L06
Practical *Operational Amplifiers*
Op-Amps 2

Chapter 14

Nonideal Effects in Operational Amplifier

Circuits*Donald A. Neamen (2009).* ***Microelectronics****: Circuit Analysis and Design,*

4th Edition, Mc-Graw-Hill

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14.2.3 Inverting Amplifier Closed-Loop Input Resistance

- The closed-loop input resistance R_{if} of the inverting amplifier is defined in Figure 14.4(a).
  - It includes the effect of feedback.
- The equivalent circuit includes:
  1. Finite open-loop gain A_{OL},
  2. Finite open-loop input differential resistance R_i,
  3. Nonzero output resistance R_o.

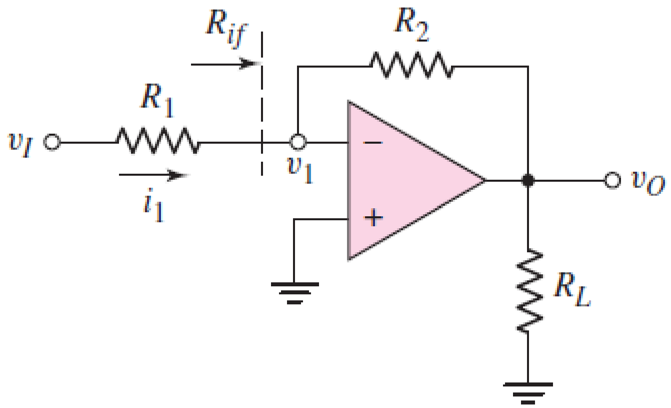


Figure 14.4(a)

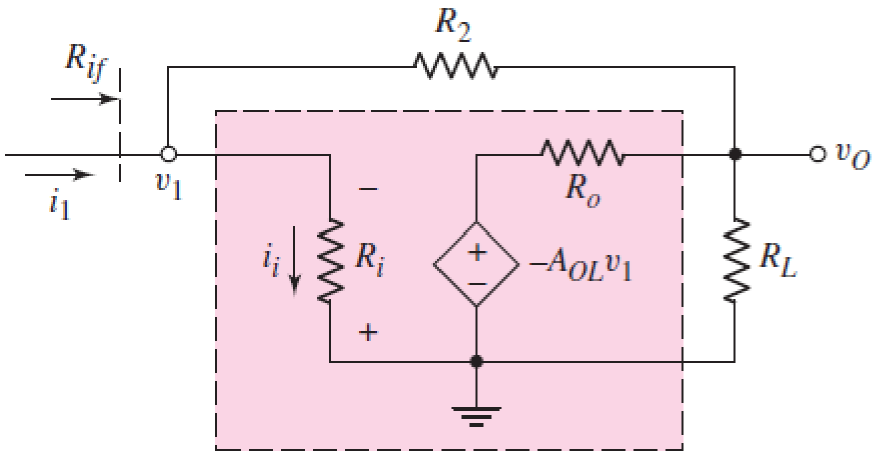


Figure 14.4(b)

14.2.3 Inverting Amplifier Closed-Loop Input Resistance

- A **KCL1** equation at the output node yields:

$$\frac{v_O}{R_L} + \frac{v_O - (-A_{OL} v_1)}{R_o} + \frac{v_o - v_1}{R_2} = 0$$

- Solving for the output voltage, we have:

$$v_O = \frac{-v_1 \left(\frac{A_{OL}}{R_o} - \frac{1}{R_2} \right)}{\frac{1}{R_L} + \frac{1}{R_o} + \frac{1}{R_2}}$$

- A second **KCL2** equation at the input node yields:

$$i_1 = \frac{v_1}{R_i} + \frac{v_1 - v_O}{R_2}$$

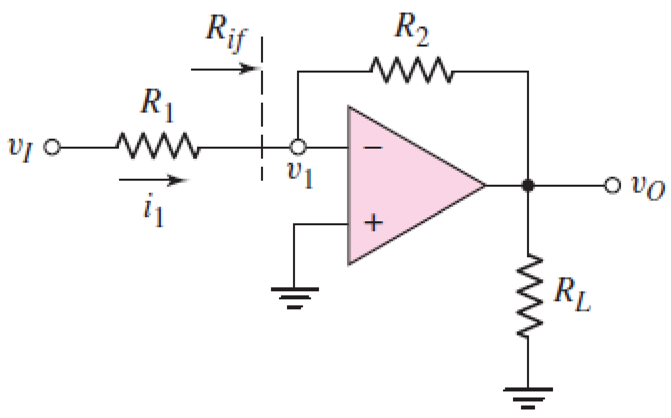


Figure 14.4(a)

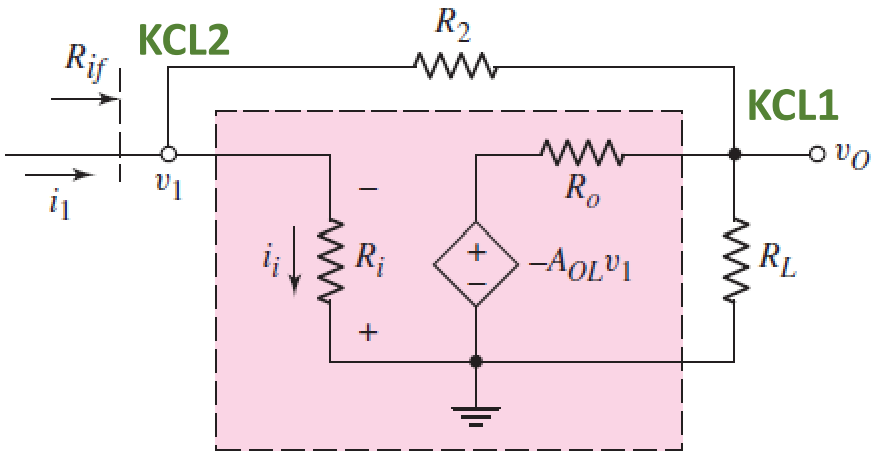


Figure 14.4(b)

14.2.3 Inverting Amplifier Closed-Loop Input Resistance

- Combining the following two equations:

$$v_O = \frac{-v_1\left(\frac{A_{OL}}{R_o} - \frac{1}{R_2}\right)}{\frac{1}{R_L} + \frac{1}{R_o} + \frac{1}{R_2}}$$

$$i_1 = \frac{v_1}{R_i} + \frac{v_1 - v_O}{R_2}$$

and rearranging terms produces:

$$\frac{i_1}{v_1} = \frac{1}{R_{if}} = \frac{1}{R_i} + \frac{1}{R_2}\frac{\left(1 + A_{OL} + \frac{R_o}{R_L}\right)}{1 + \frac{R_o}{R_L} + \frac{R_o}{R_2}}$$

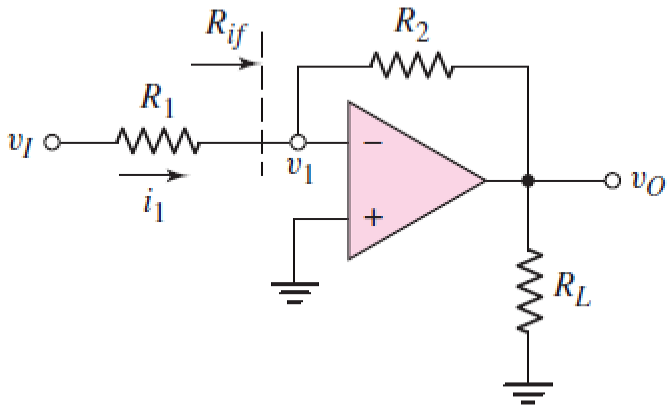


Figure 14.4(a)

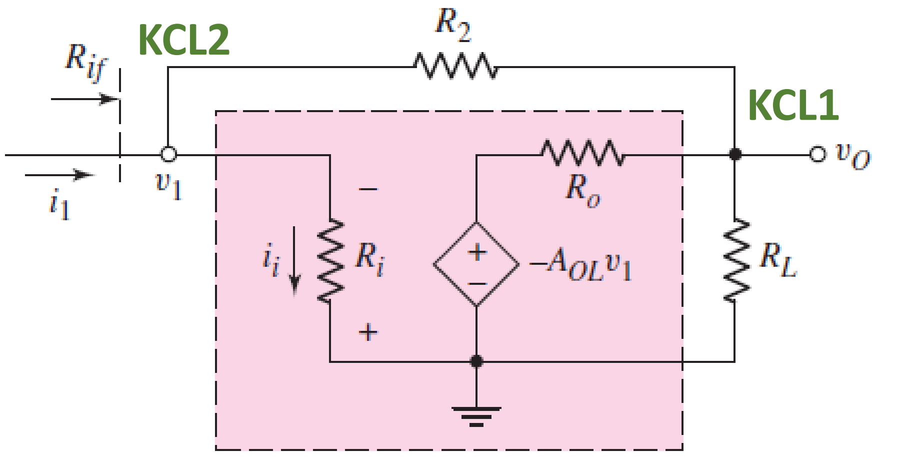


Figure 14.4(b)

14.2.3 Inverting Amplifier
Closed-Loop Input Resistance

$$\frac{i_1}{v_1} = \frac{1}{R_{if}} = \frac{1}{R_i} + \frac{1}{R_2}\frac{\left(1 + A_{OL} + \frac{R_o}{R_L}\right)}{1 + \frac{R_o}{R_L} + \frac{R_o}{R_2}}$$

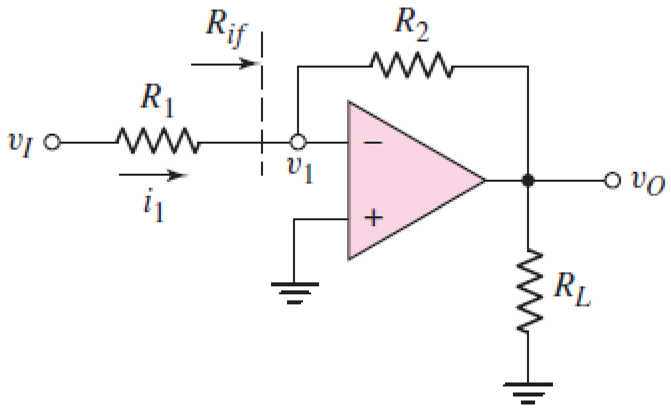


Figure 14.4(a)

- The equation above describes the closed-loop input resistance R_{if} of the inverting amplifier, with:
  1. Finite open-loop gain A_{OL},
  2. Finite open-loop input resistance R_i, and
  3. Nonzero output resistance R_o.
- In the limit as $A_{OL} \to \infty$, we see that $1/R_{if} \to \infty$, or $R_{if} \to 0$.
  - Which means that $v_1 \to 0$, or v_1 is at virtual ground.
- This is a characteristic of an ideal inverting op-amp.

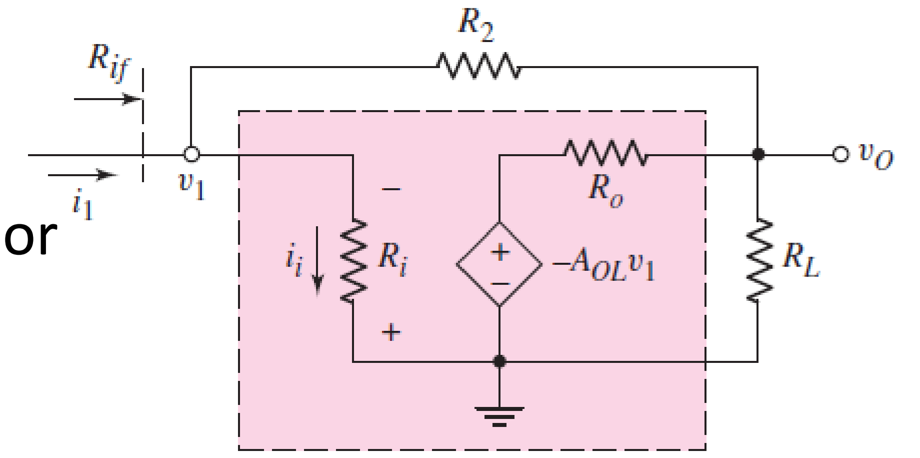
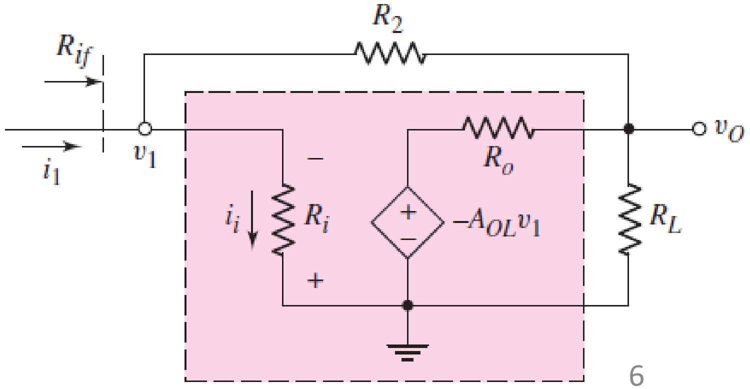


Figure 14.4(b)

EXAMPLE 14.2

- Consider an inverting amplifier with a feedback resistor $R_2 = 10k\Omega$, and an op-amp with parameters $A_{OL} = 10^5$ and $R_i = 10k\Omega$.
- Assume the output resistance R_o of the op-amp is negligible, e.g. $R_o \rightarrow 0$.
- Determine the closed-loop input resistance at the inverting terminal of an inverting amplifier.
- **Solution**: If $R_o = 0$, then:

$$\frac{1}{R_{if}} = \frac{1}{R_i} + \frac{1}{R_2}\frac{\left(1 + A_{OL} + \cancel{\frac{R_o}{R_L}}\right)}{1 + \cancel{\frac{R_o}{R_L}} + \cancel{\frac{R_o}{R_2}}} = \frac{1}{R_i} + \frac{1 + A_{OL}}{R_2} = \frac{1}{10^4} + \frac{1 + 10^5}{10^4} \approx 10^{-4} + 10$$



- The closed-loop input resistance is then $R_{if} \approx 0.1\Omega$.

Comments:

R_{if} of the inverting amplifier is a very strong function of the finite open-loop gain A_{OL}.

R_i essentially does not affect the closed-loop input resistance.

14.2.3 Inverting Amplifier Closed-Loop Input Resistance

- A nonzero R_{if} and a finite R_i implies that:
  - The signal current into the op-amp is not $zero$, as assumed in the ideal case.
- From Figure 14.4(b), we see that:

$$v_1 = i_1 R_{if}$$

- Therefore:

$$i_i = \frac{v_1}{R_i} = i_1 \left(\frac{R_{if}}{R_i} \right)$$

- The fraction of input signal current shunted away from R_2 and into the op-amp is (R_{if}/R_i).
- From the previous example, the fraction is:

$$\frac{R_{if}}{R_i} = \frac{0.1}{10\text{k}} = 10^{-5}!$$

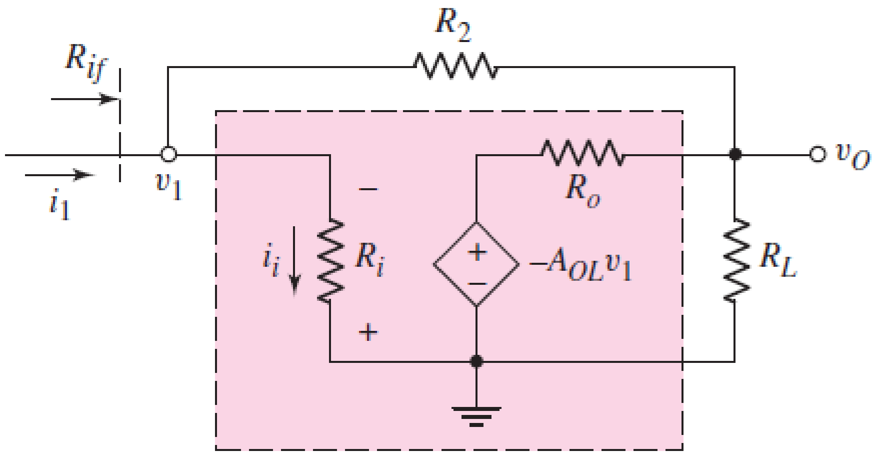


Figure 14.4(b)

14.2.4 Noninverting Amplifier Closed-Loop Input Resistance

- The input resistance seen by the signal source is designated R_{if}.
- The equivalent circuit includes:
  1. Finite open-loop gain A_{OL},
  2. Finite open-loop input differential resistance R_i,
  3. Non-zero output resistance R_o.

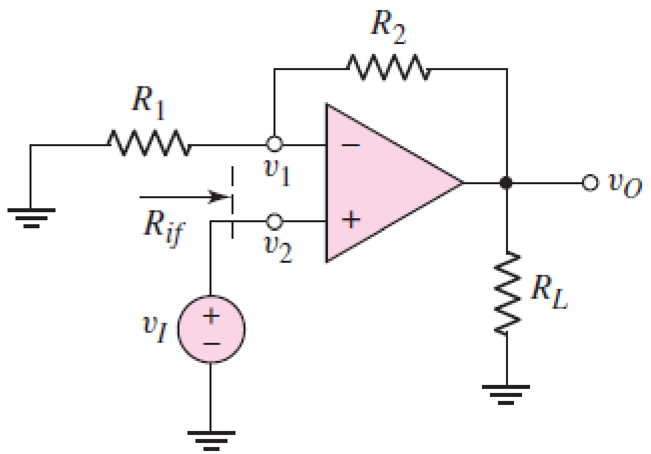


Figure 14.5(a)

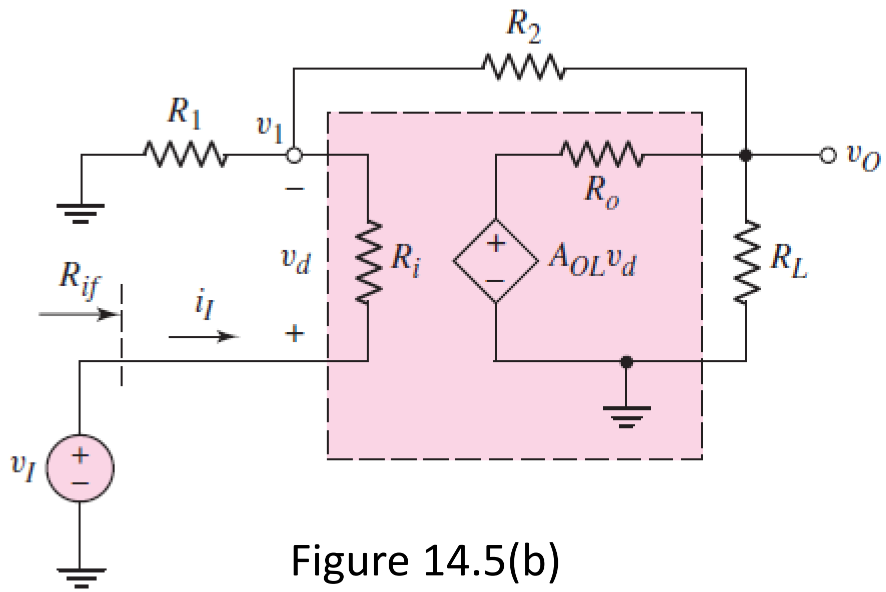


Figure 14.5(b)

14.2.4 Noninverting Amplifier Closed-Loop Input Resistance

- Writing a **KCL1** equation at the output node:

$$\frac{v_O}{R_L} + \frac{v_O - A_{OL} v_d}{R_o} + \frac{v_O - v_1}{R_2} = 0$$

- Solving for the output voltage, we have:

$$v_O = \frac{\frac{v_1}{R_2} + \frac{A_{OL} v_d}{R_o}}{\frac{1}{R_L} + \frac{1}{R_o} + \frac{1}{R_2}}$$

- Another **KCL2** equation at the v_1 node yields:

$$i_I = \frac{v_1}{R_1} + \frac{v_1 - v_O}{R_2}$$

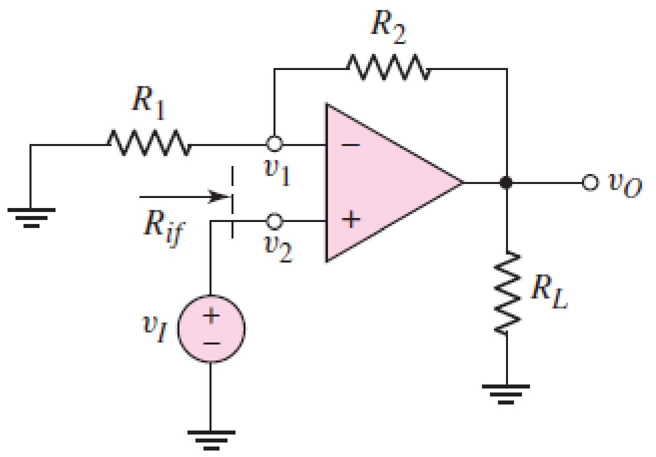
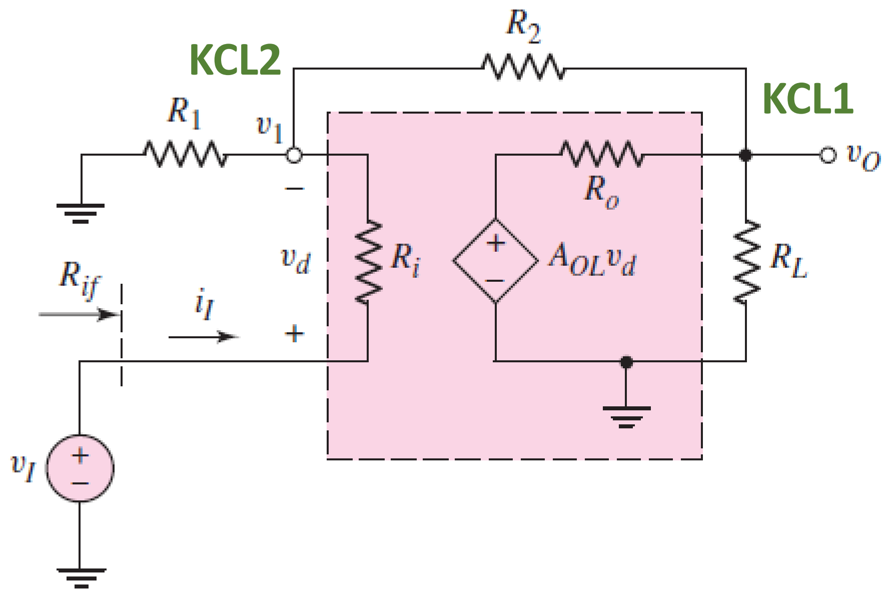


Figure 14.5(a)



14.2.4 Noninverting Amplifier Closed-Loop Input Resistance

- After substituting equations, we find that R_{if} can be written in the form:

$$R_{if} = \frac{v_I}{i_I} = \frac{R_i(1 + A_{OL}) + R_2\left(1 + \frac{R_i}{R_1}\right)}{1 + \frac{R_2}{R_1}}$$

- In the limit as $A_{OL} \to \infty$, or as the $R_i \to \infty$, we see that $R_{if} \to \infty$.
  - Which is a property of the ideal noninverting amplifier.

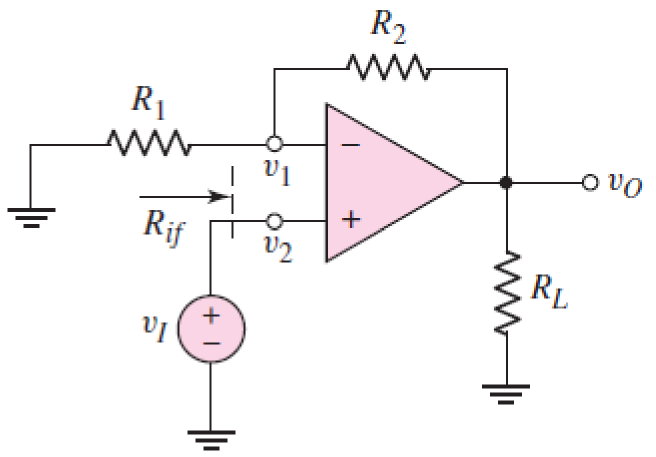


Figure 14.5(a)

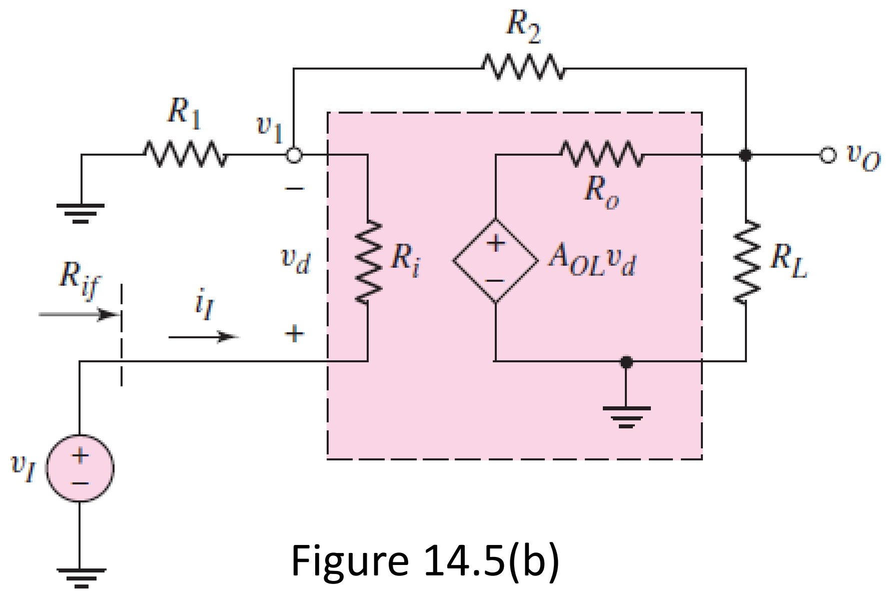


Figure 14.5(b)

EXAMPLE 14.3

- Consider an op-amp with an open-loop gain of $A_{OL} = 10^5$ and an input resistance of $R_i = 10k\Omega$ in a noninverting amplifier configuration with resistor values of $R_1 = R_2 = 10k\Omega$.
- Determine the closed-loop input resistance at the noninverting terminal of a noninverting amplifier.
- **Solution**: The input resistance is:
- $R_{if} = \frac{v_I}{i_I} = \frac{R_i(1+A_{OL})+R_2\left(1+\frac{R_i}{R_1}\right)}{1+\frac{R_2}{R_1}} = \frac{10k\left(1+10^5\right)+10k\left(1+\frac{10k}{10k}\right)}{1+\frac{10k}{10k}} \approx 500M\Omega$

EXAMPLE 14.3

$$R_{if} \approx 500M\Omega$$

- **Comments**:
  - As expected, the closed-loop input resistance of the noninverting amplifier is very large.
  - Equation:

$$R_{if} = \frac{v_I}{i_I} = \frac{R_i(1 + A_{OL}) + R_2\left(1 + \frac{R_i}{R_1}\right)}{1 + \frac{R_2}{R_1}}$$

shows that R_{if} is dominated by the term $R_i(1 + A_{OL})$.

  - The combination of a large R_i and large A_{OL} produces an extremely large input resistance, as predicted by ideal feedback theory.

14.2.5 Nonzero Output Resistance

- Since the ideal op-amp has a $zero$ output resistance
  - The output voltage is independent of the load impedance.
  - The op-amp acts as an ideal voltage source and there is no loading effect.
- An actual op-amp circuit has a $nonzero\ R_o$, which means that:
  - The output voltage and the closed-loop gain are function of the load impedance R_L.

14.2.5 Nonzero Output Resistance

- Figure 14.6 is the equivalent circuit of both an inverting and noninverting amplifier and is used to find R_o.
- The op-amp has:
  1. Finite open-loop gain A_{OL},
  2. Nonzero output resistance R_o, and
  3. An infinite input resistance R_i .
- To determine R_o:
  - We set the independent input voltages equal to $zero$.

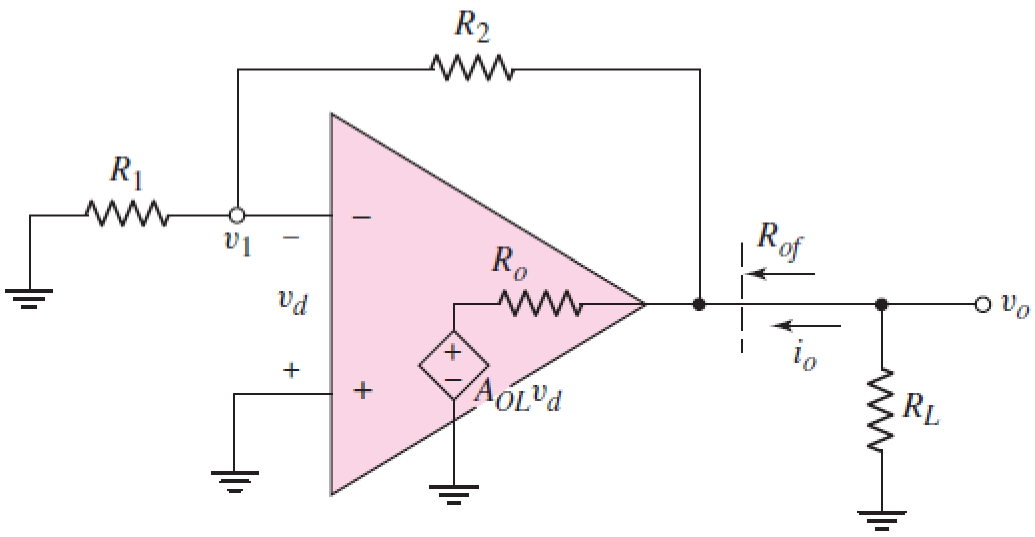


Figure 14.6

14.2.5 Nonzero Output Resistance

- A **KCL** equation at the output node yields:

$$i_o = \frac{v_o - A_{OL}v_d}{R_o} + \frac{v_o}{R_1 + R_2}$$

- The differential input voltage is:

$$v_d = v_2 - v_1 = -v_1$$

- But by voltage divider technique:

$$v_1 = \left(\frac{R_1}{R_1 + R_2}\right) v_o$$

$$v_d = -\left(\frac{R_1}{R_1 + R_2}\right) v_o$$

- Combining the two Equations above, we have:

$$i_o = \frac{v_o}{R_o} - \frac{A_{OL}}{R_o}\left[-\left(\frac{R_1}{R_1 + R_2}\right) v_o\right] + \left(\frac{v_o}{R_1 + R_2}\right)$$

$$\frac{i_o}{v_o} = \frac{1}{R_{of}} = \frac{1}{R_o}\left[1 + \frac{A_{OL}}{1 + R_2/R_1}\right] + \left(\frac{1}{R_1 + R_2}\right)$$

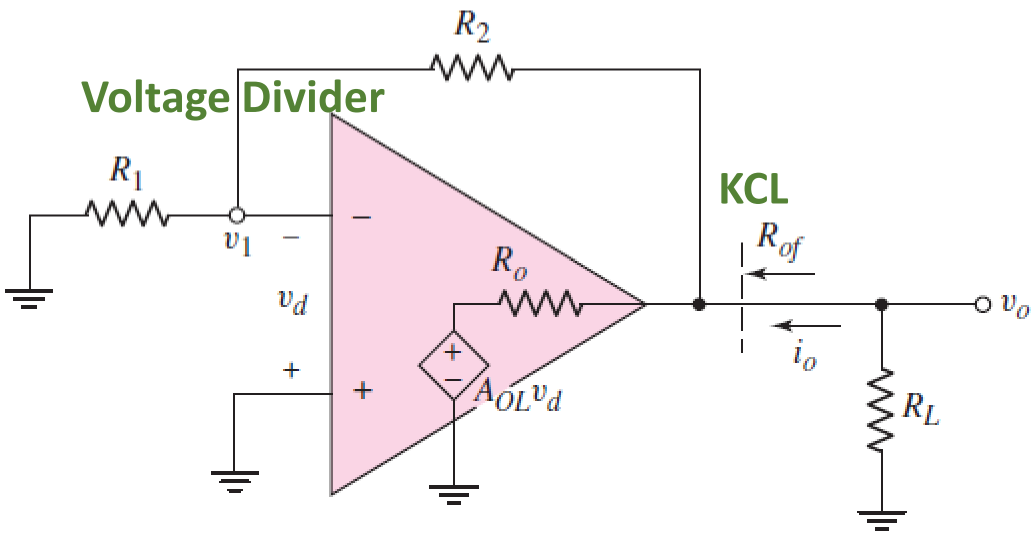


Figure 14.6

14.2.5 Nonzero Output Resistance

$$\frac{i_o}{v_o} = \frac{1}{R_{of}} = \frac{1}{R_o}\left[1 + \frac{A_{OL}}{1 + R_2/R_1}\right] + \left(\frac{1}{R_1 + R_2}\right)$$

- Since R_o is normally small and A_{OL} is normally large, the Equation above, to a good approximation, is as follows:

$$\frac{1}{R_{of}} \approx \frac{1}{R_o}\left[\frac{A_{OL}}{1 + R_2/R_1}\right]$$

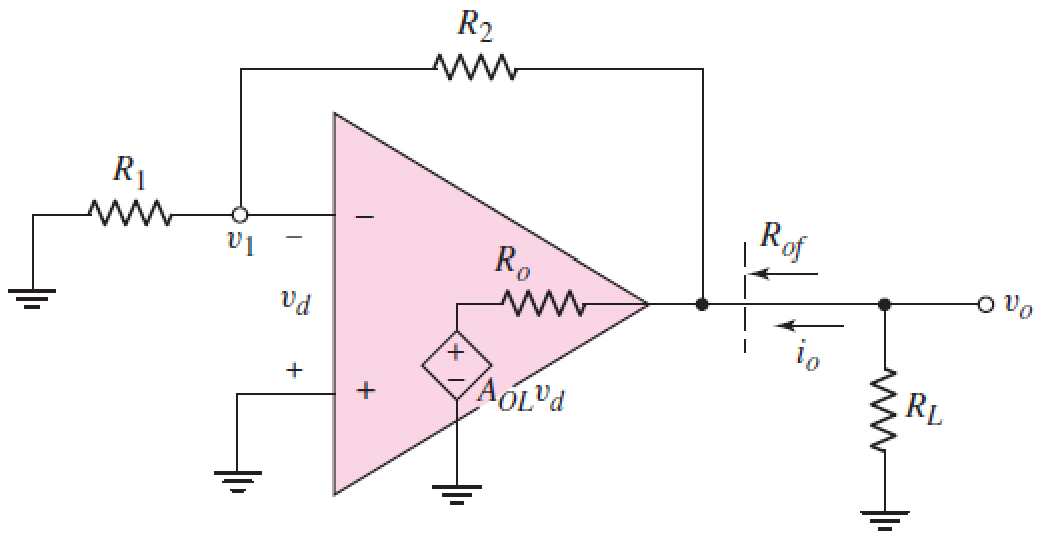


Figure 14.6

14.2.5 Nonzero Output Resistance

$$\frac{1}{R_{of}} \approx \frac{1}{R_o}\left[\frac{A_{OL}}{1 + R_2/R_1}\right]$$

- **Comments:**
  - In most op-amp circuits, the open-loop output resistance R_o is on the order of 100Ω.
  - Since A_{OL} is normally much larger than $(1 + R_2/R_1)$, the R_{of} can be very small.
  - R_{of} values in the *milliohm* range are easily attained using opamps.

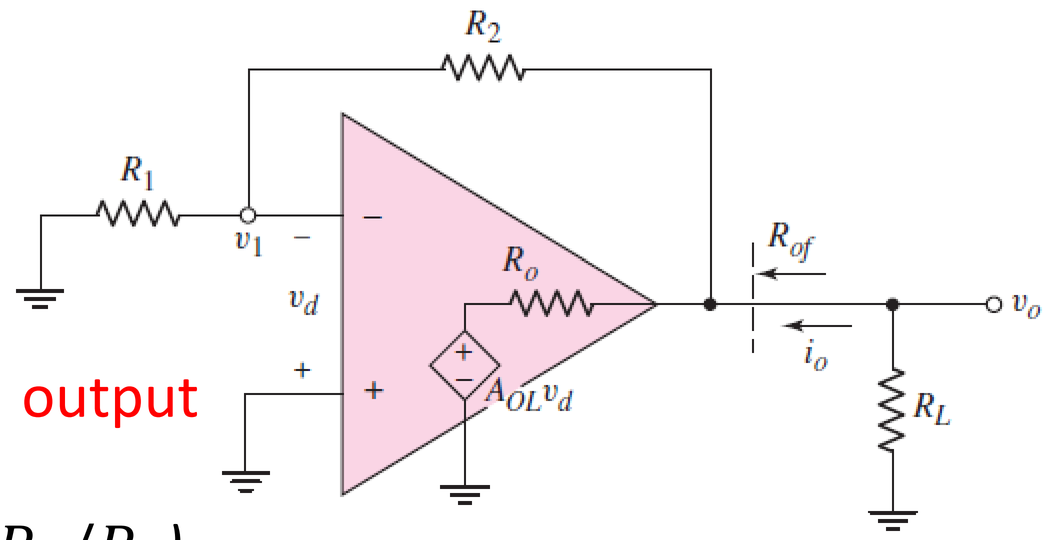


Figure 14.6

EXAMPLE 14.4

- **Objective**: Determine the output resistance of an op-amp circuit R_{of}.
- **Computer Simulation Solution**: Figure 14.7 shows an inverting amplifier circuit with a standard 741 op-amp.

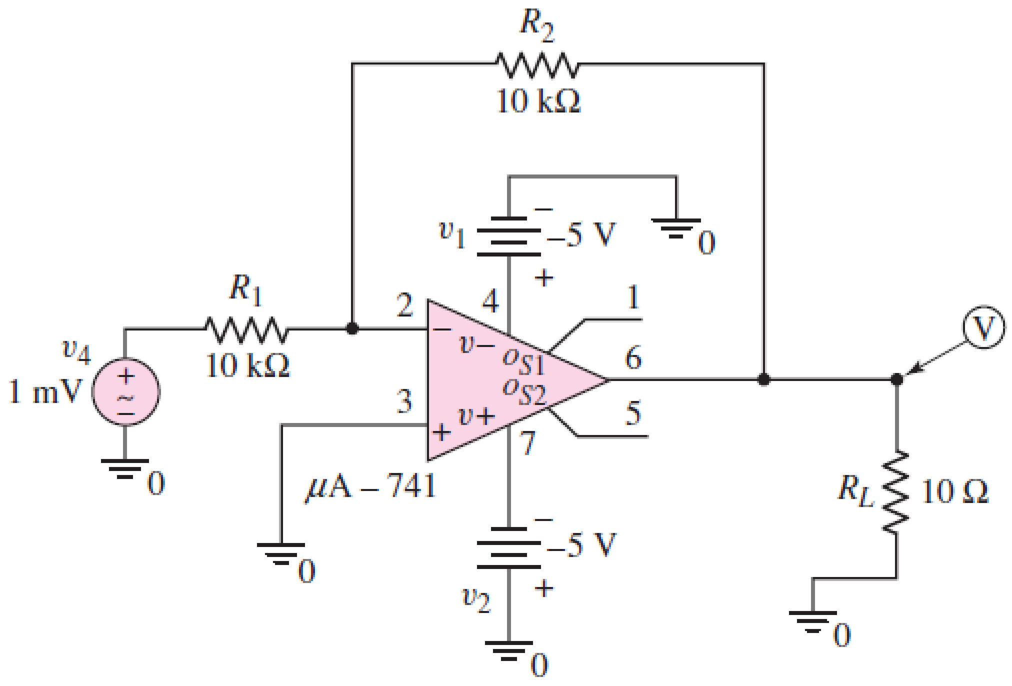


Figure 14.7

How the Output Resistance of an Opamp Circuit is Measured?

- One method of determining the output resistance is:
  - Measure the output voltage for two different values of load resistance connected to the output, i.e. R_{L1} and R_{L2}
  - Then, treating the amplifier as a Thevenin equivalent circuit with: a fixed source in series with an output resistance.

$$V_{o1} = A_v V_i \left(\frac{R_{L1}}{R_{L1} + R_{of}} \right)$$

$$V_{o2} = A_v V_i \left(\frac{R_{L2}}{R_{L2} + R_{of}} \right)$$

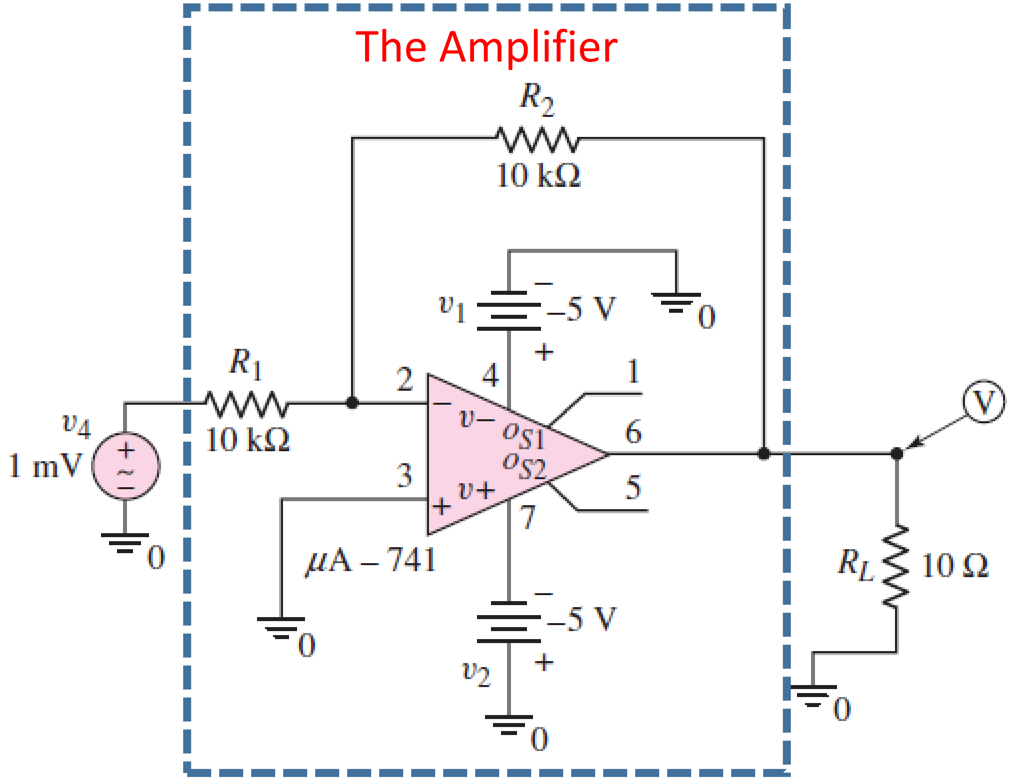


Figure 14.7

How the Output Resistance of an Opamp Circuit is Measured?

$$V_{o1} = A_v V_i \left(\frac{R_{L1}}{R_{L1} + R_{of}} \right)$$

$$V_{o2} = A_v V_i \left(\frac{R_{L2}}{R_{L2} + R_{of}} \right)$$

- Let:

$$\frac{V_{o1}}{V_{o2}} = \left(\frac{R_{L1}}{R_{L1} + R_{of}} \right) \left(\frac{R_{L2} + R_{of}}{R_{L2}} \right)$$

- Then:

$$R_{of} = \frac{R_{L1} R_{L2} \left(1 - \frac{V_{o1}}{V_{o2}} \right)}{R_{L2} \left(\frac{V_{o1}}{V_{o2}} \right) - R_{L1}}$$

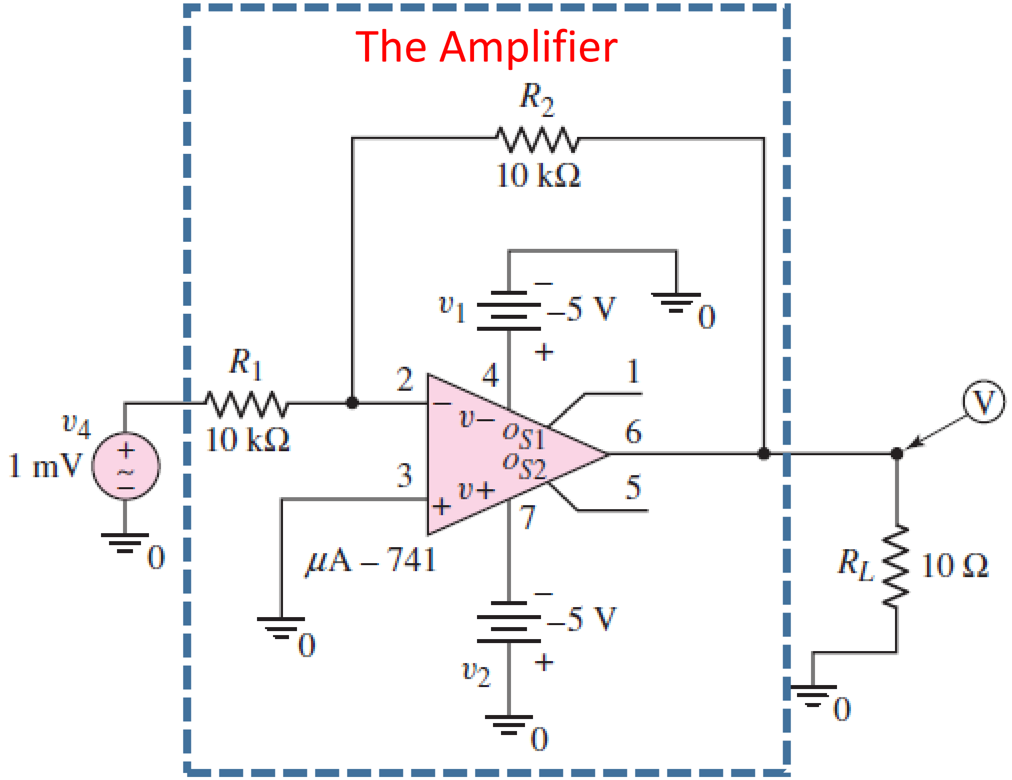


Figure 14.7

EXAMPLE 14.4

- A $1mV$ signal was applied.
  - For a 10Ω load, the output voltage is $0.999837\ mV$, and
  - For a 20Ω load, the output voltage is $0.9999132\ mV$.

$$R_{of} = \frac{R_{L1}R_{L2}\left(1 - \frac{V_{o1}}{V_{o2}}\right)}{R_{L2}\left(\frac{V_{o1}}{V_{o2}}\right) - R_{L1}}$$

$$= \frac{10 \cdot 20\left(1 - \frac{0.999837\ m}{0.9999132\ m}\right)}{20\left(\frac{0.999837\ m}{0.9999132\ m}\right) - 10}$$

- This gives an output resistance of:

$$R_{of} = 1.53\ m\Omega$$

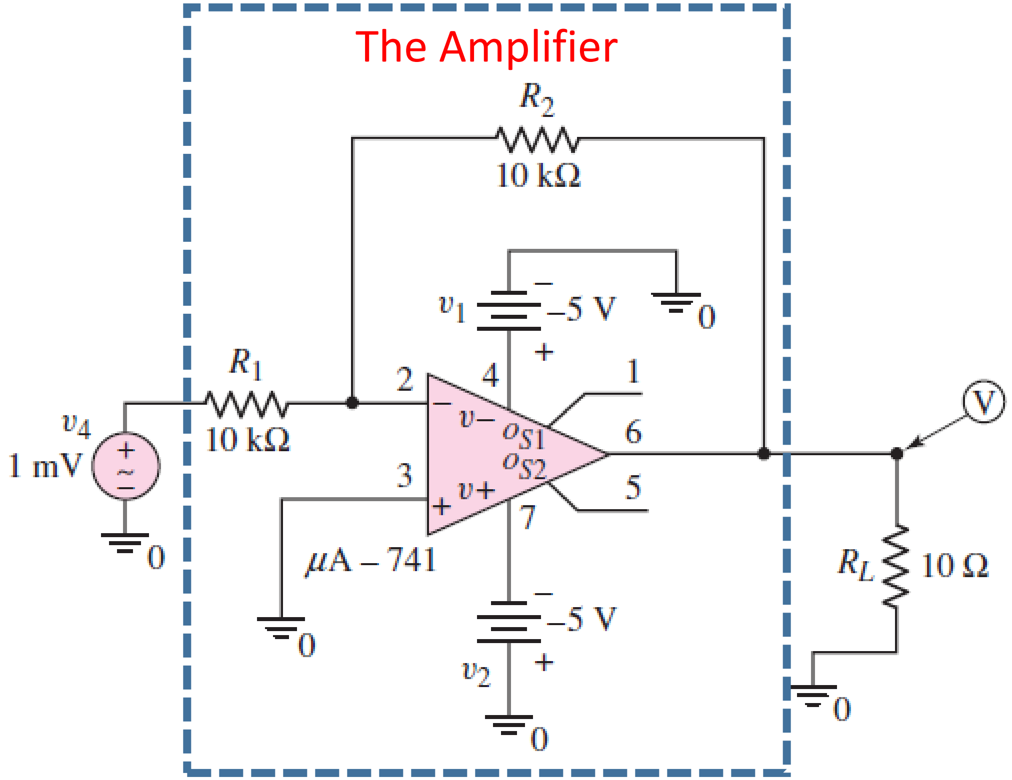


Figure 14.7

EXAMPLE 14.4

$$R_{of} = 1.53\ m\Omega$$

- **Comments**:
  - As mentioned, the output resistance of a voltage amplifier with negative feedback can be very small.
  - The ideal output resistance is *zero*, but a practical op-amp circuit can have an output resistance in the *milliohm* range.

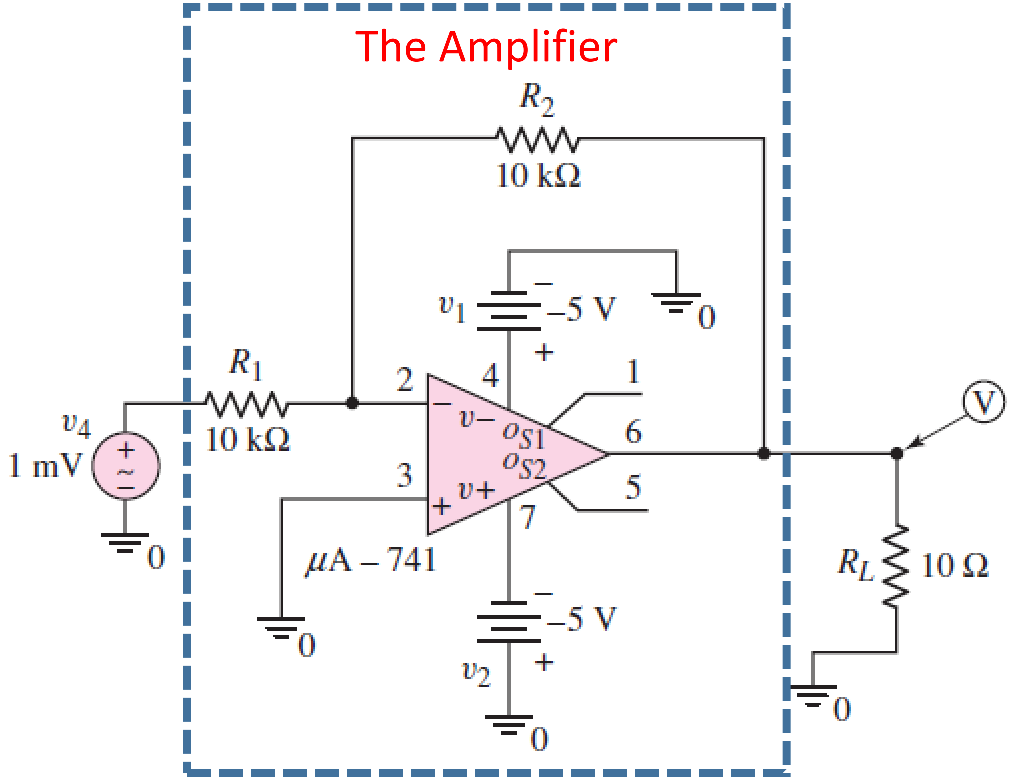


Figure 14.7

Do it yourself

- Solve all the Exercise Problems as well as the Test Your Understanding Questions.