

Question 1 (6 pts)

Design a three-port resistive divider for an equal power split and a 100 Ω system impedance.
a) If port 3 is matched, calculate the change in output power at port 3 (in dB) when port 2 is connected first to a matched load, and then to a load having a mismatch of $\Gamma = 0.3$. Assume the incident voltage from port 1 is 1 V.

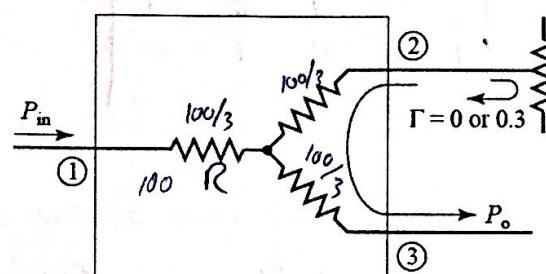
$$R = \frac{100}{3} = 33.3 \Omega$$

$$P_3 = \frac{1}{2} \frac{|V_3|^2}{Z_3} ; P_2 = P_3 = \frac{1}{4} P_1 \times \frac{1}{4}$$

$$Z_3 = R + Z_0 = 133.3 \Omega$$

$$V_3 = V_3^+ + V_3^-$$

3dB will be losted in the resistors.



*case (1): port is matched

$$\text{so } V_2^+ = V_3^+ = 0$$

$$V_3 = V_3^- \rightarrow V_2 = V_2^+ + V_2^-$$

$$V_1 = 1 \text{ volt.}$$

~~4x R / 2~~

$$V_2 = \frac{1}{2} V_1 = 0.5 \text{ volt.}$$

$$\text{so } V_2 = V_3 = \frac{1}{2} V_1 = 0.5 \text{ volt.}$$

$$\text{so } P_3 = \frac{1}{2} \times \frac{(0.5)^2}{100 + 33.3} = 9.38 \times 10^{-4} \text{ W}$$

$$P_3 = -30.3 \text{ dB}$$

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \end{bmatrix} = \begin{bmatrix} 1 & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix}$$

$$\begin{aligned} V_2^+ &= V_2^+ + V_2^- \\ V_2^- &= P |V_2^+| \\ V_2^+ &= 1 \text{ volt.} \end{aligned}$$

* Case (2):

$$V_3 = V_3^- \rightarrow V_2 = V_2^+ + V_2^- \Rightarrow V_2 = 1.3 V_2^+$$

$$V_1^- = S_{12} V_2^+$$

$$V_2^- = (0.3) V_2^+$$

$$P_3 = \frac{1}{2} \frac{|V_3^-|^2}{133.33} = \frac{|0.35|^2}{2 \times 133.33} = 41.6 \times 10^{-4} \text{ W}$$

$$P_3 = -33.4 \text{ dB}$$

5.5

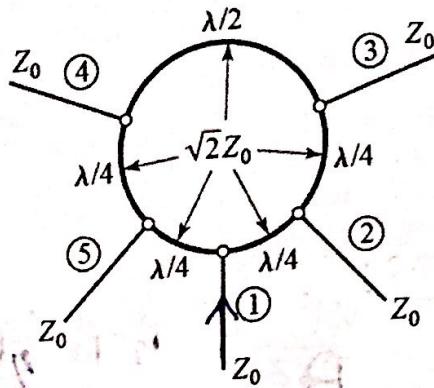
Question 2 (7 pts)

For the symmetric hybrid shown below. If port 1 is fed with an incident wave of $1 \angle 0$ V. Assume that the outputs are matched.

- a) Construct the scattering matrix.

$$[S] = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

2-5



- b) Calculate the phase between ports 3 and 4.

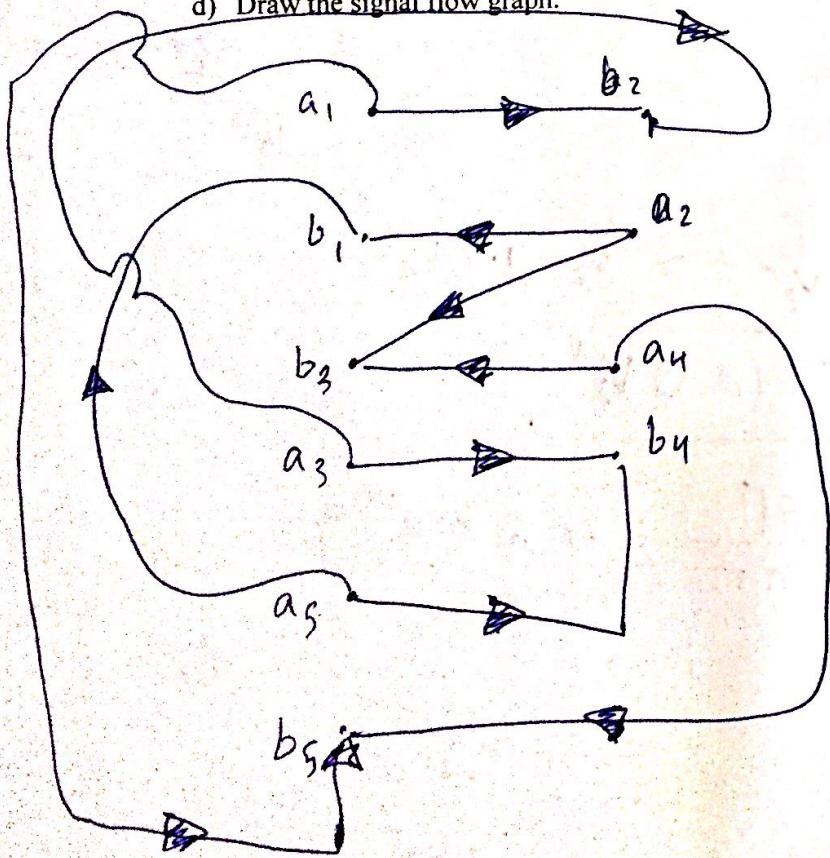
$$\text{phase shift}_{(3 \rightarrow 4)} = \lambda/2 = \underline{\underline{180^\circ}}$$

0.5

- c) What is the coupler properties?

- ① Matched for all ports. ✓
- ② The Power is divided equally. 0.5

- d) Draw the signal flow graph.



2

Question 3 (7 pts)

$$\frac{2G - 1.9G}{2} = \underline{\underline{0.05G \text{ Hz}}} = f_c$$

To suppress noise in a 50Ω digital communication system a bandpass RF filter is required with a passband from 1.9 GHz to 2 GHz. The minimum attenuation of the filter at 6 GHz and 0.475 GHz should be 25 dB. Assuming that a 0.5 dB ripple in the passband can be tolerated.

- Design a filter that will use a minimum number of components.
- Draw the circuit and label all the components.
- Plot the attenuation as a function of frequency.
- Determine the frequency in which the attenuation equal to 10 dB.

(a)

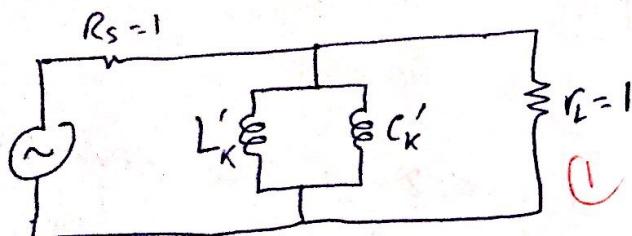
$$|\frac{\omega}{\omega_c}| - 1 = \left| \frac{0.475}{0.05G} \right| - 1 = 8.5 \quad \text{so } N=1$$

2

(b)

consider the element is Capacitor

$$g_1 = \underline{\underline{0.6986}}$$



$$L'_K' = \frac{(\Delta R_o)}{\omega_0 C_K}$$

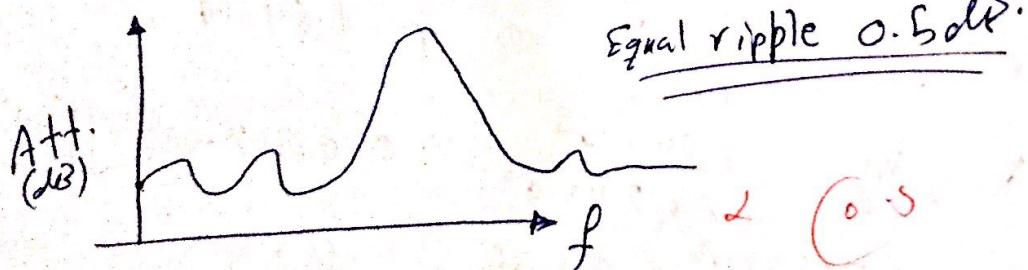
$$\Delta = 0.1$$

$$L'_K' = \frac{0.1 \times 50}{2\pi \times 50 \times 10^6 \times 0.6986} = (22.78 \text{ nH})$$

$$C'_K = \frac{C_K}{\Delta \omega_0 R_o} = \frac{0.6986}{0.1 \times 2\pi \times 50 \times 10^6 \times 50} = (0.4145 \text{ nF.})$$

2 ③

(c)



$$(d) @ 10 \text{ dB} \Rightarrow |\frac{\omega}{\omega_c}| - 1 \approx 7.2$$

$$|\frac{\omega}{\omega_c}| = 8.2 \Rightarrow \omega = 8.2 \omega_c = 8.2 \times 2\pi \times 50 \times 10^6$$

$$\Rightarrow \omega = \frac{2\pi \times 410 \times 10^6}{\text{rad/s}} \\ \text{so } f = 410 \text{ MHz}$$

3

2

Q4 Question 4 (5 pts)

Consider a microstrip resonator constructed from a $\lambda/2$ length of 50Ω open-circuited microstrip line. The substrate is FR-4 ($\epsilon_r = 4.4$, $\tan\delta = 0.02$), with a thickness of 0.78 mm, and the conductors are copper ($\sigma = 5.813 \times 10^7 \text{ S/m}$).

- Compute the required length of the line for resonance at 2.4 GHz, and the unloaded Q of the resonator. Ignore fringing fields at the end of the line.
- Sketch its equivalent RLC lumped circuit and determine their values.

$$(a) Q_0 = \frac{\beta}{2\alpha} \quad \dots \quad (1)$$

for microstrip: $A = \frac{Z_0}{60} \sqrt{\frac{\epsilon_r+1}{2}} + \frac{\epsilon_r-1}{\epsilon_r+1} \left(0.23 + \frac{0.11}{\epsilon_r} \right)$

assume $= \underline{1.53}$

$$B = \frac{3771}{2Z_0\sqrt{\epsilon_r}} = \underline{5.65}$$

Assume $\frac{W}{d} < 2$: $\frac{W}{d} = 1.9 < 2$ True Assumption.

so $W = 0.78 \text{ mm} \times 1.9 \Rightarrow W = 1.482 \text{ mm}$

$$\epsilon_e = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2} * \frac{1}{1 + \frac{12}{\frac{W}{d}}} = \underline{3.329}$$

so $V_p = \frac{c}{\epsilon_e} = \frac{3 \times 10^8}{3.329} \Rightarrow V_p = 1.64 \times 10^8$

$$L = \frac{\lambda}{2} = \frac{V_p}{2f} = \frac{1.64 \times 10^8}{2 \times 2.4 \times 10^9} = 0.03412 \text{ m} = \underline{3.42 \text{ cm}}$$

$$\beta = \frac{2\pi f}{V_p} = \frac{2\pi \times 2.4 \times 10^9}{1.64 \times 10^8} = \underline{91.95} \quad \dots \quad (2)$$

$$\alpha_c = \frac{R_s}{Z_0 W} \Rightarrow R_s = \sqrt{\frac{W M_0}{2\sigma}} = \sqrt{\frac{2\pi \times 2.4 \times 10^9 \times 4\pi \times 10^{-7}}{2 \times 5.813 \times 10^7}} = \underline{0.0128 \text{ Np/m}}$$

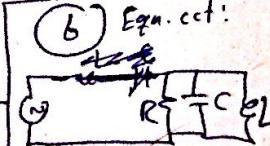
$$\alpha_d = \frac{K_0 \epsilon_r (\epsilon_e - 1) \tan\delta}{2(\epsilon_e (\epsilon_r - 1))} = \frac{50.3 \times 4.4 (3.329 - 1) (0.02)}{2 (3.329) (4.4 - 1)} = \underline{0.8309 \text{ Np/m}}$$

$$K_0 = 50.3$$

$$\text{so } \alpha_{\text{Total}} = \alpha_c + \alpha_d = 0.8309 + 0.0128 = 0.8437 \text{ Np/m.} \quad (3)$$

Substitute (3) & (2) into (1)

$$\Rightarrow Q_0 = 108.98$$



(b) Eqn. ext: $R = \frac{Z_0}{\alpha L} = \frac{1732.83}{108.98} \Omega$

$$C = \frac{\pi}{2\omega_0 Z_0} = \frac{2.083}{108.98} \text{ pF}$$

$$L = \frac{1}{\omega_0^2 C} = 2.111 \text{ nH}$$