Topics in Communications.
"Microwaves"

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Dr. Yanal Al-Faouri

By: Mohammad Abu Hashya.

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Microwave Engineering

Chapter 7

Dr. Yanal Faouri

Email: y.faouri@ju.edu.jo

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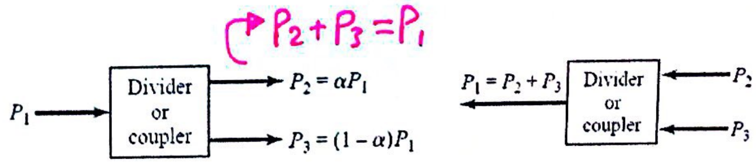
Power Dividers and Directional Couplers

- Basic Properties of Dividers and Couplers
- The T-Junction Power Divider
- The Wilkinson Power Divider
- Waveguide Directional Couplers
- The Quadrature (90°) Hybrid
- Coupled Line Directional Couplers
- The Lange Coupler
- The 180° Hybrid
- Other Types of Couplers

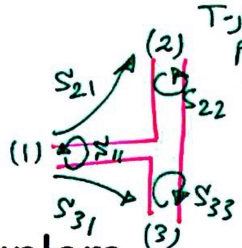
2/.

Introduction



- Power dividers and directional couplers are passive microwave components used for power division or power combining.
- In power division, an input signal is divided into two (or more) output signals of lesser power, while a power combiner accepts two or more input signals and combines them at an output port.
- The coupler or divider may have three ports, four ports, or more, and may be (ideally) lossless.
- Three-port networks take the form of T-junctions and other power dividers, while four-port networks take the form of directional couplers and hybrids.
- Power dividers usually provide in-phase output signals with an equal power division ratio (3 dB), but unequal power division ratios are also possible.
- Directional couplers can be designed for arbitrary power division, while hybrid junctions usually have equal power division.
- Hybrid junctions have either a 90° or a 180° phase shift between the output ports.

Half Power.



T-junction power divider.

Basic Properties of Dividers and Couplers Three-Port Networks (T-Junctions) (1)

- The simplest type of power combiner (divider) is a *T-junction*, which is a three-port network with two inputs (outputs) and one output (input). The scattering matrix:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

- If all ports are matched, then $S_{ii} = 0$, and if the network is reciprocal, the scattering matrix reduces to;

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- If the network is lossless;

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad |S_{12}|^2 + |S_{23}|^2 = 1 \qquad |S_{13}|^2 + |S_{23}|^2 = 1 \rightarrow \text{unity property.}$$

$$S_{13}^* S_{23} = 0 \qquad S_{23}^* S_{12} = 0 \qquad S_{12}^* S_{13} = 0 \rightarrow \text{zero property.}$$

first column. second column. third column.

Three-Port Networks (T-Junctions) (2)

- The three-port network cannot be simultaneously lossless, reciprocal, and matched at all ports. If any one of these three conditions is relaxed, then a physically realizable device is possible.
- If the three-port network is nonreciprocal, then $S_{ij} \neq S_{ji}$, and the conditions of input matching at all ports and energy conservation can be satisfied. Such a device is known as a **circulator**, and generally relies on an anisotropic material, such as ferrite, to achieve nonreciprocal behavior.
- Any matched lossless three-port network must be nonreciprocal and, thus, a circulator. The scattering matrix of a matched three-port network has the following form:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

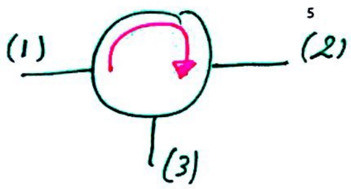
circulator.

port(1) is matched

i/p port(2)

port(3) is o/p.

$$S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



$$S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Three-Port Networks (T-Junctions) (3)

- If the network is lossless, [S] must be unitary, which implies the following conditions:

$$S_{31}^* S_{32} = 0$$

$$S_{21}^* S_{23} = 0$$

$$S_{12}^* S_{13} = 0$$

$$|S_{12}|^2 + |S_{13}|^2 = 1$$

$$|S_{21}|^2 + |S_{23}|^2 = 1$$

$$|S_{31}|^2 + |S_{32}|^2 = 1$$

These equations can be satisfied in one of two ways. Either

$$S_{12} = S_{23} = S_{31} = 0, \quad |S_{21}| = |S_{32}| = |S_{13}| = 1$$

Corresponds to a circulator that allows power flow only from port 1 to 2, or port 2 to 3, or port 3 to 1

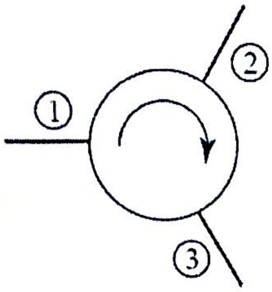
OR

$$S_{21} = S_{32} = S_{13} = 0, \quad |S_{12}| = |S_{23}| = |S_{31}| = 1$$

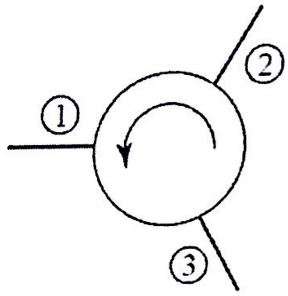
Corresponds to a circulator with the opposite direction of power flow

Three-Port Networks (T-Junctions) (4)

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



Three-Port Networks (T-Junctions) (5)

- A **lossless and reciprocal** three-port network can be physically realized if only **two** of its ports are matched. If ports 1 and 2 are the matched ports, then the scattering matrix can be written as;

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

To be lossless, the following unitarity conditions must be satisfied

$$S_{13}^* S_{23} = 0$$

$$S_{12}^* S_{13} + S_{23}^* S_{33} = 0$$

$$S_{23}^* S_{12} + S_{33}^* S_{13} = 0$$

$$|S_{12}|^2 + |S_{13}|^2 = 1$$

$$|S_{12}|^2 + |S_{23}|^2 = 1$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1$$

Microstrip Power Dividers

- Other **3-port** devices used are:

- **T-junction power divider** (uses λ/4 matching circuits)

✓**Lossless Divider** → it is lossless, reciprocal But it is NOT Matched.
✓**Resistive Divider**

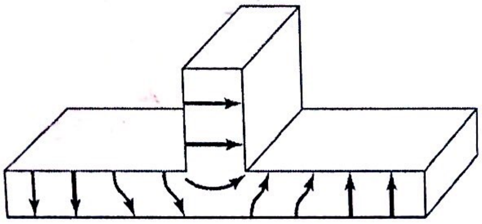
- **Wilkinson Power divider**

T-Junction Power Divider (1)

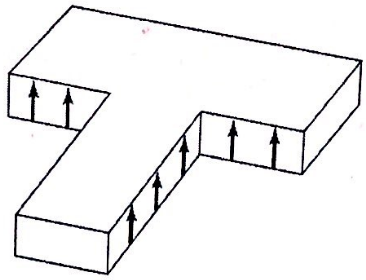
- The T-junction power divider is a simple three-port network that can be used for power division or power combining, and it can be implemented in virtually any type of transmission line medium.
- **Lossless Divider:**
- The lossless T-junction dividers can all be modeled as a junction of three transmission lines. In general, there may be fringing fields and higher order modes associated with the discontinuity at such a junction, leading to stored energy that can be accounted for by a lumped susceptance (B). In order for the divider to be matched to the input line of characteristic impedance Z_0, we must have

$$Y_{in} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

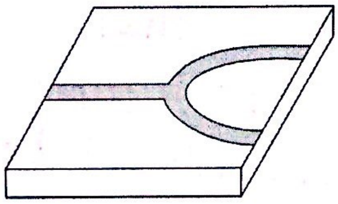
T-Junction Power Divider (2)



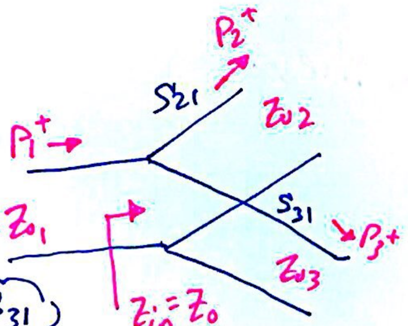
E-plane waveguide T



H-plane waveguide T



Microstrip line T-junction divider



$S_{11} = 0$

$S_{21} = S_{31}$

$S_{23} = S_{32} = 0$

$$\Rightarrow S = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & S_{22} & 0 \\ \frac{1}{\sqrt{2}} & 0 & S_{33} \end{bmatrix}$$

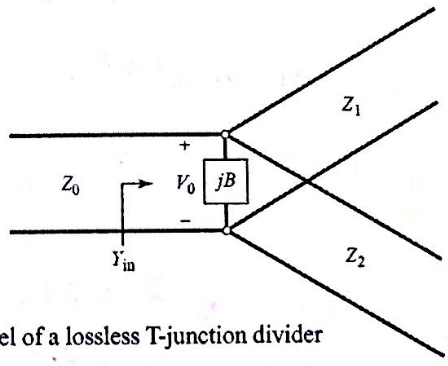
T-Junction Power Divider (3)

Behind the page

- If the transmission lines are assumed to be lossless (or of low loss), then the characteristic impedances are real. If we also assume $B = 0$, then;

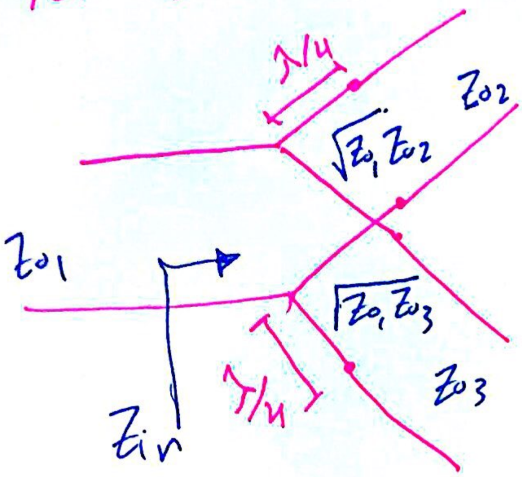
$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

In practice, if B is not negligible, some type of discontinuity compensation or a reactive tuning element can usually be used to cancel this susceptance, at least over a narrow frequency range.



Transmission line model of a lossless T-junction divider

* for $\lambda/4$ Transformers:

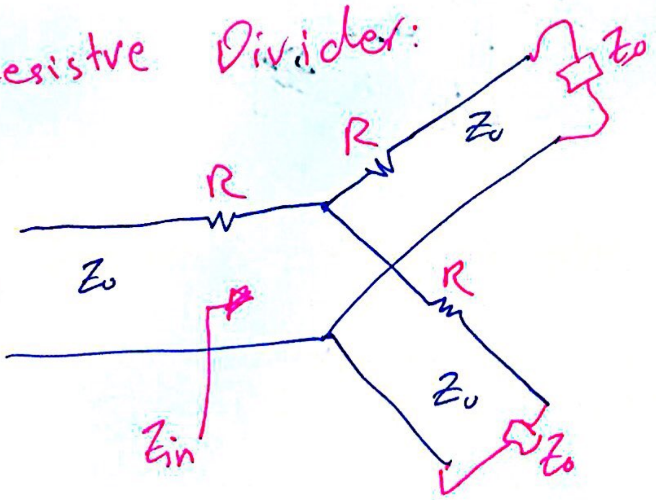


$$\Rightarrow S = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

these are coupled.

This was to prove that you can't have the three (reciprocal, lossless, matched) together..

* Resistive Divider:



$$Z_{in} = R + \left((R+Z_o) \,//\, (R+Z_o)\right)$$

$$= R + \frac{R+Z_o}{2} = \frac{3R}{2} + \frac{Z_o}{2}$$

$$\Rightarrow \frac{3R}{2} = \frac{Z_o}{2} \Rightarrow Z_o = 3R$$

$$\boxed{R = \frac{Z_o}{3}} \ \Omega$$

Example: T-Junction Power Divider

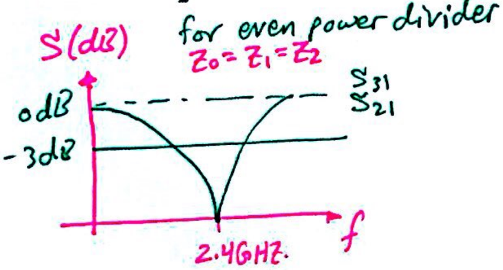
- A lossless T-junction power divider has a source impedance of 50 Ω. Find the output characteristic impedances so that the output powers are in a 2:1 ratio. Compute the reflection coefficients seen looking into the output ports

$$P_{in} = \frac{1}{2}\frac{V_0^2}{Z_0}$$

$$\left.\begin{aligned} P_1 &= \frac{1}{2}\frac{V_0^2}{Z_1} = \frac{1}{3}P_{in} \\ P_2 &= \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{2}{3}P_{in} \end{aligned}\right\} P_1+P_2=P_{in}.$$

$$Z_1 = 3Z_0 = 150\ \Omega$$

$$Z_2 = \frac{3Z_0}{2} = 75\ \Omega$$

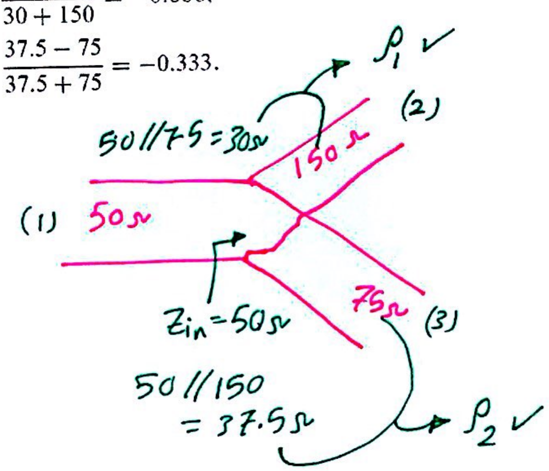


$$Z_{in} = 75||150 = 50\ \Omega$$

Looking into the 150 output line, we see an impedance of 50 || 75 = 30 Ω, while at the 75 Ω output line we see an impedance of 50 || 150 = 37.5 Ω. The reflection coefficients seen looking into these ports are:

$$\Gamma_1 = \frac{30-150}{30+150} = -0.666,$$

$$\Gamma_2 = \frac{37.5-75}{37.5+75} = -0.333.$$



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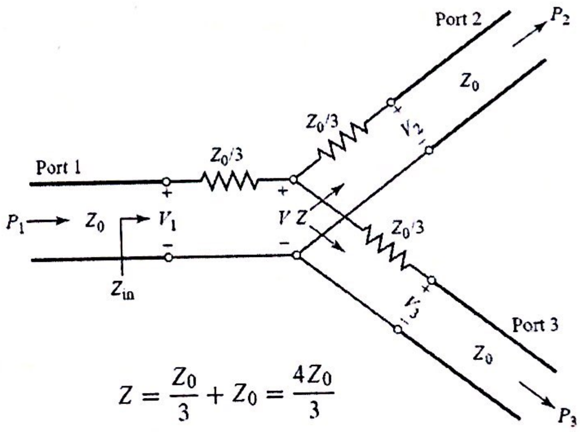
T-Junction Power Divider (4)

- **Resistive Divider:**
- If a three-port divider contains lossy components, it can be made to be matched at all ports, although the two output ports may not be isolated.
- The circuit for such a divider using lumped-element resistors. An equal-split (−3 dB) divider is used, but unequal power division ratios are also possible.



$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}$$

$$Z_{in} = \frac{Z_0}{3} + \frac{2Z_0}{3} = Z_0$$

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T-Junction Power Divider (5)

- If the voltage at port 1 is V_1, then by voltage division the voltage V at the center of the junction is

$$V = V_1 \frac{2Z_0/3}{Z_0/3 + 2Z_0/3} = \frac{2}{3}V_1$$

and the output voltages are, again by voltage division,

$$V_2 = V_3 = V \frac{Z_0}{Z_0 + Z_0/3} = \frac{3}{4}V = \frac{1}{2}V_1$$

*But Not Lossless.

Thus, $S_{21} = S_{31} = S_{23} = 1/2$, so the output powers are 6 dB below the input power level. The network is reciprocal, so the scattering matrix is symmetric

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

*reciprocal

*matched.

$$P_{in} = \frac{1}{2}\frac{V_1^2}{Z_0} \qquad P_2 = P_3 = \frac{1}{2}\frac{(1/2V_1)^2}{Z_0} = \frac{1}{8}\frac{V_1^2}{Z_0} = \frac{1}{4}P_{in}$$

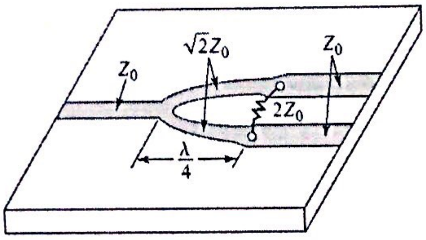
which shows that half of the supplied power is dissipated in the resistors

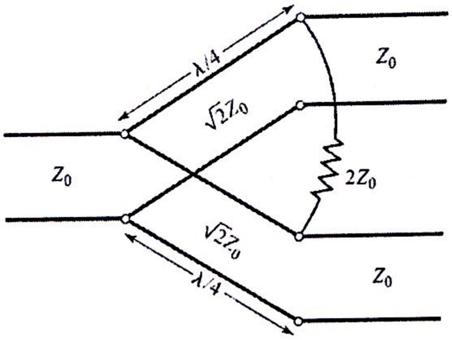
Wilkinson Power Divider (1)

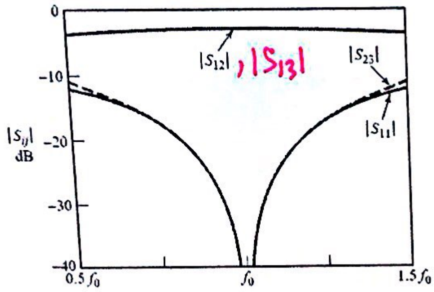
- The lossless T-junction divider suffers from the disadvantage of not being matched at all ports, and it does not have isolation between output ports.
- The resistive divider can be matched at all ports, but even though it is not lossless, isolation is still not achieved.
- A lossy three-port network can be made having all ports matched, with isolation between output ports.
- The Wilkinson power divider is such a network, with the useful property of appearing lossless when the output ports are matched; that is, only reflected power from the output ports is dissipated.
- The Wilkinson power divider can be made with arbitrary power division (even or odd modes).

$\beta l = \lambda/4$

Wilkinson Power Divider (2)





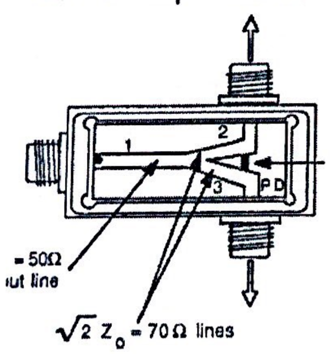


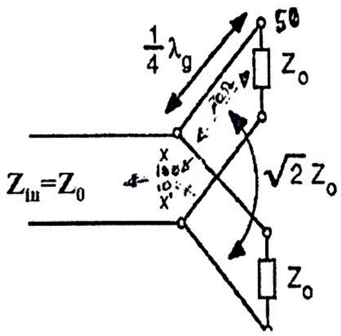
* $\frac{\lambda}{4}$ transformer make the BW. narrow band. (reduce BW).

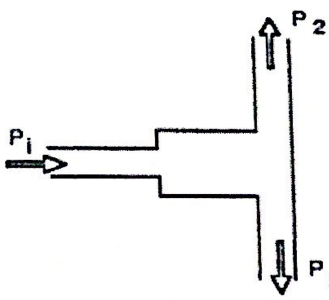
Wilkinson Power Divider (3)

for matched output ports, $\mathbf{Z}_{x\text{-}x'}$ are 100 Ω||100 Ω = 50 Ω. Again to match 50 Ω and 100 Ω of the two ends of any λ/4 line, we need the line to have Z=70 Ω.

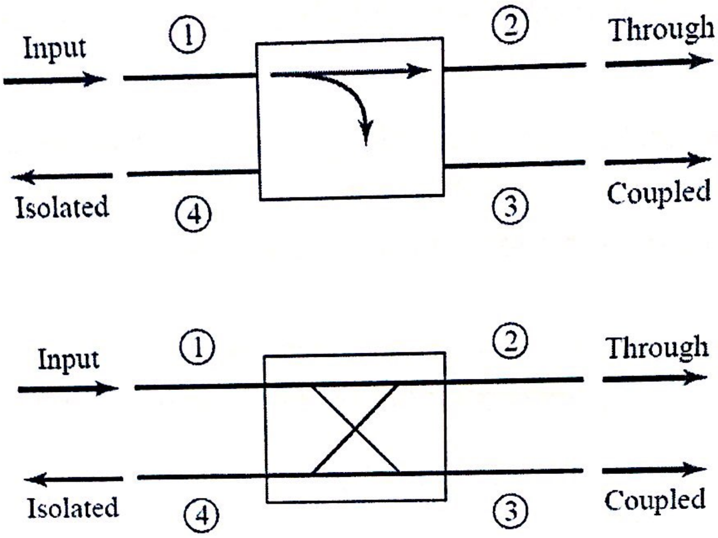
- 100 Ω chip resistor acts as an absorbed load for reflected signals.







Four-Port Networks (Directional Couplers)



Two commonly used symbols for directional couplers, and power flow conventions

it is odd power splitter (Not even).

if we input from 1 ⇒ it will go for 2 & 3 , 4 will be isolated.
" " " " 3 " " " " 1 & 4 , 2 " " ".

directional coupler ≡ odd power divider

Basic Operation

- Power incident at port 1 will couple to port 2 (the through port) and to port 3 (the coupled port), but not to port 4 (the isolated port).
- Similarly, power incident in port 2 will couple to ports 1 and 4, but not 3. Thus, ports 1 and 4 are decoupled, as are ports 2 and 3. The fraction of power coupled from port 1 to port 3 is given by C (the coupling coefficient) and the leakage of power from port 1 to port 4 is given by I (the isolation).
- Another quantity that characterizes a coupler is the directivity D = I – C (dB), which is the ratio of the powers delivered to the coupled port and the isolated port. The ideal coupler is characterized solely by the coupling factor, as the isolation and directivity are infinite.
- The ideal coupler is also lossless and matched at all ports.

Types of Couplers

- Directional couplers can be made in many different forms.

- For output power equally divided between ports: **Hybrid** Coupler (90° or 180°). 90° or 180° between the two outputs.

- For output power unequally divided between ports: **Directional** Coupler.

- **Microstrip Coupler** is a four port device used in microwave balanced amplifiers, mixers, power dividers and combiners.

Behind the page.

Directional Couplers (2)

- The scattering matrix of a reciprocal four-port network matched at all ports has the following form

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

$\rightarrow \lambda/4$

A Symmetric Coupler: $\theta = \varphi = \pi/2$. The phases of the terms having amplitude β are chosen equal

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

$\rightarrow \lambda/2$.

An Antisymmetric Coupler: $\theta = 0$, $\varphi = \pi$. The phases of the terms having amplitude β are chosen to be 180° apart

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

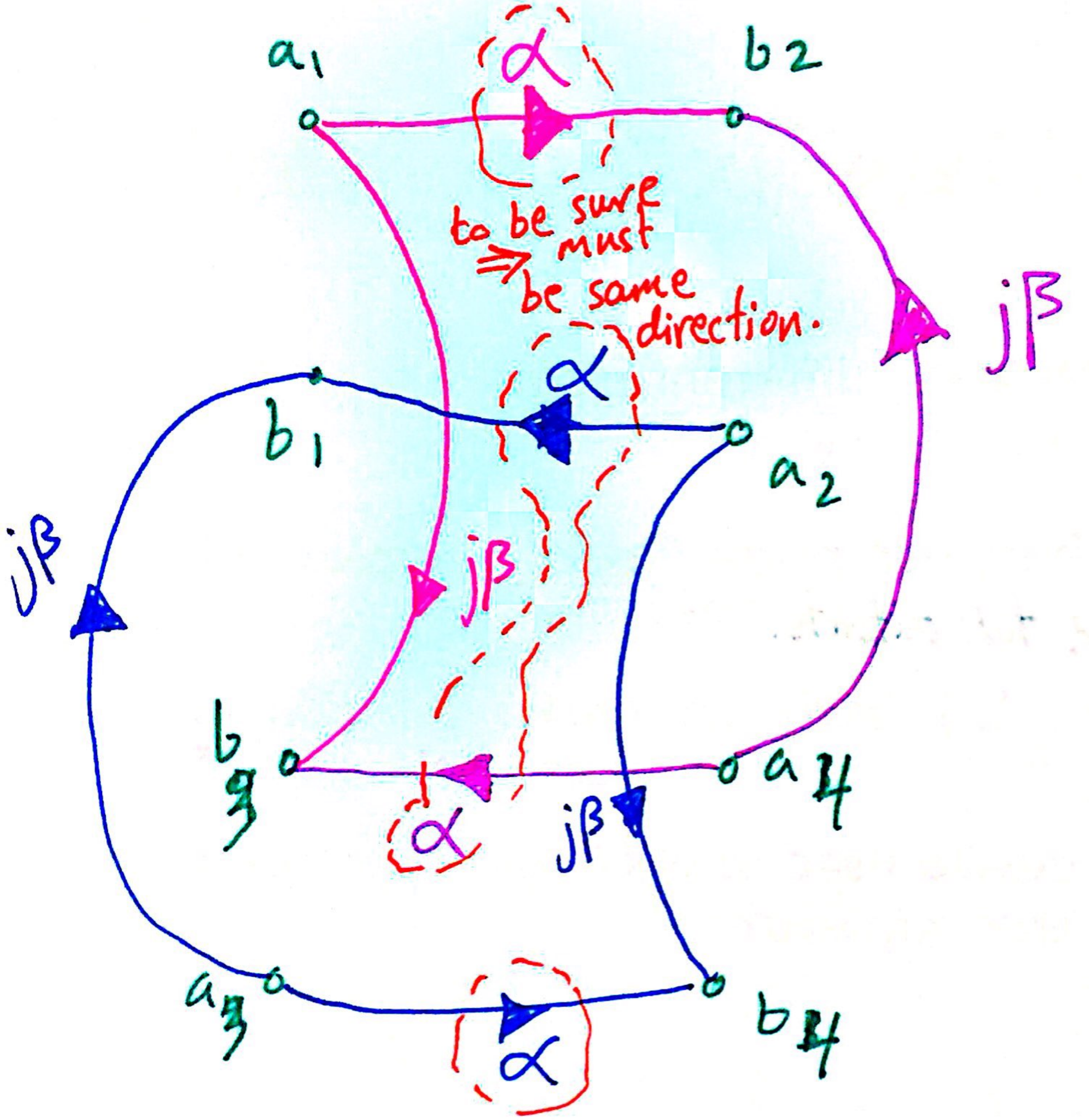
$$\alpha^2 + \beta^2 = 1$$

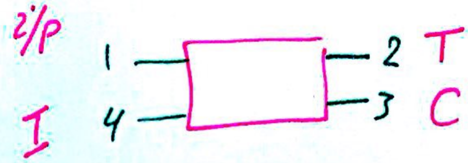
$\Downarrow$ to be lossless.

$$\alpha = \beta = \frac{1}{\sqrt{2}}$$

Behind the page.

for the symmetric coupler:





Directional Couplers (3)

for Power

for voltage we can use: $C = 20\log\frac{V_1}{V_3}$ & so on for others.

$$\text{Coupling} = C = 10\log\frac{P_1}{P_3} = -20\log\beta\ \text{dB}$$

$$= 20\log\frac{V_3}{V_1}\ \text{dB}$$

$$\text{Directivity} = D = 10\log\frac{P_3}{P_4} = 20\log\frac{\beta}{|S_{14}|}\ \text{dB}$$

$$\text{Isolation} = I = 10\log\frac{P_1}{P_4} = -20\log|S_{14}|\ \text{dB}$$

$$\text{Insertion loss} = L = 10\log\frac{P_1}{P_2} = -20\log|S_{12}|\ \text{dB}$$

Need D to be as larger as possible $\rightarrow \infty$ since it would be closer for the ideal directional coupler.

Directional Couplers (4)

- The coupling factor indicates the fraction of the input power that is coupled to the output port.
- The directivity is a measure of the coupler's ability to isolate forward and backward waves (or the coupled and uncoupled ports).
- The isolation is a measure of the power delivered to the uncoupled port.

$$I = D + C\ \text{dB}$$

- The insertion loss accounts for the input power delivered to the through port, diminished by power delivered to the coupled and isolated ports. The ideal coupler has infinite directivity and isolation ($S_{14} = 0$). Then both α and β can be determined from the coupling factor, C.

$\beta = C \Rightarrow C = \sqrt{1-\alpha^2}$

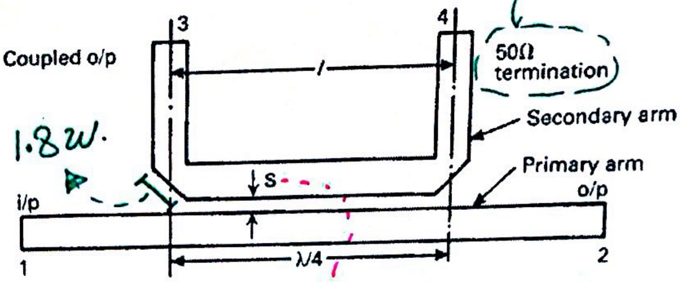
Directional Coupler (1)

- Power available on ports depends on direction of power flow. Different for forward/reverse wave

or Z_0 termination.

A) Parallel line directional coupler:

✓ Coupling depends on the gap 's'
✓ Difficult to realize for tight coupling



$C = \frac{V_3}{V_1} = 20 \log \frac{V_3}{V_1}$ (dB) $D = \frac{V_4}{V_3} = 20 \log \frac{V_4}{V_3}$ (dB) → Undesirable coupling

$I = \frac{V_4}{V_1} = 20 \log \frac{V_4}{V_1}$ (dB) $T = \frac{V_2}{V_1} = 20 \log \frac{V_2}{V_1}$ (dB)

↳ As long as S smaller ⇒ coupling Coefficient increase

S↓ ⇒ C↑

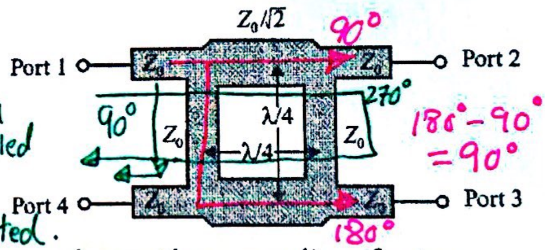
But under some Limitations.

↳ for $\frac{V_1}{V_2}$ ⇒ insertion loss.

for $\frac{V_2}{V_1}$ ⇒ Transmission.

Directional Coupler (2)

270 will be cancelled with 90 so port 4 is isolated.



180°−90° = 90°

B) Hybrid (Branch line) coupler

- It is a special cases of directional couplers, where the coupling factor is 3 dB (tight coupling, hard to achieve with parallel line case), which implies that $\alpha = \beta = 1/\sqrt{2}$. $-20\log(\frac{1}{\sqrt{2}}) = 3dB$
- There are two types of hybrids.

➢The quadrature hybrid has a 90° phase shift between ports 2 and 3 ($\theta = \phi = \pi/2$) when fed at port 1, and is an example of a symmetric coupler. Its scattering matrix has the following form:

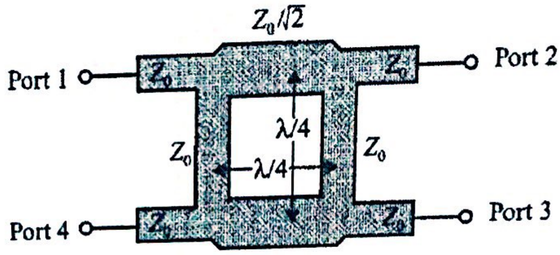
$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

(−) → delay.
(+) → advance.

↳ gives the phase shift between the output ports.

Quadrature Hybrid

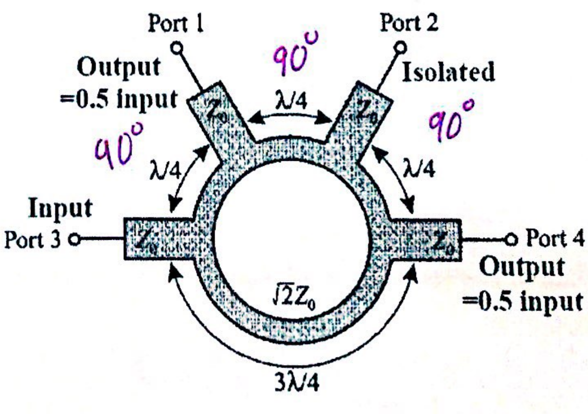
- Power feed at port 1 travel clockwise and counter clockwise to output ports 3 and 4.
- Phase difference between the two signals (via two path) arrived
- at port 4 is $\lambda_g/2$ or 180°, so they cancel each other **isolated**
- But the two signals (via two path) arrived at port 3 are in phase and add with each other (but having 180°out-of-phase with input)
- Thus, port 3 is **coupled** port of the **quadrature** hybrid coupler



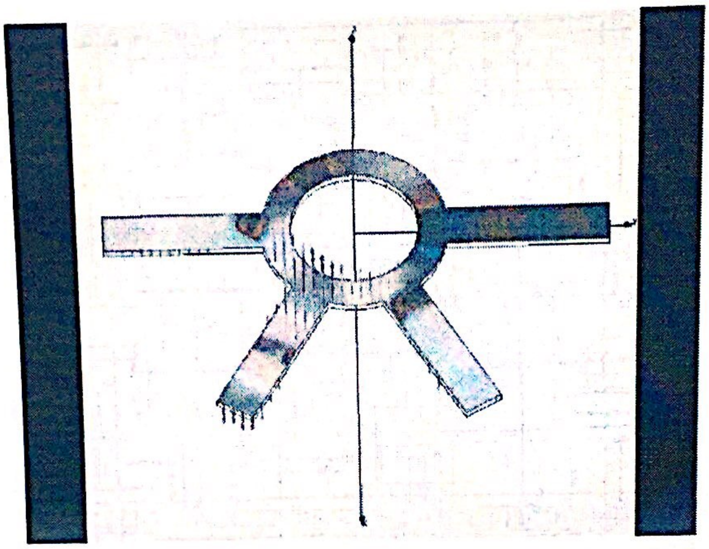
180° Hybrid Coupler (1)

➢The magic-T hybrid and the rat-race hybrid have a 180° phase difference between ports 2 and 3 when fed at port 4, and are examples of an antisymmetric coupler. Its scattering matrix has the following form:

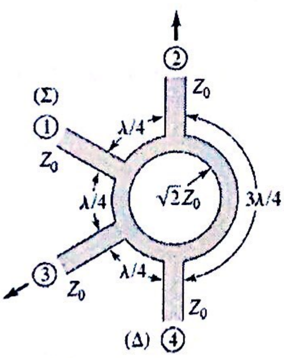
$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

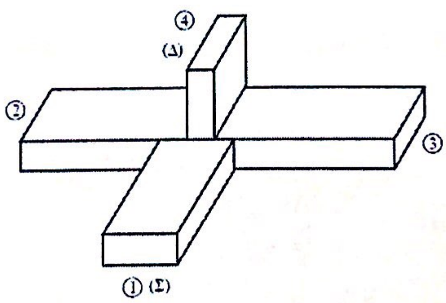


180º Hybrid Coupler (2)

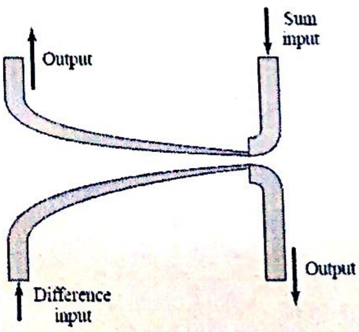


180º Hybrid Coupler (3)



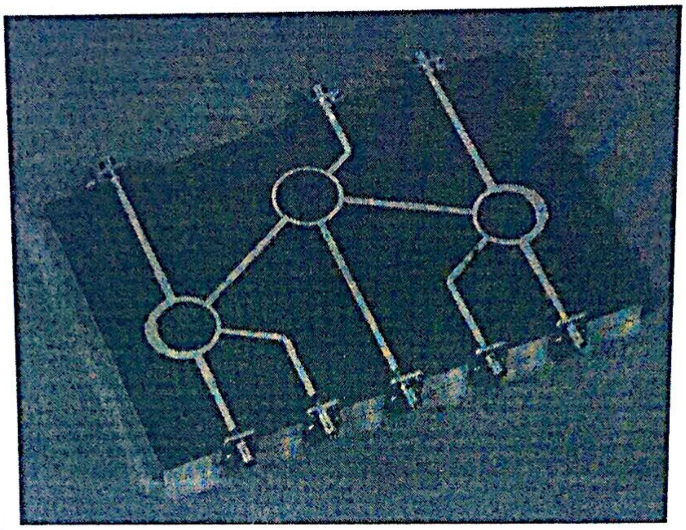


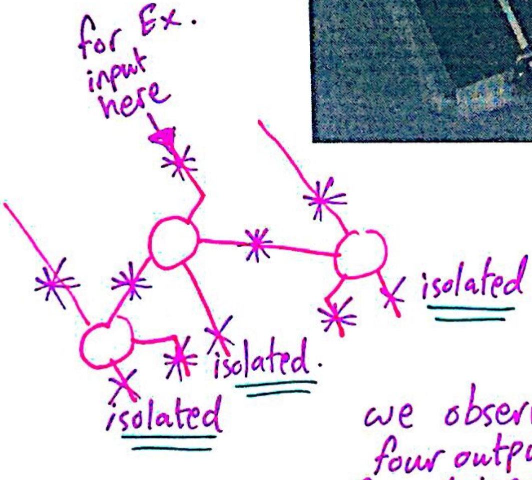
waveguide hybrid junction (magic-T)



Tapered Coupled Line Hybrid

Photograph of a microstrip power divider network using three ring hybrids





we observed four outputs for 1 input.

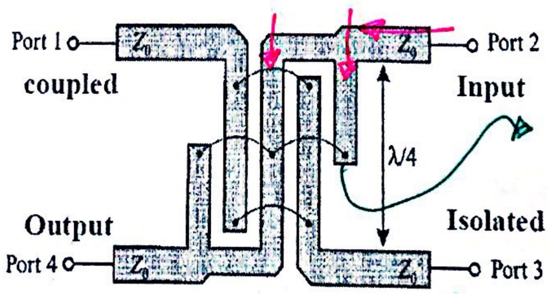
Behind page. ⇒

Directional Coupler (4)

C) 3 db Lunge coupler:

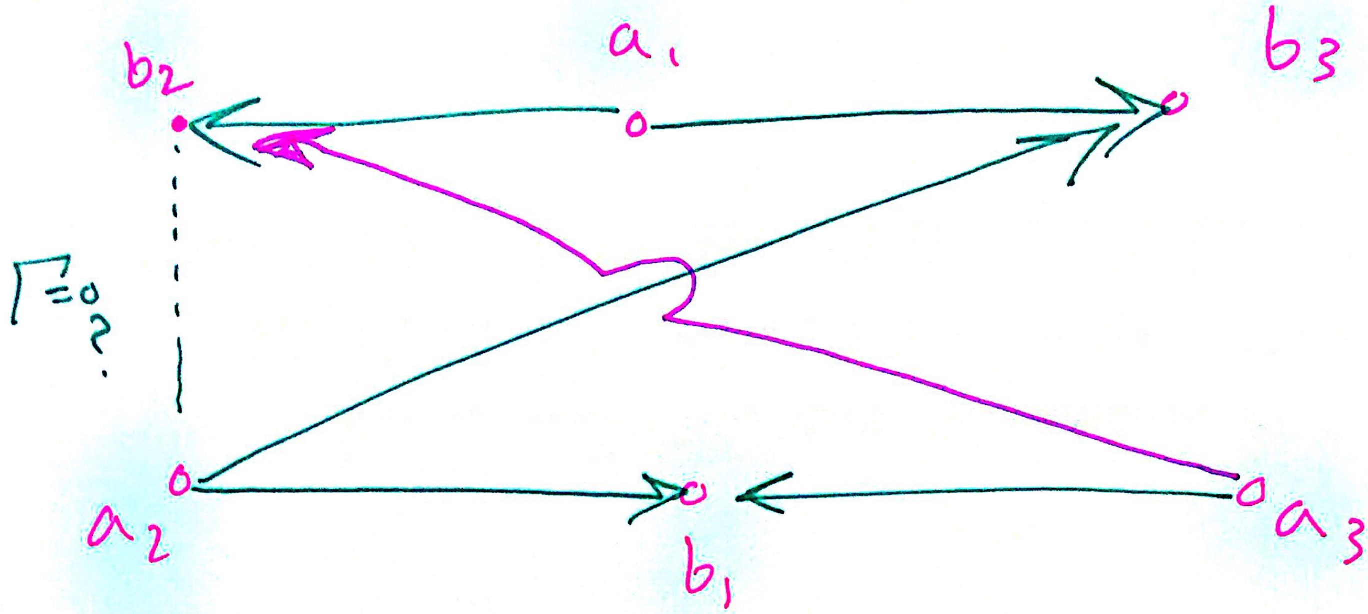
- Compact and provide tight broadband coupling
- Phase difference between output & coupled ports is 90°

(Note, bridge wire)



This is an open shunt stab.

this also called: Unfolded.



if it is lossless we put lines between a_2 & b_2, b_3 & a_3.

Topics in Communications.

"Microwaves"

Spring 2017/2018

Dr. Yanal Al-Faouri

By: Mohammad Abu Hashya.

* * *

Microwave Engineering

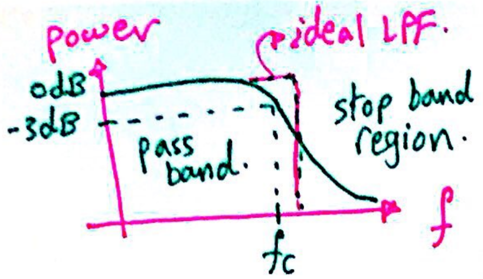
Chapter 8

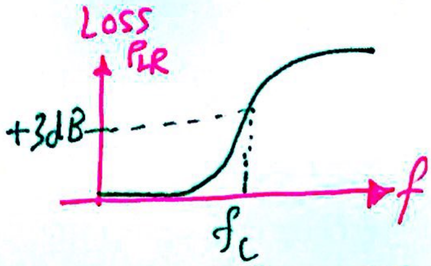
Dr. Yanal Faouri

Email: y.faouri@ju.edu.jo

Microwave Filters

- Periodic Structures
- Filter Design by the Image Parameter Method
- Filter Design by the Insertion Loss Method
- Filter Transformations
- Filter Implementation
- Stepped-Impedance Low-Pass Filters
- Coupled Line Filters
- Filters Using Coupled Resonators





Introduction (1)

these are representing LPF.

- A filter is a two-port network used to control the frequency response at a certain point in an RF or microwave system by providing transmission at frequencies within the passband of the filter and attenuation in the stopband of the filter.
- Typical frequency responses include low-pass, high-pass, bandpass, and band-reject characteristics.
- Applications can be found in virtually any type of RF or microwave communication, radar, or test and measurement system.
- Filters are indispensable components in wireless systems, used in receivers for rejecting signals outside the operating band, attenuating undesired mixer products, and for setting the IF bandwidth of the receiver. In transmitters, filters are used to control the spurious responses of up-converting mixers, to select the desired sidebands, and to limit the bandwidth of the radiated signal.

important components.

Introduction (2)

using ABCD matrix.

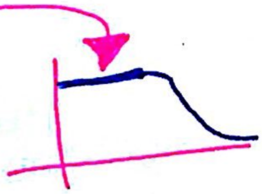
- The **image parameter method** of filter design was developed in the late 1930s and was useful for low-frequency filters in radio and telephony.
- Today, most microwave filter design is done with sophisticated computer-aided design (CAD) packages based on the **insertion loss method**.
- Because of continuing advances in network synthesis with distributed elements, the use of low temperature superconductors and other new materials, and the incorporation of active devices in filter circuits, microwave filter design remains an active research area.

Introduction (3)

- Filters designed using the **image parameter method** consist of a cascade of simpler two port filter sections to provide the desired cutoff frequencies and attenuation characteristics but do not allow the specification of a particular frequency response over the complete operating range.
- Thus, although the procedure is relatively simple, the design of filters by the image parameter method often must be iterated many times to achieve the desired results.
- A more modern procedure, called the **insertion loss method**, uses network synthesis techniques to design filters with a completely specified frequency response. The design is simplified by beginning with low-pass filter prototypes that are normalized in terms of impedance and frequency.

Introduction (4)

- Transformations are then applied to convert the prototype designs to the desired frequency range and impedance level.
- Both the image parameter and insertion loss methods of filter design lead to circuits using lumped elements (capacitors and inductors).
- Finally, **Transmission line stubs** (with dimensions computed using Richard's transformation, impedance/admittance inverters, and the Kuroda identities).
- OR **Stepped-Impedance** techniques are used to facilitate filter implementation in terms of practical microwave components.

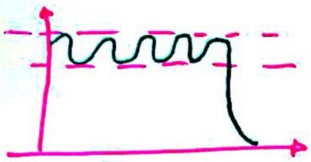


Introduction (5)

- Three types of practical filter responses are considered here:
- Binomial or Butterworth or Maximally flat response: can cater the need for minimum insertion loss by providing flattest possible pass-band response.
- Equal-ripple or Chebyshev response: can satisfy a requirement for the sharpest cutoff region.
- Linear Phase response: is important in multiplexing filters used in communication systems to avoid distortion at the cost of filters sharp-cutoff characteristics

give → flat, sharp, linear phase.

ripple



⇓

Two types
3dB ripple
or 0.5dB ripple.

Filter Design by the Insertion Loss Method

- A perfect filter would have zero insertion loss in the passband, infinite attenuation in the stopband, and a linear phase response (to avoid signal distortion) in the passband.
- Of course, such filters do not exist in practice, so compromises must be made; herein lies the art of filter design.
- The image parameter method may yield a usable filter response for some applications, but there is no methodical way of improving the design.
- The insertion loss method, however, allows a high degree of control over the passband and stopband amplitude and phase characteristics, with a systematic way to synthesize a desired response.
- The necessary design trade-offs can be evaluated to best meet the application requirements. If, for example, a minimum insertion loss is most important, a binomial response could be used; a Chebyshev response would satisfy a requirement for the sharpest cutoff. If it is possible to sacrifice the attenuation rate, a better phase response can be obtained by using a linear phase filter design.
- In addition, in all cases, the insertion loss method allows filter performance to be improved in a straightforward manner, at the expense of a higher order filter.
- For the filter prototypes to be discussed below, the order of the filter is equal to the number of reactive elements (N).

↳ real number of elements used in the design.

Characterization by Power Loss Ratio

transmitted = incident − reflected.

- In the insertion loss method a filter response (to be physically realizable) is defined by its insertion loss, or power loss ratio, P_{LR}

$$P_{LR} = \frac{\text{Power available from source}}{\text{Power delivered to load}} = \frac{P_{inc}}{P_{load}} = \frac{1}{1-|\Gamma(\omega)|^2}$$

$P_t = P_i - P_r$

$= \frac{V_0^{+2}}{2Z_0}(1-|\rho|^2)$

- The insertion loss (IL) in dB is $\text{IL} = 10 \log P_{LR}$

ρ is function of frequancy.

- Notice that specifying the power loss ratio simultaneously constrains the magnitude of the reflection coefficient, $|\Gamma(\omega)|$. We now discuss some practical filter responses.
- **Maximally flat:** This characteristic is also called the binomial or Butterworth response, and is optimum in the sense that it provides the flattest possible passband response for a given filter complexity, or order.
- For a low-pass filter, it is specified by

$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c}\right)^{2N}$$

- $K \Rightarrow$ constant.
- $\omega_c = 2\pi f_c$ (cutoff frequancy).
- N is the order of the filter.

used when we want to design a filter analytically.

But we will use another method.

$\omega_c = 1$
$N = 2$

$P_{LR} = 1 + K^2 \omega^4$

desired $P_{LR} = 3dB$ @ $\omega = \omega_c$.

$2 = 1 + K^2 (1)^4$

$\Rightarrow K = 1$

$10 \log(2) = 3dB$

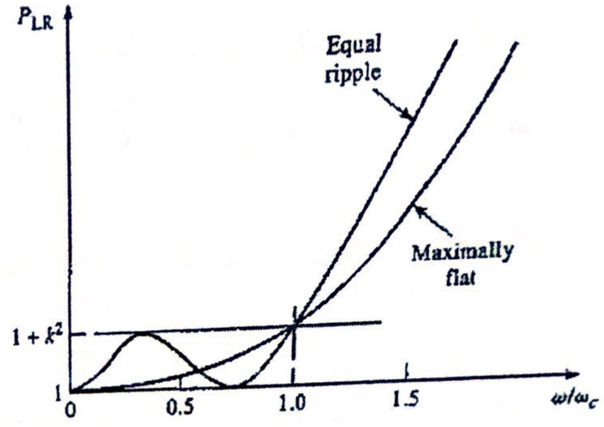
- where N is the order of the filter and ω_c is the cutoff frequency.
- The passband extends from $\omega = 0$ to $\omega = \omega_c$; at the band edge the power loss ratio is $1 + k^2$.
- If we choose this as the −3 dB point, as is common, we have $k = 1$, which we will assume from now on. ← حوديا"
- For $\omega > \omega_c$, the attenuation increases monotonically with frequency.
- For $\omega >> \omega_c$, $P_{LR} \approx k^2(\omega/\omega_c)^{2N}$, which shows that the insertion loss increases at the rate of 20N dB/decade.
- Like the binomial response for multi-section quarter-wave matching transformers, the first (2N − 1) derivatives are zero at $\omega = 0$.

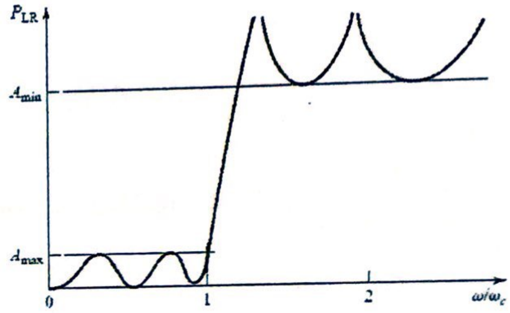
- **Equal ripple**: If a Chebyshev polynomial is used to specify the insertion loss of an N^{th} order low-pass filter as:

$$P_{LR} = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right)$$

- Then a sharper cutoff will result, although the passband response will have ripples of amplitude $1 + k^2$, since $T_N(x)$ oscillates between ±1 for $|x|^2$. Thus, k^2 determines the passband ripple level

$T_N(x)$ is Chebyshev polynomial of order 'N'
where,
$T_1(x) = x$
$T_2(x) = 2x^2 - 1$
$T_3(x) = 4x^3 - 3x$
$T_4(x) = 8x^4 - 8x^2 + 1$





- For large x, $T_N(x) \approx 1/2\,(2x)^N$, so for $\omega >> \omega_c$ the insertion loss becomes;

$$P_{LR} \simeq \frac{k^2}{4}\left(\frac{2\omega}{\omega_c}\right)^{2N}$$

* we will deal with K for 3dB or 0.5 dB ripple.

- which also increases at the rate of 20N dB/decade. However, the insertion loss for the Chebyshev case is $(2^{2N})/4$ greater than the binomial response at any given frequency where $\omega >> \omega c$.
- Elliptic function: The maximally flat and equal-ripple responses both have monotonically increasing attenuation in the stopband. In many applications it is adequate to specify a minimum stopband attenuation, in which case a better cutoff rate can be obtained.
- Such filters are called elliptic function filters, and they have equal-ripple responses in the passband as well as in the stopband. The maximum attenuation in the passband, A_{max}, can be specified, as well as the minimum attenuation in the stopband, A_{min}.
- Elliptic function filters are difficult to synthesize, so we will not consider them further.

- **Linear phase**: The above filters specify the amplitude response, but in some applications (such as multiplexing filters for communication systems) it is important to have a linear phase response in the passband to avoid signal distortion.
  - Since a sharp-cutoff response is generally incompatible with a good phase response, the phase response of a filter must be deliberately synthesized, usually resulting in an inferior attenuation characteristic.
  - A linear phase characteristic can be achieved with the following phase response:

$$\phi(\omega) = A\omega\left[1 + p\left(\frac{\omega}{\omega_c}\right)^{2N}\right] \qquad \tau_d = \frac{d\phi}{d\omega} = A\left[1 + p(2N+1)\left(\frac{\omega}{\omega_c}\right)^{2N}\right]$$

constant.

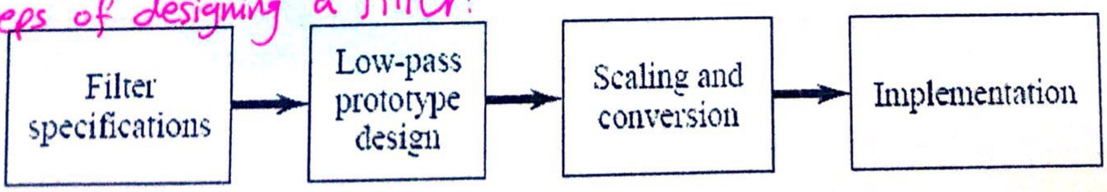
  - which shows that the group delay for a linear phase filter is a maximally flat function.

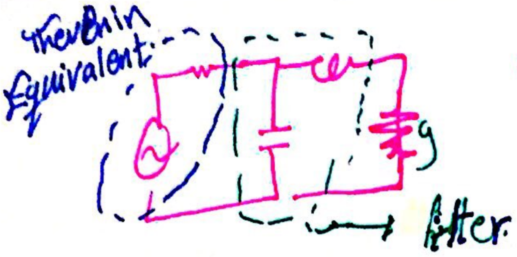
- More general filter specifications can be obtained, but the above cases are the most common. (ex. A Bessel-Thomson filter provides the best (flattest) group delay responses of all the filter types)

{Binomial
Chebyshev
linear phase.

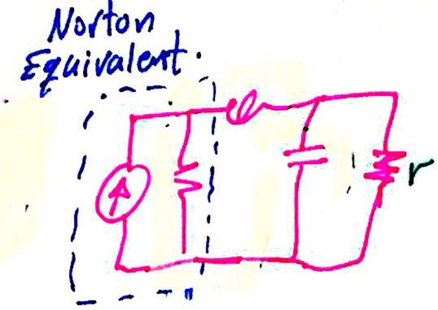
- We will next discuss the design of low-pass filter prototypes that are normalized in terms of impedance and frequency; this normalization simplifies the design of filters for arbitrary frequency, impedance, and type (low-pass, high-pass, bandpass, or bandstop).
- The low-pass prototypes are then scaled to the desired frequency and impedance, and the lumped-element components replaced with distributed circuit elements for implementation at microwave frequencies.

steps of designing a filter:





g - for parallel.
r - for series.



Maximally Flat Low-Pass Filter Prototype

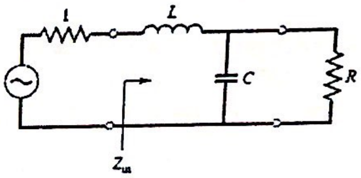
- For a normalized ($Z_S = 1\ \Omega$ and $\omega_C = 1$) two-element (N=2) low-pass filter, the input impedance and reflection coefficient are:

$$P_{LR} = 1 + \omega^4$$

$$P_{LR} = 1 + k^2\left(\frac{\omega}{\omega_c}\right)^{2N}$$

$$Z_{in} = j\omega L + \frac{R(1 - j\omega RC)}{1 + \omega^2 R^2 C^2}$$

$$\Gamma = \frac{Z_{in} - 1}{Z_{in} + 1}$$



$$P_{LR} = \frac{1}{1 - |\Gamma|^2} = \frac{1}{1 - [(Z_{in} - 1)/(Z_{in} + 1)][(Z_{in}^* - 1)/(Z_{in}^* + 1)]} = \frac{|Z_{in} + 1|^2}{2(Z_{in} + Z_{in}^*)}$$

$$Z_{in} + Z_{in}^* = \frac{2R}{1 + \omega^2 R^2 C^2}$$

$$|Z_{in} + 1|^2 = \left(\frac{R}{1 + \omega^2 R^2 C^2} + 1\right)^2 + \left(\omega L - \frac{\omega C R^2}{1 + \omega^2 R^2 C^2}\right)^2$$

$$P_{LR} = \frac{1 + \omega^2 R^2 C^2}{4R}\left[\left(\frac{R}{1 + \omega^2 R^2 C^2} + 1\right)^2 + \left(\omega L - \frac{\omega C R^2}{1 + \omega^2 R^2 C^2}\right)^2\right]$$

$$= \frac{1}{4R}(R^2 + 2R + 1 + R^2\omega^2 C^2 + \omega^2 L^2 + \omega^4 L^2 C^2 R^2 - 2\omega^2 LCR^2)$$

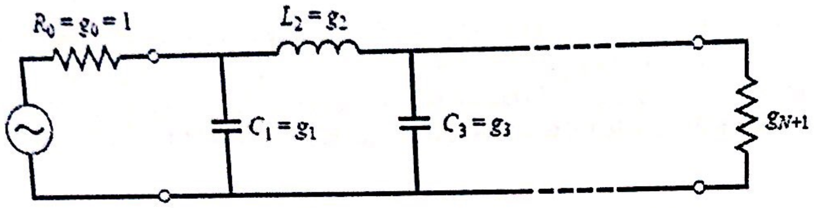
$$= 1 + \frac{1}{4R}[(1 - R)^2 + (R^2C^2 + L^2 - 2LCR^2)\omega^2 + L^2C^2R^2\omega^4]$$

Observe that this expression is a polynomial in ω^2. Comparing to the desired response; shows that R = 1, since $P_{LR} = 1$ for $\omega = 0$. In addition, the coefficient of ω^2 must vanish, so;

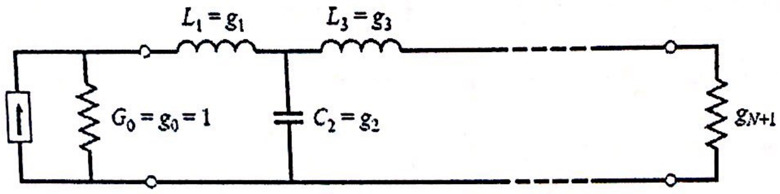
$\omega^2 = 0$ ➔ $C^2 + L^2 - 2LC = (C - L)^2 = 0$ ➔ or $L = C$

$\omega^4 = 1$ ➔ $\frac{1}{4}L^2C^2 = \frac{1}{4}L^4 = 1$ ➔ $L = C = \sqrt{2}$

- In principle, this procedure can be extended to find the element values for filters with an arbitrary number of elements N, but clearly this is not practical for large N.
- For a normalized low-pass design, where the source impedance is 1 and the cutoff frequency is $\omega_C = 1$ rad/sec, however, the element values for the ladder-type circuits can be tabulated.
- The elements alternate between series and shunt connections.



(a)



(b)

Ladder circuits for low-pass filter prototypes and their element definitions.
(a) Prototype beginning with a shunt element.
(b) Prototype beginning with a series element.

The element values are numbered from g_0 at the generator impedance to g_{N+1} at the load impedance for a filter having N reactive elements.

where: $g_0 = \begin{cases} \text{generator resistance for the network of Fig a.} \\ \text{generator conductance for the network of Fig b.} \end{cases}$

$g_k \; (k = 1 \text{ to } N) = \begin{cases} \text{inductance for series inductors} \\ \text{capacitor for shunt capacitors} \end{cases}$

$g_{N+1} = \begin{cases} \text{load resistance if } g_N \text{ is a shunt capacitor} \\ \text{load conductance if } g_N \text{ is a series inductor} \end{cases}$

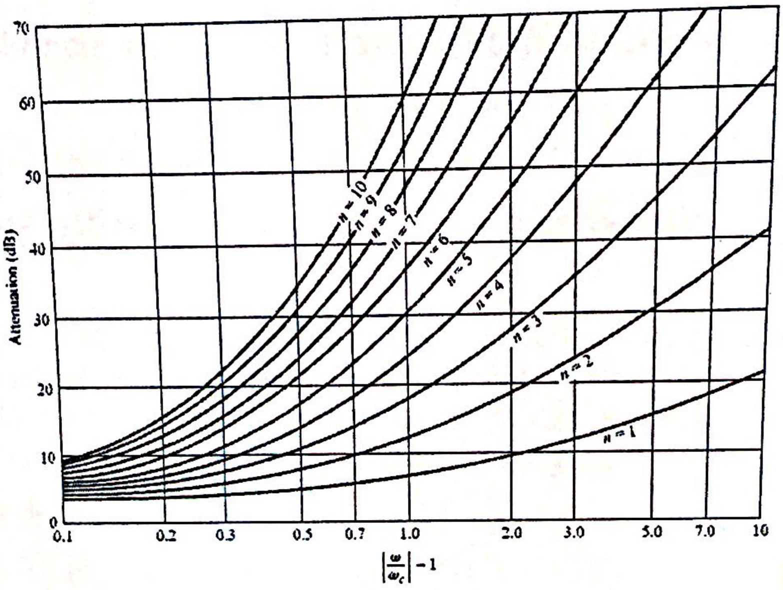
Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, N = 1 to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

this last number for the load.

an advantage for Binomial always Load is 1.

- Finally, as a matter of practical design procedure, it will be necessary to determine the size, or order, of the filter. This is usually dictated by a specification on the insertion loss at some frequency in the stopband of the filter. The figure below shows the attenuation characteristics for various N versus normalized frequency.
- If a filter with N > 10 is required, a good result can usually be obtained by cascading two designs of lower order.



⇒ for $\alpha = 20$ dB

$f_c = 2$ GHz

F = 4GHz

↳ operating freq.

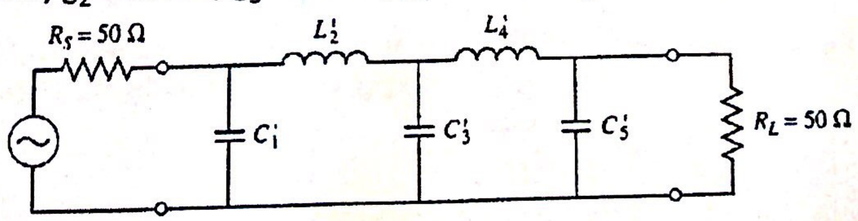
$\left|\frac{\omega}{\omega_c}\right| = d$

2-1 ⇒ ①

has N = 4

Example 1:

- A maximally-flat low-pass filter is to be designed with a cut-off frequency of 2 GHz and a minimum attenuation of 15 dB at 3 GHz
- (a) Find the number (N) of required filter elements.
- (b) Find the un-scaled values of the filter reactive elements.
- **Solution:**
- Given, f = 3 GHz and f_c = 2 GHz. Thus, $|\omega/\omega_c|-1 = 0.5$. From the chart we get, **N ≥ 5** to achieve minimum attenuation of 15 dB and from the table, the un-scaled values of the filter reactive elements are;
- $g_1 = 0.6180$; $g_2 = 1.618$; $g_3 = 2.000$; $g_4 = 1.618$; $g_5 = 0.618$;



Equal-Ripple Low-Pass Filter Prototype

- For an equal-ripple low-pass filter with a cutoff frequency $\omega_c = 1$ rad/sec

$$P_{LR} = 1 + k^2 T_N^2(\omega)$$

- where $1 + k^2$ is the ripple level in the passband.
- Since the Chebyshev polynomials have the property that;

$$T_N(0) = \begin{cases} 0 & \text{for } N \text{ odd,} \\ 1 & \text{for } N \text{ even,} \end{cases}$$

- Which shows that the filter will have a unity power loss ratio at $\omega = 0$ for N odd, but a power loss ratio of $1 + k^2$ at $\omega = 0$ for N even. Thus, there are two cases to consider, depending on N. For N = 2; $T_2(x) = 2x^2 - 1$

$$1 + k^2(4\omega^4 - 4\omega^2 + 1) = 1 + \frac{1}{4R}[(1-R)^2 + (R^2C^2 + L^2 - 2LCR^2)\omega^2 + L^2C^2R^2\omega^4]$$

at $\omega = 0$ we have that → $k^2 = \frac{(1-R)^2}{4R}$

$$R = 1 + 2k^2 \pm 2k\sqrt{1+k^2} \quad \text{(for } N \text{ even)}$$

- Equating coefficients of ω^2 and ω^4 yields the additional relations;

$$-4k^2 = \frac{1}{4R}(R^2C^2 + L^2 - 2LCR^2) \qquad 4k^2 = \frac{1}{4R}L^2C^2R^2$$

- which can be used to find L and C.
- Note that the value for R is not unity, so there will be an impedance mismatch if the load has a unity (normalized) impedance; this can be corrected with a quarter-wave transformer, or by using an additional filter element to make N odd.
- For odd N, it can be shown that R = 1. (This is because there is a unity power loss ratio at $\omega = 0$ for N odd)

- Tables exist for designing equal-ripple low-pass filters with a normalized source impedance and cutoff frequency ($\omega'_c = 1$ rad/sec), and these can be applied to either of the ladder circuits shown before.
- The design data depends on the specified passband ripple level.
- Notice that the load impedance $g_{N+1} \neq 1$ for even N.
- If the stopband attenuation is specified, the curves can be used to determine the necessary value of N for these ripple values.

Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, N = 1 to 10, 0.5 dB ripple)

0.5 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

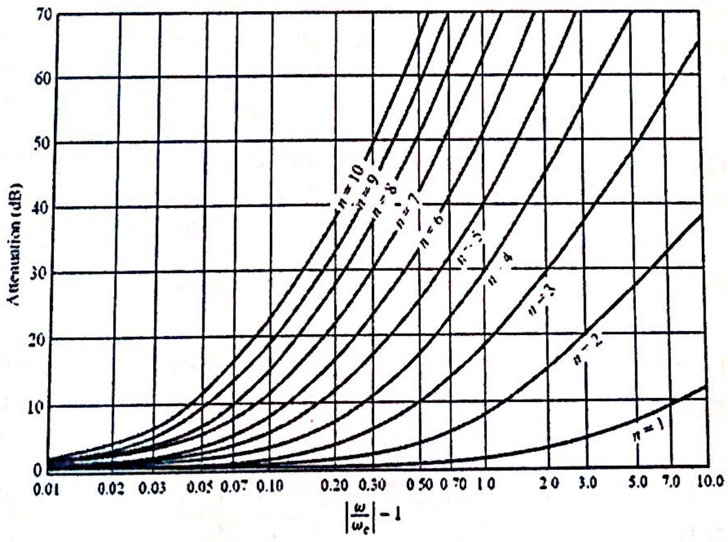
disadvantage for chebyshev for even number of elements Load ≠ 1 (matching have). to be done

Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, N = 1 to 10, 3 dB ripple)

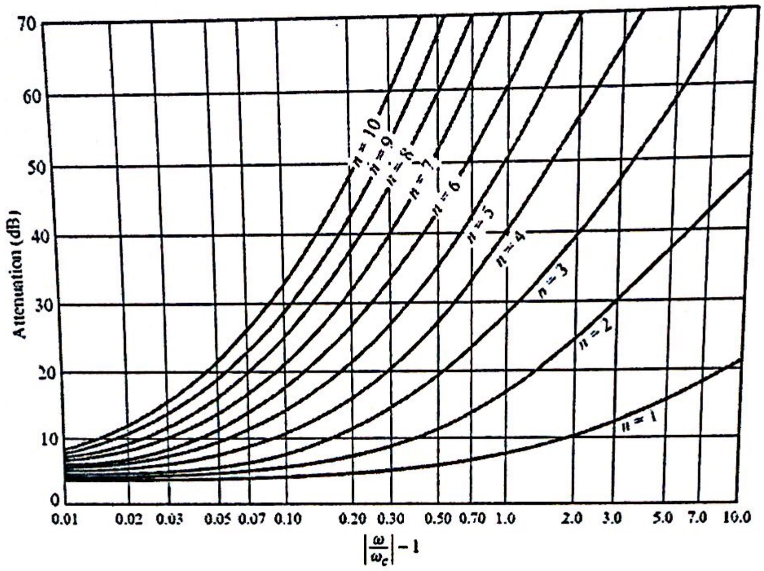
3.0 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Attenuation versus normalized frequency for equal-ripple filter prototypes. (0.5 dB ripple level)



Attenuation versus normalized frequency for equal-ripple filter prototypes. (3.0 dB ripple level)



Linear Phase Low-Pass Filter Prototypes

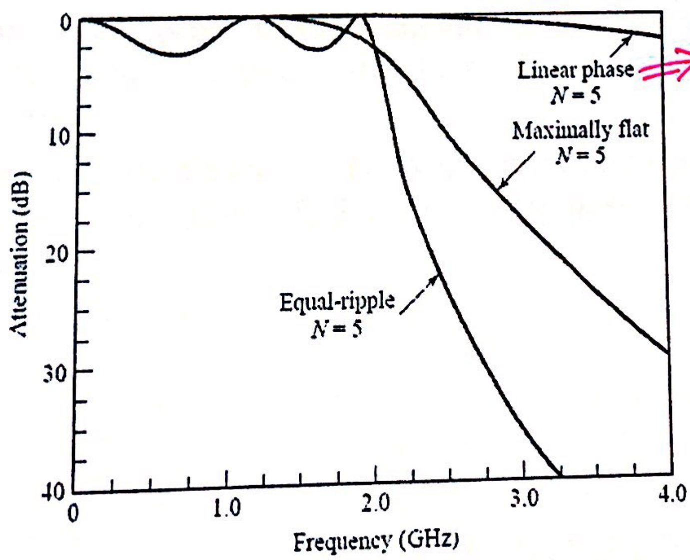
- Filters having a maximally flat time delay, or a linear phase response, can be designed in the same way, but things are somewhat more complicated because the phase of the voltage transfer function is not as simply expressed as is its amplitude.
- Design values have been derived for such filters; for the ladder circuits, and they are tabulated for a normalized source impedance and cutoff frequency ($\omega'_c = 1$ rad/sec).
- The resulting normalized group delay in the passband will be $\tau_d = 1/\omega'_c = 1$ sec.
- The group delay of a filter is a function of many things besides the type of filter. Group delay increases as the order of a filter is increased and if the bandwidth decreases.

$$\phi(\omega) = A\omega\left[1 + p\left(\frac{\omega}{\omega_c}\right)^{2N}\right] \qquad \tau = \frac{d\phi}{d\omega} = A\left[1 + p(2N+1)\left(\frac{\omega}{\omega_c}\right)^{2N}\right]$$

Element Values for Maximally Flat Time Delay Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

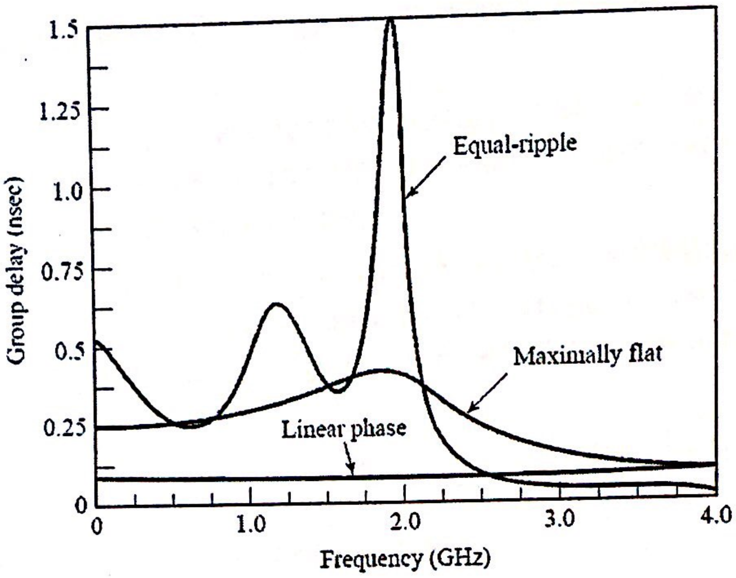
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.5774	0.4226	1.0000								
3	1.2550	0.5528	0.1922	1.0000							
4	1.0598	0.5116	0.3181	0.1104	1.0000						
5	0.9303	0.4577	0.3312	0.2090	0.0718	1.0000					
6	0.8377	0.4116	0.3158	0.2364	0.1480	0.0505	1.0000				
7	0.7677	0.3744	0.2944	0.2378	0.1778	0.1104	0.0375	1.0000			
8	0.7125	0.3446	0.2735	0.2297	0.1867	0.1387	0.0855	0.0289	1.0000		
9	0.6678	0.3203	0.2547	0.2184	0.1859	0.1506	0.1111	0.0682	0.0230	1.0000	
10	0.6305	0.3002	0.2384	0.2066	0.1808	0.1539	0.1240	0.0911	0.0557	0.0187	1.0000

Comparison of the three responses in terms of attenuation



⇒ flattest.

Comparison of the three responses in terms of phase



Filter Transformations

- The low-pass filter prototypes designed before were normalized designs having a source impedance of $R_s = 1\ \Omega$ and a cutoff frequency of $\omega_c = 1$ rad/sec.
- These designs can be scaled in terms of impedance and frequency, and converted to give high-pass, bandpass, or bandstop characteristics.

Impedance Scaling

- Scaling convert normalized filters to filters with prescribed impedance and ω_c
- Impedance scaling: In the prototype design, the source and load resistances are unity (except for equal-ripple filters with even N, which have non-unity load resistance).
- A source resistance of R_0 can be obtained by multiplying all the impedances of the prototype design by R_0. Thus, if we let primes denote impedance scaled quantities, the new filter component values are given by;

$$L' = R_0 L \qquad C' = \frac{C}{R_0} \qquad R_s' = R_0 \overset{=1}{R_s} \qquad R_L' = R_0 R_L$$

where L, C, and R_L are the component values for the original prototype.

Frequency Scaling

- Frequency scaling for low-pass filters: To change the cutoff frequency of a low-pass prototype from unity to ω_c requires that we scale the frequency dependence of the filter by the factor $1/\omega_c$, which is accomplished by replacing ω by ω/ω_c:

$$\omega \leftarrow \frac{\omega}{\omega_c} \qquad P'_{LR}(\omega) = P_{LR}\left(\frac{\omega}{\omega_c}\right)$$

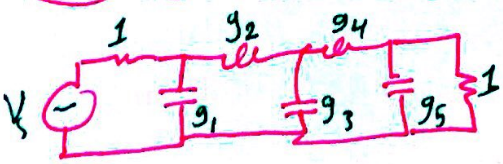
The new element values are given by

$$jX_k = j\frac{\omega}{\omega_c}L_k = j\omega L'_k \qquad jB_k = j\frac{\omega}{\omega_c}C_k = j\omega C'_k \rightarrow L'_k = \frac{L_k}{\omega_c} \qquad C'_k = \frac{C_k}{\omega_c}$$

When both impedance and frequency scaling are required

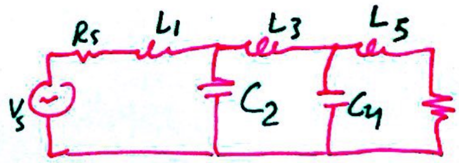
$$L'_k = \frac{R_0 L_k}{\omega_c} \qquad C'_k = \frac{C_k}{R_0 \omega_c}$$

→ write it in formula sheet.



$g_1 \rightarrow C_1 \rightarrow C_1'$
$g_2 \rightarrow L_2$
$g_3 \rightarrow C_3$
$g_4 \rightarrow L_4$
$g_5 \rightarrow C_5$

OR



Example 2:

$\omega_c = 2\pi \times 2G$ rad/s.

- If the maximally-flat low pass filter of example 1 (see slide 20) has an impedance of 50 Ω, then; $g_1 = 0.6180$; $g_2 = 1.618$; $g_3 = 2.000$; $g_4 = 1.618$; $g_5 = 0.618$

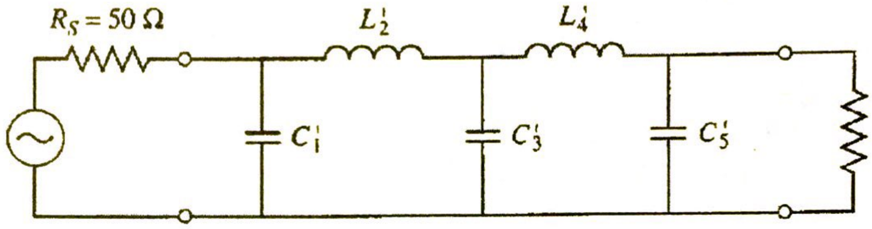
(1) find the scaled value of the reactive elements.

(2) Draw the circuit.

symmetrical for odd # of elements.

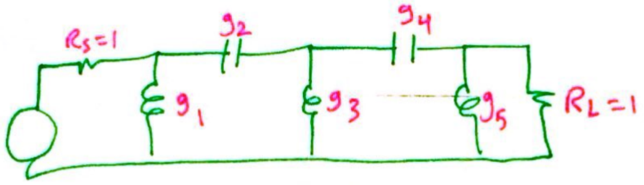
Solution:

$C'_1 = 0.984$ pF, $L'_2 = 6.438$ nH, $C'_3 = 3.183$ pF, $L'_4 = 6.438$ nH, $C'_5 = 0.984$ pF



5 elements so C'_1 & C'_5 are symmetrical also L'_2 & L'_4 are symmetrical.

in the Bottom of the previous page. LPF. HPF.



Low-pass to High-pass Transformation

- The frequency substitution: $\omega \leftarrow -\dfrac{\omega_c}{\omega}$
- This substitution maps ω = 0 to ω = ±∞, and vice versa; cutoff occurs when $\omega = \pm\omega_c$.
- The negative sign is needed to convert inductors (and capacitors) to realizable capacitors (and inductors).

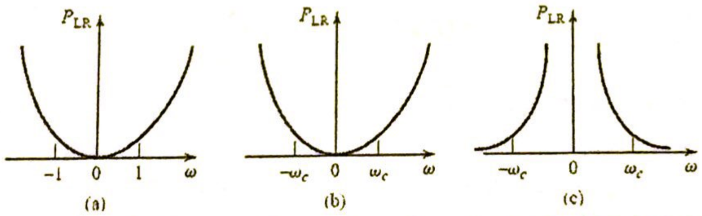
$$jX_k = -j\frac{\omega_c}{\omega}L_k = \frac{1}{j\omega C'_k} \qquad jB_k = -j\frac{\omega_c}{\omega}C_k = \frac{1}{j\omega L'_k}$$

$$C'_k = \frac{1}{\omega_c L_k}$$

$$L'_k = \frac{1}{\omega_c C_k}$$

$$C'_k = \frac{1}{R_0 \omega_c L_k}$$

$$L'_k = \frac{R_0}{\omega_c C_k}$$



Frequency scaling for low-pass filters and transformation to a high-pass response.
(a) Low-pass filter prototype response for $\omega_c = 1$ rad/sec.
(b) Frequency scaling for low-pass response.
(c) Transformation to high-pass response.

Do the last example considering HPF.

Bandpass and Bandstop Transformations

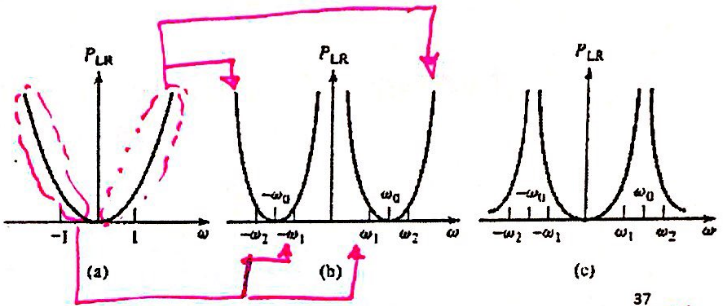
- Low-pass prototype filter designs can also be transformed to have the bandpass or bandstop responses.
- If ω_1 and ω_2 denote the edges of the passband, then a bandpass response can be obtained using the following frequency substitution:

$$\omega \leftarrow \frac{\omega_0}{\omega_2-\omega_1}\left(\overbrace{\frac{\omega}{\omega_0}}^{\text{LPF part}} - \underbrace{\frac{\omega_0}{\omega}}_{\text{HPF part.}}\right) = \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

$\Delta = \frac{\omega_2-\omega_1}{\omega_0}$ is the fractional bandwidth of the passband

$$\omega_0 = \sqrt{\omega_1\omega_2}.$$

Bandpass and bandstop frequency transformations.
(a) Low-pass filter prototype response for $\omega_c = 1$.
(b) Transformation to bandpass response.
(c) Transformation to bandstop response



Bandwidth $= \omega_2 - \omega_1$

$\omega_0 \equiv$ resonance frequancy.

- Then the transformation maps the bandpass characteristics to the low-pass response as follows:

$$\Delta = \frac{\omega_2-\omega_1}{\omega_0}, \quad \omega_0^2 = \omega_1\omega_2$$

When $\omega = \omega_0$, $\frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = 0.$

When $\omega = \omega_1$, $\frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega_1^2-\omega_0^2}{\omega_0\omega_1}\right) = -1.$

$$\Rightarrow \frac{\omega_0}{\omega_2-\omega_1}\left(\frac{\omega_1^2-\omega_1\omega_2}{\omega_0\omega_1}\right) = -1$$

When $\omega = \omega_2$, $\frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega_2^2-\omega_0^2}{\omega_0\omega_2}\right) = 1.$

$$jX_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)L_k = \underbrace{j\frac{\omega L_k}{\Delta\omega_0}}_{\text{inductor}} - \underbrace{j\frac{\omega_0 L_k}{\Delta\omega}}_{\text{capacitor.}} = j\omega L'_k - j\frac{1}{\omega C'_k}$$

$$jB_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)C_k = j\frac{\omega C_k}{\Delta\omega_0} - j\frac{\omega_0 C_k}{\Delta\omega} = j\omega C'_k - j\frac{1}{\omega L'_k}$$

$$L'_k = \frac{L_k}{\Delta\omega_0} \qquad C'_k = \frac{\Delta}{\omega_0 L_k}$$

Similarly

$$L'_k = \frac{\Delta}{\omega_0 C_k} \qquad C'_k = \frac{C_k}{\Delta\omega_0}$$

- The low-pass filter elements are thus converted to series resonant circuits (having a low impedance at resonance) in the series arms, and to parallel resonant circuits (having a high impedance at resonance) in the shunt arms.
- Notice that both series and parallel resonator elements have a resonant frequency of ω_0.

- The inverse transformation can be used to obtain a bandstop response. Thus,

$$\omega \leftarrow -\Delta\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^{-1}$$

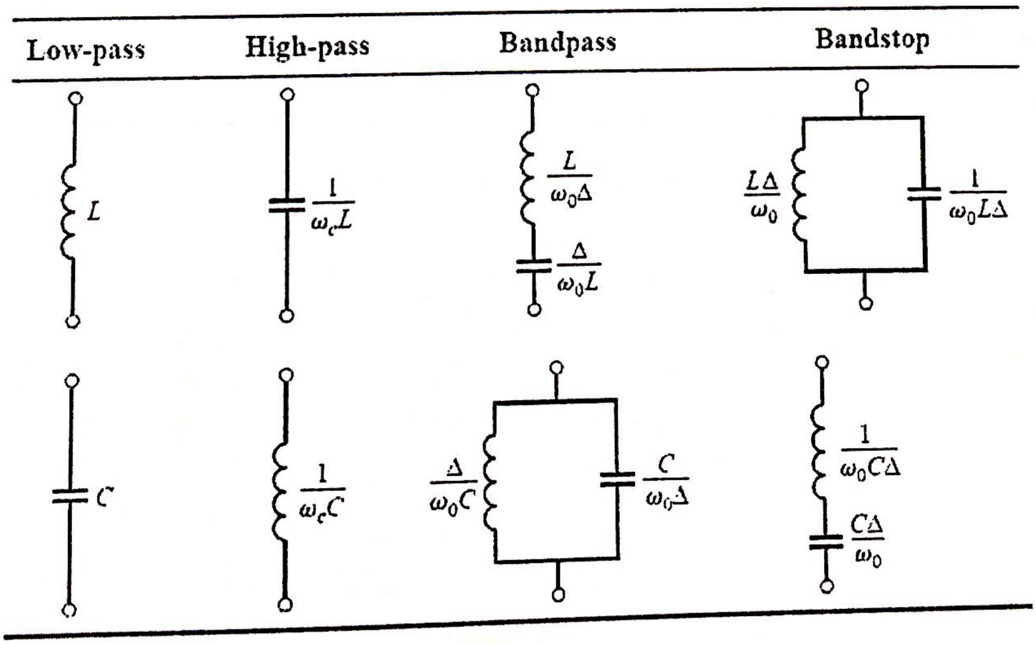
- Then series inductors of the low-pass prototype are converted to parallel LC circuits having element values given by:

$$L'_k = \frac{\Delta L_k}{\omega_0} \qquad C'_k = \frac{1}{\omega_0 \Delta L_k}$$

- The shunt capacitor of the low-pass prototype is converted to series LC circuits having element values given by:

$$L'_k = \frac{1}{\omega_0 \Delta C_k} \qquad C'_k = \frac{\Delta C_k}{\omega_0}$$

Summary of Prototype Filter Transformations $\left(\Delta = \frac{\omega_2 - \omega_1}{\omega_0}\right)$



Example: Bandpass Filter Design

- Design a bandpass filter having a 0.5 dB equal-ripple response, with N = 3. The center frequency is 1 GHz, the bandwidth is 10%, and the impedance is 50 Ω.

$\to \Delta$.

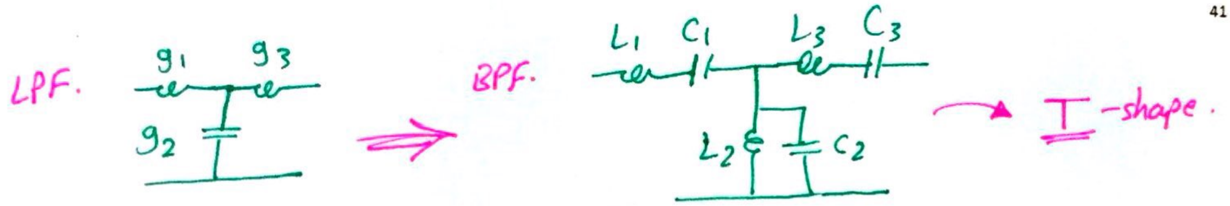
$\omega = 2\pi \times 1G$

- **Solution:** since odd Number of elements ⇒ symmetric ⇒ elements for g_3 same for g_1.

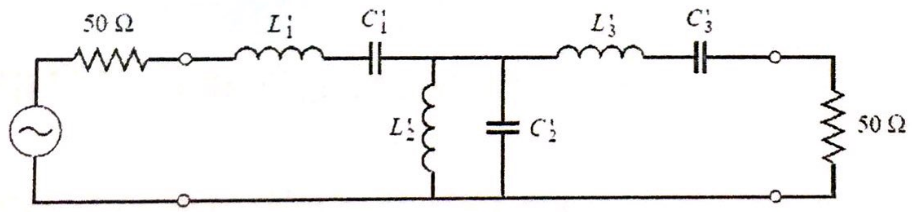
$g_1 = 1.5963 = L_1$, $g_2 = 1.0967 = C_2$, $g_3 = 1.5963 = L_3$, $g_4 = 1.000 = R_L$.

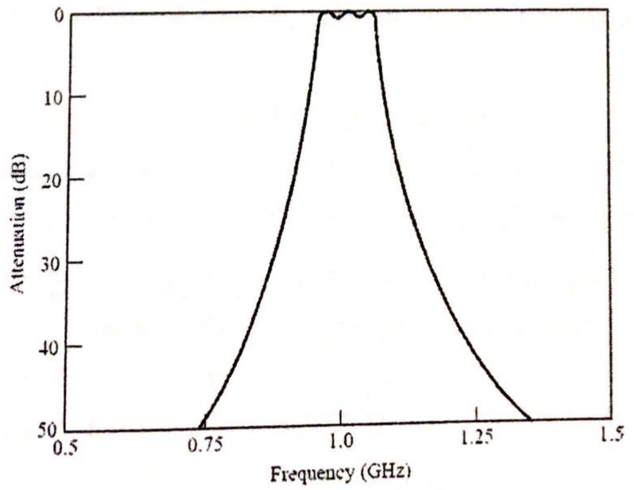
$$L_1' = \frac{L_1 R_0}{\omega_0 \Delta} = 127.0 \text{ nH}, \qquad L_2' = \frac{\Delta R_0}{\omega_0 C_2} = 0.726 \text{ nH}, \qquad L_3' = \frac{L_3 R_0}{\omega_0 \Delta} = 127.0 \text{ nH},$$

$$C_1' = \frac{\Delta}{\omega_0 L_1 R_0} = 0.199 \text{ pF}, \qquad C_2' = \frac{C_2}{\omega_0 \Delta R_0} = 34.91 \text{ pF}, \qquad C_3' = \frac{\Delta}{\omega_0 L_3 R_0} = 0.199 \text{ pF}.$$



if we have shunt (LPF) ⇒ it will result π-shape BPF.





Filter Implementation

- The lumped-element filter designs generally work well at low frequencies, but two problems arise at higher RF and microwave frequencies.

✓ First, lumped-element inductors and capacitors are generally available only for a limited range of values, and can be difficult to implement at microwave frequencies. Distributed elements, such as open-circuited or short-circuited transmission line stubs, are often used to approximate ideal lumped elements.

✓ In addition, at microwave frequencies the distances between filter components is not negligible.

- The first problem is treated with **Richards' transformation**, which can be used to convert lumped elements to transmission line sections.
- **Kuroda's identities** can then be used to physically separate filter elements by using transmission line sections.
- Because such additional transmission line sections do not affect the filter response, this type of design is called **redundant filter synthesis**.
- It is possible to design microwave filters that take advantage of these sections to improve the filter response; such non-redundant synthesis does not have a lumped-element counterpart.

Recall:

$Z_{in} = jZ_0 \tan\beta\ell$ SC

$Z_{in} = -jZ_0 \cot\beta\ell$ OC

Richards' Transformation

- The transformation $\Omega = \tan\beta\ell = \tan\left(\frac{\omega\ell}{v_p}\right)$

maps the ω plane to the Ω plane, which repeats with a period of $\omega\ell/v_p = 2\pi$.

- This transformation was introduced by P. Richards to synthesize an LC network using open- and short-circuited transmission line stubs.

$$jX_L = j\Omega L = jL\tan\beta\ell \qquad jB_C = j\Omega C = jC\tan\beta\ell$$

compare with

$\frac{1}{jC\tan\beta\ell} = -j\left(\frac{1}{C}\right)\cot\beta\ell$

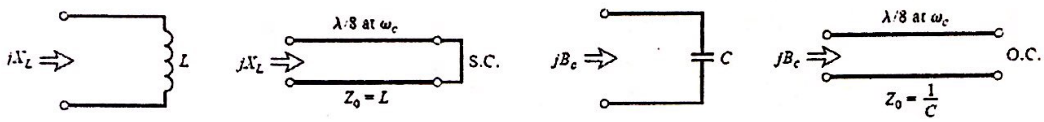
$Z_0 = \frac{1}{C}$ in case OC.

- These results indicate that an inductor can be replaced with a short-circuited stub of length βℓ and characteristic impedance L, while a capacitor can be replaced with an open-circuited stub of length βℓ and characteristic impedance 1/C. A unity filter impedance is assumed.

$\beta\ell = \frac{\pi}{4}$

$\ell = \frac{\pi}{4} \cdot \frac{\lambda}{2\pi} = \frac{\lambda}{8}$ → 90°

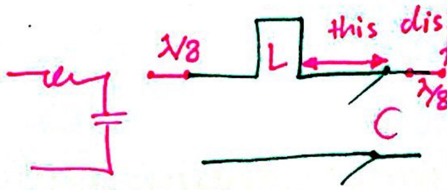
- Cutoff occurs at unity frequency for a low-pass filter prototype; to obtain the same cutoff frequency for the Richards'-transformed filter ($\Omega = 1 = \tan \beta\ell$) ➔ $\beta\ell = \pi/4$
- which gives a stub length of $\ell = \lambda/8$, where λ is the wavelength of the line at the cutoff frequency (ω_c).
- At the frequency $\omega_0 = 2\omega_c$, the lines will be $\lambda/4$ long, and an attenuation pole will occur.
- At frequencies away from ω_c, the impedances of the stubs will no longer match the original lumped-element impedances, and the filter response will differ from the desired prototype response. In addition, the response will be periodic in frequency, repeating every $4\omega_c$ ($\ell = \lambda/2$).
- Since the electrical lengths of all the stubs are the same ($\lambda/8$ at ω_c), these lines are called **commensurate lines**.



inductor
⇓
series
short.

capacitor
⇓
shunt
open.

* we add λ/8 before & after the components.



this distance found from Kuroda's Identities.

Method (1): (using stub)

Kuroda's Identities

* most of the time we will convert series $\xrightarrow{\text{to}}$ shunt.

- The four Kuroda identities use redundant transmission line sections to achieve a more practical microwave filter implementation by performing any of the following operations:

✓ Physically separate transmission line stubs

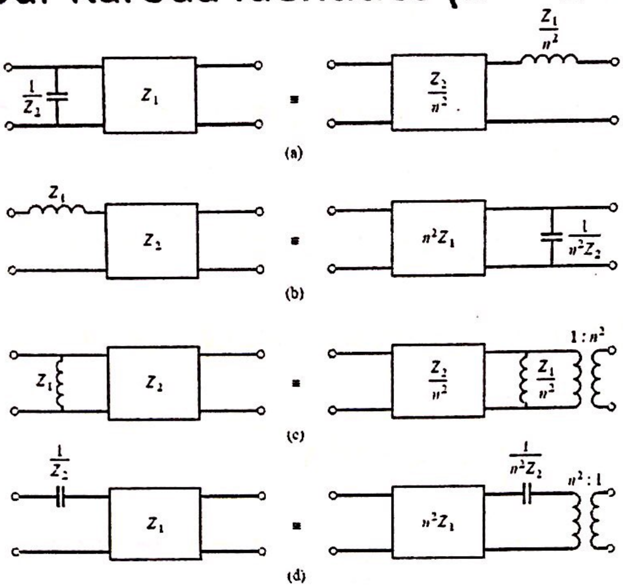
✓ Transform series stubs into shunt stubs, or vice versa

✓ Change impractical characteristic impedances into more realizable values

- The additional transmission line sections are called unit elements and are $\lambda/8$ long at ω_c; the unit elements are thus commensurate with the stubs used to implement the inductors and capacitors of the prototype design.

The Four Kuroda Identities ($n^2 = 1 + Z_2/Z_1$)

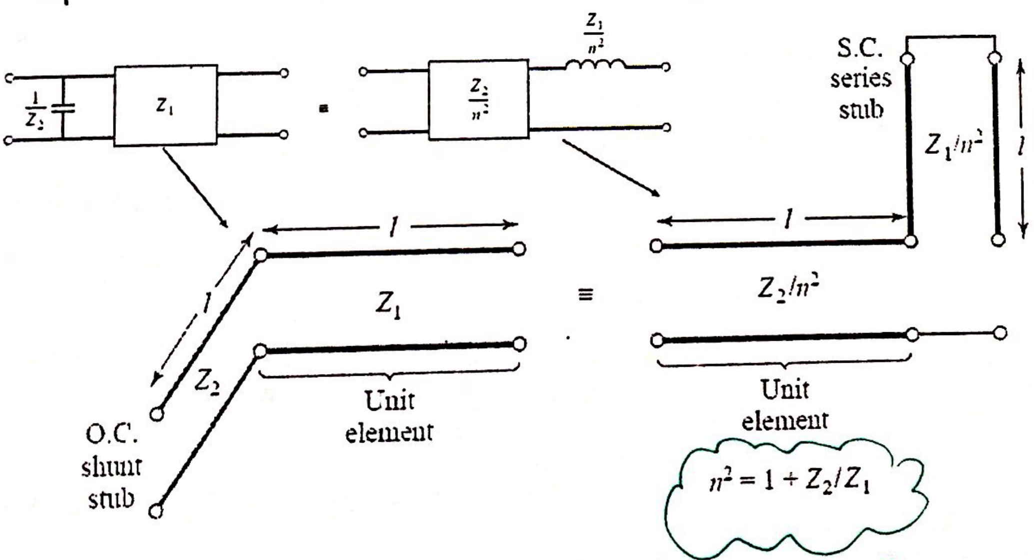
each box represents a unit element, or transmission line, of the indicated characteristic impedance and length ($\lambda/8$ at ω_c)



→ Draw them on formula sheet. ✓.

Not included in this course.

Equivalent circuits illustrating Kuroda identity



n is a factor used during the Transformation.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta\ell & jZ_1\sin\beta\ell \\ \frac{j}{Z_1}\sin\beta\ell & \cos\beta\ell \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}}\begin{bmatrix} 1 & j\Omega Z_1 \\ \frac{j\Omega}{Z_1} & 1 \end{bmatrix}.$$

divide By $\cos\beta\ell$

The open-circuited shunt stub in the first circuit has an impedance of $-jZ_2 \cot\beta\ell = -j\,Z_2/\Omega$, so the ABCD matrix of the entire circuit is:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_L = \begin{bmatrix} 1 & 0 \\ \frac{j\Omega}{Z_2} & 1 \end{bmatrix}\begin{bmatrix} 1 & j\Omega Z_1 \\ \frac{j\Omega}{Z_1} & 1 \end{bmatrix}\frac{1}{\sqrt{1+\Omega^2}} = \frac{1}{\sqrt{1+\Omega^2}}\begin{bmatrix} 1 & j\Omega Z_1 \\ j\Omega\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right) & 1-\Omega^2\frac{Z_1}{Z_2} \end{bmatrix}$$

The short-circuited series stub in the second circuit has an impedance of $j\,(Z_1/n^2)\tan\beta\ell = j\Omega Z_1/n^2$, so the ABCD matrix of the entire circuit is:

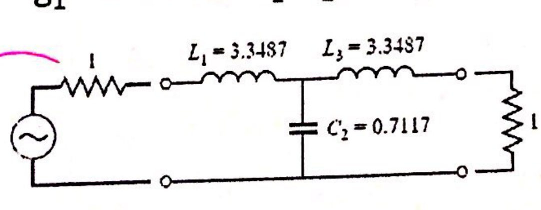
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_R = \begin{bmatrix} 1 & j\frac{\Omega Z_2}{n^2} \\ \frac{j\Omega n^2}{Z_2} & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{j\Omega Z_1}{n^2} \\ 0 & 1 \end{bmatrix}\frac{1}{\sqrt{1+\Omega^2}} = \frac{1}{\sqrt{1+\Omega^2}}\begin{bmatrix} 1 & \frac{j\Omega}{n^2}(Z_1+Z_2) \\ \frac{j\Omega n^2}{Z_2} & 1-\Omega^2\frac{Z_1}{Z_2} \end{bmatrix}$$

The results are identical if we choose $n^2 = 1 + Z_2/Z_1$

Example: Low-Pass Filter Design Using Stubs

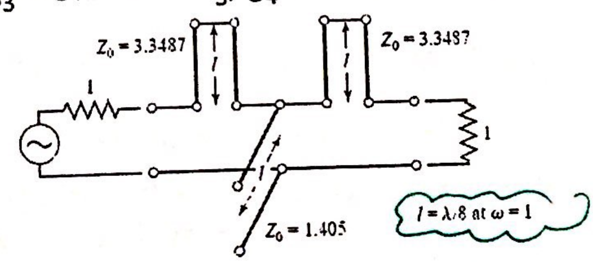
- Design a low-pass filter for fabrication using microstrip lines. The specifications include a cutoff frequency of 4 GHz, an impedance of 50 Ω, and a third-order 3 dB equal-ripple passband response.
- **Solution:**

$g_1 = 3.3487 = L_1$, $g_2 = 0.7117 = C_2$, $g_3 = 3.3487 = L_3$, $g_4 = 1.0000 = R_L$

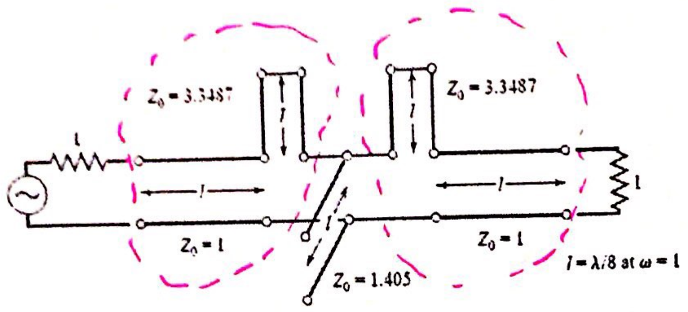


1. Lumped-element low-pass filter prototype

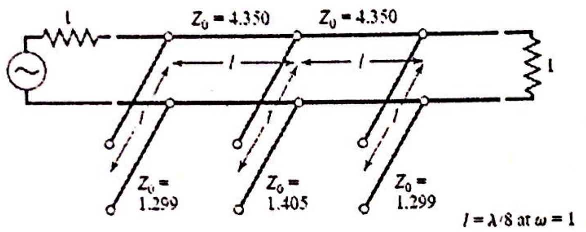
always solve using Normalization.
So you can use λ/8.



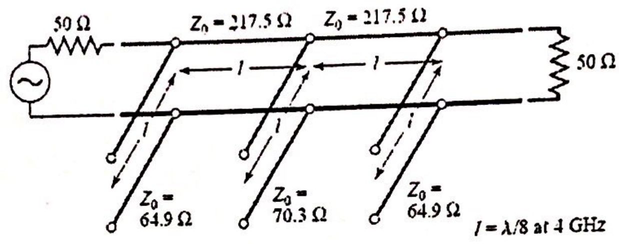
2. Using Richards' transformations to convert inductors and capacitors to series and shunt stubs



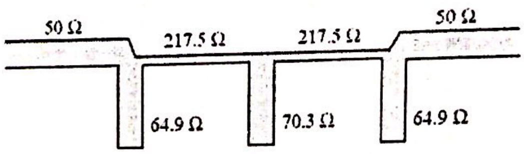
3. Adding unit elements at the ends of the filter



4. Applying the second Kuroda identity (b)

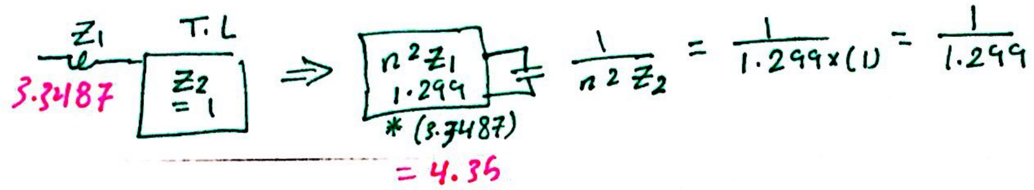


5. After impedance and frequency scaling



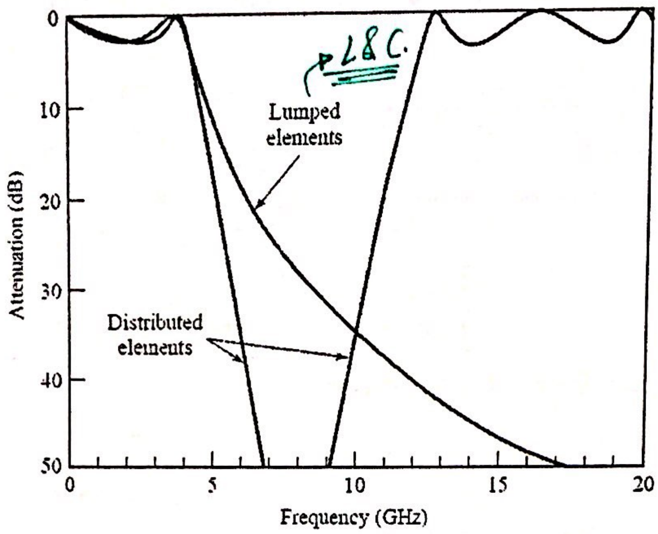
6. Microstrip fabrication of the final filter

(It is usually most convenient to work with normalized quantities until the last step in the design)



- The series stubs would be very difficult to implement in microstrip line form, so we will use one of the Kuroda identities to convert these to shunt stubs.
- First we add unit elements at either end of the filter.
- These redundant elements do not affect filter performance since they are matched to the source and load ($Z_0 = 1$).
- Then we can apply the second Kuroda identity (b) to both ends of the filter.
- In both cases we have that $n^2 = 1 + \frac{Z_2}{Z_1} = 1 + \frac{1}{3.3487} = 1.299$
- Finally, we scale both the impedance and frequency of the circuit, which simply involves multiplying the normalized characteristic impedances by 50 Ω and choosing the line and stub lengths to be $\lambda/8$ at 4 GHz.
- Similar procedures can be used for bandstop filters, but the Kuroda identities are **not useful** for high-pass or bandpass filters.

Amplitude response



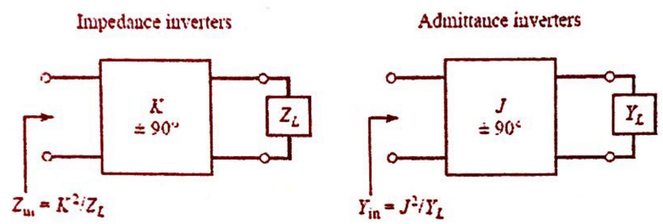
- Note that the passband characteristics are very similar up to 4 GHz ($\approx \omega_c$), but the distributed element filter has a sharper cutoff.
- Also notice that the distributed-element filter has a response that repeats every 16 GHz (4 ω_c), as a result of the periodic nature of Richards' transformation.

Method(2): (using stub)

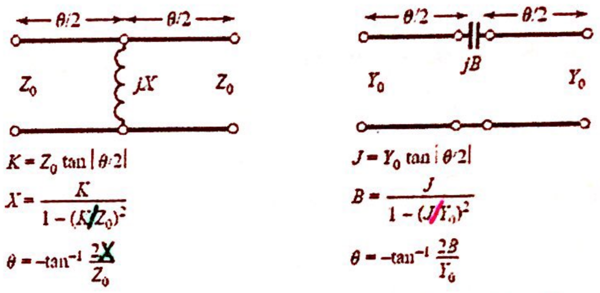
Impedance and Admittance Inverters (1)

- As we have seen, it is often desirable to use only series, or only shunt, elements when implementing a filter with a particular type of transmission line.
- The Kuroda identities can be used for conversions of this form, but another possibility is to use impedance (K) or admittance (J) inverters.
- Such inverters are especially useful for bandpass or bandstop filters with narrow (<10%) bandwidths.
- These inverters essentially form the inverse of the load impedance or admittance, they can be used to transform series-connected elements to shunt-connected elements, or vice versa.
- In its simplest form, an impedance or admittance inverter can be constructed using a quarter-wave transformer of the appropriate characteristic impedance.
- Inverters of this form turn out to be useful for modeling the coupled resonator filters.
- The lengths (θ/2) of the transmission line sections are generally required to be negative for this type of inverter, but this poses no problem if these lines can be absorbed into connecting transmission lines on either side.

Impedance and Admittance Inverters (2)

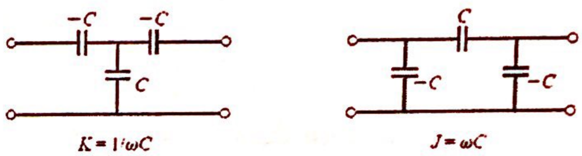


Operation of impedance and admittance inverters



Implementation using transmission lines and reactive elements

Implementation as quarter-wave transformers



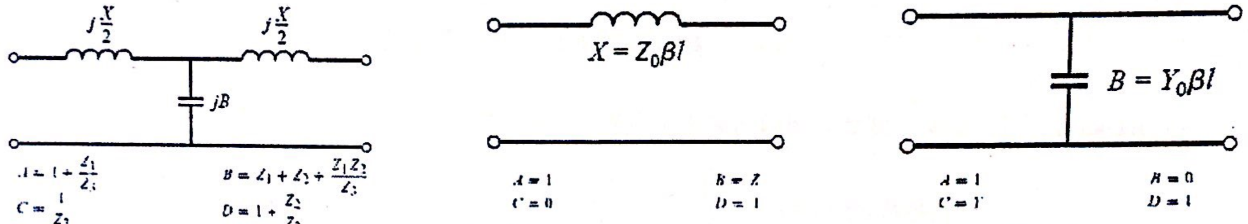
Implementation using capacitor networks

Method(3): Stepped-Impedance Low-Pass Filters

- A relatively easy (but inefficient) way to implement low-pass filters in microstrip or stripline is to use alternating sections of very high and very low characteristic impedance lines.
- Such filters are usually referred to as stepped-impedance, or hi-Z, low-Z filters, and are popular because they are easier to design and take up less space than a similar low-pass filter using stubs.
- Because of the approximations involved, however, their electrical performance is not as good, so the use of such filters is usually limited to applications where a sharp cutoff is not required (for instance, in rejecting out-of-band mixer products).

The following ABCD matrices Can be found as we learned in CH4.

Approximate Equivalent Circuits for Short Transmission Line Sections



$$\frac{X}{2} = Z_0 \tan\left(\frac{\beta\ell}{2}\right). \quad \text{for very small angle.} \quad X \simeq Z_0\beta\ell, \quad B \simeq 0.$$

$$B = \frac{1}{Z_0}\sin\beta\ell.$$

$$X \simeq 0, \quad B \simeq Y_0\beta\ell.$$

$\frac{R_0}{Z_h}$ is a factor.

$$\beta\ell = \frac{LR_0}{Z_h} \text{ (inductor)} \rightarrow \text{in deg.} \qquad \beta\ell = \frac{CZ_\ell}{R_0} \text{ (capacitor)} \rightarrow \text{in deg.}$$

in Radian $\left(* \frac{\pi}{180^\circ}\right)$

where R_0 is the filter impedance and L and C are the normalized element values (the g_k) of the low-pass prototype 57

$X = Z_0 \beta L$

(H) $\Rightarrow L' = \frac{L R_0}{\omega_c}$ (g) normalized.

$\beta L = \frac{X}{Z_0} = \frac{\omega L}{Z_0}$

$\Rightarrow \beta L = \frac{\cancel{\omega} g \cancel{R_0}}{\cancel{Z_0} \cancel{\omega_c}}$ @ $\omega = \omega_c$, $R_0 = Z_0$ (lossless) $\Rightarrow \underline{\beta L = g}$

- For a transmission line with ($\ell < \lambda/4$ or $\beta\ell < \pi/2$) and ($Z_0 >> 1$):

➔ The series elements have a positive reactance (inductors), while the shunt element has a negative reactance (capacitor).

Thus, $X = \omega L = Z_0 \sin(\theta) = Z_0 \sin(\beta\ell) \cong Z_0 \beta\ell$ and $B \cong 0$

- Using scaling equation (for impedance and frequency) gives:

$$\omega L = \omega\frac{L_k R_0}{\omega_c} \approx Z_0\beta l \quad \text{at } \omega = \omega_c \qquad \beta l = \frac{L_k R_0}{Z_{0h}}$$

$\beta = \frac{2\pi}{\lambda}$ & $\omega = 2\pi f$

where $Z_0 = Z_{0h}$ or impedance of thin transmission line section and R_0 is equivalent to filter impedance.

- Similarly from, $\omega L \approx Z_0\beta l \Rightarrow L \approx \frac{Z_{0h}\beta l}{\omega} = \frac{Z_{0h} l}{\lambda_g f}$ (shown before)

- Typically, the impedance of the inductive microstrip line ($Z_{0h} \approx Z_h$) is set to be as high as physically possible (thinnest practically fabricated line)
- To get the best response near cutoff, these lengths should be evaluated at $\omega = \omega_c$

- For a transmission line with ($\ell < \lambda/4$ or $\beta\ell < \pi/2$) and ($Z_0 << 1$); B'+' and X 'small' and the transmission line approximates a shunt capacitive impedance

$$\text{Thus; } B = \omega C = Y_0 \sin(\theta) = Y_0 \sin(\beta\ell) \cong Y_0\, \beta\ell \text{ and } X \cong 0$$

- Using scaling equation (for impedance & frequency) to un-scale gives:

$$\omega C = \omega \frac{C_k}{\omega_c R_0} \approx \frac{\beta l}{Z_0} \qquad \text{at } \omega = \omega_c \qquad \beta l = \frac{C_k Z_{0l}}{R_0}$$

where $Z_0 = Z_{0\ell}$ or impedance of thick transmission line section and R_0 is equivalent to filter impedance.

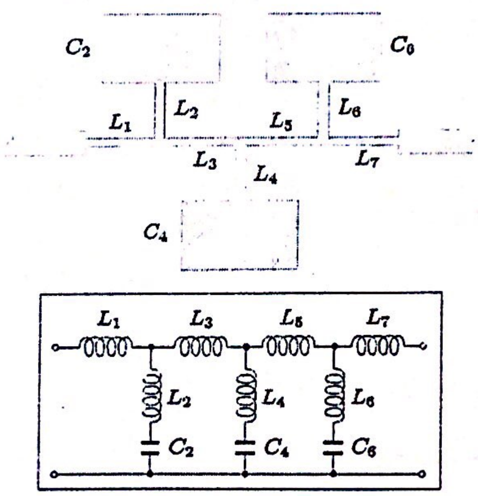
- Similarly from,

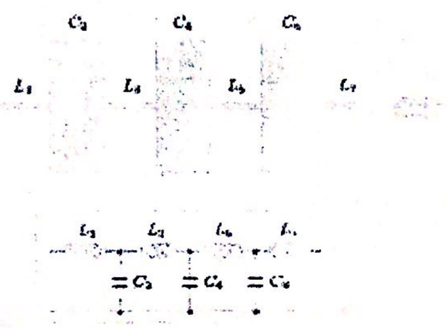
$$\omega C \approx \frac{\beta l}{Z_0} \Rightarrow C \approx \frac{\beta l}{\omega Z_{0l}} = \frac{l}{\lambda_g f Z_{0l}} \quad \text{(shown before)}$$

$\beta \to \frac{2\pi}{\lambda}$, $\omega \to 2\pi f$

- Typically, the impedance of the capacitive microstrip line ($Z_{0\ell} \approx Z_\ell$) is set to be as low as possible (thickest fabricated line the circuit size allowed)
- Typically, the ratio Z_h/Z_ℓ should be as high as possible (practically fabricated)

Stepped Impedance Filters Examples

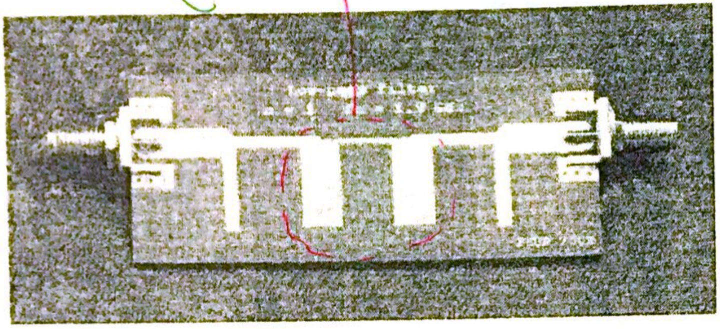


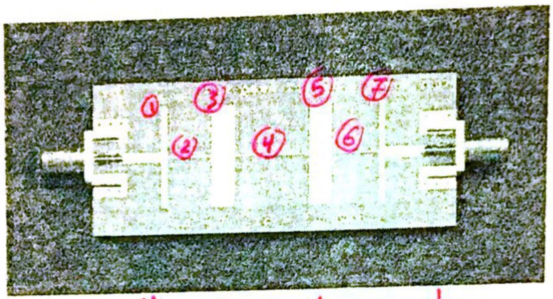


they considered 2 elements only if there was an inductor Between them. "narrow Line".

Filters Prototypes

These are 2 capacitors in parallel represent one element together.





Richards' Transformation & Kuroda's Identities

Stepped Impedance

⇒ 7 elements.

Between 2 narrow Lines ⇒ Capacitor.

Example: Stepped-Impedance Filter Design

$f_c \Rightarrow \omega_c = 2\pi \times 2.5G$

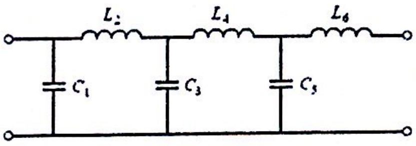
- Design a stepped-impedance low-pass filter having a maximally flat response and a cutoff frequency of 2.5 GHz. It is desired to have more than 20 dB insertion loss at 4 GHz. The filter impedance is 50 Ω; the highest practical line impedance is 120 Ω, and the lowest is 20 Ω. Consider the effect of losses when this filter is implemented with a microstrip substrate having d = 0.158 cm, $\varepsilon_r = 4.2$, tan δ = 0.02, and copper conductors of 0.5 mil thickness.
- **Solution:**

$$\frac{\omega}{\omega_c} - 1 = \frac{4.0}{2.5} - 1 = 0.6$$

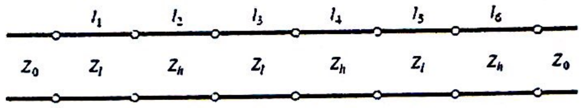
from the curve: 20dB, 0.6 → N=6

$$g_1 = 0.517 = C_1,\ g_2 = 1.414 = L_2,\ g_3 = 1.932 = C_3,$$
$$g_4 = 1.932 = L_4,\ g_5 = 1.414 = C_5,\ g_6 = 0.517 = L_6$$

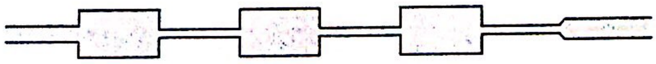
Solution:



Low-pass filter prototype circuit



Stepped impedance implementation



Microstrip layout of final filter

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$$\beta \ell_C = \frac{g\, Z_{0\ell}}{R_0} = \frac{(0.517) * 20}{50} * \frac{180}{\pi} = 11.8$$

C_K

for $\beta \ell_L = \dfrac{g\, R_0}{Z_{0h}}$

from CH8

Section	$Z_i = Z_\ell$ or Z_h (Ω)	$\beta\ell_i$ (deg)	W_i (mm)	ℓ_i (mm)
1	20	11.8	11.3	2.05
2	120	33.8	0.428	6.63
3	20	44.3	11.3	7.69
4	120	46.1	0.428	9.04
5	20	32.4	11.3	5.63
6	120	12.3	0.428	2.41

$$C \simeq \frac{\beta L}{\omega Z_{0\ell}} = \frac{\ell}{\lambda f Z_{0\ell}}$$

CH3. find ϵ_e then λ_e.

microstrip line lengths

electrical line lengths

from CH3.

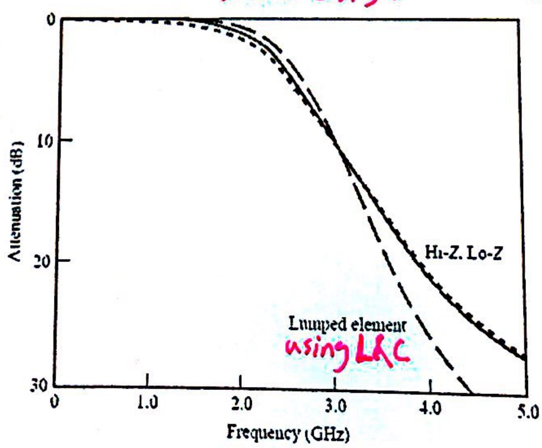
microstrip line widths

The effect of loss is to increase the passband attenuation to about 1 dB at 2 GHz.

The lumped-element filter gives more attenuation at higher frequencies.

This is because the stepped-impedance filter elements depart significantly from the lumped-element values at higher frequencies.

The stepped impedance filter may have other passbands at higher frequencies, but the response will not be perfectly periodic because the lines are not commensurate.



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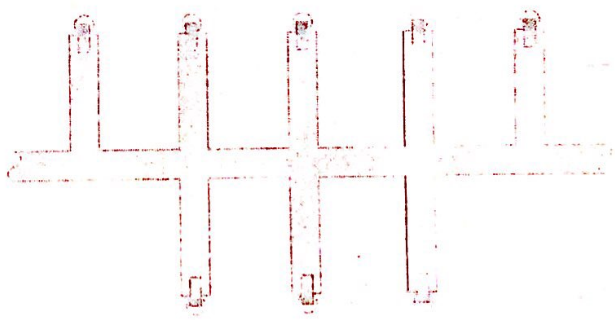
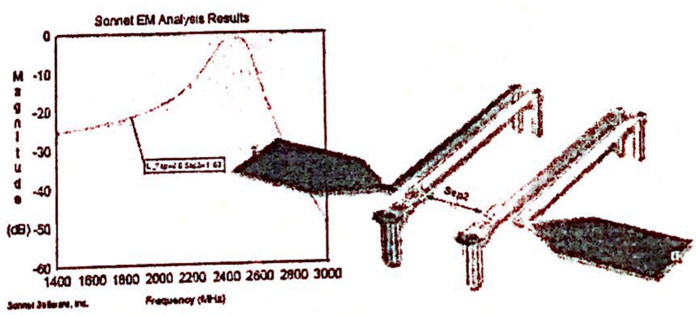
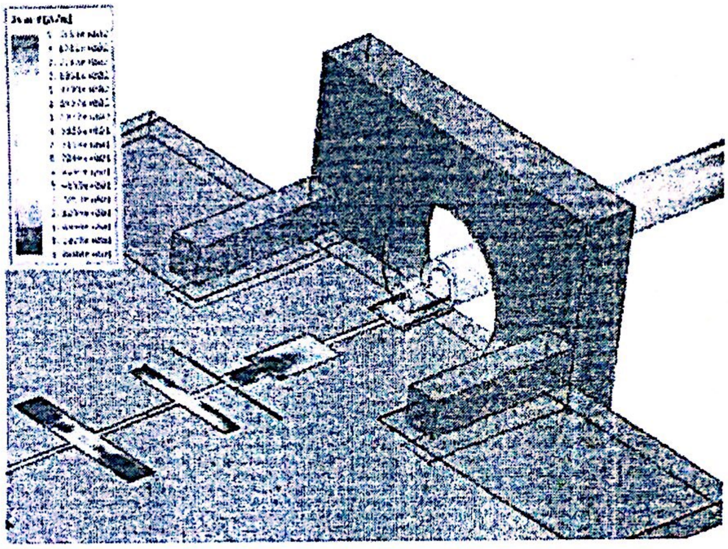
Coupled Line Filters

- The parallel coupled transmission lines (for directional couplers) can be used to construct many types of filters.
- Fabrication of multisection bandpass or bandstop coupled line filters is particularly easy in microstrip or stripline form for bandwidths less than about 20%.
- Wider bandwidth filters generally require very tightly coupled lines, which are difficult to fabricate.
- We will first study the filter characteristics of a single quarter-wave coupled line section, and then show how these sections can be used to design a bandpass filter.





4 GHz BPF Surface Currents



Topics in Communications.
"Microwaves"

Spring 2017/2018

Dr. Yanal Al-Faouri

By: Mohammad Abu Hashya.

* * *

Microwave Engineering

Chapter 9

Dr. Yanal Faouri

Email: y.faouri@ju.edu.jo

Theory and Design of Ferrimagnetic Components

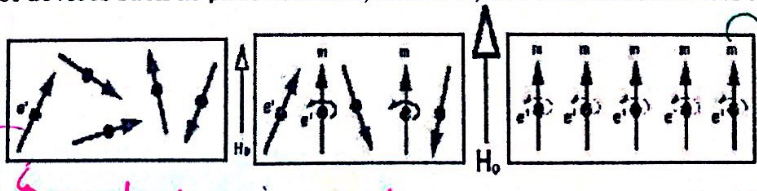
- **Basic Properties of Ferrimagnetic Materials**
- **Plane Wave Propagation in a Ferrite Medium**
- **Ferrite Isolators**
- **Ferrite Phase Shifters**
- **Ferrite Circulators**

Introduction (1)

- The components and networks discussed up to this point have all been reciprocal. That is, the response between any two ports i and j of a component did not depend on the direction of signal flow (thus, $S_{ij} = S_{ji}$).
- This will always be the case when the component is passive and consists of only isotropic materials, but if the component contains either **active devices** or **anisotropic material**, nonreciprocal behavior can be obtained.
- In some cases nonreciprocity is a useful property (e.g., circulators, isolators), while in other cases nonreciprocity is an ancillary property (e.g., transistor amplifiers, ferrite phase shifters).

- Some of the most practical anisotropic materials for microwave applications are ferrimagnetic compounds, also known as ferrites, such as yttrium iron garnet (YIG) and materials composed of iron oxides and various other elements such as aluminum, cobalt, manganese, and nickel.
- In contrast to ferromagnetic materials (e.g., iron, steel), ferrimagnetic compounds have high resistivity and a significant amount of anisotropy at microwave frequencies.
- The magnetic anisotropy of a ferrimagnetic material is actually induced by applying a DC magnetic bias field.
- This field aligns the magnetic dipoles in the ferrite material to produce a net (nonzero) magnetic dipole moment, and causes the magnetic dipoles to precess (rotate) at a frequency controlled by the strength of the bias field.
- A microwave signal circularly polarized in the same direction as this precession will interact strongly with the dipole moments, while an oppositely polarized field will interact less strongly.
- Since, for a given direction of rotation, the sense of polarization changes with the direction of propagation, a microwave signal will propagate through a magnetically biased ferrite differently in different directions.
- This effect can be utilized to fabricate directional devices such as isolators, circulators, and gyrators.
- Another useful characteristic of ferrimagnetic materials is that the interaction with an applied microwave signal can be controlled by adjusting the strength of the bias field. This effect leads to a variety of control devices such as phase shifters, switches, and tunable resonators and filters.

dipole moment



magnetic moment.

Summation of all dipoles moment ≡ Total magnetization = zero.
(De-magnetized)

start to magnetize.

we need to reach magnetization without Breakdown the material.

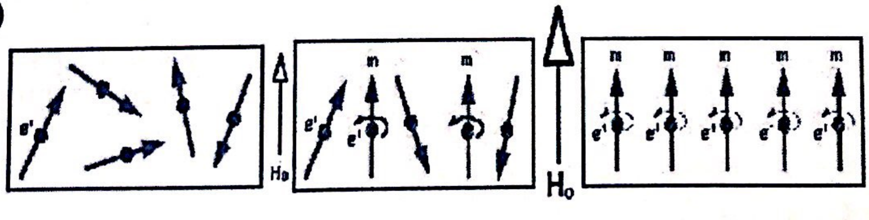
Ferrimagnetic vs. Paraelectric Materials

- It is interesting to compare ferrimagnetic materials to paraelectric materials, which are almost the dual of ferrimagnetic materials.
- Certain ceramic compounds, such as lithium niobate and barium titanate, have the property that their dielectric permittivity can be controlled with the application of a DC bias electric field.
- Paraelectric materials can therefore be used for variable phase shifters and other control components.
- Unlike ferrimagnetic materials, paraelectric materials are isotropic, and therefore paraelectric devices are reciprocal.
- Paraelectric materials typically have very high dielectric constants and loss tangents when used in bulk form, so modern applications generally use thin films of paraelectric material layered on a substrate.
- An important advantage of paraelectric devices over ferrite devices is that the need for a large and heavy magnet or biasing coil is eliminated.

Ferrite (1)

- Microwave control devices are fabricated utilizing the interaction between electromagnetic waves and magnetic material properties of a magnetized ferrite material.
- Ferrite: a magnetic material with Tensor permeability $\{[\mu_r] \approx (50 \text{ to } 3000)\}$and scalar permittivity $\{\varepsilon_r \approx (10 \text{ to } 20)\}$
- Auxiliary functions are: $\mathbf{D} = \varepsilon_o \varepsilon_r \mathbf{E}$ and $\mathbf{B} = \mu_o[\mu_r]\mathbf{H}$
- Magnetism in ferrite: magnetic moments (due to the spin motion of electrons) and inter-exchange force (depends on the inter-atomic distance) create spontaneous magnetized domains.
- In the absence of external magnetization, the magnetic domains of ferrite material are arranged in such a way that they cancel each others effect (Not Magnet)

it is Tensor: $\mu_r = \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix}$



Scanned by CamScanner

Ferrite (2)

- When magnetized by an axial magnetic field ($\mathbf{H_0}$), the magnetic moments (associate with spin motion of electrons) of ferrite material starts to rotate around the axis of $\mathbf{H_0}$.
- The natural frequency of rotation of magnetic moments are called Uniform Precession Resonance frequency (ω_0)
- Direction and frequency of this rotation depends on $\mathbf{H_0}$.
- But due to **dumping losses** of ferrite, the rotation can't be sustained and magnetic moments spirally align itself with $\mathbf{H_0}$.
- If all the magnetic dipoles are aligned in the direction of $\mathbf{H_0}$, ferrite material is said to be saturated. **(magnetization)**

Basic Properties of Ferrimagnetic Materials

1. The Permeability Tensor:

- The permeability tensor for a ferrimagnetic material can be deduced from a relatively simple microscopic view of the atom.
- The magnetic properties of a material are due to the existence of magnetic dipole moments, which arise primarily from electron spin.
- From quantum mechanical considerations, the magnetic dipole moment (**m**) of an electron due to its spin is given by

$m = IS\hat{a}_n \ (A\cdot m^2)$

$$m = \frac{q\hbar}{2m_e} = 9.27 \times 10^{-24} \text{ A-m}^2$$

where $\hbar$ is Planck's constant divided by 2π called the reduced Plank's constant, q is the electron charge, and m_e is the mass of the electron.

$$h = 6.626\,070\,040(81) \times 10^{-34} \text{ J·s} = 4.135\,667\,662(25) \times 10^{-15} \text{ eV·s.}$$

$$\hbar = \frac{h}{2\pi} = 1.054\,571\,800(13) \times 10^{-34} \text{ J·s} = 6.582\,119\,514(40) \times 10^{-16} \text{ eV·s.}$$

- An electron in orbit around a nucleus gives rise to an effective current loop and thus an additional magnetic moment, but this effect is generally insignificant compared to the magnetic moment due to spin.
- The Land´e g factor is a measure of the relative contributions of the orbital moment and the spin moment to the total magnetic moment; g = 1 when the moment is due only to orbital motion, and g = 2 when the moment is due only to spin. For most microwave ferrite materials, g is in the range 1.98 – 2.01, so g = 2 is a good approximation.
- In most solids, electron spins occur in pairs with opposite signs, so the overall magnetic moment is negligible.
- In a magnetic material, however, a large fraction of the electron spins are unpaired (more left-hand spins than right-hand spins, or vice versa), but are generally oriented in random directions so that the net magnetic moment is still small.

for Max magnetized:

$\underline{\theta = 0}$

it rotates like a helix

- An external magnetic field, however, can cause the dipole moments to align in the same direction to produce a large overall magnetic moment.
- The existence of <u>exchange forces</u> can keep adjacent electron spins aligned after the external field is removed; the material is then said to be permanently magnetized.
- An electron has a spin **angular momentum** (s) given in terms of Planck's constant as;

normalized

$$s = \frac{\hbar}{2} = \frac{h}{4\pi}$$

- The vector direction of this momentum is opposite the direction of the spin magnetic dipole moment.
- The ratio of the spin magnetic moment to the spin angular momentum is a constant called the **gyromagnetic ratio**:

m const. & s const. $\Rightarrow \gamma$ const.

$$\gamma = \frac{m}{s} = \frac{q}{m_e} = 1.759 \times 10^{11} \text{ C/kg} \qquad \bar{m} = -\gamma\bar{s}$$

where the negative sign is due to the fact that these vectors are oppositely directed

$$m = -\frac{m}{s} \Rightarrow \vec{m} = -\gamma\vec{s}$$

$$\gamma = -\frac{m}{s} = \frac{q\,\cancel{\hbar}/2m_e}{\cancel{\hbar}/\cancel{2}} = q/m_e \equiv \text{C/Kg}$$

Equation of Motion

$\bar{S} = \frac{-1}{\gamma} \bar{m}$

$\frac{dS}{dt} = \frac{-1}{\gamma} \frac{d\bar{m}}{dt} = \mu_0 \bar{m} \times \bar{H}_0$

- When a magnetic bias field $\mathbf{H_0} = H_0\, \mathbf{a_z}$ is present, a torque will be exerted on the magnetic dipole:

$$\bar{T} = \bar{m} \times \bar{B}_0 = \mu_0 \bar{m} \times \bar{H}_0 = -\mu_0 \gamma \bar{s} \times \bar{H}_0$$

- Since torque is equal to the time rate of change of angular momentum, we have;

$$\frac{d\bar{s}}{dt} = \frac{-1}{\gamma}\frac{d\bar{m}}{dt} = \bar{T} = \mu_0 \bar{m} \times \bar{H}_0 \qquad \frac{d\bar{m}}{dt} = -\mu_0 \gamma \bar{m} \times \bar{H}_0$$

- This is **the equation of motion** for the magnetic dipole moment (**m**). The solution for this equation shows that the magnetic dipole precesses around the H_0-field vector, similar to a spinning top precessing around a vertical axis.
- The equation of motion in terms of its three vector components gives;

$$\frac{dm_x}{dt} = -\mu_0 \gamma m_y H_0 \qquad \frac{dm_y}{dt} = \mu_0 \gamma m_x H_0 \qquad \frac{dm_z}{dt} = 0$$

$\hookrightarrow m_z = \text{constant.}$

$\frac{\partial^2 m_x}{dt^2} + \omega_0^2 m_s = 0$ $\Rightarrow$ sol. $\to A_1 e^{j\omega t} + A_2 e^{-j\omega t}$

$D^2 + \omega_0^2 = 0$ $\qquad A_1 \cos\omega t + A_2 \sin\omega t.$

$D = \pm j\omega_0$ (imaginary)

Equation of Motion Solution:

- To obtain two equations for m_x and m_y:

$$\frac{d^2 m_x}{dt^2} + \omega_0^2 m_x = 0 \qquad \frac{d^2 m_y}{dt^2} + \omega_0^2 m_y = 0 \qquad \omega_0 = \mu_0 \gamma H_0$$

- ω_0 is called the **Larmor (or precession) frequency**. One solution to the equation of motion is given by;

$$m_x = A \cos\omega_0 t \qquad m_y = A \sin\omega_0 t$$

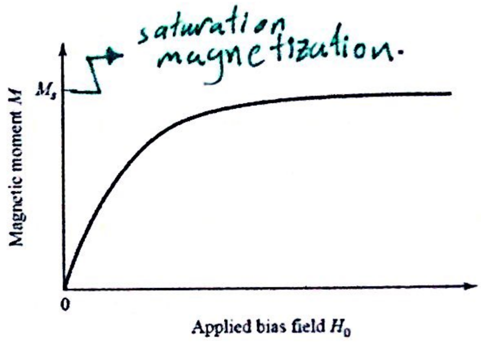
- Where m_z is a constant, and the magnitude of **m** is also a constant, so we have the relation that

$$|\bar{m}|^2 = \left(\frac{q\hbar}{2m_e}\right)^2 = m_x^2 + m_y^2 + m_z^2 = A^2 + m_z^2$$

- Thus the precession angle, θ, between **m** and $\mathbf{H_0}$(the z-axis) is given by

$$\sin\theta = \frac{\sqrt{m_x^2 + m_y^2}}{|\bar{m}|} = \frac{A}{|\bar{m}|}$$

- The projection of **m** on the xy plane (m_x and m_y) shows that **m** traces a circular path in this plane.
- The position of this projection at time t is given by $\varphi = \omega_0 t$, so the angular rate of rotation is $d\varphi/dt = \omega_0$, the precession frequency.
- In the absence of any damping forces, the actual precession angle will be determined by the initial position of the magnetic dipole, and the dipole will precess about $\mathbf{H_0}$ at this angle indefinitely (free precession).
- In reality, however, the existence of damping forces will cause the magnetic dipole moment to spiral in from its initial angle until **m** is aligned with $\mathbf{H_0}$ ($\theta = 0$).



Magnetic moment of a ferrimagnetic material versus bias field, H_0

- Now assume that there are N unbalanced electron spins (magnetic dipoles) per unit volume, so that the total magnetization **(M)** is:

$$\bar{M} = N\bar{m}$$

equation of motion becomes ➔ $\dfrac{d\bar{M}}{dt} = -\mu_0 \gamma \bar{M} \times \bar{H}$

<u>where **H** is the internal applied field, don't confuse between **M** here and **M** for magnetic current.</u>

- As the strength of the bias field H_0 is increased, more magnetic dipole moments will align with H_0 until all are aligned, and **M** reaches an upper limit.
- The material is then said to be **magnetically saturated**, and M_s is denoted as the saturation magnetization.
- M_s is thus a physical property of the ferrite material, and it typically ranges from $4\pi M_s = 300 - 5000$ G.
- Below saturation, ferrite materials can be **very lossy** at microwave frequencies, and the **RF interaction is reduced**. For this reason ferrites are usually operated in the saturated state.

*كل مازادت مساحة ال B-H curve ⇐ زادت

!

- **The saturation magnetization** of a material is a strong function of temperature, **decreasing as temperature increases**. This effect can be understood by noting that the vibrational energy of an atom increases with temperature, making it more difficult to align all the magnetic dipoles.
- At a high enough temperature the thermal energy is greater than the energy supplied by the internal magnetic field, and a zero net magnetization results. This temperature is called the **Curie temperature** (T_C).

Now for AC:

- Now consider the interaction of a small AC (microwave) magnetic field with a magnetically saturated ferrite material. Such a field will cause a forced precession of the dipole moments around the $\mathbf{H_0(a_z)}$ axis at the frequency of the applied AC field, much like the operation of an AC synchronous motor. The small-signal approximation will be applied.
- If **H** is the applied AC field, the total magnetic field is

$$\bar{H}_t = H_0\hat{z} + \bar{H}, \qquad |\bar{H}| \ll H_0$$

This field produces a total magnetization in the ferrite material $\bar{M}_t = M_s\hat{z} + \bar{M}$ where M_s is the (DC) saturation magnetization and **M** is the additional (AC) magnetization (in the xy plane) caused by **H**

$$\frac{dM_x}{dt} = -\mu_0\gamma M_y(H_0 + H_z) + \mu_0\gamma(M_s + M_z)H_y,$$

$$\frac{dM_y}{dt} = \mu_0\gamma M_x(H_0 + H_z) - \mu_0\gamma(M_s + M_z)H_x,$$

$$\frac{dM_z}{dt} = -\mu_0\gamma M_x H_y + \mu_0\gamma M_y H_x.$$

(separable) it is constant w.r.t Z.

since $dM_S/dt = 0$. Since $|\mathbf{H}| << H_0$, we have $|\mathbf{M}||\mathbf{H}| << |\mathbf{M}|H_0$ and $|\mathbf{M}||\mathbf{H}| << \mathbf{M_s}|\mathbf{H}|$, so we can ignore MH products, reduces to;

$$\frac{dM_x}{dt} = -\omega_0 M_y + \omega_m H_y,$$

$$\frac{dM_y}{dt} = \omega_0 M_x - \omega_m H_x,$$

$$\frac{dM_z}{dt} = 0,$$

where $\omega_0 = \mu_0\gamma H_0$ and $\omega_m = \mu_0\gamma M_s$. Solving for M_x and M_0 gives the following equations:

$$\frac{d^2M_x}{dt^2} + \omega_0^2 M_x = \omega_m \frac{dH_y}{dt} + \omega_0\omega_m H_x. \qquad (\omega_0^2 - \omega^2)M_x = \omega_0\omega_m H_x + j\omega\omega_m H_y, \text{ --- (1)}$$

$$\frac{d^2M_y}{dt^2} + \omega_0^2 M_y = -\omega_m \frac{dH_x}{dt} + \omega_0\omega_m H_y \qquad (\omega_0^2 - \omega^2)M_y = -j\omega\omega_m H_x + \omega_0\omega_m H_y, \text{ --- (2)}$$

real complex.

$$\bar{M} = [\chi]\bar{H} = \begin{bmatrix} \chi_{xx} & \chi_{xy} & 0 \\ \chi_{yx} & \chi_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \bar{H}$$

These are the equations of motion for the forced precession of the magnetic dipoles, assuming small-signal conditions. **It is now an easy step to arrive at the permeability tensor for ferrites**; to gain some physical insight into the magnetic interaction process by considering circularly polarized AC fields.

row (1) found from equ. (1)
row (2) found from equ (2)

$$X_{xx} = \frac{\omega_0\omega_m}{\omega_0^2 - \omega^2}$$

$$X_{xy} = j\frac{\omega\omega_m}{\omega_0^2 - \omega 2}$$

$\vdots$

Permeability Tensor for Ferrites

$$\bar{B} = \mu_0(\bar{M} + \bar{H}) = [\mu]\bar{H}$$

$$[\mu] = \begin{bmatrix} \mu_0 & 0 & 0 \\ 0 & \mu & j\kappa \\ 0 & -j\kappa & \mu \end{bmatrix} \quad (\hat{x} \text{ bias})$$

$$[\mu] = \begin{bmatrix} \mu & 0 & -j\kappa \\ 0 & \mu_0 & 0 \\ j\kappa & 0 & \mu \end{bmatrix} \quad (\hat{y} \text{ bias})$$

$$[\mu] = \begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} \quad (\hat{z} \text{ bias})$$

μ_r

$$\mu = \mu_0(1 + \chi_{xx}) = \mu_0(1 + \chi_{yy}) = \mu_0\left(1 + \frac{\omega_0\omega_m}{\omega_0^2 - \omega^2}\right)$$

$$\kappa = -j\mu_0\chi_{xy} = j\mu_0\chi_{yx} = \mu_0\frac{\omega\omega_m}{\omega_0^2 - \omega^2}$$

$$\mu + \kappa = \mu_0\left(1 + \frac{\omega_m}{\omega_0 - \omega}\right),$$

$$\mu - \kappa = \mu_0\left(1 + \frac{\omega_m}{\omega_0 + \omega}\right).$$

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$\chi_{xx} = \frac{\omega_0\omega_m}{\omega_0^2 - \omega^2}$

$\chi_{xy} = \frac{j\omega\omega_m}{\omega_0^2 - \omega^2}$

$\chi_{xz} = 0$

μ_r

$\mu = \mu_0(1 + \chi)$

$\mu_{xx} = \mu_0(1 + \chi_{xx})$

$= \mu_0\left(1 + \frac{\omega_0\omega_m}{\omega_0^2 - \omega^2}\right)$

$K + \mu = \mu_0\left(\frac{\omega\omega_m}{\omega_0^2 - \omega^2} + 1 + \frac{\omega_0\omega_m}{\omega_0^2 - \omega^2}\right)$

$= \mu_0\left(1 + \frac{\omega_m(\omega + \omega_0)}{(\omega_0 - \omega)(\omega_0 + \omega)}\right)$

$K = -j\left(\frac{j\omega\omega_m}{\omega_0^2 - \omega^2}\right)\mu_0$

$K = \mu_0\frac{\omega\omega_m}{\omega_0^2 - \omega^2}$

Note about Units:

- A comment must be made about units:
- By tradition most practical work in magnetics is done with CGS units, with:

✓magnetization measured in gauss (1 gauss [G] = 10^{-4} weber/m^2)

✓field strength measured in oersteds ($4\pi \times 10^{-3}$ oersted [Oe] = 1 A/m)

✓Larmor frequency can be expressed as:

$$f_0 = \omega_0/2\pi = \mu_0\gamma H_0/2\pi = (2.8 \text{ MHz/Oe}) \times (H_0 \text{ oersted})$$

✓Thus, μ_0 = 1 G/Oe in CGS units, implying that B and H have the same numerical values in a nonmagnetic material.

✓$f_m = \omega_m/2\pi = \mu_0\gamma M_s/2\pi = (2.8 \text{ MHz/Oe}) \times (4\pi M_s \text{ gauss})$

- Saturation magnetization is usually expressed as $4\pi M_s$ gauss; the corresponding MKS value is then $\mu_0 M_s$ weber/m^2 = 10^{-4} ($4\pi M_s$ gauss).
- In practice, these units are convenient and easy to use.

Sub. $H^+ = H_0^+(\hat{a_x} - j\hat{a_y})$

$(\omega_0^2-\omega^2)M_x = \omega_0\omega_m M_x + j\omega\omega_m H_y$

$H_x = H_0^+$

$H_y = -jH_0^+$

$\Rightarrow (\omega_0^2-\omega^2)M_x = \omega_0\omega_m H_0^+ + \omega\omega_m H_0^+$

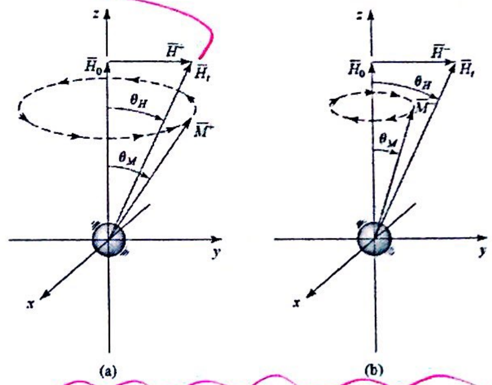
$M_x = \left(\frac{\omega_0\omega_m}{\omega_0^2-\omega^2} + \frac{\omega\omega_m}{\omega_0^2-\omega^2}\right)H_0^+$

$M_x^+ = \frac{\omega_m H_0^+}{\omega_0-\omega}$

$\vec{H_t} = \vec{H_0} + \vec{H^+}$ → + means right handed, − means left handed.

write on formula sheet.

• 2. Circularly Polarized Fields



Forced precession of a magnetic dipole with circularly polarized fields. (a) RHCP, $\theta_M > \theta_H$. (b) LHCP $\theta_M < \theta_H$.

A right-hand circularly polarized (RHCP):

$$\bar{H}^+ = H^+(\hat{x} - j\hat{y})$$

$$\bar{\mathcal{H}}^+ = \mathrm{Re}\{\bar{H}^+ e^{j\omega t}\} = H^+(\hat{x}\cos\omega t + \hat{y}\sin\omega t)$$

proved above.

$$M_x^+ = \frac{\omega_m}{\omega_0-\omega}H^+ \qquad M_y^+ = \frac{-j\omega_m}{\omega_0-\omega}H^+$$

$$\bar{M}^+ = M_x^+\hat{x} + M_y^+\hat{y} = \frac{\omega_m}{\omega_0-\omega}H^+(\hat{x} - j\hat{y})$$

$$\mu^+ = \mu_0\left(1 + \frac{\omega_m}{\omega_0-\omega}\right) = \mu + \kappa$$

$$\tan\theta_M = \frac{|\mathcal{M}^+|}{M_s} = \frac{\omega_m H^+}{(\omega_0-\omega)M_s} = \frac{\omega_0 H^+}{(\omega_0-\omega)H_0}$$

$$\tan\theta_H = \frac{|\mathcal{H}^+|}{H_0} = \frac{H^+}{H_0}$$

LHCP

$$\bar{H}^- = H^-(\hat{x} + j\hat{y})$$

$$\bar{\mathcal{H}}^- = \mathrm{Re}\{\bar{H}^- e^{j\omega t}\} = H^-(\hat{x}\cos\omega t - \hat{y}\sin\omega t)$$

$$M_x^- = \frac{\omega_m}{\omega_0+\omega}H^- \qquad M_y^- = \frac{j\omega_m}{\omega_0+\omega}H^-$$

$$\bar{M}^- = M_x^-\hat{x} + M_y^-\hat{y} = \frac{\omega_m}{\omega_0+\omega}H^-(\hat{x} + j\hat{y})$$

$$\mu^- = \mu_0\left(1 + \frac{\omega_m}{\omega_0+\omega}\right) = \mu - \kappa$$

$$\tan\theta_M = \frac{|\bar{\mathcal{M}}^-|}{M_s} = \frac{\omega_m H^-}{(\omega_0+\omega)M_s} = \frac{\omega_0 H^-}{(\omega_0+\omega)H_0}$$

$$\tan\theta_H = \frac{|\mathcal{H}^-|}{H_0} = \frac{H^-}{H_0}$$

$\omega_0 = \mu_0\gamma H_0$

$\omega_m = \mu_0\gamma M_s$

$\frac{\omega_m}{M_s} = \frac{\omega_0}{H_0} = \mu_0\gamma$

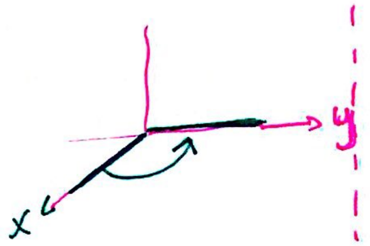
$H_s^+ = H^+(\hat{a_x} - j\hat{a_y})$

$H^+(t) = \mathrm{Re}\{H^+(\hat{a_x} - j\hat{a_y})e^{j\omega t}\}$ → $\cos\omega t + j\sin\omega t$.

$H^+(t) = H^+\cos\omega t\,\hat{a_x} + \sin\omega t\,\hat{a_y}$

@ $\omega t = 0$: $H^+(t) = H^+\hat{a_x}$

@ $\omega t = \pi/2$: $H^+(t) = H^+\hat{a_y}$



• 3. Effect of Loss

As with other resonant systems, loss can be accounted for by making the resonant frequency complex:

$\mu = \mu' - j\mu''$

$\varepsilon = \varepsilon' - \varepsilon''$

$\kappa = \kappa' - j\kappa''$

$\varepsilon'' = \tan\delta$

$$\mu' = \left\{1 + \frac{\omega_o\omega_m(\omega_o^2-\omega^2)}{(\omega_o^2-\omega^2)^2 + 4(\omega\omega_o\alpha)^2}\right\} \xrightarrow{\alpha=0} \left\{1 + \frac{\omega_o\omega_m}{(\omega_o^2-\omega^2)}\right\}$$

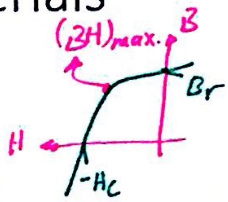
$$\mu'' = \left\{\frac{\omega\omega_m\alpha(\omega_o^2-\omega^2)}{(\omega_o^2-\omega^2)^2 + 4(\omega\omega_o\alpha)^2}\right\} \xrightarrow{\alpha=0} \{0\}$$

$$\kappa' = \left\{1 + \frac{\omega\omega_m(\omega_o^2-\omega^2)}{(\omega_o^2-\omega^2)^2 + 4(\omega\omega_o\alpha)^2}\right\} \xrightarrow{\alpha=0} \left\{\frac{\omega\omega_m}{(\omega_o^2-\omega^2)}\right\}$$

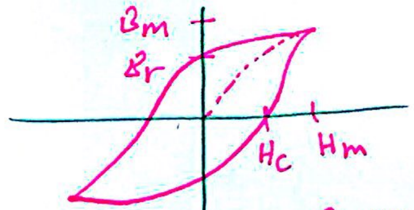
$$\kappa'' = \left\{\frac{2\omega_o\omega_m\omega^2\alpha}{(\omega_o^2-\omega^2)^2 + 4(\omega\omega_o\alpha)^2}\right\} \xrightarrow{\alpha=0} \{0\}$$

$$\omega_0 \leftarrow \omega_0 + j\alpha\omega$$

Most Common Permanent Magnet Materials



Material	Composition	B_r (Oe)	H_c (G)	$(BH)_{max}$ (G-Oe) $\times 10^6$
ALNICO 5	Al, Ni, Co, Cu	12,000	720	5.0
ALNICO 8	Al, Ni, Co, Cu, Ti	7100	2000	5.5
ALNICO 9	Al, Ni, Co, Cu, Ti	10,400	1600	8.5
Remalloy	Mo, Co, Fe	10,500	250	1.1
Platinum cobalt	Pt, Co	6450	4300	9.5
Ceramic	$BaO_6Fe_2O_3$	3950	2400	3.5
Cobalt samarium	Co, Sm	8400	7000	16.0



- we need area of BH curve as large as possible.

Derivation Not required.

Plane Wave Propagation in a Ferrite Medium

- Once we have this macroscopic description of the ferrite material, we can solve Maxwell's equations for wave propagation in various geometries involving ferrite materials.
- **Propagation in Direction of Bias (Faraday Rotation)**
- Consider an infinite ferrite-filled region with a DC magnetic bias field given by $\mathbf{H_0} = H_0\,\mathbf{a_z}$, and a tensor permittivity [μ] given by:
- Maxwell's equations can be written as

$$[\mu] = \mu_0([U] + [\chi]) = \begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} \quad (\hat{z} \text{ bias})$$

$$\nabla \times \bar{E} = -j\omega[\mu]\bar{H},$$

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E},$$

$$\nabla \cdot \bar{D} = 0,$$

$$\nabla \cdot \bar{B} = 0.$$

$$\beta_\pm = \omega\sqrt{\epsilon(\mu \pm \kappa)}.$$

$$Y = \frac{H_y}{E_x} = \frac{-H_x}{E_y} = \frac{\omega\epsilon}{\beta}$$ → Wave admittance

$$Y_+ = \frac{\omega\epsilon}{\beta_+} = \sqrt{\frac{\epsilon}{\mu + \kappa}}$$ → RHCP

$$Y_- = \frac{\omega\epsilon}{\beta_-} = \sqrt{\frac{\epsilon}{\mu - \kappa}}$$ → LHCP

write on formula sheet.

Example: Plane Wave Propagation in a Ferrite Medium

- Consider an infinite ferrite medium with $4\pi M_s = 1800$ G, $\Delta H = 75$ Oe, $\varepsilon_r = 14$, and $\tan\delta = 0.001$. If the bias field strength is $H_0 = 3570$ Oe, calculate the phase and attenuation constants for RHCP and LHCP plane waves for f = 0 to 20 GHz.
- **Solution:** $\epsilon'' = \tan\delta = 0.001$

$$\omega_0 \leftarrow \omega_0 + j\frac{\mu_0 \gamma \Delta H}{2}$$

$$f_0 = \frac{\omega_0}{2\pi} = (2.8 \text{ MHz/Oe})(3570 \text{ Oe}) = 10.0 \text{ GHz}.$$

$$f_0 \leftarrow f_0 + j\frac{(2.8 \text{ MHz/Oe})(75 \text{ Oe})}{2} = (10.0 + j0.105) \text{ GHz}$$

$$f_m = \frac{\omega_m}{2\pi} = (2.8 \text{ MHz/Oe})\ (1800 \text{ G}) = 5.04 \text{ GHz}.$$

$$\gamma_\pm = \alpha_\pm + j\beta_\pm = j\omega\sqrt{\epsilon(\mu \pm \kappa)},$$

$$\epsilon = \epsilon_0\epsilon_r(1 - j\tan\delta)$$

α_+ & $\beta_+ \Rightarrow$ Right handed.

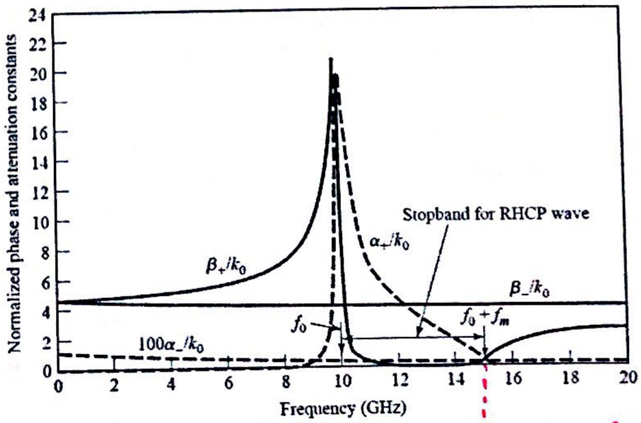
α_- & $\beta_- \Rightarrow$ Left handed.

Normalized phase and attenuation constants for circularly polarized plane waves in the ferrite medium

A circularly polarized plane wave rotating in the same direction as the precessing magnetic dipoles of a ferrite medium will have a strong interaction with the material, while a circularly polarized wave rotating in the opposite direction will have a weaker interaction.

The attenuation of a circularly polarized wave was very large near the gyromagnetic resonance of the ferrite, while the attenuation of a wave propagating in the opposite direction was very small.

This effect can be used to construct an isolator; such isolators must operate near gyromagnetic resonance and so are called resonance isolators.



10G + 5G = 15 GHz = $f_0 + f_m$

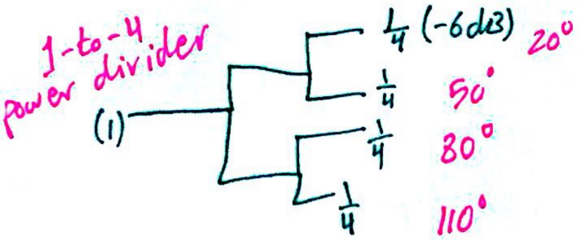
Ferrite Isolators

- One of the most useful microwave ferrite components is the isolator, which is a two-port device having unidirectional transmission characteristics.
- The scattering matrix for an ideal isolator has the form: $[S] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

➔matched, not unitary, lossy, nonreciprocal component.

✓A common application uses an isolator between a high-power source and a load to prevent possible reflections from damaging the source.

✓An isolator can be used in place of a matching or tuning network, but it should be realized that any power reflected from the load will be absorbed by the isolator, as opposed to being reflected back to the load, which is the case when a matching network is used.



Ferrite Phase Shifters (1)

any active component is non reciprocal.

- Another important application of ferrite materials is in phase shifters, which are two-port components that provide variable phase shift (where the output signal is controlled to have some desired phase relationship to the input) by changing the bias field of the ferrite. (Microwave diodes and FETs can also be used to implement phase shifters).
- Phase shifters find application in test and measurements systems, but the most significant use is in phased array antennas where the antenna beam can be steered in space by electronically controlled phase shifters.
- Applications:

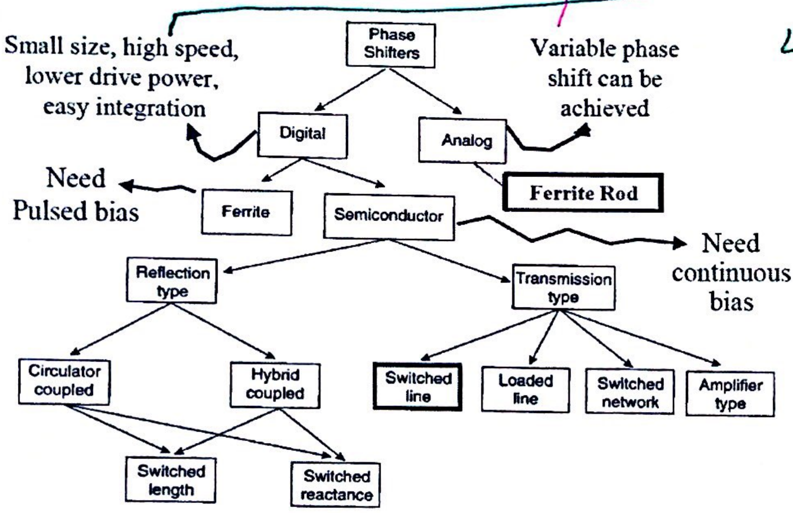
✓Used in a variety of communication and radar systems

✓In microwave instrumentation and measurement systems

✓In industrial applications.

Classification of Phase Shifters

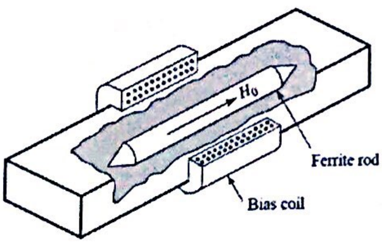
disadvantages for Digital.



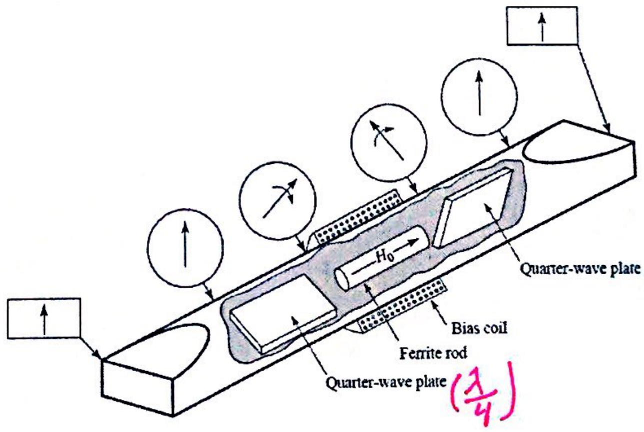
Disadvantages for Analog.

Analog Ferrite Phase Shifters

- In an **Analog phase shifter,** differential phase shift can be varied in a continuous manner using a control signal (external H_0).



Reggia-Spencer reciprocal phase shifter



$(\lambda/4)$

$(\lambda/4)$

Nonreciprocal Faraday rotation phase shifter

advantage: variable phase shift.

disadvantage: need very Large Baising coil.

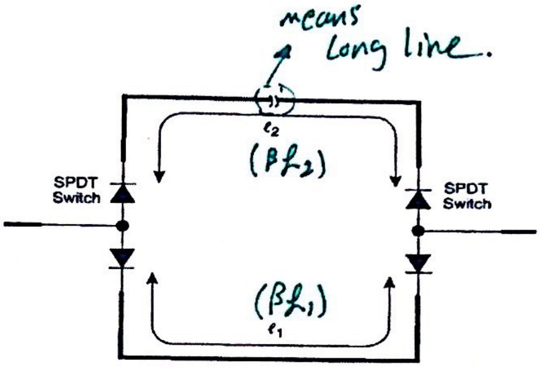
Digital Phase Shifters (1)

- In a **Digital phase shifter,** differential phase shift is changed by a few predetermined phase-shift values 22.5°,45°,90°,180°

- Transmission type Switched line phase shifter: consist of two SPDT switches and two transmission lines of different lengths.

- **Disadvantage:** is due to **resonance** in the line that is off, if its length is near a multiple of λ/2 at the frequency of operation.

λ/2. ⇒ extension for the Line.



Binary phase shifter with:

$$\Delta\phi = \beta(\ell_2 - \ell_1) = \beta\Delta\ell$$

$$\Delta\ell = \ell_2 - \ell_1$$

$$\beta = \omega / V_P$$

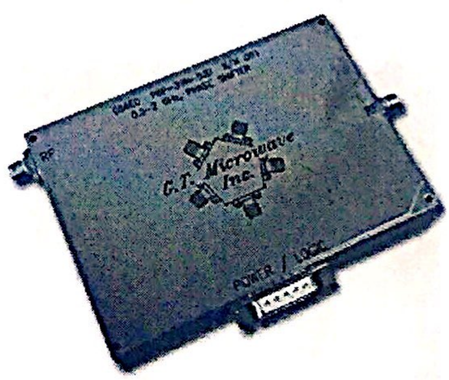
$$\Delta\phi = \omega\Delta\ell / V_P$$

$$\tau_d = \Delta\phi / \omega = \Delta\ell / V_P$$

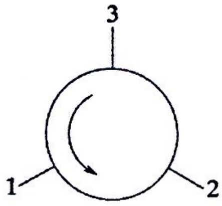
Digital Phase Shifters (2)

Specification of Digital Phase Shifter

Frequency	0.5 to 2.0 GHz
Phase Accuracy	+/- 10
Amplitude Balance	+/- 1.0 dB
VSWR	1.7:1
Noise Figure	7 dB (High noise).
Input power	0 dBm CW ~ +2- dBm Max
Size	4.95 x 3.38 x 1.0 inch
Switching Speed	500 nSec (very high switching speed).



Ferrite Circulators (1)



- A circulator is a three-port microwave device that can be **lossless** and **matched** at all ports; by using the unitary properties of the scattering matrix we were able to show that such a device must be **nonreciprocal**.
- The scattering matrix for an ideal circulator has the following form:

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

transmission loss, 1 to 2 $= 10\log_{10}\frac{P_2}{P_1} \approx -0.5\,\text{dB}$ → ideal = 0

isolation, 1 to 3 $= 10\log_{10}\frac{P_3}{P_1} < -20\,\text{dB}$ → ideal = $-\infty$

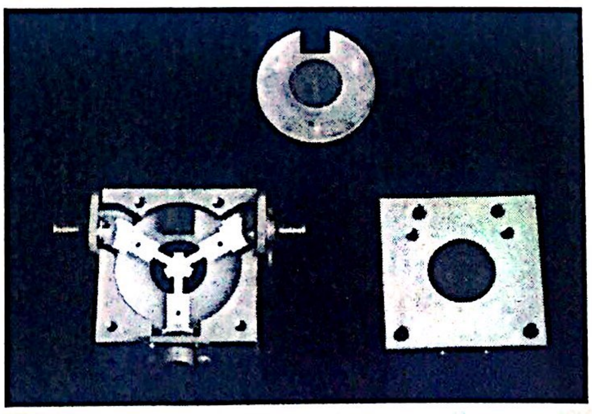
VSWR<1.2

which shows that power can flow from port 1 to port 2, port 2 to port 3, and port 3 to port 1, but not in the reverse directions.

Ferrite Circulators (2)

For a ferrite circulator, this result can be produced by changing the **polarity** of the magnetic bias field. Most circulators use permanent magnets for the bias field, but if an electromagnet is used the circulator can operate in a latching (remanent) mode as a single-pole double-throw (SPDT) switch.

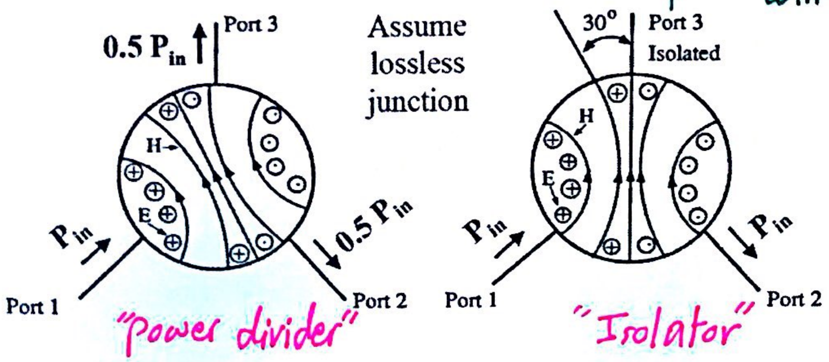
Photograph of a disassembled ferrite junction circulator, showing the stripline conductor, the ferrite disks, and the bias magnet. The middle port of the circulator is terminated with a matched load, so this circulator is actually configured as an isolator.
Note the change in the width of the stripline conductors due to the different dielectric constants of the ferrite and the surrounding plastic material.



* To have the circulator working as an isolator make one port terminated with a matched load.

Ferrite Junction Circulator

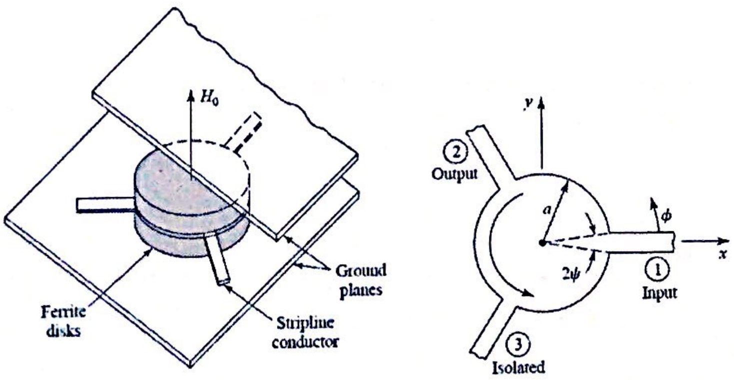
this port will be matched.



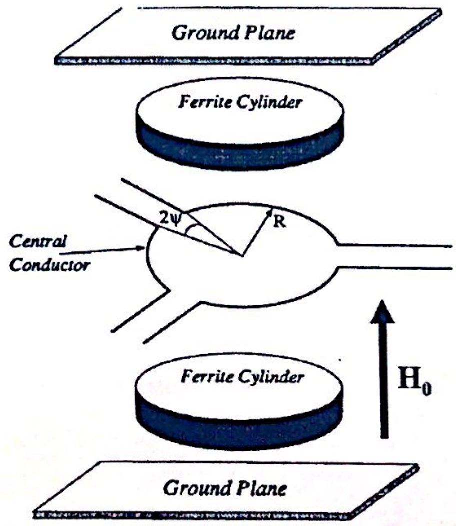
Thus one circulation condition is obtained by operating within **two split frequencies** (achieved by fixing the radius 'R' of the disk) and another circulation condition is met by **adjusting the splitting** (achieved by fixing H_0), until the standing wave pattern of the E-field is rotated through 30°. Thus, Port 3 isolated.

Stripline or Microstrip Y-junction Circulator

Consists of two ferrite disc resonators separated by a conductor disk coupled by symmetrical and 120 degree apart Y-junction transmission lines.

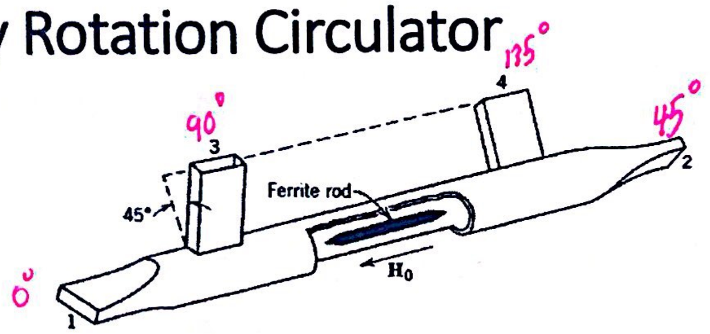


(a) (b)



$$S = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Faraday Rotation Circulator



<u>In</u>	➔	<u>out</u>
Port 1	➔	Port 2
Port 2	➔	Port 3
Port 3	➔	Port 4

$4 \rightarrow 1$

Ferrite rod is magnetically biased to give 45° rotation. TE_{10} mode of rectangular waveguide entering **port 1** is transformed into TE_{11} mode of the circular guide.

This mode is then rotated by 45° degrees, as it passes through the ferrite rod and is changed back into the TE_{10} mode before leaving from **port 2**.

Very little energy is coupled out from the other **ports** because the field distributions in the circular guide are orthogonal to the optimum orientation for these ports.

magic T ≡ 4 port power divider.

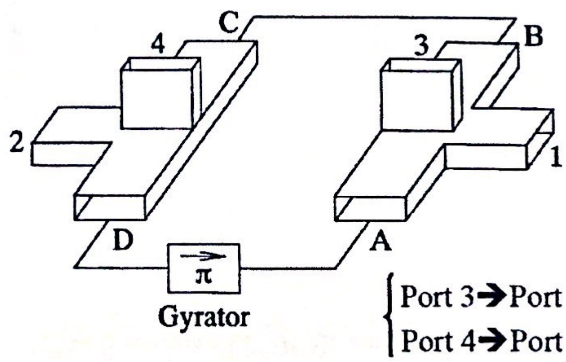
Differential Phase Shift Circulator

Consists of two magic T's and a directional 180° phase-shifter to change path electrical length in one direction.

A wave incident in **port 1** will split into two equal-amplitude in-phase waves coming out of arms A and B.

Since the phase-shifter does not change the phase in this path or direction of propagation, waves arrive at D and C are in phase, and hence they combine and emerge from **port 2**.

BUT, wave incident at **port 2** comes out in phase from D and C and since the signal phase of path D-A is changed by 180°, the waves at port A and B are 180° out-of-phase. Thus, they combine and exit from **port 3** (due to property of **magic T**).



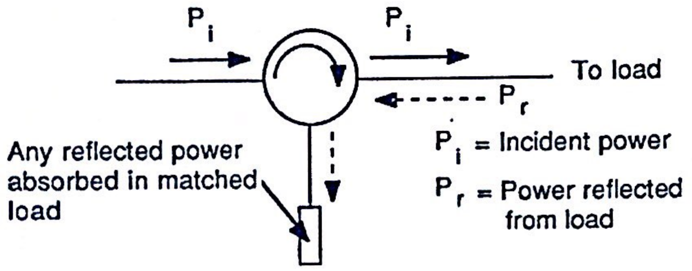
Port 3➔Port 4
Port 4➔Port 1

$1 \rightarrow 2$

$2 \rightarrow 3$

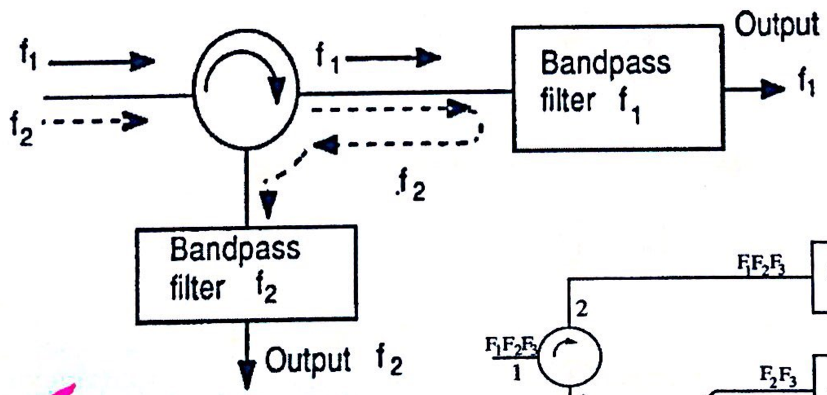
Application: Isolator

A circulator can also be used as an isolator by terminating one of the ports with a matched load.

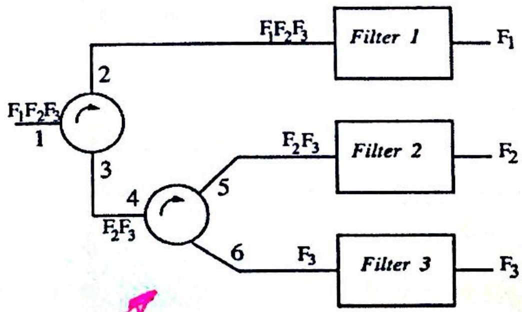


Three port isolator has an advantage over two port isolator is that reflected power is absorbed by the matched load and **not** the material inside the isolator. Thus easy to design the cooling system for the device.

Application: Channel Separator



- Here circulator is used as a **channel separator**.
- Incoming signals of frequency f_1, f_2 and f_3 are separated.

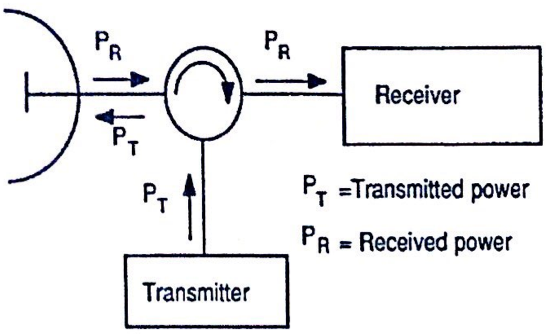


To separate into 2 signals.

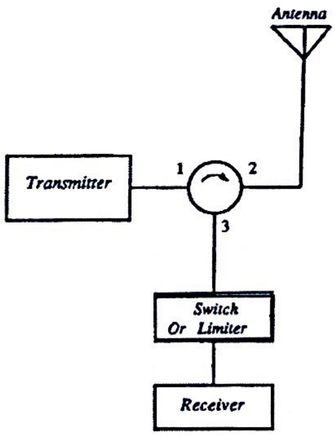
To separate into 3 signals.
(we added a circulator)

& so on for more separation.

Application: Duplexer



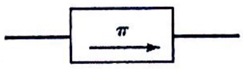
This enable the use of antenna for both reception and transmission.



Typically due to lack of isolation (reflected signals from antenna), additional protection for the receiver is required

The Gyrator (used to have an isolator).

- It is a two-port device having a 180° differential phase shift



Symbol for a gyrator, which has a differential phase shift of 180°.

$$[S] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

An **isolator** constructed with a gyrator and two quadrature hybrids.
The forward wave (→) is passed, while the reverse wave (←) is absorbed in the matched load of the first hybrid