{i,b} \\ V_{i,c} \end{array} \right\}$ unbalanced system.

$$V_i = V_{i,a}^{(0)} + V_{i,a}^{(1)} + V_{i,a}^{(2)}$$

$$V_{i,a}^{(1)} = V_f - Z_{15}^{(1)} I_{fa}^{(1)}$$

$$V_{i,a}^{(2)} = 0 - Z_{15}^{(2)} I_{fa}^{(2)}$$

$$V_{i,a}^{(0)} = 0 - Z_{15}^{(0)} I_{fa}^{(0)}$$



$$\begin{bmatrix} V_{i,a} \\ V_{i,b} \\ V_{i,c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{i,a}^{(0)} \\ V_{i,a}^{(1)} \\ V_{i,a}^{(2)} \end{bmatrix}$$