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EE 529: Selected Topics in Communications  
First Exam (Spring 2018) March 13<sup>th</sup>, 2018

Question 1 (6 pts)

A 3-port network is completely characterized at some frequency  $\omega$  by the scattering matrix:

$$S = \begin{bmatrix} 0 & 0.2 & 0.5 \\ 0.5 & 0 & 0.2 \\ 0.5 & 0.5 & 0 \end{bmatrix}$$

If a matched load is attached to port 2, while a short circuit has been placed at port 3, then;

- Find the reflection coefficient at port 1
- Find the transmission coefficient from port 1 to port 2

a)  $[V^-] = [S][V^+]$

$$V_1^- = V_1^+ S_{11} + V_2^+ S_{12} + V_3^+ S_{13}$$

$$V_2^- = V_1^+ S_{21} + V_2^+ S_{22} + V_3^+ S_{23}$$

$$V_3^- = V_1^+ S_{31} + V_2^+ S_{32} + V_3^+ S_{33}$$

Need to find  $\frac{V_1^-}{V_1^+}$  ?!  
other info:

$$V_3 = 0 = V_3^+ + V_3^-$$

$$V_3^+ = -V_3^-$$

Matched load @ port 2:  
means that

$$S_{21} = 0.5 = \frac{V_2^-}{V_1^+}$$

$$V_2^- = 0.5 V_1^+$$

$$\frac{V_1^-}{V_1^+} = \frac{V_2^+}{V_1^+} S_{12} + \frac{V_3^+}{V_1^+} S_{13}$$

the equations are:

$$V_1^- = V_2^+ S_{12} + V_3^+ S_{13}$$

$$V_2^- = V_1^+ S_{21} - V_3^- S_{23} = 2V_2^- S_{21} - V_3^- S_{23}$$

$$V_3^- = V_1^+ S_{31} + V_2^+ S_{32} = 2V_2^- S_{31} + V_2^+ S_{32}$$

$$\frac{V_1^-}{V_1^+} = \frac{V_2^+}{V_1^+} S_{12} + \frac{V_3^-}{V_1^+} S_{13} \quad \text{--- (1)}$$

$$\frac{V_2^+}{V_1^+} S_{32} + \frac{2V_2^-}{V_1^+} S_{31} = \frac{V_3^-}{V_1^+} \quad \text{--- (2)}$$

$$(1 + 2S_{21}) V_2^- = V_3^- S_{23}$$

$$\frac{V_3^-}{V_2^-} = \frac{S_{23}}{2S_{21} - 1} = \frac{2V_3^-}{V_1^+} \quad \text{--- (3)}$$

sub. (1) into (2):  
Now (2) becomes

$$0.5 \frac{S_{23}}{2S_{21} - 1} = \frac{V_2^+}{V_1^+} S_{32} + \frac{2V_2^-}{V_1^+} S_{31}$$

so  $\frac{V_2^+}{V_1^+} S_{32} = \frac{0.5 S_{23}}{2S_{21} - 1} - S_{31} \Rightarrow \frac{V_2^+}{V_1^+} = \frac{0.5 S_{23}/S_{32}}{2S_{21} - 1} - \frac{S_{31}}{S_{32}} \quad \text{--- (4)}$

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Question 2 (5 pts)

C.S

A microwave antenna feed network operating at 5 GHz requires a 50 Ω printed transmission line that is 16 λ long. A copper microstrip, with  $d = 0.16$  cm,  $\epsilon_r = 2.20$ ,  $\sigma = 5.8 \times 10^7$  S/m and  $\tan \delta = 0.001$  is used. Calculate the total loss in using this transmission line.

$$f_0 = 5 \text{ GHz}$$

$$Z_0 = 50 \Omega$$

$$L = 16\lambda$$

$$A = \frac{Z_0}{60} \sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1} \left( 0.23 + \frac{0.11}{\epsilon_r} \right) \Rightarrow \boxed{A = 1.16}$$

$$B = \frac{377\pi}{2Z_0\epsilon_r} \Rightarrow \boxed{B = 7.99}$$

Assume  $\frac{W}{d} < 2$ :

$$\frac{W}{d} \approx \frac{8e^A}{e^{2A} - 2} = 3.12 \text{ it is NOT } < 2$$

So we are sure that  $\frac{W}{d} > 2$ :

$$\frac{W}{d} = \frac{2}{\pi} \left[ B - 1 - \ln(2B - 1) + \frac{\epsilon_r - 1}{2\epsilon_r} \left\{ \ln(B - 1) + 0.39 - \frac{0.61}{\epsilon_r} \right\} \right]$$

$$\boxed{\frac{W}{d} = 3.08}$$

$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \times \frac{1}{\sqrt{1 + 12\frac{d}{W}}} = 1.87 \Rightarrow \boxed{\epsilon_e = 1.87}$$

$$\alpha_d = \frac{K_0 \epsilon_r (\epsilon_e - 1) \tan \delta}{2\sqrt{\epsilon_e} (\epsilon_r - 1)} ; K_0 = \frac{\omega}{c} = \frac{2\pi \times 5 \times 10^9}{3 \times 10^8} = 104.72$$

$$\boxed{\alpha_d = 0.061} \text{ Np/m}$$

$$\alpha_c = \frac{R_s}{Z_0 W} ; R_s = \sqrt{\frac{\omega \mu_0}{2\sigma}} = 0.0184 \Omega ; W = \frac{W}{d} \cdot d = (3.08)(0.16) = 0.4928$$

$$\text{so } \boxed{\alpha_c = 7.47 \times 10^{-4}} \text{ Np/cm} = 7.47 \times 10^{-2} \text{ Np/m}$$

$$\alpha = \alpha_c + \alpha_d = \frac{0.1357}{0.06175} \text{ Np/m}$$

$$= 0.5363 \text{ dB}$$

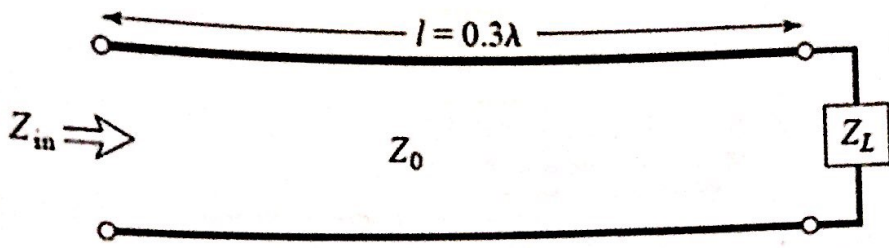
$$\boxed{1.1787}$$



Question 3 (6 pts)

9.5

A lossless transmission line of electrical length  $l = 0.3\lambda$  has a characteristic impedance of  $100\Omega$  is terminated with a complex load has a reflection coefficient at the load of  $\Gamma = 0.803e^{-j94.76^\circ}$  as shown in the accompanying figure.



- A. Find the load impedance  $Z_L$ , the reflection coefficient at the input of the line, and the input impedance to the line.
- B. Design a lossless L-section matching circuit to match the load to the transmission line at 3 GHz. (Draw the equivalent circuit and determine the value of these elements).

$Z_0 = 100 \Omega$   
 $l = 0.3\lambda$

$\rho = 0.803 \angle -95^\circ$

A)  $SWR = \frac{1+|\rho|}{1-|\rho|} = 9.15$

$\Gamma_{in} \rightarrow ? \rightarrow 0.803$   
 $\Rightarrow 6.1 \text{ cm} \rightarrow \text{for } \rho_L$

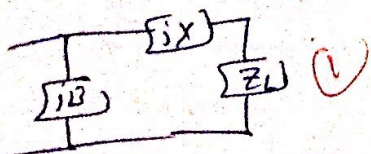
from the smith chart:

$(Z_L)_N = 0.225 - j0.875$

so  $Z_L = (Z_L)_N \times 100 = \boxed{22.5 - j87.5} \Omega$

@ input:  $Z_{in} = 0.7 + j2 \Rightarrow Z_{in} = 70 + j200 \Omega$   
 $\rho_{in} = 0.803 e^{+j49^\circ}$

B)  $Z_L = 0.225 - j0.875$  (outside  $1 + jx$  circle)



$X = \pm 42.8 \ \& \ B = \pm 0.019$

take  $X = 42.8, B = 0.019$

$X = \omega L \Rightarrow L = \frac{42.8}{2\pi f} = 2.27 \text{ nH}$

$B = 0.019 = \omega C \Rightarrow C = \frac{0.019}{\omega} = 1.01 \text{ pF}$

take  $X = -42.8 \ \& \ B = -0.019$

$X = \frac{1}{\omega C} \Rightarrow C = 1.24 \text{ pF} \ \& \ B = \frac{1}{\omega L} \Rightarrow L = 2.79 \text{ nH}$

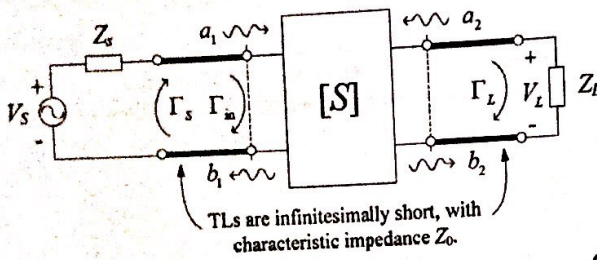
$X = \pm \sqrt{R_L(Z_0 - R_L) - X_L}$

$B = \pm \frac{(Z_0 - R_L)/R_L}{Z_0}$

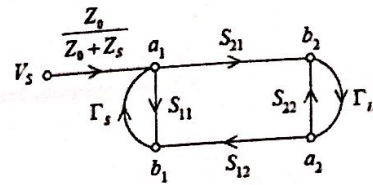


Question 4 (6 pts)

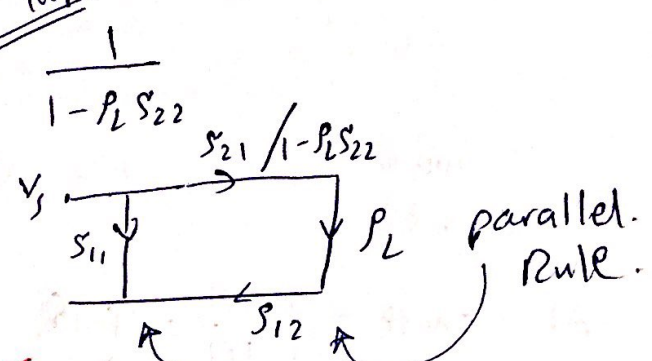
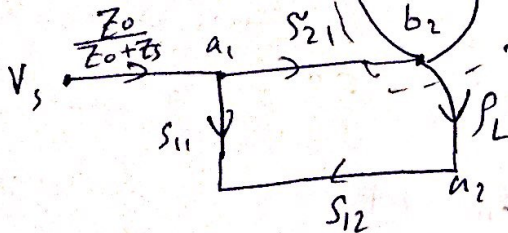
- a) [3 pts.] Determine  $\Gamma_{in}$   
 b) [3 pts.] Determine  $V_L$  using only signal flow graph if the load is located at  $z=0$ .



$V_L = V_0 + \sum_{n=1}^{\infty} (-j)^n \rho_L^n$   
 $= V_0 + (1 + \rho_L)$   
 $= b_2 (1 + \rho_L)$  just find  $V_s$



a) for  $P_{in}$ : self loop rule.



so  $P'_{in} = S_{11} + \frac{S_{12} S_{21} \rho_L}{1 - \rho_L S_{22}}$

(series Rule).

where  $\rho_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

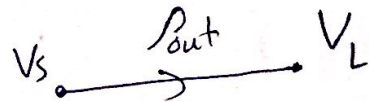
$\Rightarrow P_{in}$  is the series of  $\frac{Z_0}{Z_0 + Z_s}$  &  $P'_{in}$

$P_{in} = \left( \frac{Z_0}{Z_0 + Z_s} \right) \left( S_{11} + \frac{S_{12} S_{21} \rho_L}{1 - \rho_L S_{22}} \right)$  #

b) The same for  $P_{out}$  it could be found that:

$P_{out} = \left( \frac{Z_0}{Z_0 + Z_s} \right) \left( S_{22} + \frac{S_{12} S_{21} \rho_L}{1 - \rho_L S_{11}} \right)$

so  $V_L$  is given by:



$V_L = P_{out} V_s$



Question 5 (2 pts)

0-5

a) What is a Quasi TEM and why the microstrip line has this type of modes.

it is a type close to the exact TEM mode  
are in  $\mu$ strip, since there is TE or TM mode  
could be for the corresponding  $\mu$ strip, then like  
this type Quasi TEM could use between them.

b) Why the waveguide has a low bandwidth at microwave frequencies.

since it has a larger size.



Continue Q<sub>1</sub>:

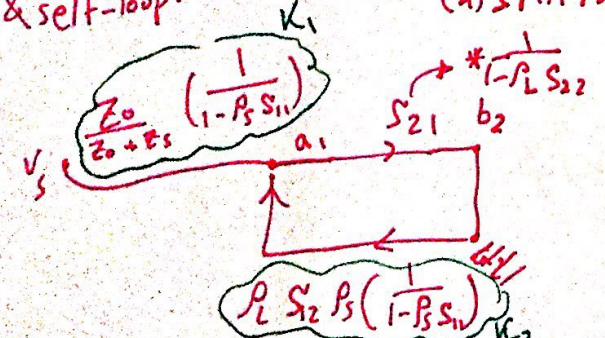
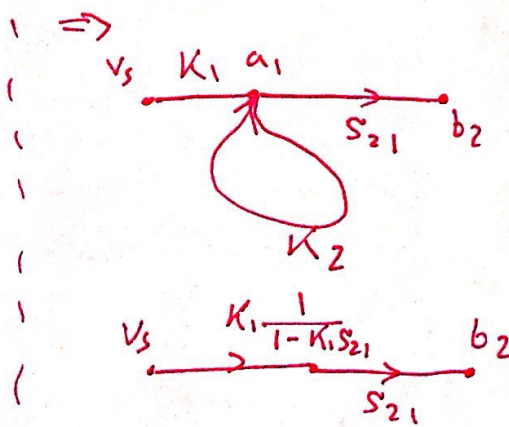
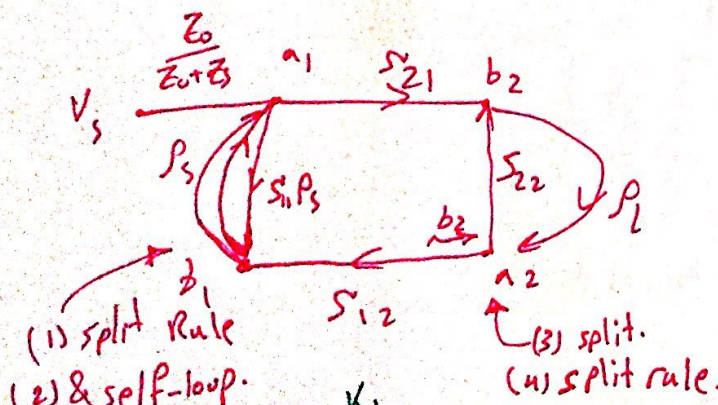
sub. (4) & (3) into (1):

$$\frac{V_1^-}{V_1^+} = \left( \frac{0.5 S_{23}/S_{32}}{2S_{21}-1} - \frac{S_{31}}{S_{32}} \right) S_{12} - \frac{0.5 S_{23} S_{13}}{2S_{21}-1}$$

substituting the values:

⇒ reflection @ port 1 = ✓

b). Trans. coef. part (1) →  $\frac{2}{3} = \frac{V_2^-}{V_1^+} = \underline{\underline{0.5}}$

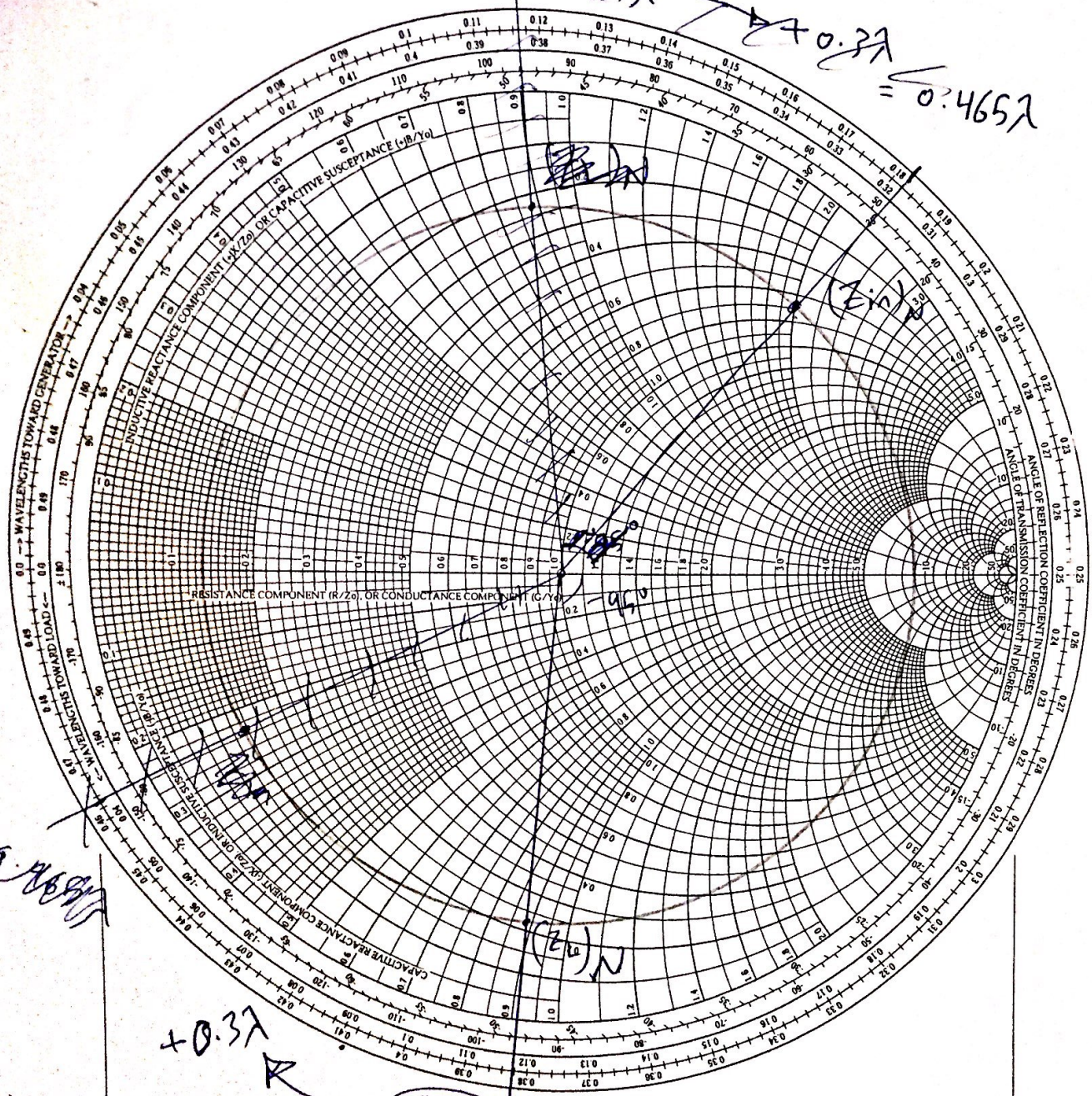


$$b_2 = K_1 \frac{1}{1-K_2 S_{21}} \times S_{21}$$

$$V_L = b_2 (1 + \rho_L)$$



$0.165\lambda$   
 $+0.3\lambda$   
 $= 0.465\lambda$



$0.465\lambda$

$+0.3\lambda$   
 $= 0.682\lambda$

$0.382\lambda$

