Topics in Communications.

"Microwaves"

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Microwave Engineering

Chapter 2

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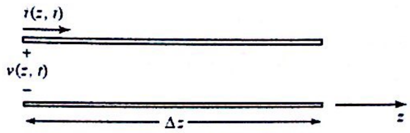
Transmission Line THEORY

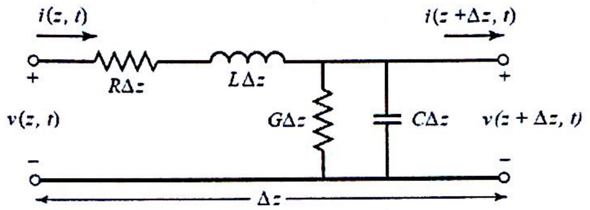
- The Lumped-Element Circuit Model for a Transmission Line
- Field Analysis of Transmission Lines
- The Terminated Lossless and Lossy Transmission Line
- The Smith Chart
- The Quarter-Wave Transformer
- Generator and Load Mismatches
- Transients on Transmission Lines
- The Terminated Lossy Transmission Line
- Transients on Transmission Lines
- Types of Transmission Lines
- Wave Velocities and Dispersion

THE LUMPED-ELEMENT CIRCUIT MODEL FOR A TRANSMISSION LINE

Low Frequency →



High Frequency →



This Type is: L- Type.
also there is other types: π & T.

KVL → $v(z,t) - R\Delta z i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z, t) = 0$

KCL → $i(z,t) - G\Delta z v(z+\Delta z, t) - C\Delta z \frac{\partial v(z+\Delta z, t)}{\partial t} - i(z+\Delta z, t) = 0$

Telegrapher Equations

- Dividing by Δz and taking the limit as Δz → 0 gives;

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t},$$
$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}.$$

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z),$$
$$\frac{dI(z)}{dz} = -(G + j\omega C)V(z).$$

Similar →

$$\nabla \times \bar{E} = -j\omega\mu\bar{H},$$
$$\nabla \times \bar{H} = j\omega\epsilon\bar{E},$$

Wave Propagation on a Transmission Line

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0,$$
$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0,$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Solution ➔
$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z},$$
$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z},$$

$$I(z) = \frac{\gamma}{R + j\omega L}\left(V_o^+ e^{-\gamma z} - V_o^- e^{\gamma z}\right)$$

Characteristics Impedance ➔ $Z_0 = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}.$

- The voltage and current on the line can be related as follows;

$$\frac{V_o^+}{I_o^+} = Z_0 = \frac{-V_o^-}{I_o^-}$$

$$I(z) = \frac{V_o^+}{Z_0} e^{-\gamma z} - \frac{V_o^-}{Z_0} e^{\gamma z}$$

$$v(z,t) = |V_o^+| \cos(\omega t - \beta z + \phi^+) e^{-\alpha z} + |V_o^-| \cos(\omega t + \beta z + \phi^-) e^{\alpha z}.$$

$$\lambda = \frac{2\pi}{\beta} \qquad v_p = \frac{\omega}{\beta} = \lambda f.$$

The Lossless Line (R = 0 = G)

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}, \qquad \beta = \omega\sqrt{LC}, \qquad \alpha = 0 \qquad Z_0 = \sqrt{\frac{L}{C}}.$$

Real.

always Lossless Line has a real char. impedance.

Solution ➔

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z},$$
$$I(z) = \frac{V_o^+}{Z_0} e^{-j\beta z} - \frac{V_o^-}{Z_0} e^{j\beta z}.$$

FIELD ANALYSIS OF TRANSMISSION LINES

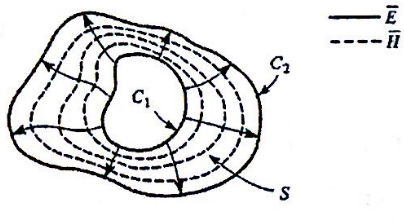
$W_m = \frac{1}{4} L I^2$

$$W_m = \frac{\mu}{4}\int_S \bar{H}\cdot\bar{H}^* ds \qquad L = \frac{\mu}{|I_o|^2}\int_S \bar{H}\cdot\bar{H}^* ds \text{ H/m}.$$

$$W_e = \frac{\epsilon}{4}\int_S \bar{E}\cdot\bar{E}^* ds. \qquad C = \frac{\epsilon}{|V_o|^2}\int_S \bar{E}\cdot\bar{E}^* ds \text{ F/m}.$$

$$R = \frac{R_s}{|I_o|^2}\int_{C_1+C_2} \bar{H}\cdot\bar{H}^* dl \ \Omega\text{/m}.$$

$$P_d = \frac{\omega\epsilon''}{2}\int_S \bar{E}\cdot\bar{E}^* ds \qquad G = \frac{\omega\epsilon''}{|V_o|^2}\int_S \bar{E}\cdot\bar{E}^* ds \text{ S/m}.$$

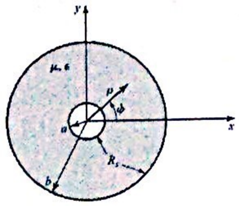


Example: Transmission Line Parameters of a Coaxial Line

$$\bar{E} = \frac{V_o \hat{\rho}}{\rho \ln b/a} e^{-\gamma z}, \qquad \epsilon = \epsilon' - j\epsilon''$$

$$\bar{H} = \frac{I_o \hat{\phi}}{2\pi\rho} e^{-\gamma z}, \qquad \mu = \mu_0 \mu_r$$

$ds = \rho d\rho d\phi$



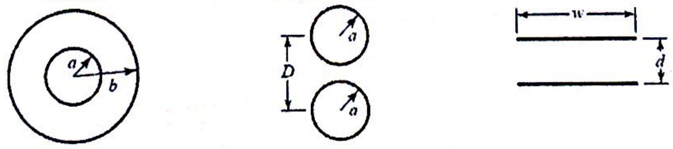
$$L = \frac{\mu}{(2\pi)^2} \int_{\phi=0}^{2\pi} \int_{\rho=a}^{b} \frac{1}{\rho^2} \rho d\rho d\phi = \frac{\mu}{2\pi} \ln b/a \text{ H/m.}$$

$$C = \frac{\epsilon'}{(\ln b/a)^2} \int_{\phi=0}^{2\pi} \int_{\rho=a}^{b} \frac{1}{\rho^2} \rho d\rho d\phi = \frac{2\pi\epsilon'}{\ln b/a} \text{ F/m,}$$

$$R = \frac{R_s}{(2\pi)^2} \left\{ \int_{\phi=0}^{2\pi} \frac{1}{a^2} a d\phi + \int_{\phi=0}^{2\pi} \frac{1}{b^2} b d\phi \right\} = \frac{R_s}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right) \Omega\text{/m,}$$

$$G = \frac{\omega\epsilon''}{(\ln b/a)^2} \int_{\phi=0}^{2\pi} \int_{\rho=a}^{b} \frac{1}{\rho^2} \rho d\rho d\phi = \frac{2\pi\omega\epsilon''}{\ln b/a} \text{ S/m.}$$

Transmission Line Parameters for Some Common Lines



	COAX	TWO-WIRE	PARALLEL PLATE
L	$\frac{\mu}{2\pi} \ln \frac{b}{a}$	$\frac{\mu}{\pi} \cosh^{-1}\left(\frac{D}{2a}\right)$	$\frac{\mu d}{w}$
C	$\frac{2\pi\epsilon'}{\ln b/a}$	$\frac{\pi\epsilon'}{\cosh^{-1}(D/2a)}$	$\frac{\epsilon' w}{d}$
R	$\frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$	$\frac{R_s}{\pi a}$	$\frac{2R_s}{w}$
G	$\frac{2\pi\omega\epsilon''}{\ln b/a}$	$\frac{\pi\omega\epsilon''}{\cosh^{-1}(D/2a)}$	$\frac{\omega\epsilon'' w}{d}$

TEM ≡ Transverse ElectroMagnetic.

same T.L equations.

The Telegrapher Equations Derived from Field Analysis of a Coaxial Line (➔TEM)

The fields should satisfy: $\nabla \times \bar{E} = -j\omega\mu\bar{H},$ $\nabla \times \bar{H} = j\omega\epsilon\bar{E},$

$$-\hat{\rho}\frac{\partial E_\phi}{\partial z} + \hat{\phi}\frac{\partial E_\rho}{\partial z} + \hat{z}\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho E_\phi) = -j\omega\mu(\hat{\rho}H_\rho + \hat{\phi}H_\phi),$$

$$-\hat{\rho}\frac{\partial H_\phi}{\partial z} + \hat{\phi}\frac{\partial H_\rho}{\partial z} + \hat{z}\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho H_\phi) = j\omega\epsilon(\hat{\rho}E_\rho + \hat{\phi}E_\phi).$$

Boundary Condition used here.

$$E_\phi = \frac{f(z)}{\rho} \qquad H_\phi = \frac{g(z)}{\rho}$$

$$\frac{\partial E_\rho}{\partial z} = -j\omega\mu H_\phi \qquad \frac{\partial H_\phi}{\partial z} = -j\omega\epsilon E_\rho$$

$$E_\rho = \frac{h(z)}{\rho}$$

$$\frac{\partial h(z)}{\partial z} = -j\omega\mu g(z),$$

$$\frac{\partial g(z)}{\partial z} = -j\omega\epsilon h(z).$$

$$V(z) = \int_{\rho=a}^{b} E_\rho(\rho, z)d\rho = h(z)\int_{\rho=a}^{b}\frac{d\rho}{\rho} = h(z)\ln\frac{b}{a},$$

$$I(z) = \int_{\phi=0}^{2\pi} H_\phi(a, z)a\,d\phi = 2\pi g(z)$$

$$\frac{\partial V(z)}{\partial z} = -j\frac{\omega\mu\ln b/a}{2\pi}I(z),$$

$$\frac{\partial I(z)}{\partial z} = -j\omega(\epsilon' - j\epsilon'')\frac{2\pi V(z)}{\ln b/a}$$

$$\frac{\partial V(z)}{\partial z} = -j\omega L I(z),$$

$$\frac{\partial I(z)}{\partial z} = -(G + j\omega C)V(z)$$

TEM ⇛ "E" ⊥ "H" ⊥ "propagation Direction".

Propagation Constant, Impedance, and Power Flow for the Lossless Coaxial Line

$$\frac{\partial^2 E_\rho}{\partial z^2} + \omega^2\mu\epsilon E_\rho = 0 \qquad \gamma^2 = -\omega^2\mu\epsilon \qquad \beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{LC}$$

$$Z_w = \frac{E_\rho}{H_\phi} = \frac{\omega\mu}{\beta} = \sqrt{\mu/\epsilon} = \eta$$ ≡ Intrinsic Impedance.

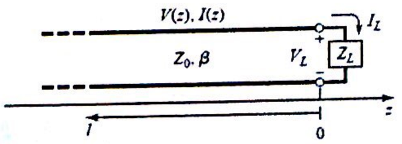
$$Z_0 = \frac{V_o}{I_o} = \frac{E_\rho \ln b/a}{2\pi H_\phi} = \frac{\eta \ln b/a}{2\pi} = \sqrt{\frac{\mu}{\epsilon}}\frac{\ln b/a}{2\pi}$$

$$P = \frac{1}{2}\int_s \bar{E} \times \bar{H}^* \cdot d\bar{s} = \frac{1}{2}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b}\frac{V_o I_o^*}{2\pi\rho^2 \ln b/a}\rho\, d\rho\, d\phi = \frac{1}{2}V_o I_o^*$$

THE TERMINATED LOSSLESS TRANSMISSION LINE

$0 \leq \Gamma \leq 1$

- This problem will illustrate wave reflection on transmission lines, a fundamental property of distributed systems.



$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z} \qquad I(z) = \frac{V_o^+}{Z_0}e^{-j\beta z} - \frac{V_o^-}{Z_0}e^{j\beta z}$$

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-}Z_0$$

reflection Coeficient

$$V_o^- = \frac{Z_L - Z_0}{Z_L + Z_0}V_o^+ \qquad \Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$V(z) = V_o^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right).$$

$$I(z) = \frac{V_o^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right).$$

$$P_{avg} = \frac{1}{2}\mathrm{Re}\{V(z)I(z)^*\} = \frac{1}{2}\frac{|V_o^+|^2}{Z_0}\mathrm{Re}\{1 - \Gamma^* e^{-2j\beta z} + \Gamma e^{2j\beta z} - |\Gamma|^2\}.$$

$$P_{avg} = \frac{1}{2}\frac{|V_o^+|^2}{Z_0}(1 - |\Gamma|^2)$$

Max. Power Transfer.
@ Γ = zero.

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$$P_{avg} = \underbrace{\frac{|V_o^+|^2}{2Z_o}}_{\text{incident power}} - \underbrace{\frac{|V_o^+|^2\,|\Gamma|^2}{2Z_o}}_{\text{reflected power}}$$

- The average power flow is constant at any point on the line and that the total power delivered to the load (P_{avg}) is equal to the incident power ($|V_o^+|^2/2Z_o$) minus the reflected power ($|V_o|^2|\Gamma|^2/2Z_o$).
- If $\Gamma = 0$, maximum power is delivered to the load, while no power is delivered for $|\Gamma| = 1$.
- The above discussion assumes that the generator is matched, so that there is no re-reflection of the reflected wave from $z < 0$. → $Z_g = Z_o$
- When the load is mismatched, not all of the available power from the generator is delivered to the load. This "loss" is called return loss (RL), and is defined (in dB) as;

$$\mathrm{RL} = -20\log|\Gamma|\ \mathrm{dB}$$

always +ve answer. "loss".

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20: since we dealing with voltage ⇒ $\Gamma = \frac{V_o^-}{V_o^+}$

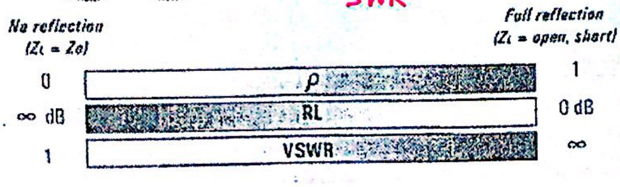
Reflection Parameters

Reflection Coefficient $\Gamma = \dfrac{V_{reflected}}{V_{incident}} = \rho \angle \Phi = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

Return loss $\triangleq -20 \log(\rho)$, (Sign) $\rho = |\Gamma|$

Voltage Standing Wave Ratio

VSWR or SWR $= \dfrac{E_{max}}{E_{min}} = \dfrac{1+\rho}{1-\rho}$



$$|V(z)| = |V_o^+||1 + \Gamma e^{2j\beta z}| = |V_o^+||1 + \Gamma e^{-2j\beta\ell}|$$
$$= |V_o^+||1 + |\Gamma| e^{j(\theta - 2\beta\ell)}|,$$

$$V_{max} = |V_o^+|(1 + |\Gamma|)$$
$$V_{min} = |V_o^+|(1 - |\Gamma|)$$
$$SWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

و $\rho = |\Gamma|$

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• Load could be: ① Mismatched. ② Short circuit. ③ Open Circuit. ④ Matched.

Mismatched Load ($Z_o \neq Z_L$)

- When the load is mismatched, however, the presence of a reflected wave leads to standing waves, and the magnitude of the voltage on the line is not constant. At z = -ℓ (ℓ is the length of the line);

$$\Gamma(\ell) = \frac{V_o^- e^{-j\beta\ell}}{V_o^+ e^{j\beta\ell}} = \Gamma(0) e^{-2j\beta\ell}$$

- At a distance z = −ℓ from the load, the input impedance seen looking toward the load is:

$$Z_{in} = \frac{V(-\ell)}{I(-\ell)} = \frac{V_o^+ (e^{j\beta\ell} + \Gamma e^{-j\beta\ell})}{V_o^+ (e^{j\beta\ell} - \Gamma e^{-j\beta\ell})} Z_0 = \frac{1 + \Gamma e^{-2j\beta\ell}}{1 - \Gamma e^{-2j\beta\ell}} Z_0$$

$$Z_{in} = Z_0 \frac{(Z_L + Z_0)e^{j\beta\ell} + (Z_L - Z_0)e^{-j\beta\ell}}{(Z_L + Z_0)e^{j\beta\ell} - (Z_L - Z_0)e^{-j\beta\ell}}$$
$$= Z_0 \frac{Z_L \cos\beta\ell + jZ_0 \sin\beta\ell}{Z_0 \cos\beta\ell + jZ_L \sin\beta\ell}$$
$$= Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}.$$

← transmission line impedance equation OR impedance transformation

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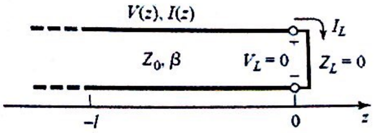
→ Z_{in} pure imaginary.

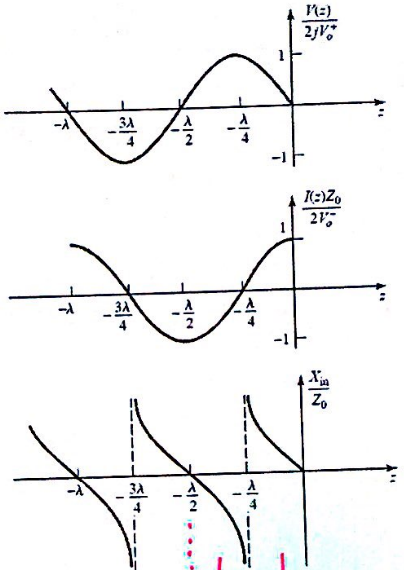
Short Circuit (Z_L=0)



$$V(z) = V_o^+(e^{-j\beta z} - e^{j\beta z}) = -2jV_o^+ \sin\beta z,$$

$$I(z) = \frac{V_o^+}{Z_0}(e^{-j\beta z} + e^{j\beta z}) = \frac{2V_o^+}{Z_0}\cos\beta z,$$

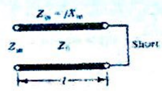
$$Z_{in} = jZ_0 \tan\beta\ell$$



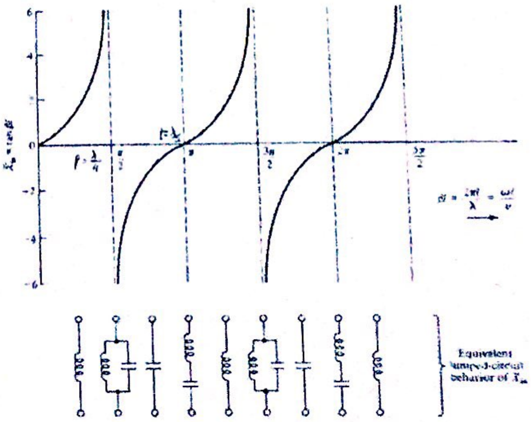
stub is inductive.

stub is capacitive.

stub is resistive.



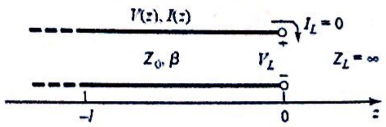
- For T.L. with Z_L = 0 or S/C; we know that Z_{in} = $jZ_0\tan\beta\ell$ *or* **X_{in}**= $\tan\beta\ell$;
- Here if 0 < ℓ < $\lambda/4$, X_{in}=>'+' and Input impedance (Z_{in}) is Inductive.
- But if $\lambda/4 < \ell < \lambda/2$, X_{in}=>'-' and Input impedance (Z_{in}) is Capacitive.



Equivalent lumped-circuit behavior of X_{in}

Scanned by CamScanner

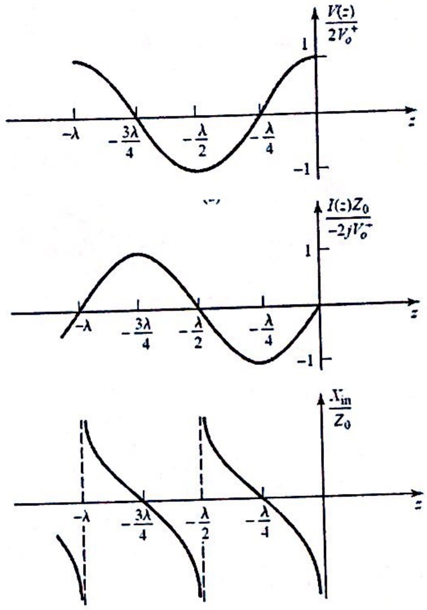
Open Circuit ($Z_L = \infty$)



$$V(z) = V_o^+(e^{-j\beta z} + e^{j\beta z}) = 2V_o^+ \cos\beta z.$$

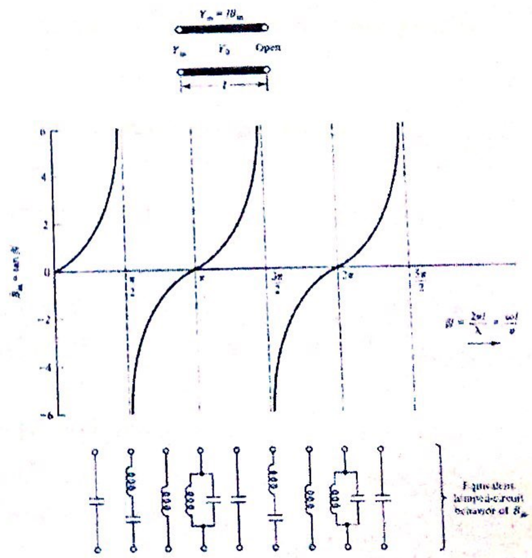
$$I(z) = \frac{V_o^+}{Z_0}(e^{-j\beta z} - e^{j\beta z}) = \frac{-2jV_o^+}{Z_0}\sin\beta z,$$

$$Z_{in} = -jZ_0 \cot\beta\ell$$

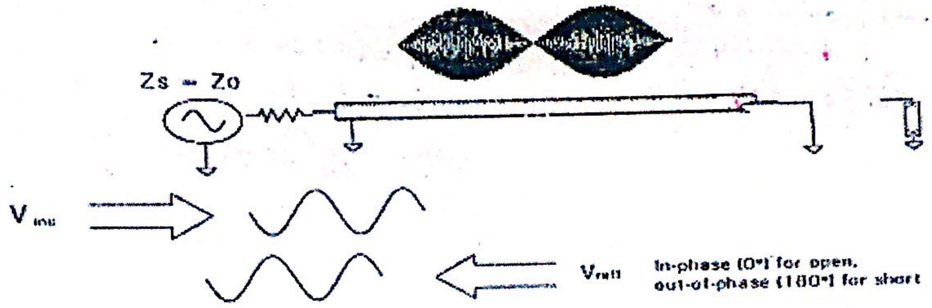


$$\Rightarrow \quad Z_0 = \sqrt{Z_{in_{sc}} \cdot Z_{in_{oc}}}$$

- T.L. with $Z_L = \infty$ or $Y_L = 0$; we know that $Y_{in} = jY_0 \tan\beta\ell$ or $B_{in} = \tan\beta\ell$;
- Here if $0 < \ell < \lambda/4$, B_{in}=>'+' and Input impedance (Z_{in}) is Capacitive.
- But if $\lambda/4 < \ell < \lambda/2$, B_{in}=>'-' and Input impedance (Z_{in}) is Inductive.
- These O/C or S/C lines can be used as STUBS for matching TL's.



Transmission Line Terminated with Short, Open

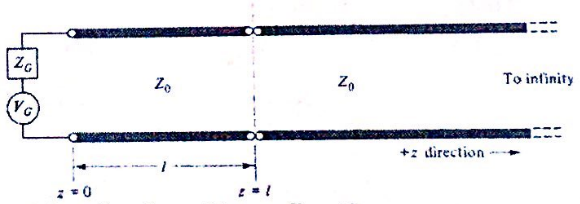


For reflection, a transmission line terminated in a short or open reflects all power back to source

if short: $\Gamma = -1 = 1\angle 180°$ phase shift is $180°$ "out of phase"

if open: $\Gamma = 1 = 1\angle 0$ phase shift is zero. "in-phase"

Matched Load ($Z_{in} = Z_L$)

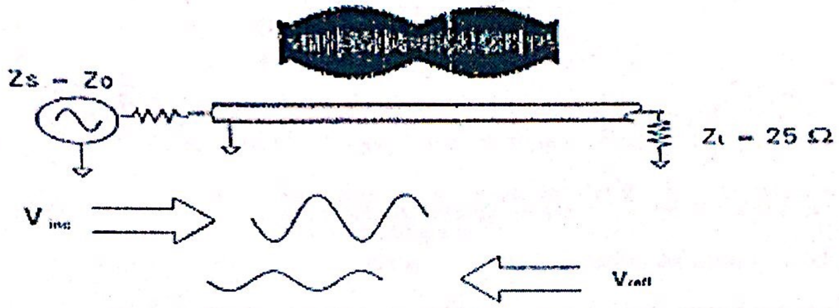


- It occurs if $\ell = \lambda/2$
- If '$Z_L = Z_o$' of the transmission line ⇒ Matched ⇒ No reflection occurs ⇒ ≈ infinite line or flat.
- A half-wavelength line (or any multiple of $\lambda/2$) does not alter or transform the load impedance, regardless of its characteristic impedance.
- Such a line is known as a quarter-wave transformer because it has the effect of transforming the load impedance in an inverse manner, depending on the characteristic impedance of the line. $Z_{in} = \frac{Z_0^2}{Z_L}$

infinite Line means NO Reflection.

Behind the page.

Transmission Line Terminated with 25 Ω

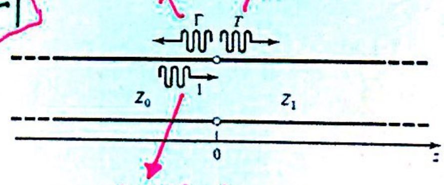


Standing wave pattern does not go to zero as with short or open

Insertion Loss (IL)

"Return loss" $\boxed{20 \log |\Gamma|}$

reflected. transmitted.



incident.

$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$V(z) = V_o^+(e^{-j\beta z} + \Gamma e^{j\beta z}), \quad z < 0 \qquad V(z) = V_o^+ T e^{-j\beta z} \quad \text{for } z > 0$$

$$T = 1 + \Gamma = 1 + \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{2Z_1}{Z_1 + Z_0}$$

$$\text{IL} = -20 \log |T| \text{dB} \qquad 10 \log \frac{P_1}{P_2} \text{ dB} \qquad 20 \log \frac{V_1}{V_2} \text{ dB}$$

$$\ln \frac{V_1}{V_2} \text{ Np} \qquad \frac{1}{2} \ln \frac{P_1}{P_2} \text{ Np}$$

$$10 \log \frac{P_1}{1 \text{ mW}} \text{ dBm}$$

Insertion loss. $\boxed{-20 \log |T|}$

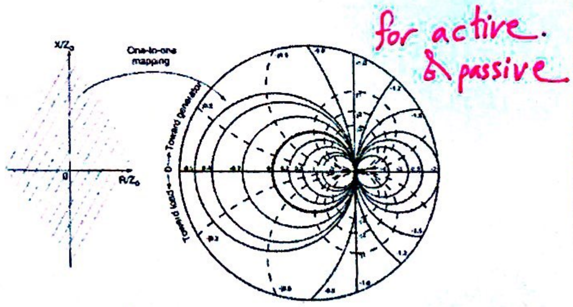
$$1 \text{ Np} = 10 \log e^2 = 8.686 \text{ dB}$$

THE SMITH CHART

- It is a graphical aid that can be very useful for solving transmission line problems.
- It was developed in 1939 by P. Smith at the Bell Telephone Laboratories.
- the Smith chart is an integral part of much of the current CAD software and test equipment for microwave design that provides a useful way of visualizing transmission line phenomenon without the need for detailed numerical calculations.
- It is based on a polar plot of the voltage reflection coefficient, Γ
- Let the reflection coefficient be expressed in magnitude and phase (polar) form as $\Gamma = |\Gamma| e^{j\theta}$.
- Then the magnitude $|\Gamma|$ is plotted as a radius ($|\Gamma| \le 1$) from the center of the chart, and the angle θ ($-180^\circ \le \theta \le 180^\circ$) is measured counterclockwise from the right-hand side of the horizontal diameter.
- Any passively realizable ($|\Gamma| \le 1$) reflection coefficient can then be plotted as a unique point on the Smith chart.
- The real utility of the Smith chart, however, lies in the fact that it can be used to convert from reflection coefficients to normalized impedances (or admittances) and vice versa by using the impedance (or admittance) circles printed on the chart.
- When dealing with impedances on a Smith chart, normalized quantities are generally used, which we will denote by lowercase letters.
- The normalization constant is usually the characteristic impedance of the transmission line. Thus, $z = Z/Z_o$ represents the normalized version of the impedance Z.

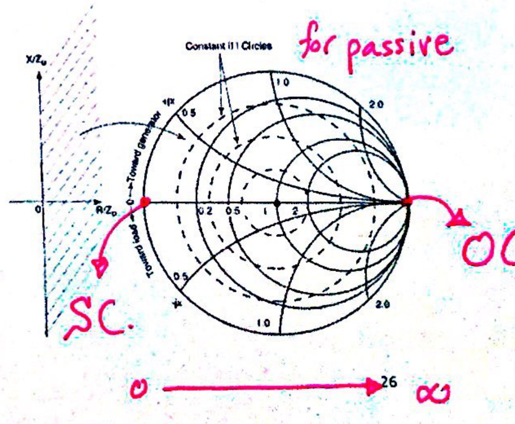
Compressed Smith Chart:

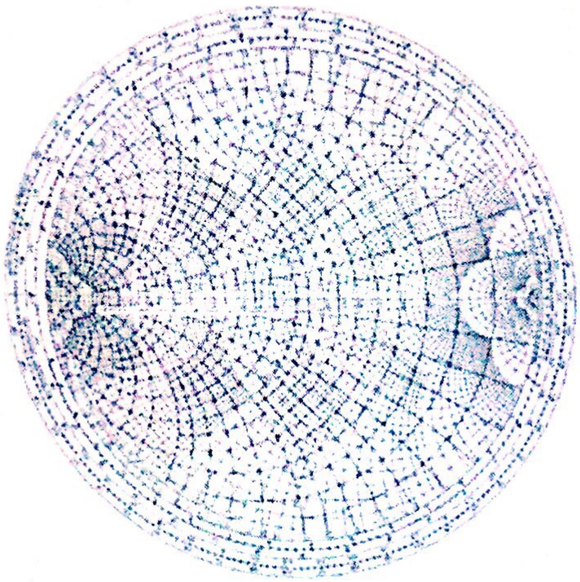
- Plotting two families of circles for all values for (r, x) creates the entire smith chart: "Compressed"
- Applies to active & passive circuits
- Impractical and seldom used



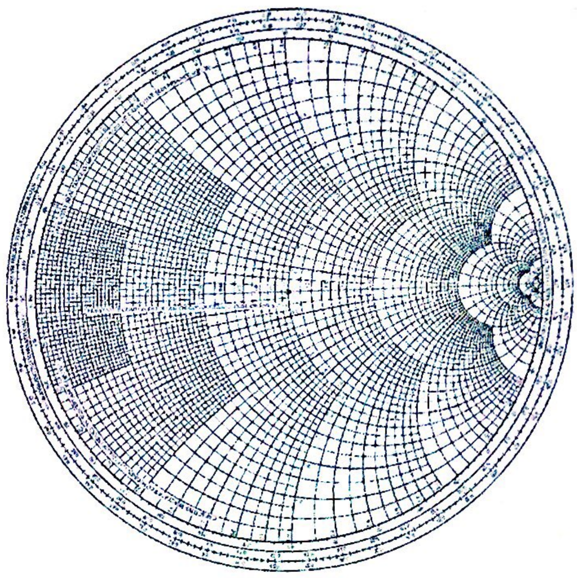
Standard Smith Chart:

- If two families of circles are plotted only for r ≥ 0: "Standard"
- If '$x \ge 0$' ⇒Positive reactance
- If '$x \le 0$' ⇒Negative reactance
- Heavily used for Passive circuits.
- Reflection Coefficient Plane with '$|\Gamma| \le 1$'





IMPEDANCE AND ADMITTANCE



IMPEDANCE

Resistance and Reactance Circles

$$\Gamma = \frac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta} \qquad z_L = \frac{1 + |\Gamma| e^{j\theta}}{1 - |\Gamma| e^{j\theta}}$$

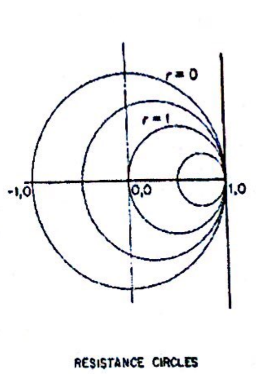
$$r_L + j x_L = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i}$$

$$r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

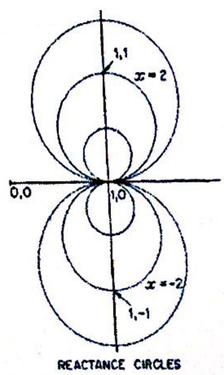
$$x_L = \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

$$\left(\Gamma_r - \frac{r_L}{1 + r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1 + r_L}\right)^2,$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2,$$



RESISTANCE CIRCLES



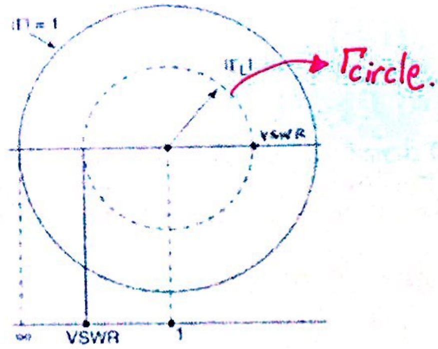
REACTANCE CIRCLES

- For example, the $r_L = 1$ circle has its center at $\Gamma_r = 0.5$, $\Gamma_i = 0$, and has a radius of 0.5 and so it passes through the center of the Smith chart.
- All of the resistance circles have centers on the horizontal $\Gamma_i = 0$ axis and pass through the $\Gamma = 1$ point on the right-hand side of the chart.
- The centers of all of the reactance circles lie on the vertical $\Gamma_r = 1$ line (off the chart), and these circles also pass through the $\Gamma = 1$ point. The resistance and reactance circles are orthogonal.
- The Smith chart can also be used to graphically solve the transmission line impedance equation since this can be written in terms of the generalized reflection coefficient as;

$$Z_{in} = Z_0 \frac{1 + \Gamma e^{-2j\beta\ell}}{1 - \Gamma e^{-2j\beta\ell}}$$

Determine *VSWR* from known Z_L:

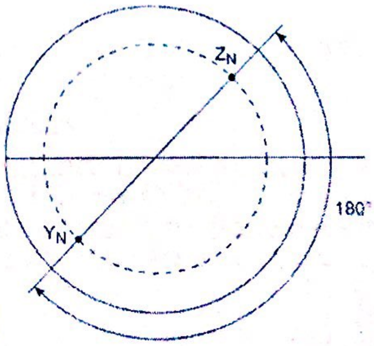
1. Plot the normalized impedance $(Z_L)_N$
2. Draw constant *VSWR* circle through $(Z_L)_N$
   - From the intersection of circle and left-hand horizontal axis, drop a line on the bottom scale to read *VSWR* value
   - Or use intersection of the circle & θ=0 axis



Determine Y_N from known Z_N: *(vice versa)*

1. Plot the normalized impedance $[(Z)_N = Z/Z_0)]$ on the standard Smith chart.
2. Draw constant *VSWR* circle through $(Z)_N$
3. Draw a line from $(Z)_N$ via the center of the of constant *VSWR* circle
4. $(Y)_N$ is the interaction of the line and circle

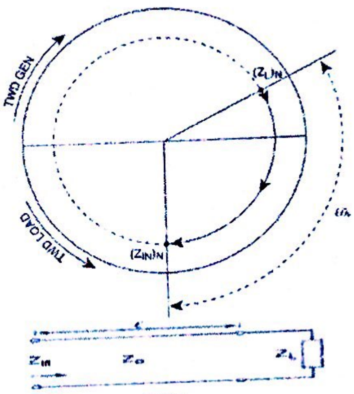
* Prove this using relation between $(Z)_N$ & $|\Gamma_N|$



every closed cycle $= 0.5\lambda$.

Determine Z_{IN} from known Z_L: *(or vice versa)*

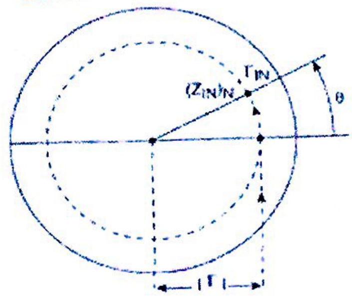
1. Plot the normalized load impedance $[(Z_L)_N = Z_L/Z_0)]$ on the standard Smith chart.
2. Draw constant *VSWR* circle through $(Z_L)_N$
3. From $(Z_L)_N$, move to a distance "l/λ" 'toward generator' on constant *VSWR* circle
4. Read the normalized input impedance value $[(Z_{IN})_N = Z_{IN}/Z_0)]$ from the smith chart.



Determine Z_{IN} from known Γ: *(for $|\Gamma_{IN}| \le 1$)*

1. For any point of TL. plot $\Gamma_{IN} = |\Gamma_{IN}|\, e^{j\theta}$,
   - Use the bottom scale to plot $|\Gamma_{IN}|$ value
   - Use circular scale to pot the angle 'θ'.
2. Read the normalized input impedance value $(Z_{IN})_N$ from the smith chart.

* Conversely find 'Γ' from known 'Z_{IN}'



Examples

(1) A load impedance of 40 + j70 Ω terminates a 100 Ω transmission line that is 0.3λ long. Find the reflection coefficient at the load, the reflection coefficient at the input to the line, the input impedance, the standing wave ratio on the line, and the return loss.

$z_L = 0.4+j0.7$, $|\Gamma| = 0.59$, $\theta_{in} = 104°$, SWR = 3.87, RL = 4.6 dB, $Z_{in} = 36.5 - j61.1\ \Omega$, $\theta_{in} = 248°$

→ found depending on the scale of the used smith chart.

(2) A load of $Z_L = 100 + j50\ \Omega$ terminates a 50 Ω line. What are the load admittance and input admittance if the line is 0.15λ long?

$z_L = 2 + j1$, $y_L = 0.4 - j0.2$, $Y_L = 0.008 - j0.004$ S

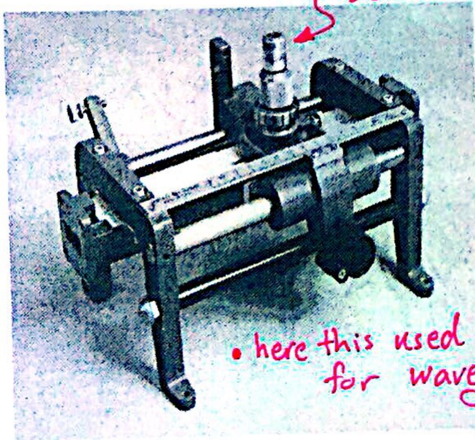
Then, on the WTG scale, the load admittance is seen to have a reference position of 0.214λ. Moving 0.15λ past this point brings us to 0.364λ. A radial line at this point on the WTG scale intersects the SWR circle at an admittance of y = 0.61 + j0.66. The actual input admittance is then Y = 0.0122 + j0.0132 S.

* distance between two minimum values $= \lambda/2$.

The Slotted Line

- A slotted line is a transmission line configuration (usually a waveguide or coaxial line) that allows the sampling of the electric field amplitude of a standing wave on a terminated line.
- With this device the SWR and the distance of the first voltage minimum from the load can be measured, and from these data the load impedance can be determined.
- Note that because the load impedance is, in general, a complex number (with two degrees of freedom), two distinct quantities must be measured with the slotted line to uniquely determine this impedance.

Detector.



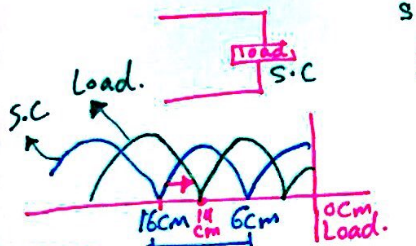
• here this used for waveguide.

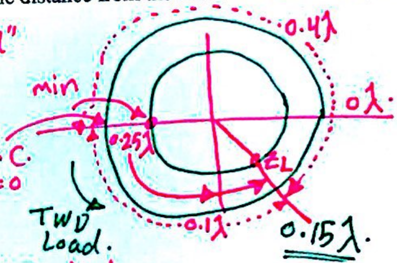
$$|\Gamma| = \frac{\text{SWR} - 1}{\text{SWR} + 1} \qquad \theta = \pi + 2\beta\ell_{\min}$$

$\ell_{\min}$ is the distance from the load to the first voltage minimum

S.C → load "TWD load"

$\frac{2\,cm}{20\,cm} = 0.1\lambda$





if we move from 6cm → 14cm TWD Gen.

$\frac{8\,cm}{20\,cm} = 0.4\lambda$

$\lambda = \frac{c}{f}$; so λ is constant. (c constant, f constant)

$\lambda = 20\,cm$; $\lambda/2$

IMPEDANCE MEASUREMENT WITH A SLOTTED LINE

The following two-step procedure has been carried out with a 50 Ω coaxial slotted line to determine an unknown load impedance:

- 1. A short circuit is placed at the load plane, resulting in a standing wave on the line with infinite SWR and sharply defined voltage minima. On the arbitrarily positioned scale on the slotted line, voltage minima are recorded at z = 0.2 cm, 2.2 cm, 4.2 cm.
- 2. The short circuit is removed and replaced with the unknown load. The standing wave ratio is measured as SWR = 1.5, and voltage minima, which are not as sharply defined as those in step 1, are recorded at z = 0.72 cm, 2.72 cm, 4.72 cm.
- Find the load impedance.

come from
2.2 − 0.72
= 1.48

or 4.2 − 2.72
= 1.48

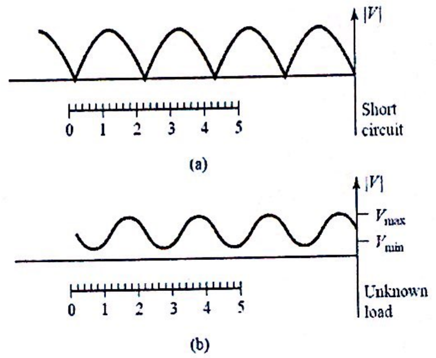
$$|\Gamma| = \frac{1.5-1}{1.5+1} = 0.2,$$

value @ min.

$$\theta = \pi + \frac{4\pi}{4.0}(1.48) = 86.4°,$$

$$\Gamma = 0.2e^{j86.4°} = 0.0126 + j0.1996$$

$$Z_L = 50\left(\frac{1+\Gamma}{1-\Gamma}\right) = 47.3 + j19.7\Omega$$



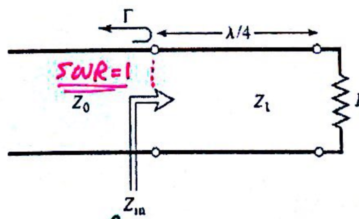
THE QUARTER-WAVE TRANSFORMER

- Which is the geometric mean of the load and source impedances. Then there will be no standing waves on the feedline (SWR = 1), although there will be standing waves on the λ/4 matching section.
- The length of the matching section is λ/4 or an odd multiple of λ/4, long, so that a perfect match may be achieved at one frequency, but impedance mismatch will occur at other frequencies.

$$Z_{in} = Z_1\frac{R_L + jZ_1\tan\beta\ell}{Z_1 + jR_L\tan\beta\ell}$$

$$Z_{in} = \frac{Z_1^2}{R_L}$$

$$Z_1 = \sqrt{Z_0 R_L}$$



need $Z_{in} = Z_o$

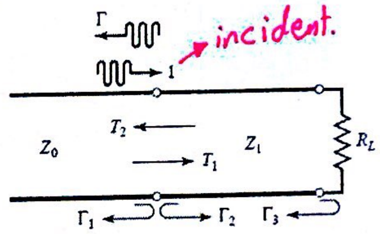
Disadvantage: only used for Resistive Load.

The Multiple-Reflection Viewpoint

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0} \qquad T_1 = \frac{2Z_1}{Z_1 + Z_0}$$

$$\Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1 \qquad T_2 = \frac{2Z_0}{Z_1 + Z_0}$$

$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1}$$



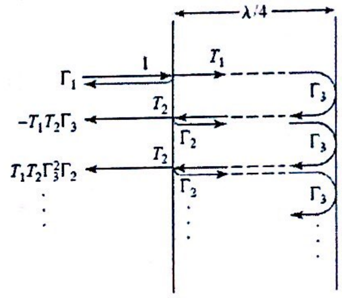
$$\Gamma = \Gamma_1 - T_1T_2\Gamma_3 + T_1T_2\Gamma_2\Gamma_3^2 - T_1T_2\Gamma_2^2\Gamma_3^3 + \cdots$$

$$= \Gamma_1 - T_1T_2\Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2\Gamma_3)^n.$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \quad \text{for } |x| < 1$$

$$\Gamma = \Gamma_1 - \frac{T_1T_2\Gamma_3}{1 + \Gamma_2\Gamma_3} = \frac{\Gamma_1 + \Gamma_1\Gamma_2\Gamma_3 - T_1T_2\Gamma_3}{1 + \Gamma_2\Gamma_3} = \frac{2(Z_1^2 - Z_0R_L)}{(Z_1 + Z_0)(R_L + Z_1)}$$

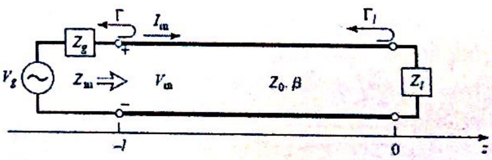
which is seen to vanish if we choose; $Z_1 = \sqrt{Z_0R_L}$ then is Γ zero and the line is matched.



Generator and Load Mismatches

- Because both the generator and load are mismatched, multiple reflections can occur on the line, as in the problem of the quarter-wave transformer. The present circuit could thus be analyzed using an infinite series to represent the multiple bounces.
- The input impedance looking into the terminated transmission line from the generator end is;

$$Z_{in} = Z_0\frac{1 + \Gamma_\ell e^{-2j\beta\ell}}{1 - \Gamma_\ell e^{-2j\beta\ell}} = Z_0\frac{Z_\ell + jZ_0\tan\beta\ell}{Z_0 + jZ_\ell\tan\beta\ell} \qquad \Gamma_\ell = \frac{Z_\ell - Z_0}{Z_\ell + Z_0} \qquad \Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0}$$



$$V(-\ell) = V_g\frac{Z_{in}}{Z_{in} + Z_g} = V_o^+\left(e^{j\beta\ell} + \Gamma_\ell e^{-j\beta\ell}\right)$$

$$V_o^+ = V_g\frac{Z_{in}}{Z_{in} + Z_g}\frac{1}{\left(e^{j\beta\ell} + \Gamma_\ell e^{-j\beta\ell}\right)} \qquad V_o^+ = V_g\frac{Z_0}{Z_0 + Z_g}\frac{e^{-j\beta\ell}}{\left(1 - \Gamma_\ell\Gamma_g e^{-2j\beta\ell}\right)}$$

write these laws on the formula sheet.

$$P = \frac{1}{2}\text{Re}\{V_{in}I_{in}^*\} = \frac{1}{2}|V_{in}|^2\text{Re}\left\{\frac{1}{Z_{in}}\right\} = \frac{1}{2}|V_g|^2\left|\frac{Z_{in}}{Z_{in}+Z_g}\right|^2\text{Re}\left\{\frac{1}{Z_{in}}\right\}$$

- **Load Matched to Line ($Z_L = Z_0$):** $P = \frac{1}{2}|V_g|^2\frac{Z_0}{(Z_0+R_g)^2+X_g^2}$

- **Generator Matched to Loaded Line:** $P = \frac{1}{2}|V_g|^2\frac{R_g}{4(R_g^2+X_g^2)}$

- **Conjugate Matching:** $Z_{in} = Z_g^*$ $\quad P = \frac{1}{2}|V_g|^2\frac{1}{4R_g}$ $\Rightarrow$ Max. Power Transfer.

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* lossy Line Can't be solved By Smith Chart.

LOSSY TRANSMISSION LINES (1)

- **The Low-Loss Line: (G ≈ 0), (R << ωL)**

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}, \qquad \gamma = \sqrt{(j\omega L)(j\omega C)\left(1+\frac{R}{j\omega L}\right)\left(1+\frac{G}{j\omega C}\right)}$$

$$= j\omega\sqrt{LC}\sqrt{1-j\left(\frac{R}{\omega L}+\frac{G}{\omega C}\right)-\frac{RG}{\omega^2 LC}}.$$

$$\alpha \simeq \frac{1}{2}\left(R\sqrt{\frac{C}{L}}+G\sqrt{\frac{L}{C}}\right) = \frac{1}{2}\left(\frac{R}{Z_0}+GZ_0\right) \qquad Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \simeq \sqrt{\frac{L}{C}}.$$

$$\beta \simeq \omega\sqrt{LC}.$$

$$\gamma = \{R\sqrt{(C/L)}\}/2 + j\omega\sqrt{(LC)} \qquad Z_0 = \sqrt{(L/C)} - \{jR\sqrt{(1/LC)}\}/(2\omega)$$

$\gamma = \alpha + j\beta$

@ High freq.:

$\Rightarrow Z_0 \simeq \sqrt{\frac{L}{C}}$

These equations are known as the **high-frequency, low-loss approximations** for transmission lines, and they are important because they show that the propagation constant and characteristic impedance for a low-loss line can be closely approximated by considering the line as lossless.

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LOSSY TRANSMISSION LINES (2)

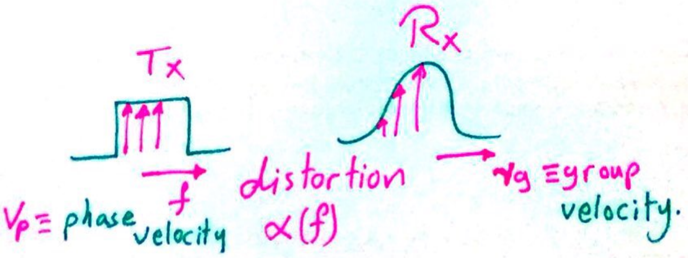
- **The Distortionless Line** $\frac{R}{L} = \frac{G}{C}$
- A special case, of a lossy line that has a linear phase factor as a function of frequency. Such a line is called a **distortionless line.**
- The theory for the distortionless line was first developed by Oliver Heaviside (1850–1925), who solved many problems in transmission line theory and reworked Maxwell's original theory of electromagnetism into the modern version that we are familiar with today.

$$\begin{aligned}\gamma &= j\omega\sqrt{LC}\sqrt{1 - 2j\frac{R}{\omega L} - \frac{R^2}{\omega^2 L^2}} \\ &= j\omega\sqrt{LC}\left(1 - j\frac{R}{\omega L}\right) \\ &= R\sqrt{\frac{C}{L}} + j\omega\sqrt{LC} = \alpha + j\beta,\end{aligned}$$

$Z_o = \sqrt{\frac{L}{C}}$ for the Distortionless line.

LOSSY TRANSMISSION LINES (3)

- **The Terminated Lossy Line**
- β is generally not exactly a linear function of frequency. then the phase velocity $v_p = \omega/\beta$ will vary with frequency.
- The implication of this is that the various frequency components of a wideband signal will travel with different phase velocities and so arrive at the receiver end of the transmission line at slightly different times.
- This will lead to ***dispersion***, a distortion of the signal, and is generally an undesirable effect. ↳ since the signal won't move in one speed.
- The departure of β from a linear function may be quite small, but the effect can be significant if the line is very long. This effect leads to the concept of **group velocity**.



$V_p = \frac{\omega}{\beta}$

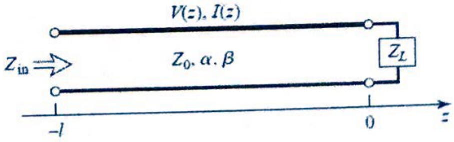
$V_g = \frac{\partial \omega}{\partial \beta}$

- The voltage and current wave on a lossy line are given;

$$V(z) = V_o^+\left(e^{-\gamma z} + \Gamma e^{\gamma z}\right) \qquad I(z) = \frac{V_o^+}{Z_0}\left(e^{-\gamma z} - \Gamma e^{\gamma z}\right)$$

- The reflection coefficient at a distance ℓ from the load is:

$$\Gamma(\ell) = \Gamma e^{-2j\beta\ell} e^{-2\alpha\ell} = \Gamma e^{-2\gamma\ell}$$



- The input impedance Zin at a distance ℓ from the load is then;

$$Z_{in} = \frac{V(-\ell)}{I(-\ell)} = Z_0 \frac{Z_L + Z_0 \tanh \gamma\ell}{Z_0 + Z_L \tanh \gamma\ell}$$

- The power delivered to the input of the terminated line at $z = -\ell$ as;

$$P_{in} = \frac{1}{2}\text{Re}\{V(-\ell)I^*(-\ell)\} = \frac{|V_o^+|^2}{2Z_0}\left(e^{2\alpha\ell} - |\Gamma|^2 e^{-2\alpha\ell}\right) = \frac{|V_o^+|^2}{2Z_0}\left(1 - |\Gamma(\ell)|^2\right)e^{2\alpha\ell}$$

- The power actually delivered to the load is:

$$P_L = \frac{1}{2}\text{Re}\{V(0)I^*(0)\} = \frac{|V_o^+|^2}{2Z_0}\left(1 - |\Gamma|^2\right)$$

- The difference in these powers corresponds to the power lost in the line:

$$P_{loss} = P_{in} - P_L = \frac{|V_o^+|^2}{2Z_0}\left[\left(e^{2\alpha\ell} - 1\right) + |\Gamma|^2\left(1 - e^{-2\alpha\ell}\right)\right]$$

write it in formula sheet.

- The first term accounts for the power loss of the incident wave, while the second term accounts for the power loss of the reflected wave; note that both terms increase as α increases.

the change between Lossless & Low-Loss.

The Perturbation Method for Calculating Attenuation

- A useful and standard technique for finding the attenuation constant of a low-loss line which avoids the use of the transmission line parameters L, C, R, and G and instead relies on the fields of the lossless line, with the assumption that the fields of the lossy line are not greatly different from the fields of the lossless line; hence it is called **perturbation method**.
- The power flow along a lossy transmission line, in the absence of reflections, is of the form;

$$P(z) = P_o e^{-2\alpha z}$$

- where P_o is the power at the $z = 0$ plane and α is the attenuation constant we wish to determine. Now define the power loss per unit length along the line as;

$$P_\ell = -\frac{\partial P}{\partial z} = 2\alpha P_o e^{-2\alpha z} = 2\alpha P(z)$$

"+ve answer"

$\alpha = \alpha_c + \alpha_d$ — conductor, dielectric.

- where the negative sign on the derivative was introduced so that P_L would be a positive quantity. From this, the attenuation constant can be determined as;

$$\alpha = \frac{P_\ell(z)}{2P(z)} = \frac{P_\ell(z=0)}{2P_o}$$

- This equation states that α can be determined from P_o, the power on the line, and P_L, the power loss per unit length of line. It is important to realize that P_L can be computed from the fields of the lossless line and can account for both conductor loss and dielectric loss.

EXAMPLE

- Use the perturbation method to find the attenuation constant of a coaxial line having a lossy dielectric and lossy conductors.

$$\bar{E} = \frac{V_o \hat{\rho}}{\rho \ln b/a} e^{-j\beta z} \qquad \bar{H} = \frac{V_o \hat{\phi}}{2\pi \rho Z_0} e^{-j\beta z} \qquad \text{where } Z_0 = (\eta/2\pi) \ln b/a$$

Rs is the real part of η.

$$P_o = \frac{1}{2}\mathrm{Re}\int_S \bar{E} \times \bar{H}^* \cdot d\bar{s} = \frac{|V_o|^2}{2Z_0}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi}\frac{\rho d\rho d\phi}{2\pi\rho^2 \ln b/a} = \frac{|V_o|^2}{2Z_0}$$

$$P_{\ell c} = \frac{R_s}{2}\int_S |\bar{H}_t|^2 ds = \frac{R_s}{2}\int_{z=0}^{1}\left\{\int_{\phi=0}^{2\pi}|H_\phi(\rho=a)|^2 a\, d\phi + \int_{\phi=0}^{2\pi}|H_\phi(\rho=b)|^2 b\, d\phi\right\} dz = \frac{R_s |V_o|^2}{4\pi Z_0^2}\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$P_{\ell d} = \frac{\overset{\sigma_d}{\omega\epsilon''}}{2}\int_V |\bar{E}|^2 dV = \frac{\omega\epsilon''}{2}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi}\int_{z=0}^{1}|E_\rho|^2 \rho\, d\rho\, d\phi\, dz = \frac{\pi\omega\epsilon''}{\ln b/a}|V_o|^2$$

$$\alpha = \frac{P_{\ell c}+P_{\ell d}}{2P_o} = \frac{R_s}{4\pi Z_0}\left(\frac{1}{a}+\frac{1}{b}\right) + \frac{\pi\omega\epsilon'' Z_0}{\ln b/a} = \frac{R_s}{2\eta \ln b/a}\left(\frac{1}{a}+\frac{1}{b}\right) + \frac{\omega\epsilon''\eta}{2}$$

$\eta = \sqrt{\mu/\epsilon'}$

$\epsilon'' = \frac{\sigma_d}{\omega}$

so $\boxed{\omega\epsilon'' = \sigma_d}$

$P_{\ell c}$ ≡ Power Loss in the Conductor.

$P_{\ell d}$ ≡ Power loss in the Dielectric.

it is the closed form integral for Coaxial Cable.

Not included.

The Wheeler Incremental Inductance Rule

- Another useful technique for the practical evaluation of attenuation due to conductor loss for TEM or quasi-TEM lines is the Wheeler incremental inductance rule.
- This method is based on the similarity of the equations for the inductance per unit length and resistance per unit length of a transmission line, respectively.
- In other words, the conductor loss of a line is due to current flow inside the conductor, which is directly related to the tangential magnetic field at the surface of the conductor and thus to the inductance of the line.
- The power loss into a cross section S of a good (but not perfect) conductor is

$$P_\ell = \frac{R_s}{2}\int_S |\bar{J}_s|^2 ds = \frac{R_s}{2}\int_S |\bar{H}_t|^2 ds \ \mathrm{W/m^2}$$

$$L = \frac{\mu}{|I_o|^2}\int_S \bar{H}\cdot\bar{H}^* ds \ \mathrm{H/m}.$$

$$R = \frac{R_s}{|I_o|^2}\int_{C_1+C_2} \bar{H}\cdot\bar{H}^* dl \ \Omega/\mathrm{m}.$$

X

- The power loss per unit length of a uniform transmission line is

$$P_\ell = \frac{R_s}{2}\int_C |\bar{H}_t|^2 d\ell \text{ W/m}$$

- The line integral is over the cross-sectional contours of both conductors. The inductance per unit length of the line; which is computed assuming the conductors are lossless is;

$$L = \frac{\mu}{|I|^2}\int_S |\bar{H}|^2 ds$$

- When the conductors have a small loss, the **H** field in the conductor is no longer zero, and this field contributes a small additional "incremental" inductance, ΔL. The fields inside the conductor decay exponentially, so that the integration into the conductor dimension can be evaluated as;

$$\Delta L = \frac{\mu_0\delta_s}{2|I|^2}\int_C |\bar{H}_t|^2 d\ell \qquad \int_0^\infty e^{-2z/\delta_s}dz = \delta_s/2 \qquad \delta_s = \sqrt{2/\omega\mu\sigma}$$

$$P_\ell = \frac{R_s|I|^2\Delta L}{\mu_0\delta_s} = \frac{|I|^2\Delta L}{\sigma\mu_0\delta_s^2} = \frac{|I|^2\omega\Delta L}{2}\text{W/m}$$

$$R_s = \sqrt{\omega\mu_0/2\sigma} = 1/\sigma\delta_s$$

$$\alpha_c = \frac{P_\ell}{2P_o} = \frac{\omega\Delta L}{2Z_0} \qquad \alpha_c = \frac{\beta\Delta Z_0}{2Z_0}$$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{L}{\sqrt{LC}} = Lv_p$$

where _Z_0 is the change in characteristic impedance when all conductor walls recede by an amount $\delta_s/2$. Yet another form of the incremental inductance rule can be obtained by using the first two terms of a Taylor series expansion for Z_0. Thus,

$$Z_0\left(\frac{\delta_s}{2}\right) \simeq Z_0 + \frac{\delta_s}{2}\frac{dZ_0}{d\ell}$$

$$\Delta Z_0 = Z_0\left(\frac{\delta_s}{2}\right) - Z_0 = \frac{\delta_s}{2}\frac{dZ_0}{d\ell}$$

$$\alpha_c = \frac{\beta\delta_s}{4Z_0}\frac{dZ_0}{d\ell} = \frac{R_s}{2Z_0\eta}\frac{dZ_0}{d\ell}$$

EXAMPLE

- Calculate the attenuation due to conductor loss of a coaxial line using the Wheeler incremental inductance rule.

The characteristic impedance of the coaxial line is;

$$Z_0 = \frac{\eta}{2\pi}\ln\frac{b}{a}$$

From the incremental inductance rule, the attenuation due to conductor loss is:

$$\alpha_c = \frac{R_s}{2Z_0\eta}\frac{dZ_0}{d\ell} = \frac{R_s}{4\pi Z_0}\left(\frac{d\ln b/a}{db} - \frac{d\ln b/a}{da}\right) = \frac{R_s}{4\pi Z_0}\left(\frac{1}{b}+\frac{1}{a}\right)$$

which is seen to be in agreement with the result of Example 2.7. The negative sign on the second differentiation in this equation is because the derivative for the inner conductor is in the −ρ direction (receding wall).

$$\alpha_c' = \alpha_c\left[1 + \frac{2}{\pi}\tan^{-1}1.4\left(\frac{\Delta}{\delta_s}\right)^2\right]$$

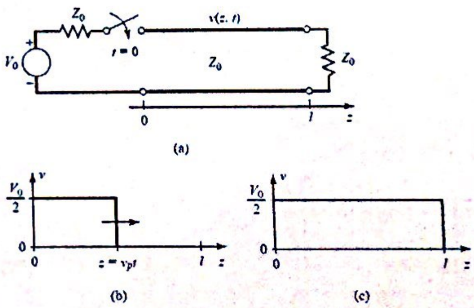
where α_c is the attenuation due to perfectly smooth conductors, α_c is the attenuation corrected for surface roughness, Δ is the rms surface roughness, and δ_s is the skin depth of the conductors.

TRANSIENTS ON TRANSMISSION LINES

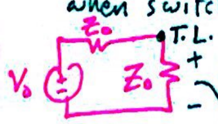
- So far we have concentrated on the behavior of transmission lines at a single frequency, and in many cases of practical interest this viewpoint is entirely satisfactory.
- In some situations, however, where short pulses or very wideband signals are propagating on a transmission line, it is useful to consider wave propagation from a transient, or time domain, point of view.
- We want to determine the voltage response on the transmission line as a function of time and position.
- **Reflection of Pulses from a Terminated Transmission Line**
- **Reflection of Pulses from a Short Circuit Terminated Transmission Line**
- **Reflection of Pulses from an Open Circuit Terminated Transmission Line**

Reflection of Pulses from a Terminated Transmission Line (Z_G=Z_0=Z_L)

- A DC source is switched on at t = 0
- Assume v(z, t) = 0 for all z, for t < 0
- Because of the finite transit time of the line, $Z_{in} = Z_0$ for $t < 2\ell/v_p$
- The initial voltage on the line is thus $V_0/2$ according to VDR
- The leading edge of the pulse will be at position z on the line at time $t = z/v_p$
- The pulse reaches the load at time $t = \ell/v_p$
- The circuit is now in a steady-state condition, and voltage on the line is constant: $v(z, t) = V_0/2$ for all $t > \ell/v_p$



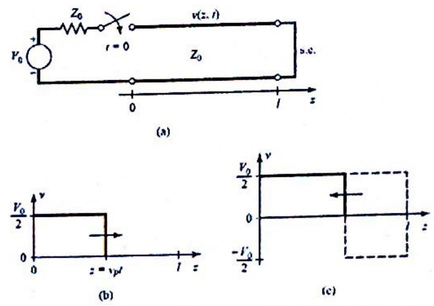
when switch is closed:



$\Rightarrow$ By voltage division: $\dfrac{V_0}{2}$

Reflection of Pulses from a Short Circuit Terminated Transmission Line ($Z_L = 0$)

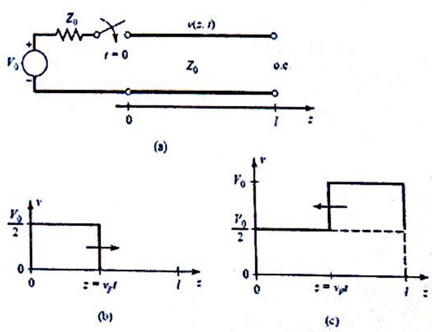
- Initially, the input impedance of the line again appears as Z_0, and the initial incident pulse again has an amplitude of $V_0/2$
- The short-circuit load has $\Gamma = -1$, which has the effect of inverting the reflected pulse as it travels back toward the source
- The superposition of the forward and reverse traveling pulses leads to cancellation, for the period where $\ell/v_p < t < 2\ell/v_p$
- When the return pulse reaches the source, at $t = 2\ell/v_p$, it will not be re-reflected because the source is matched to the line.
- The circuit is then in steady state, with zero voltage everywhere on the line.



The voltage waveform at a fixed point z on the line will consist of a rectangular pulse of amplitude $V_0/2$ existing only over the time period $z/v_p < t < (2\ell - z)/v_p$

Reflection of Pulses from an Open Circuit Terminated Transmission Line ($Z_L = \infty$)

- As in previous cases, the input impedance of the line initially appears as Z_0, and the initial incident pulse has an amplitude of $V_0/2$
- The open-circuit load has $\Gamma = 1$, which reflects the incident waveform with the same polarity toward the source.
- The amplitudes of the forward and reverse pulses add to create a wave with an amplitude of V_0
- At $t = 2\ell/v_p$ the return pulse reaches the source, but it is not re-reflected since the source is matched to the line.
- The circuit is then in steady state, with a constant voltage of V_0 on the line.

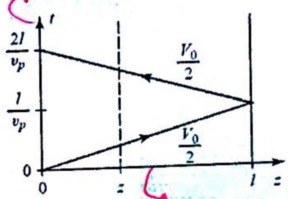


Space-Time Diagram.

Bounce Diagrams for Transient Propagation

- An alternative way of viewing the progress of a pulse propagating in time and position along a transmission line is with a bounce diagram.
- As an example, the bounce diagram for the transient circuit of an open circuit transmission line.
- The horizontal axis represents position on the line, while the vertical axis represents time.
- The ray representing the incident wave begins at t = z = 0 and travels to the right (increasing z) and up (for increasing t).
- This ray is labeled with the amplitude of the incident wave, $V_0/2$. At $t = l/v_p$ the incident wave reaches the open-circuit load and is reflected to produce a wave of amplitude $V_0/2$ traveling back to the source.
- The ray for this reflected wave thus moves to the left and up, until it reaches the source at z = 0 and $t = 2l/v_p$, at which point steady state is reached. The total voltage at any position z and time t can be easily found by drawing a vertical line through the point z and extending up from t = 0 to t.
- The total voltage is found by adding the voltages of each forward or reverse traveling wave component, as represented by the rays that intersect this vertical line.

Time.

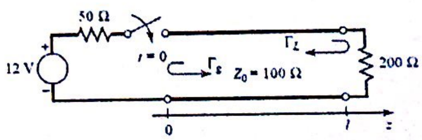


Space.

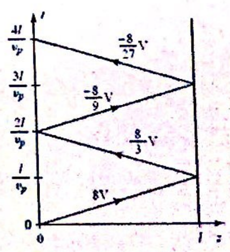
EXAMPLE

- Draw the bounce diagram for the transient circuit shown, including the first three reflections.

$v^+ = 12\frac{100}{50+100} = 8.0\,\text{V}$ $\quad \Gamma_g = \frac{50-100}{50+100} = -1/3$ $\quad \Gamma_L = \frac{200-100}{200+100} = 1/3$



The amplitude of the wave reflected from the load is 8/3 V. When this wave reaches the source, it will be reflected to form a wave of amplitude –8/9 V. The next reflection from the load will have an amplitude of –8/27 V.



$V_\infty = 12\ \frac{200}{200+50} = 9.6$ Volt.

(Final Value)

Topics in Communications.

"Microwaves"

Spring 2017/2018

Dr. Yanal Al-Faouri

By: Mohammad Abu Hashya.

* * *

Microwave Engineering

Chapter 3

Transmission Line and Waveguides

Milestones

- The development of waveguide and other transmission lines for the low-loss transmission of power at high frequencies. Although Heaviside considered the possibility of propagation of electromagnetic waves inside a closed hollow tube in 1893, he rejected the idea because he believed that two conductors were necessary for the transfer of electromagnetic energy.
- In 1897, Lord Rayleigh (John William Strutt) mathematically proved that wave propagation in waveguides was possible for both circular and rectangular cross sections. Rayleigh also noted the infinite set of waveguide modes of the TE and TM type that were possible and the existence of a cutoff frequency, but no experimental verification was made at the time. The waveguide was then essentially forgotten until it was rediscovered independently in 1936 by two researchers.
- After preliminary experiments in 1932, George C. Southworth of the AT&T Company in New York presented a paper on the waveguide in 1936. At the same meeting, W. L. Barrow of MIT presented a paper on the circular waveguide, with experimental confirmation of propagation.

- Early RF and microwave systems relied on waveguides, two-wire lines, and coaxial lines for transmission.
- Waveguides have the advantage of high power-handling capability and low loss **but** are bulky and expensive, especially at low frequencies.
- Two-wire lines are inexpensive **but** lack shielding.
- Coaxial lines are shielded **but** are a difficult medium in which to fabricate complex microwave components.

Planar transmission lines

- Planar transmission lines provide an alternative, in the form of **stripline, microstrip lines, slotlines, coplanar waveguides,** and several other types of related geometries. Such transmission lines are compact, low in cost, and capable of being easily integrated with active circuit devices, such as diodes and transistors, to form microwave integrated circuits.
- The first planar transmission line may have been a flat-strip coaxial line, similar to a stripline, used in a production power divider network in World War II, but planar lines did not see intensive development until the 1950s.

- **Microstrip** lines were developed at ITT (International Telephone & Telegraph) laboratories and were competitors of stripline. The first microstrip lines used a relatively thick dielectric substrate, which accentuated the non-TEM mode behavior and frequency dispersion of the line.
- This characteristic made it less desirable than stripline until the 1960s, when much thinner substrates began to be used. This reduced the frequency dependence of the line, and now **microstrip lines are often the preferred medium for microwave integrated circuits**.

Single vs. more-conductors Transmission Line

- Transmission lines that consist of two or more conductors may support transverse electromagnetic (TEM) waves, characterized by the lack of longitudinal field components. Such lines have a uniquely defined voltage, current, and characteristic impedance.

- Waveguides, often consisting of a single conductor, support transverse electric (TE) and/or transverse magnetic (TM) waves, characterized by the presence of longitudinal magnetic or electric field components where the characteristics impedance not uniquely defined.

Guided Transmission Media

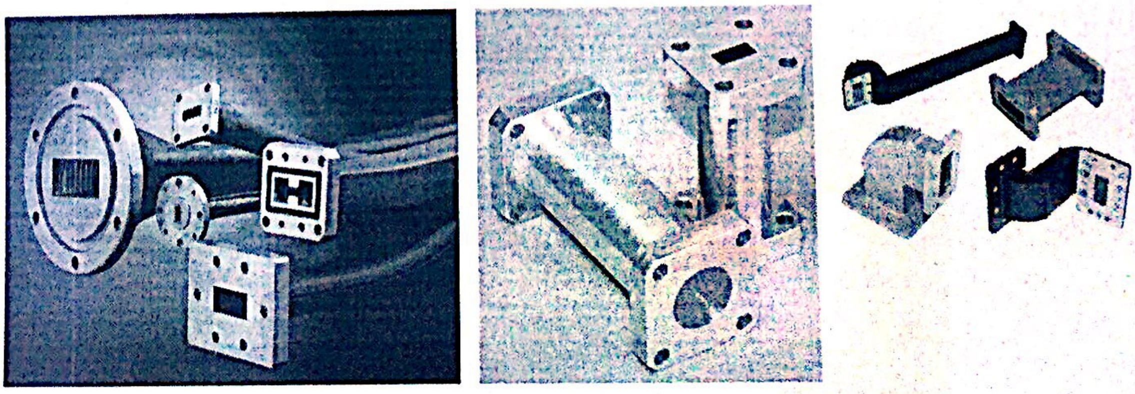
- Coaxial TL: Low radiation, frequency ranges up to 3 GHz, support TEM mode.
- Two-wire TL: Low radiation, frequency up to 300 MHz, support TEM mode.
- Waveguide: For high frequency/power signals, Support TE/TM modes.
- Microstrip: Lossy, quasi-TEM modes, high bandwidth, easy integration.
- Stripline: Less lossy, TEM, high bandwidth, low power capacity.
- Suspended-substrate stripline: easy for device integration.
- Slot line: Very useful for specific applications.
- Coplanar line: Conductor and GND is in the same plane.

Not important in the course.

* We prefer to use microstrip rather than stripline (due to its small size).

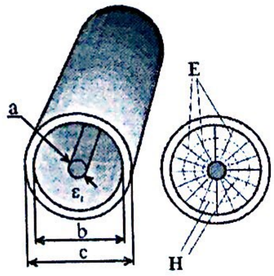
Types of Transmission Lines (1)

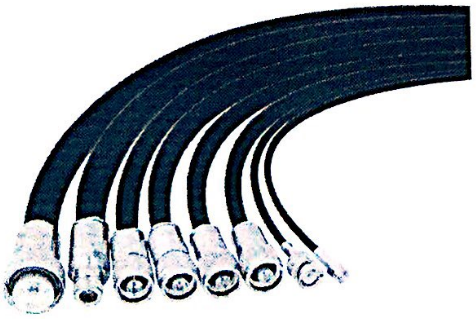
- Waveguides

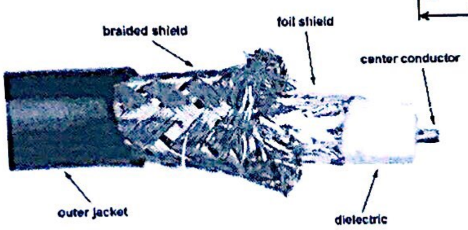


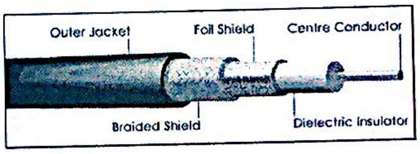
Types of Transmission Lines (2)

- Coaxial Cable



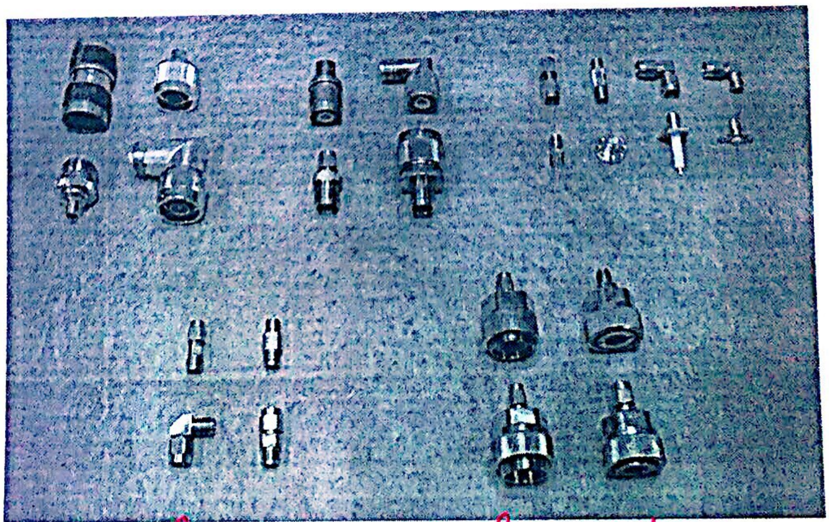






* For Bigger Coaxial Cable ⇛ Higher Impedance.

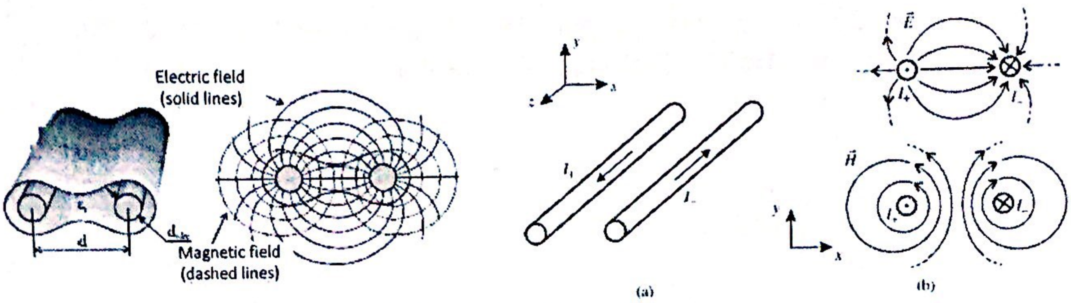
Coaxial Connectors

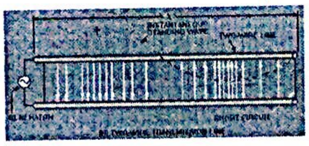


- we don't prefer to use alot of connecters since it cause more losses for the Power.

Types of Transmission Lines (3)

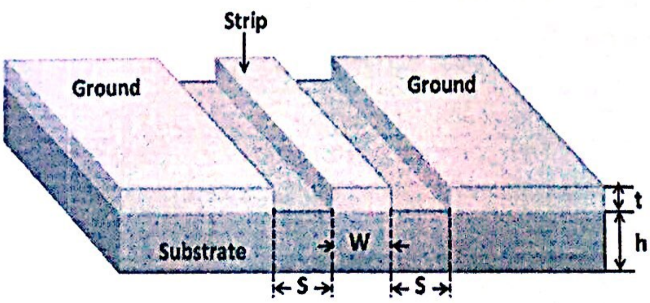
- Two-Wire

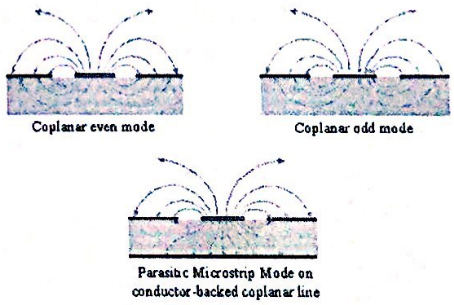




Types of Transmission Lines (4)

- Coplanar line



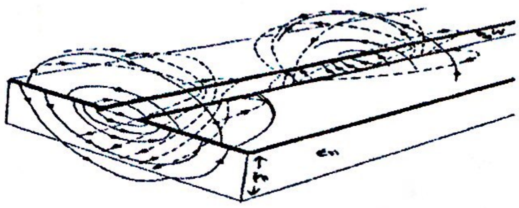


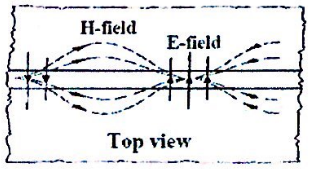
- **Substrate**: it is the dielectric material that carry the conductor.

Types of Transmission Lines (5)

- Slotline

Composed of two conducting half planes separated by a slot on one side of the high-permittivity dielectric substrate. Mode of operation ➔ TE (approximate)



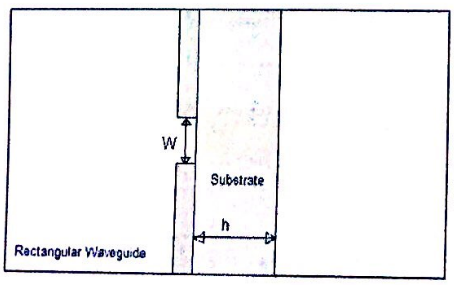


Types of Transmission Lines (6)

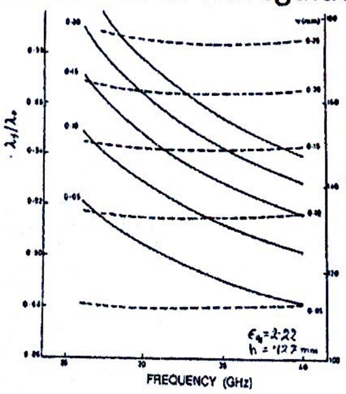
- Finline

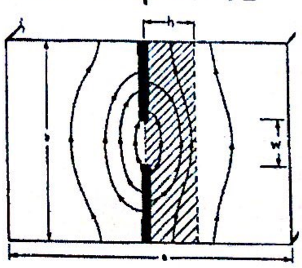
Composed of a slotline on the axis of a rectangular waveguide with substrate parallel to the shorter wall of waveguide. Mode ➔ quasi-TE

$E_z = 0.1 * E_{X,Y}$ or $H_{X,Y}$



Finline

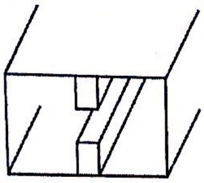




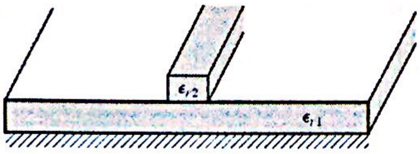
• Any planer line contain a substrate with a conductor.

Other Types of Lines and Guides

Ridge waveguide



Dielectric waveguide



Covered microstrip

leading to increased bandwidth and better (more constant) impedance characteristics

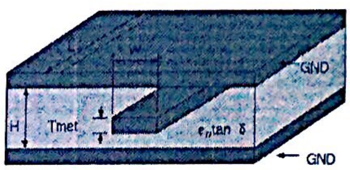
used for impedance matching purposes

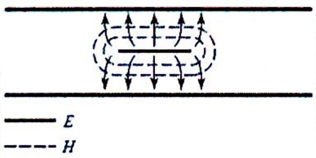
convenient for miniaturization and integration with active devices.
Its small size makes it useful for millimeter wave to optical frequencies, although it can be very lossy at bends or junctions in the ridge line

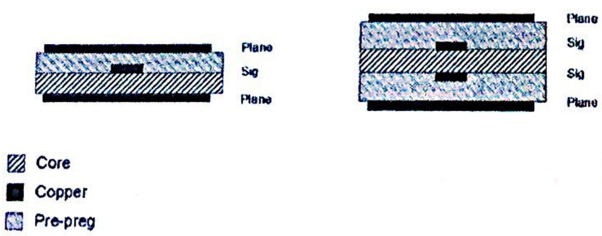
is convenient for miniaturization and integration with active devices. Its small size makes it useful for millimeter wave to optical frequencies, although it can be very lossy at bends or junctions in the ridge line

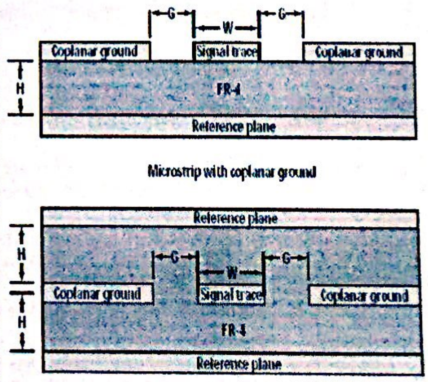
Types of Transmission Lines (7)

- Stripline









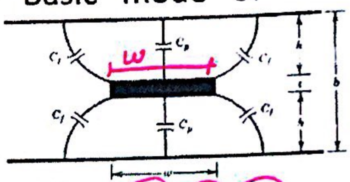
$C_p \equiv$ Parallel Plate Capacitance.

$C_f \equiv$ Fringing Field Capacitance.

Stripline

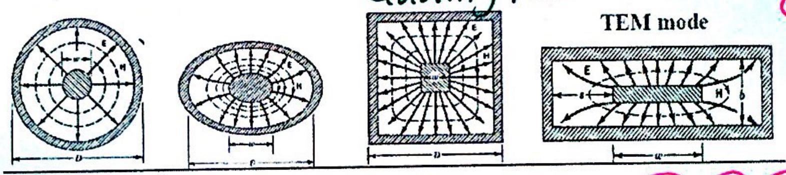
- Consists of a center conductor embedded in a dielectric material that is sandwiched between two conducting plates. Basic mode of operation of stripline is **TEM** mode.
- Advantages: lightweight, miniature, easy-to-fabricate, cost effective, Large band-width, etc.
- Disadvantages: High line-loss, Low power capability; Poor mutual isolation; Low unloaded Q, etc.
- Evolution of Striplines:

Quality Factor.



$C_t = 2C_p + 4C_f$

Total Capacitance.



* photo-Litho-graphic technique.

$$C = \frac{\epsilon A}{d}$$

$d = h$.

- Stripline is a planar type of transmission line that lends itself well to microwave integrated circuitry, miniaturization, and photolithographic fabrication.
- A thin conducting strip of width W is centered between two wide conducting ground planes of separation b, and the region between the ground planes is filled with a dielectric material.
- In practice stripline is usually constructed by etching the center conductor on a grounded dielectric substrate of thickness b/2 and then covering with another grounded substrate.
- Variations of the basic geometry of a stripline include stripline with differing dielectric substrate thicknesses (**asymmetric stripline**) or different dielectric constants (**inhomogeneous stripline**). Air dielectric is sometimes used when it is necessary to minimize loss.
- Because stripline has two conductors and a homogeneous dielectric, it supports a TEM wave, and this is the usual mode of operation. Like parallel plate guide and coaxial line, however, stripline can also support higher order waveguide modes. (TE or TM)
- These can usually be avoided in practice by restricting both the ground plane spacing and the sidewall width to less than $\lambda_d/2$.
- Shorting vias between the ground planes are often used to enforce this condition relative to the sidewall width.
- Shorting vias should also be used to eliminate higher order modes that can be generated when an asymmetry is introduced between the ground planes (e.g., when a surface-mounted coaxial transition is used).

write all of them on the formula sheet.

$K = \sqrt{K_x^2 + K_y^2 + K_z^2}$

loss tangent $\tan\delta = \frac{\sigma}{\omega\epsilon} = \frac{\epsilon''}{\epsilon} = \frac{|J_s|}{|J_{ds}|}$

$K = \frac{\omega\sqrt{\epsilon_r}}{c}$

Formulas for Propagation Constant, Characteristic Impedance, and Attenuation

$v_p = 1/\sqrt{\mu_0\epsilon_0\epsilon_r} = c/\sqrt{\epsilon_r}$ $\quad \beta = \frac{\omega}{v_p} = \omega\sqrt{\mu_0\epsilon_0\epsilon_r} = \sqrt{\epsilon_r}k_0$

$Z_0 = \sqrt{\frac{L}{C}} = \frac{\sqrt{LC}}{C} = \frac{1}{v_p C}$ → used just if C was given.

$$Z_0 = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{b}{W_e + 0.441b}$$

$$\frac{W_e}{b} = \frac{W}{b} - \begin{cases} 0 & \text{for } \frac{W}{b} > 0.35 \\ (0.35 - W/b)^2 & \text{for } \frac{W}{b} < 0.35 \end{cases}$$

$$\frac{W}{b} = \begin{cases} x & \text{for } \sqrt{\epsilon_r}Z_0 < 120\ \Omega \\ 0.85 - \sqrt{0.6 - x} & \text{for } \sqrt{\epsilon_r}Z_0 > 120\ \Omega \end{cases}$$

$$x = \frac{30\pi}{\sqrt{\epsilon_r}Z_0} - 0.441$$

$\alpha_d = \frac{k^2 \tan\delta}{2\beta}$ Np/m (TE or TM waves) $\quad \alpha_d = \frac{k\tan\delta}{2}$ Np/m (TEM waves) → $\beta = K$ for TEM

$$\alpha_c = \begin{cases} \frac{2.7\times10^{-3} R_s \epsilon_r Z_0}{30\pi(b-t)} A & \text{for } \sqrt{\epsilon_r}Z_0 < 120\ \Omega \\ \frac{0.16 R_s}{Z_0 b} B & \text{for } \sqrt{\epsilon_r}Z_0 > 120\ \Omega \end{cases} \quad \text{Np/m}$$

$R_s = \frac{1}{\sigma_c \delta}$

$$A = 1 + \frac{2W}{b-t} + \frac{1}{\pi}\frac{b+t}{b-t}\ln\left(\frac{2b-t}{t}\right).$$

$$B = 1 + \frac{b}{(0.5W + 0.7t)}\left(0.5 + \frac{0.414t}{W} + \frac{1}{2\pi}\ln\frac{4\pi W}{t}\right)$$

$\alpha = \alpha_c + \alpha_d$

W_e = Effictive width.

$W_e > W$

it is found if Z_0 given.

$\alpha(dB) = 20\log e^{\alpha(NB)}$

$R_s = Re\{\eta\} = \frac{1}{\sigma_c\delta}$

$\delta = \frac{1}{\sqrt{\pi f \mu \sigma_c}}$

EXAMPLE: STRIPLINE DESIGN

TEM

σ_c is known.

- Find the width for a 50 Ω copper stripline conductor with b = 0.32 cm and ε_r = 2.20. If the dielectric loss tangent is 0.001 and the operating frequency is 10 GHz, calculate the attenuation in dB/λ. Assume a conductor thickness of t = 0.01 mm.

$\sqrt{\epsilon_r}Z_0 = \sqrt{2.2}(50) = 74.2 < 120$

$x = 30\pi/(\sqrt{\epsilon_r}Z_0) - 0.441 = 0.830$

gives the strip width as W = bx = (0.32) (0.830) = 0.266 cm

At 10 GHz, the wave number is;

$k = \frac{2\pi f\sqrt{\epsilon_r}}{c} = 310.6\ \text{m}^{-1}$ rad/m $\quad \lambda = \frac{c}{\sqrt{\epsilon_r} f} = 2.02$ cm

$\alpha_d = \frac{k\tan\delta}{2} = \frac{(310.6)(0.001)}{2} = 0.155$ Np/m

$\alpha_c = \frac{2.7\times10^{-3} R_s \epsilon_r Z_0 A}{30\pi(b-t)} = 0.122$ Np/m

since A = 4.74

The total attenuation constant is

$\alpha = \alpha_d + \alpha_c = 0.277$ Np/m.

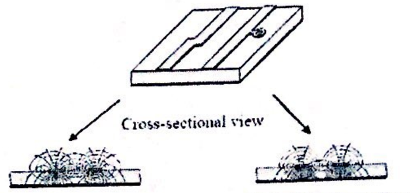
In dB,

α (dB) = 20 log e^{α} = 2.41 dB/m

so in terms of wavelength the attenuation is;

α (dB) = (2.41) (0.0202) = 0.049 dB/λ

Types of Transmission Lines (8)



- Microstrip line

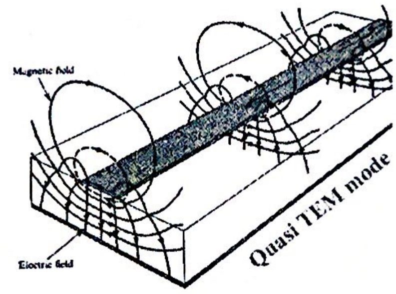
Consist of a thin conducting strip placed above a dielectric material (substrate), which is supported on its bottom by a conducting plate (ground plane). Alumina and Droid are common μ-wave substrates.

- Advantages:

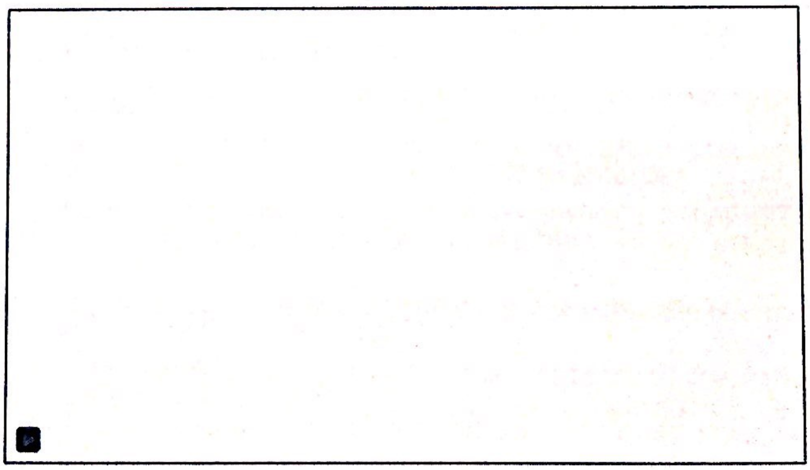
Lightweight, Miniature, easy to fabricate, cost effective, Large band-width, etc.

- Disadvantages:

High line-loss, Low power capability; Poor mutual isolation; Low unloaded Q, etc.



Electric Field on a Microstrip Line



- Microstrip line is one of the most popular types of planar transmission lines primarily because it can be fabricated by photolithographic processes and is easily miniaturized and integrated with both passive and active microwave devices.
- A conductor of width W is printed on a thin, grounded dielectric substrate of thickness d and relative permittivity ε_r
- If the dielectric substrate were not present ($\varepsilon_r = 1$), we would have a two-wire line consisting of a flat strip conductor over a ground plane, embedded in a homogeneous medium (air).
- This would constitute a simple TEM transmission line with phase velocity $v_p = c$ and propagation constant $\beta = k_0$.
- https://www.youtube.com/watch?v=HLW0hPRhvUY

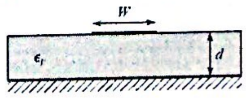
- The presence of the dielectric, particularly the fact that the dielectric does not fill the region above the strip ($y > d$), complicates the behavior and analysis of microstrip line.
- Unlike stripline, where all the fields are contained within a homogeneous dielectric region, microstrip has some (usually most) of its field lines in the dielectric region between the strip conductor and the ground plane and some fraction in the air region above the substrate.
- For this reason, microstrip line cannot support a pure TEM wave since the phase velocity of TEM fields in the dielectric region would be $c/\sqrt{\varepsilon_r}$, while the phase velocity of TEM fields in the air region would be c, so a phase-matching condition at the dielectric–air interface would be impossible to enforce.
- In actuality, the exact fields of a microstrip line constitute a hybrid TM-TE wave and require more advanced analysis techniques than we are prepared to deal with here.
- In most practical applications, however, the dielectric substrate is electrically very thin ($d << \lambda$), and so the fields are quasi-TEM.
- In other words, the fields are essentially the same as those of the static (DC) case. Thus, good approximations for the phase velocity, propagation constant, and characteristic impedance can be obtained from static, or quasi-static, solutions.

Formulas for Effective Dielectric Constant, Characteristic Impedance, and Attenuation

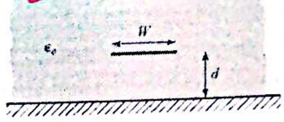
$v_p = \frac{c}{\sqrt{\epsilon_e}}$ $\qquad \beta = k_0\sqrt{\epsilon_e}$ $\quad \to k_0 = \frac{\omega}{c}$

$\boxed{1 \leq \epsilon_e \leq \epsilon_r}$ always.

$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12d/W}}$$



(a)



(b)

given

$$Z_0 = \begin{cases} \dfrac{60}{\sqrt{\epsilon_e}}\ln\left(\dfrac{8d}{W} + \dfrac{W}{4d}\right) & \text{for } W/d \leq 1 \\ \dfrac{120\pi}{\sqrt{\epsilon_e}\,[W/d + 1.393 + 0.667\ln(W/d + 1.444)]} & \text{for } W/d \geq 1 \end{cases}$$

found from Assumption

$$\frac{W}{d} = \begin{cases} \dfrac{8e^A}{e^{2A} - 2} & \text{for } W/d < 2 \quad \text{Narrow Line} \\ \dfrac{2}{\pi}\left[B - 1 - \ln(2B - 1) + \dfrac{\epsilon_r - 1}{2\epsilon_r}\left\{\ln(B - 1) + 0.39 - \dfrac{0.61}{\epsilon_r}\right\}\right] & \text{for } W/d > 2 \quad \text{Wide Line} \end{cases}$$

$$A = \frac{Z_0}{60}\sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1}\left(0.23 + \frac{0.11}{\epsilon_r}\right)$$

$$B = \frac{377\pi}{2Z_0\sqrt{\epsilon_r}}.$$

A & B are found at first.

$$\alpha_d = \frac{k_0\epsilon_r(\epsilon_e - 1)\tan\delta}{2\sqrt{\epsilon_e}(\epsilon_r - 1)} \text{ Np/m}$$

$$\frac{\epsilon_r(\epsilon_e - 1)}{\epsilon_e(\epsilon_r - 1)} \Leftarrow \text{Filling Factor}$$

$$\alpha_c = \frac{R_s}{Z_0 W} \text{ Np/m} \qquad R_s = \frac{1}{\delta_s\sigma} = \sqrt{\frac{\omega\mu}{2\sigma}}$$

start by assuming $\frac{W}{d} < 2$.

$\Rightarrow$ if it is < 2 then Assumption is true.
if > 2 then wrong assumption, so take the other expression.

DC $\qquad Z_c$ same Z_0

@ $t=0 \Rightarrow \frac{w_{eff}(0)}{h} = \frac{w}{h}$ in both cases.

Analysis: Given 'w/h', 't/h' & 'ε', find 'Z_0' & 'ε_{eff}' of the μ-strip:

(At f = 0 Hz)

$\frac{w_{eff}(0)}{h} \leq 1$:

$$Z_c(0) = Z_c(f = 0) = \frac{60}{\sqrt{\varepsilon_{r,eff}(0)}}\ln\left[\frac{8h}{w_{eff}(0)} + \frac{w_{eff}(0)}{4h}\right]$$

$$\varepsilon_{r,eff}(0) = \varepsilon_{r,eff}(f = 0) = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \times \left\{\left[1 + 12\frac{h}{w_{eff}(0)}\right]^{-1/2} + 0.04\left[1 - \frac{w_{eff}(0)}{h}\right]^2\right\}$$

$\frac{w_{eff}(0)}{h} > 1$:

$$Z_c(0) = Z_c(f = 0) = \frac{\dfrac{120\pi}{\sqrt{\varepsilon_{r,eff}(0)}}}{\dfrac{w_{eff}(0)}{h} + 1.393 + 0.667\ln\left[\dfrac{w_{eff}(0)}{h} + 1.444\right]}$$

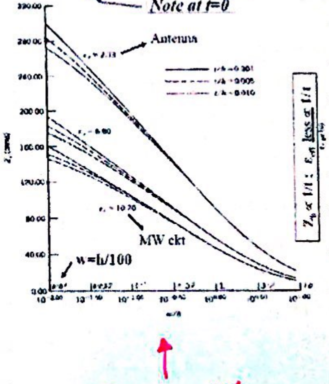
$$\varepsilon_{r,eff}(0) = \varepsilon_{r,eff}(f = 0) = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{w_{eff}(0)}\right]^{-1/2}$$

where, 'w_{eff}' is the effective width due to fringing fields, and 'ε_{eff}' is effective dielectric constant resulted due to two different dielectric.

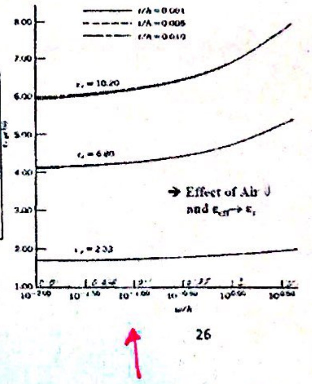
where, $$\frac{w_{eff}(0)}{h} = \frac{w_{eff}(f = 0)}{h} = \boxed{\frac{w}{h}} + \frac{t}{h}\left[1 + \ln\left(\frac{2h}{t}\right)\right] \quad \text{for } \frac{w}{h} \geq \frac{1}{2\pi}$$

$$\frac{w_{eff}(0)}{h} = \frac{w_{eff}(f = 0)}{h} = \boxed{\frac{w}{h}} + \frac{t}{h}\left[1 + \ln\left(\frac{4\pi w}{t}\right)\right] \quad \text{for } \frac{w}{h} < \frac{1}{2\pi} \approx 0.16$$

Note at t=0



Z vs. $\frac{W}{h}$



$\epsilon_{r\,eff}^{(0)}$ vs. $\frac{W}{h}$

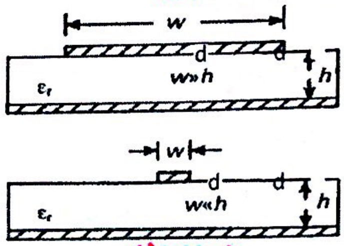
n i p .

- Impedance of microstrip line depends on 'd', 'w' & 'substrate ε_r'
- Narrow line is more depressive (⇑ Z) $\Rightarrow W\downarrow\ Z\uparrow$
- Increasing 'ε_r' leads to more field being concentrated in the substrate. $\varepsilon_r\uparrow \Rightarrow Z\downarrow$

It also binds fringing fields more tightly to the center of the conductor which reduce radiation loss (Z ⇓)

- Increasing 'h' or 'd' leads EM field in substrate to be loosely bound & can cause more radiation. It causes unwanted surface wave (Z ⇑) $d\uparrow \Rightarrow Z\uparrow$
- $\boxed{\text{Synthesis}}$: Given 'Z_0' & 'ε', find 'w/d' ratio of the microstrip line.
- $\boxed{\text{Analysis}}$: Given 'w/d', 't/d' & 'ε', find 'Z_0' & 'ε_e' of the μ-strip.

Wide Line.



Narrow Line

EXAMPLE: MICROSTRIP LINE DESIGN

- Design a microstrip line on a 0.5 mm (h) alumina substrate (ε_r = 9.9, tan δ = 0.001) for a 50 Ω characteristic impedance. Find the length of this line required to produce a phase delay of 270° at 10 GHz, and compute the total loss on this line, assuming copper conductors.

First find W/d for Z_0 = 50 Ω, and initially guess that W/d < 2.
A = 2.142, W/d = 0.9654.

So the condition that W/d < 2 is satisfied; otherwise we would use the expression for W/d > 2. Then the required line width is W = 0.9654d = 0.483 mm.
The effective dielectric constant is ε_e = 6.665. The line length, ℓ, for a 270° phase shift is found as;

convert it to radian.

$$\phi = 270° = \beta\ell = \sqrt{\epsilon_e}k_0\ell,$$

$$k_0 = \frac{2\pi f}{c} = 209.4\ \text{m}^{-1},$$

$$\ell = \frac{270°(\pi/180°)}{\sqrt{\epsilon_e}k_0} = 8.72\ \text{mm}$$

Attenuation due to dielectric loss is α_d = 0.255 Np/m = 0.022 dB/cm.
The surface resistivity for copper at 10 GHz is 0.026 Ω.
The attenuation due to conductor loss is α_c = 0.0108 Np/cm = 0.094 dB/cm.
The total loss on the line is then 0.101 dB.

$1 \le \epsilon_e \le \varepsilon_r$

$\sigma_c = 5.7\times10^7$

* we find f_c :
- if f_c had a value Not in the range 3 to 10 GHZ then use the previous relations.
- But if $[3 \le f_c \le 10]$ GHZ then use the following relations.

Frequency-Dependent Effects and Higher Order Modes

By finding f_c.

You have to see at first is it dispersive or Non dispersive.

- Dispersive media: 'v_p' of the EM wave is a function of frequency.
- microstrip is dispersive at: $f_c \ge 0.3\sqrt{\frac{Z_c(0)}{h}\frac{1}{\sqrt{\varepsilon_r - 1}}} \times 10^9$ (where h is in cm)

  Typically, $f_c \approx 3$ to 10 GHz.

  center Freq.
- Dispersive characteristic of μ-strip TL; find '$Z_0(f)$, $v_p(f)$ & $\varepsilon_e(f)$:

$$Z_c(f) = Z_c(0)\sqrt{\frac{\varepsilon_{r,\text{eff}}(0)}{\varepsilon_{r,\text{eff}}(f)}} \qquad W_{\text{eff}}(f) = W + \frac{W_{\text{eff}}(0) - W}{1 + (f/f_t)^2}$$

$$\varepsilon_{r,\text{eff}}(f) = \varepsilon_r - \left[\frac{\varepsilon_r - \varepsilon_{r,\text{eff}}(0)}{1 + \frac{\varepsilon_{r,\text{eff}}(0)}{\varepsilon_r}\left(\frac{f}{f_t}\right)^2}\right]$$

$\varepsilon_{\text{eff}} = \varepsilon_0 \, \varepsilon_{\text{reff}}$

$$v_p(f) = \frac{1}{\sqrt{\mu\varepsilon_{\text{eff}}(f)}} = \frac{1}{\sqrt{\mu_r\mu_0\varepsilon_0\varepsilon_{r,\text{eff}}(f)}} = \frac{v_0}{\sqrt{\mu_r\varepsilon_{r,\text{eff}}(f)}}$$

$$f_t = \frac{Z_c(0)}{2\mu_0 h}$$

$$\lambda_g(f) = \frac{v_p(f)}{f} = \frac{v_0}{f\sqrt{\mu_r\varepsilon_{r,\text{eff}}(f)}} = \frac{\lambda_0}{\sqrt{\mu_r\varepsilon_{r,\text{eff}}(f)}}$$

Comparison of Common Transmission Lines and Waveguides

Characteristic	Coax	Waveguide	Stripline	Microstrip
Modes: Preferred	TEM	TE_{10}	TEM	Quasi-TEM
Other	TM,TE	TM,TE	TM,TE	Hybrid TM,TE
Dispersion	None	Medium	None	Low
Bandwidth	High	Low	High	High
Loss	Medium	Low	High	High
Power capacity	Medium	High	Low	Low
Physical size	Large	Large	Medium	Small
Ease of fabrication	Medium	Medium	Easy	Easy
Integration with	Hard	Hard	Fair	Easy

why Micro strip is prefered over others?
high BW, small size, Easy to fabricate, and easy to integrate.

WAVE VELOCITIES AND DISPERSION

- The speed of light in a medium is the velocity at which a plane wave would propagate in that medium, while the phase velocity is the speed at which a constant phase point travels.
- For a TEM plane wave, these two velocities are identical, but for other types of guided wave propagation the phase velocity may be greater or less than the speed of light.
- If the phase velocity and attenuation of a line or guide are constants that do not change with frequency, then the phase of a signal that contains more than one frequency component will not be distorted.
- If the phase velocity is different for different frequencies, then the individual frequency components will not maintain their original phase relationships as they propagate down the transmission line or waveguide, and signal distortion will occur.
- Such an effect is called **dispersion** since different phase velocities allow the "faster" waves to lead in phase relative to the "slower" waves, and the original phase relationships will gradually be dispersed as the signal propagates down the line. In such a case, there is no single phase velocity that can be attributed to the signal as a whole.

Group Velocity

- However, if the bandwidth of the signal is relatively small or if the dispersion is not too severe, a group velocity can be defined in a meaningful way. This velocity can be used to describe the speed at which the signal propagates.

- The physical interpretation of group velocity is the velocity at which a narrowband signal propagates.

$$\beta_o = \beta(\omega_o) \qquad \beta'_o = \left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_o} \qquad v_g = \frac{1}{\beta'_o} = \left.\left(\frac{d\beta}{d\omega}\right)^{-1}\right|_{\omega=\omega_o}$$

Topics in Communications.
"Microwaves"

Spring 2017/2018

Dr. Yanal Al-Faouri

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* * *

Microwave Engineering

Chapter 4

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Microwave Network Analysis

- Impedance and Equivalent Voltages and Currents
- Impedance and Admittance Matrices
- The Scattering Matrix
- The Transmission *(ABCD)* Matrix
- Signal Flow Graphs
- Discontinuities and Modal Analysis
- Excitation of Microstrip Lines

Field Analysis vs. Circuit Analysis

- Field analysis gives us much more information about the particular problem under consideration than we really want or need. That is, because the solution to Maxwell's equations for a given problem is **complete**, it gives the electric and magnetic fields at all points in space. However, usually we are only interested in the voltage or current at a set of terminals, the power flow through a device, or some other type of "terminal" quantity, as opposed to a minute description of the fields at all points in space.

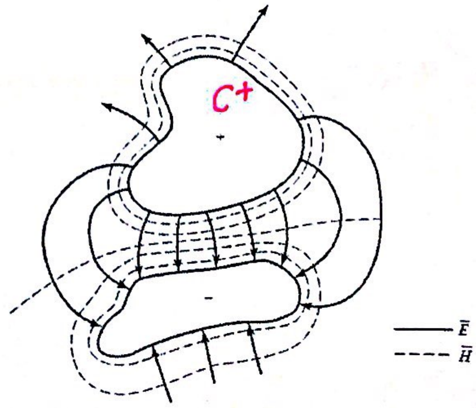
for i/p & o/p.

- Another reason for using circuit or network analysis is that it is then **very easy** to modify the original problem, or combine several elements together and find the response, without having to reanalyze in detail the behavior of each element in combination with its neighbors.

Equivalent Voltages and Currents

At microwave frequencies the measurement of voltage or current is difficult (or impossible), unless a clearly defined terminal pair is available.

Such a terminal pair may be present in the case of TEM-type lines (such as coaxial cable, microstrip line, or stripline), but does not strictly exist for non-TEM lines (such as rectangular, circular, or surface waveguides).



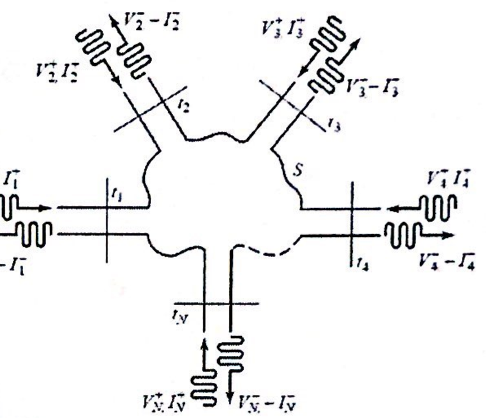
$$V = \int_{+}^{-} \bar{E} \cdot d\bar{\ell} \qquad I = \oint_{C+} \bar{H} \cdot d\bar{\ell} \qquad Z_0 = \frac{V}{I}$$

IMPEDANCE AND ADMITTANCE MATRICES

Impedance Matrix.

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & & & \vdots \\ \vdots & & & \vdots \\ Z_{N1} & \cdots & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} \Longrightarrow [V] = [Z][I]$$

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & & & \vdots \\ \vdots & & & \vdots \\ Y_{N1} & \cdots & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \Longrightarrow [I] = [Y][V]$$



$$[Y] = [Z]^{-1}$$

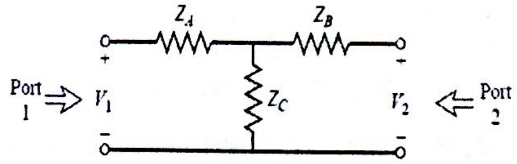
$$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{I_k=0 \text{ for } k \neq j} \qquad Y_{ij} = \left.\frac{I_i}{V_j}\right|_{V_k=0 \text{ for } k \neq j}$$

To make $V_K = 0$ $\Rightarrow$ S/C on the terminal.

To make this $I_K = 0$ $\Rightarrow$ O/C on the terminal.

EXAMPLE: EVALUATION OF IMPEDANCE PARAMETERS:

- Find the Z parameters of the two-port T-network shown



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_A + Z_C$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{V_2}{I_2}\frac{Z_C}{Z_B + Z_C} = Z_C$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_B + Z_C$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$Z_{21} = Z_{12}$, indicating that the circuit is reciprocal.

THE SCATTERING MATRIX

- Analytical technique for multi-port network → end @ 3GHz → start @ 3GHZ.
- Provide a complete characterization of RF/microwave network
- Relates incident and scattered waves at various ports of the network
- Diagonal elements of [S] represents 'Γ_k' and off-diagonal elements represent 'T_k'

reflection coeficient at port 1.

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & & & \vdots \\ S_{N1} & \cdots & & S_{NN} \\ \vdots & & & \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

Transmission Coeficient from 2 ~~to~~ 1.

$\Gamma_K \equiv$ Reflection Coeficient.

$T_K \equiv$ Transmission Coeficient.

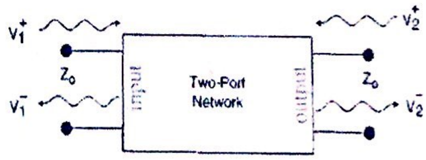
$$[V^-] = [S][V^+]$$

$$S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{V_k^+ = 0 \text{ for } k \neq j}$$

↳ No input from the other ports. " No incident".

S-Parameters of a Two Port Network (1)

- Z_0: Characteristic Impedance of both ports
- V_1^+ & V_2^+ are incident waves at port 1 and 2
- V_1^- & V_2^- are scattered waves from port 1 and 2
- S-parameters are defined to describe the linear relationship between incident and scattered wave



$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+$$
$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+$$

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} \qquad [S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

- scattered wave : include Transmission & Reflection.

S-Parameters of a two port network (2)

$$S_{11} = \frac{V_1^-}{V_1^+}\bigg|_{V_2^+=0} = \Gamma_{IN}$$

- Input reflection coefficient when output port is terminated in a matched load (Z_L=Z_0).

$$S_{21} = \frac{V_2^-}{V_1^+}\bigg|_{V_2^+=0}$$

- Forward transmission coefficient when output port is terminated in a matched load (Z_L=Z_0).

$$S_{12} = \frac{V_1^-}{V_2^+}\bigg|_{V_1^+=0}$$

- Reverse transmission coefficient when input port is terminated in a matched load (Z_G=Z_0).

$$S_{22} = \frac{V_2^-}{V_2^+}\bigg|_{V_1^+=0} = \Gamma_{OUT}$$

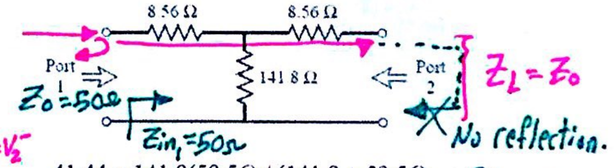
- Output reflection coefficient when input port is terminated in a matched load (Z_G=Z_0).

Example: Evaluation of Scattering Parametres

* Given the characteristic impedance for the two-ports = 50 Ω

$Z_{in}^{(1)}$ = 8.56 + [141.8(8.56 + 50)]/(141.8 + 8.56 + 50) = 50 Ω, so S_{11} = 0
Because of the symmetry of the circuit, S_{22} = 0.

$$S_{11} = \frac{V_1^-}{V_1^+}\bigg|_{V_2^+=0} = \Gamma^{(1)}\big|_{V_2^+=0} = \frac{Z_{in}^{(1)} - Z_0}{Z_{in}^{(1)} + Z_0}\bigg|_{Z_0 \text{ on port 2}}$$



41.44 = 141.8(58.56) / (141.8 + 58.56)

$S_{11} = 0$

$$S_{21} = \frac{V_2^-}{V_1^+}\bigg|_{V_2^+=0}$$

$\cancel{V_2^+}^{\,0} + V_2^- = V_2 = V_2^-$

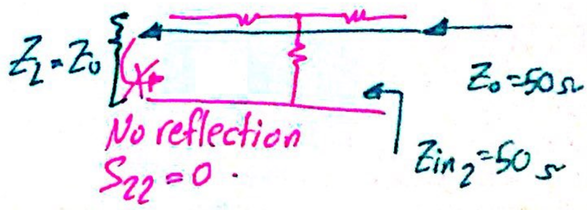
$\to Z_{in} = 8.56 + 41.44$
$= \underline{50\,\Omega}$.

using Voltage Division:

$$V_2^- = V_2 = V_1\left(\frac{41.44}{41.44 + 8.56}\right)\left(\frac{50}{50 + 8.56}\right) = 0.707V_1$$

Thus, $S_{12} = S_{21} = 0.707$.
If the input power is $|V_1^+|^2/2Z_0$, then the output power is $|V_2^-|^2/2Z_0 = |S_{21}V_1^+|^2/2Z_0 = |S_{21}|^2|V_1^+|^2/2Z_0 = |V_1^+|^2/4Z_0$, which is one-half (−3 dB) of the input power

- Note: if the network symmetrical
  Then it is Reciprocal
  But if it was Reciprocal, not necessary
  To be symmetrical.



• remember: $\tau = 1 - \rho$

• To be Reciprocal it must be: upper triangle = lower triangle in the corresponding matrix.

Properties of S-Parameters (1)

- For **reciprocal** Networks: $S_{12} = S_{21}$ or for N-port network; $S_{ij} = S_{ji}$ for $i \neq j$ and $i, j=1,..N$
- For **symmetrical** Networks: $S_{11} = S_{22}$ or for N-port network; $S_{ii} = S_{jj}$ for $i \neq j$ and $i, j= 1,..N$
- **Unity property** of lossless networks:

➢For N-port network: $\sum_{i=1}^{N} S_{ij}S_{ij}^{*} = 1$, for $j=1,\ldots,N$

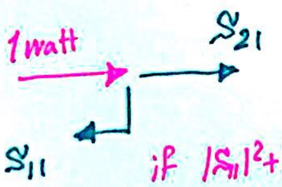
➢For 2-port:

$S_{11} S_{11}^{*}+S_{21} S_{21}^{*}=1 \Rightarrow |S_{11}|^2+|S_{21}|^2=1$ and;

$S_{12} S_{12}^{*}+S_{22} S_{22}^{*}=1 \Rightarrow |S_{12}|^2+|S_{22}|^2=1$

- If the 2-port network is reciprocal: $S_{12} = S_{21} \Rightarrow |S_{11}| = |S_{22}|$

* if one column hadn't applied to the condition ($\neq 1$) you don't have to continue (it is not Lossless Network).



if $|S_{11}|^2+|S_{21}|^2 = 1$ Lossless.

if it is less than 1 then there is some power lost in the Network.

• Note: if the Network is Lossless & Reciprocal Then it is Symmetrical

• 2 conditions to assure the symmetrical.

Properties of S-Parameters (2)

- **Zero property** of lossless networks:

➢For N-port network; $\sum_{k=1}^{N} S_{ki}S_{kj}^{*} = 0$ for $i \neq j$; $i,j=1\ldots N$

* it means that we dealing with Transmission Coeficient.

➢For 2-port: $S_{11} S_{12}^{*}+S_{21} S_{22}^{*}= 0$ and $S_{12} S_{11}^{*}+S_{22} S_{21}^{*}= 0$

- if the network is reciprocal: $S_{12} = S_{21} \Rightarrow S_{11} S_{21}^{*}+S_{21} S_{22}^{*}= 0$

- **Attenuation**: Ratio of rms voltages (currents) at input and output ports.
- **Return loss (dB)**: is defined as the ratio of the incident power to the reflected power at any point on the transmission line, expressed in dB.

RL (dB) = $10 \log_{10} (P_i/P_0) = 10 \log_{10} (|V^+|^2/|V^-|^2) = 10 \log_{10} (1/|\Gamma|^2) =$

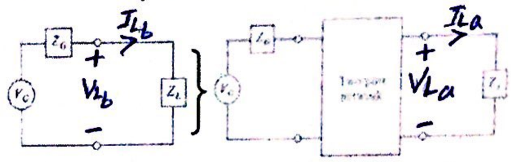
$-20 \log_{10} |\Gamma|$ (+ve answer).

$P_{Lb} \equiv$ Power to the load (before).
$P_{La} \equiv$ Power to the load (after).

Properties of S-Parameters (3)

- **Insertion Loss** (in dB): Attenuation resulting from inserting a passive circuit between source and load.

  ↳ $P_{La} < P_{Lb}$
- IL (dB) = $10 \log_{10} (P_{Lb}/P_{La}) = 20 \log_{10} (V_{Lb}/V_{La})$
- Note: unlike attenuation, insertion loss is expressed as the power ratio at the same terminals (Z_L)

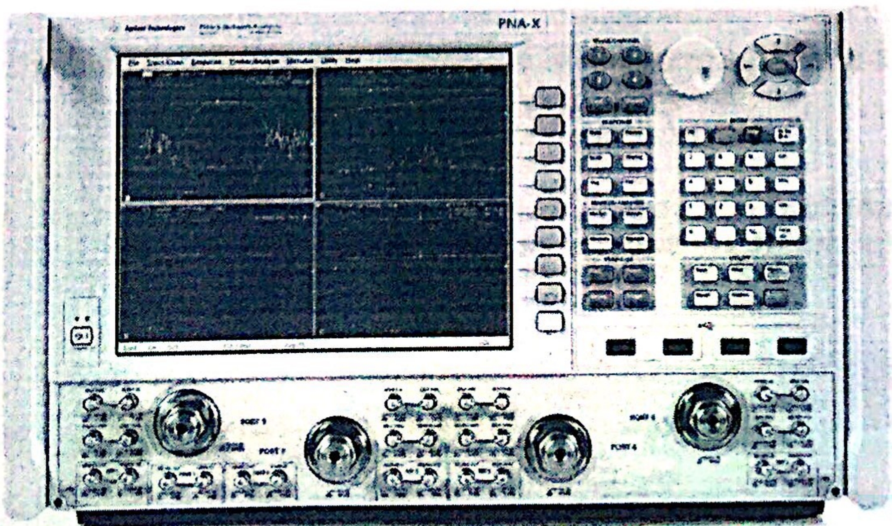


- **Insertion Phase (θ_I)**: Difference between the phase of the load voltage (or current) before the network is inserted (θ_b) and after the network is inserted (θ_a). Thus : $\theta_I = \theta_b - \theta_a$.
- When Z_G and Z_L are real, $\theta_b = 0$ → $\theta_I = -\theta_a$. Thus a positive value of θ_I indicates **phase delay** due to network (or, θ_a negative) and negative value indicates **phase advance** (or, θ_a positive).

phase delay ≡ Lag.

phase advance ≡ Lead.

Network Analyzer



↳ it can find the S-parameters for any component.

↳ it can draw the smith chart.

Practical measurements of S-parameter

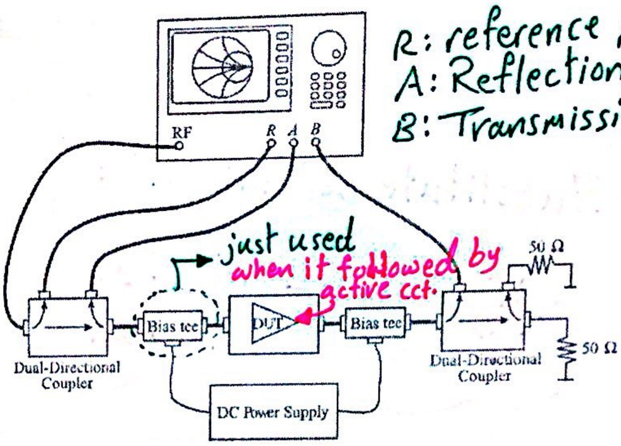
- Vector network analyzer can measure the voltages (magnitude and phase) of incident & reflected waves.
- Output port use internal source to provide RF/MW signal to sweep over specified frequency range.
- The measurement channel 'R' is a reference port and is employed for measure the incident waves.
- Channels 'A' and 'B' are used to measure the reflected and transmitted waves.

In the measurement setup of the figure, S_{11}=A/R and S_{21}=B/R. To measure S_{12} and S_{22} we have to reverse the DUT.

- In this setup, directional coupler allows the separation of the incident and reflected waves at the input port of DUT.

In the output side, unused ports of the coupler is terminated in matched load

- The bias tees are employed to provide necessary biasing conditions, such as a Q-point for DUT.
- In order to measure the DUT response accurately, we need to calibrate the device using known loads of open, short and matched loads. But such system has its disadvantages.



R: reference point.
A: Reflection.
B: Transmission.

just used when it followed by active cct.

DUT ≡ Device Under Test.

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This will find S_{11} & S_{21}

To find S_{22} & S_{12} just flip the DUT.

Example: Application of Scattering Parameters

A two-port network is known to have the following scattering matrix

$$[S] = \begin{bmatrix} 0.15\angle 0° & 0.85\angle -45° \\ 0.85\angle 45° & 0.2\angle 0° \end{bmatrix}$$

Determine if the network is reciprocal and lossless. If port 2 is terminated with a matched load, what is the return loss seen at port 1? If port 2 is terminated with a short circuit, what is the return loss seen at port 1?

Because [S] is not symmetric, the network is not reciprocal

$|S_{11}|^2 + |S_{21}|^2 = (0.15)^2 + (0.85)^2 = 0.745 \neq 1$ so the network is not lossless.

one column is enough to know is it not Lossless.

When port 2 is terminated with a matched load, the reflection coefficient seen at port 1 is $\Gamma = S_{11} = 0.15$. So the return loss is RL = −20 log |Γ| = −20 log (0.15) = 16.5 dB.

this equation proved below

$$\Gamma = \frac{V_1^-}{V_1^+} = S_{11} - S_{12}\frac{V_2^-}{V_1^+} = S_{11} - \frac{S_{12}S_{21}}{1+S_{22}} = 0.15 - \frac{(0.85\angle -45°)(0.85\angle 45°)}{1+0.2} = -0.452$$

RL = −20 log |Γ| = −20 log (0.452) = 6.9 dB.

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if $V_2 = 0$

$\Rightarrow V_2 = V_2^+ + V_2^-$

$\Rightarrow V_2^- = -V_2^+$

$V_1^- = S_{11}V_1^+ + S_{12}V_2^+$

$V_2^- = S_{21}V_1^+ + S_{22}V_2^+$

$S_{11} + S_{12}\frac{V_2^+}{V_1^+} = \frac{V_1^-}{V_1^+}$

$\Rightarrow V_2^- = -V_2^+$

so, $\frac{V_1^-}{V_1^+} = S_{11} - S_{12}\frac{V_2^-}{V_1^+}$

Continue ...

$$\Rightarrow \quad \boxed{\frac{V_1^-}{V_1^+} = S_{11} + S_{12}\frac{V_2^-}{V_1^+}} \quad \cdots (1)$$

$$\frac{V_2^-}{V_1^+} = S_{21} - S_{22}\frac{V_2^-}{V_1^+} \Rightarrow \frac{V_2^-}{V_1^+}(1 + S_{22}) = S_{21}$$

$$\Rightarrow \quad \boxed{\frac{V_2^-}{V_1^+} = \frac{S_{21}}{1 + S_{22}}} \quad \cdots (2)$$

Substitute (2) into (1):

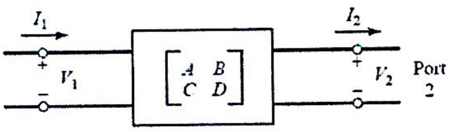
$$\frac{V_1^-}{V_1^+} = S_{11} - \frac{S_{12}S_{21}}{1 + S_{22}} \quad \#$$

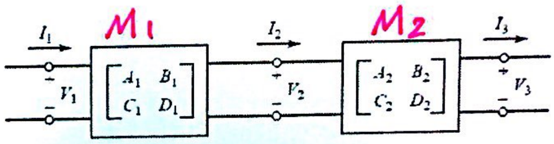
THE TRANSMISSION (ABCD) MATRIX

- The Z, Y, and S parameter representations can be used to characterize a microwave network with an arbitrary number of ports, but in practice many microwave networks consist of a cascade connection of two or more two-port networks.
  ↳ parallel.
- In this case it is convenient to define a 2 × 2 transmission, or ABCD, matrix, for each two-port network.
- The ABCD matrix of the cascade connection of two or more two-port networks can be easily found by multiplying the ABCD matrices of the individual two-ports.

$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$



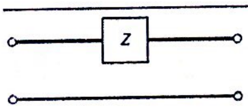
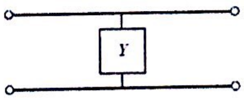
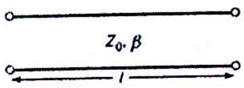
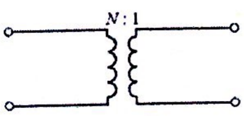
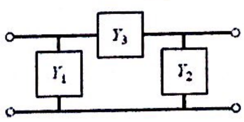
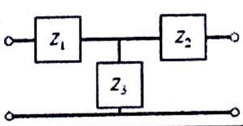


$M_{eq} = M_1 \cdot M_2$

Units of ABCD:

A → unitless.
B → Ω.
C → $(\Omega)^{-1}$ or S.
D → unitless.

ABCD Parameters of Some Useful Two-Port Circuits

Circuit	ABCD Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos\beta\ell$ $C = jY_0 \sin\beta\ell$	$B = jZ_0 \sin\beta\ell$ $D = \cos\beta\ell$
	$A = N$ $C = 0$	$B = 0$ $D = \frac{1}{N}$
	$A = 1 + \frac{Y_2}{Y_3}$ $C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$	$B = \frac{1}{Y_3}$ $D = 1 + \frac{Y_1}{Y_3}$
	$A = 1 + \frac{Z_1}{Z_3}$ $C = \frac{1}{Z_3}$	$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$ $D = 1 + \frac{Z_2}{Z_3}$

EXAMPLE: EVALUATION OF ABCD PARAMETERS

- Find the ABCD parameters of the shown two-port network between ports 1 and 2

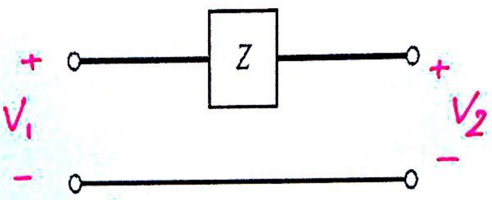
$V_1 = V_2$

$$A = \frac{V_1}{V_2}\bigg|_{I_2=0} \quad \text{Thus} \rightarrow A=1$$ "unitless"

$$B = \frac{V_1}{I_2}\bigg|_{V_2=0} = \frac{V_1}{V_1/Z} = Z$$ "Ω".

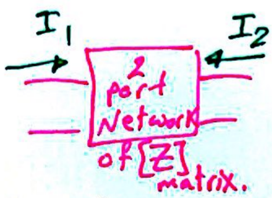
$$C = \frac{I_1}{V_2}\bigg|_{I_2=0} = 0$$ "since No admittance" in the circuit.

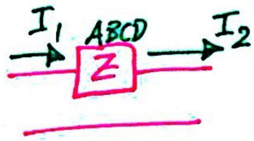
$$D = \frac{I_1}{I_2}\bigg|_{V_2=0} = \frac{I_1}{I_1} = 1$$ "unitless"



$A = 1 \quad B = Z$

$C = 0 \quad D = 1$





Relation to Impedance Matrix

negative sign since the direction of the current is reversed comparing with the two port network of $[Z]_{matrix}$ as shown above.

$$V_1 = I_1 Z_{11} - I_2 Z_{12}$$
$$V_2 = I_1 Z_{21} - I_2 Z_{22}$$

$$A = \frac{V_1}{V_2}\bigg|_{I_2=0} = \frac{I_1 Z_{11}}{I_1 Z_{21}} = Z_{11}/Z_{21},$$

$$B = \frac{V_1}{I_2}\bigg|_{V_2=0} = \frac{I_1 Z_{11} - I_2 Z_{12}}{I_2}\bigg|_{V_2=0} = Z_{11}\frac{I_1}{I_2}\bigg|_{V_2=0} - Z_{12}$$

$$= Z_{11}\frac{I_1 Z_{22}}{I_1 Z_{21}} - Z_{12} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}},$$

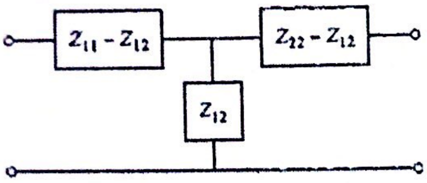
$$C = \frac{I_1}{V_2}\bigg|_{I_2=0} = \frac{I_1}{I_1 Z_{21}} = 1/Z_{21}.$$

$$D = \frac{I_1}{I_2}\bigg|_{V_2=0} = \frac{I_2 Z_{22}/Z_{21}}{I_2} = Z_{22}/Z_{21}.$$

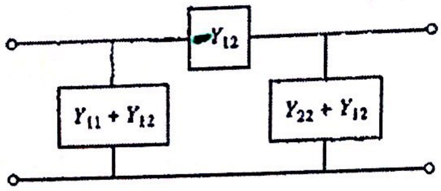
If the network is reciprocal, then $Z_{12} = Z_{21}$ and $AD - BC = 1$

Equivalent Circuits for Two-Port Networks

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$



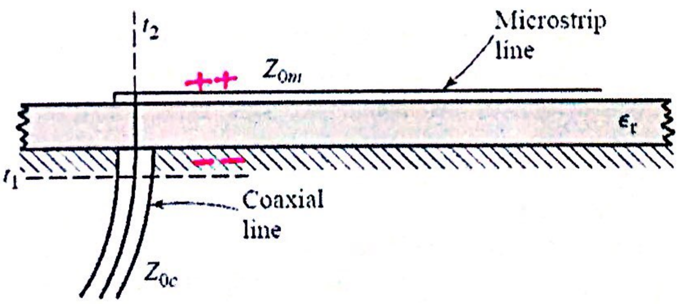
$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$



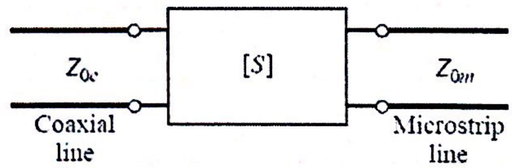
- If the network is reciprocal, then $Z_{12} = Z_{21}$ and $Y_{12} = Y_{21}$. These representations lead naturally to the T and π equivalent circuits
- If the network is reciprocal, there are six degrees of freedom (the real and imaginary parts of three matrix elements), so the equivalent circuit should have six independent parameters.
- A nonreciprocal network cannot be represented by a passive equivalent circuit using reciprocal elements.
- The impedance or admittance matrix elements are purely imaginary for a lossless network. This reduces the degrees of freedom for such a network to three, and implies that the T and π equivalent circuits can be constructed from purely reactive elements.

* Non reciprocal occur in case of Active Circuits.

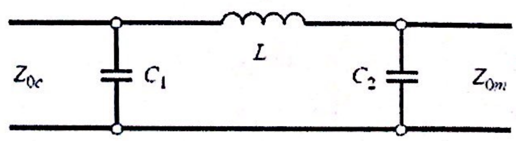
A coax-to-microstrip transition



Geometry of the transition

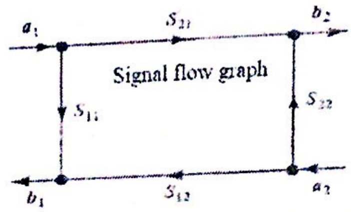


Representation of the transition by a "black box"



A possible equivalent circuit for the transition

a for icident.
b for reflected.



SIGNAL FLOW GRAPHS

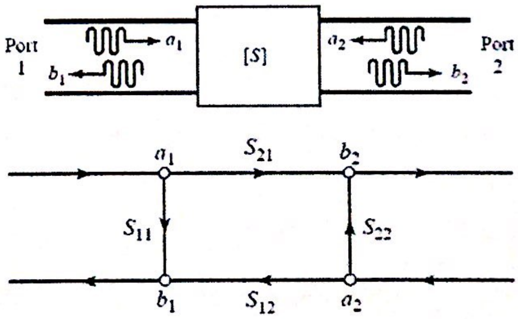
- It is an additional technique that is very useful for the analysis of microwave networks in terms of transmitted and reflected waves.
- The primary components of a signal flow graph are nodes and branches:

➢Nodes: Each port i of a microwave network has two nodes, a_i and b_i. Node a_i is identified with a wave entering port i, while node b_i is identified with a wave reflected from port i. The voltage at a node is equal to the sum of all signals entering that node.

➢Branches: A branch is a directed path between two nodes representing signal flow from one node to another. Every branch has an associated scattering parameter or reflection coefficient.

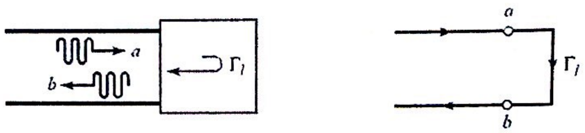
Example

- For example, a wave of amplitude a_1 incident at port 1 is split, with part going through S_{11} and out port 1 as a reflected wave, and part transmitted through S_{21} to node b_2.
- At node b_2, the wave goes out port 2; if a load with nonzero reflection coefficient is connected at port 2, this wave will be at least partly reflected and reenter the two-port network at node a_2.
- Part of this wave can be reflected back out port 2 via S_{22}, and part can be transmitted out port 1 through S_{12}

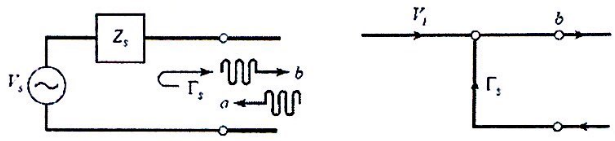


One-Port Network

Load End



Source End

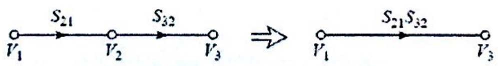


→ see the e-learning file for examples.

Decomposition of Signal Flow Graphs

Rule 1 (Series Rule)

"multiplication"

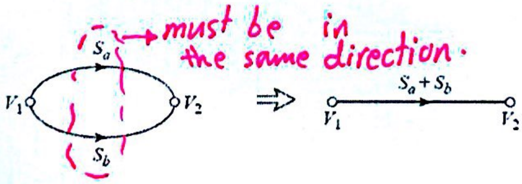


$V_3 = S_{32}V_2 = S_{32}S_{21}V_1$

Rule 2 (Parallel Rule)

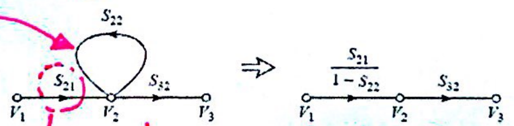
"Summation"

→ must be in the same direction.



$V_2 = S_aV_1 + S_bV_1 = (S_a + S_b)V_1$

Rule 3 (Self-Loop Rule)



$V_2 = S_{21}V_1 + S_{22}V_2,$ ➡ $V_3 = \frac{S_{32}S_{21}}{1 - S_{22}}V_1$

$V_3 = S_{32}V_2.$

$\rightarrow * \frac{1}{1-\text{self loop}}$ ⇒ it will result series rule.

Rule 4 (Splitting Rule)

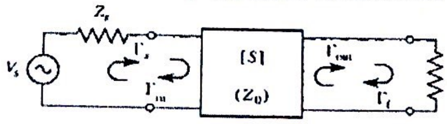
$V_4 = S_{42}V_2 = S_{21}S_{42}V_1$

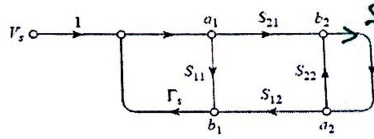
$V_3 = S_{32}V_2 = S_{21}S_{32}V_1$

"No more simplifying for it"

Example: Application of Signal Flow Graph

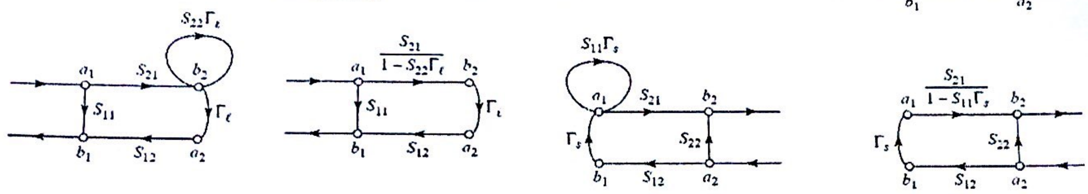
- Use signal flow graphs to derive expressions for Γ_{in} and Γ_{out} for the microwave network shown:





$S_{21}V_1^+ + S_{22}V_2^+$

The rest will go to the load.
if the load is matched
$\Gamma_L = 0$
"o.C".



$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_\ell}{1 - S_{22}\Gamma_\ell} \qquad \Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}$$

$$\Gamma_S = \frac{Z_S - Z_o}{Z_S + Z_o}$$

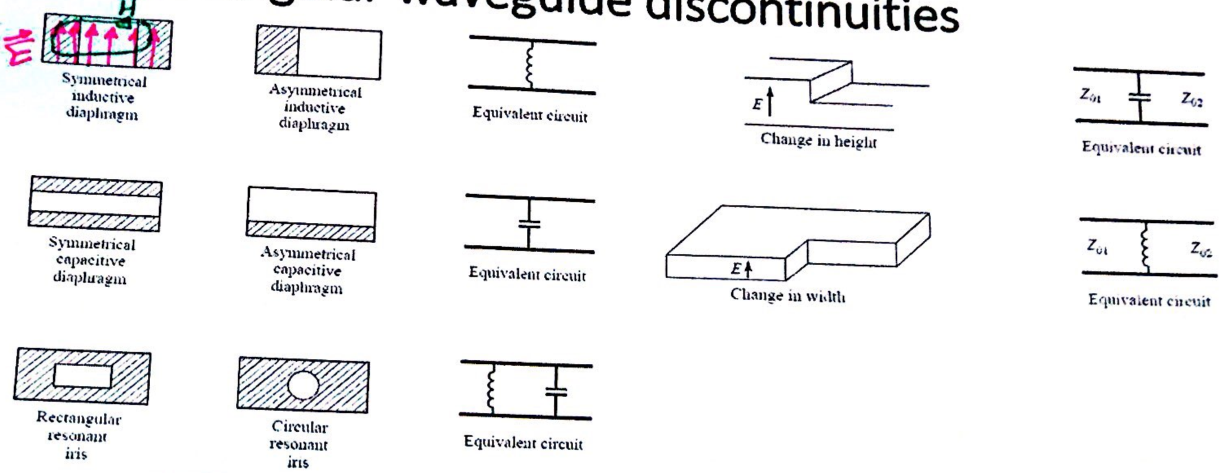
$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

DISCONTINUITIES AND MODAL ANALYSIS

- By either necessity or design, microwave circuits and networks often consist of transmission lines with various types of discontinuities.
- In some cases, discontinuities are an unavoidable result of mechanical or electrical transitions from one medium to another (e.g., a junction between two waveguides, or a coax-to-microstrip transition), and the discontinuity effect is unwanted but may be significant enough to warrant characterization.
- In other cases, discontinuities may be deliberately introduced into the circuit to perform a certain electrical function (e.g., reactive diaphragms in waveguide, or stubs on a microstrip line for matching or filter circuits).
- In any event, a transmission line discontinuity can be represented as an **equivalent circuit** at some point on the transmission line. Depending on the type of discontinuity, the equivalent circuit may be a simple shunt or series element across the line or, in the more general case, a T- or π-equivalent circuit may be required.
- The component values of an equivalent circuit depend on the (1) **parameters of the line** and (2) **the discontinuity**, as well as on the (3) **frequency of operation**.

parallel with the narrower edge.

Rectangular waveguide discontinuities



* effecting on Magnetic field ⇒ inductor.
* effecting on Electric field ⇒ Capacitor.

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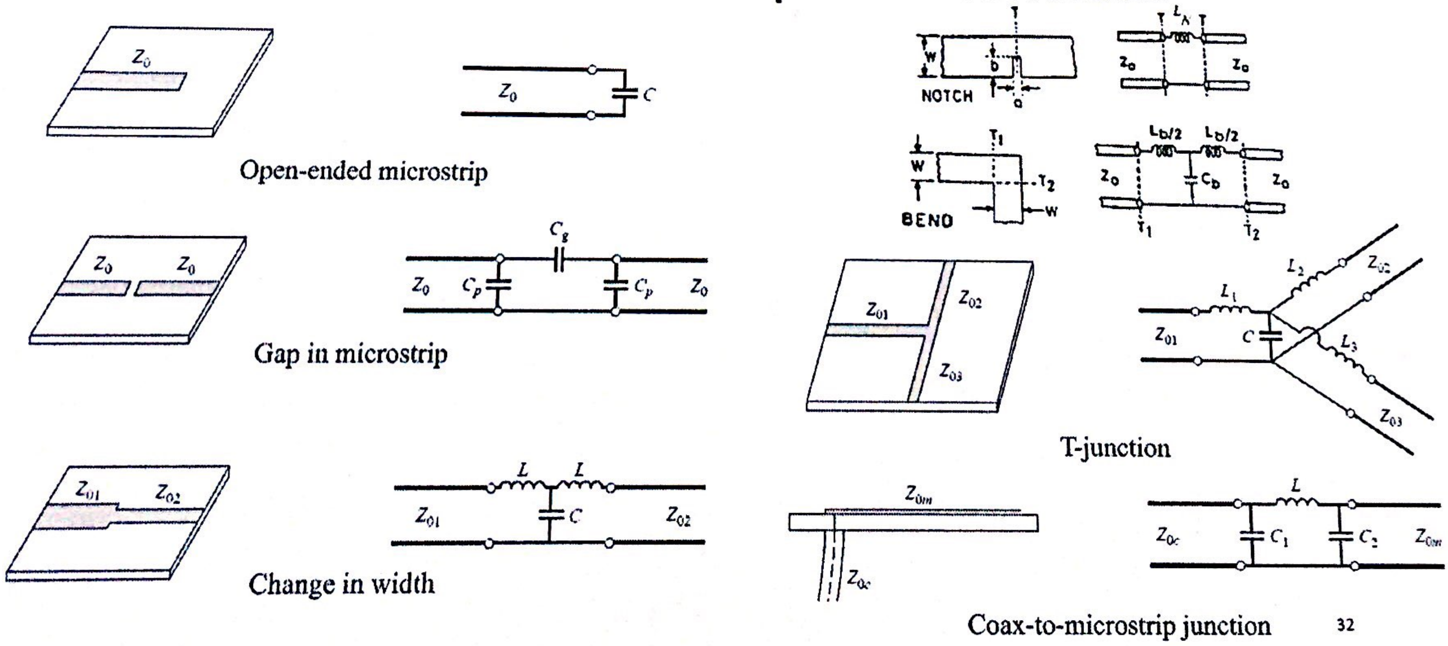
Some common microstrip discontinuities



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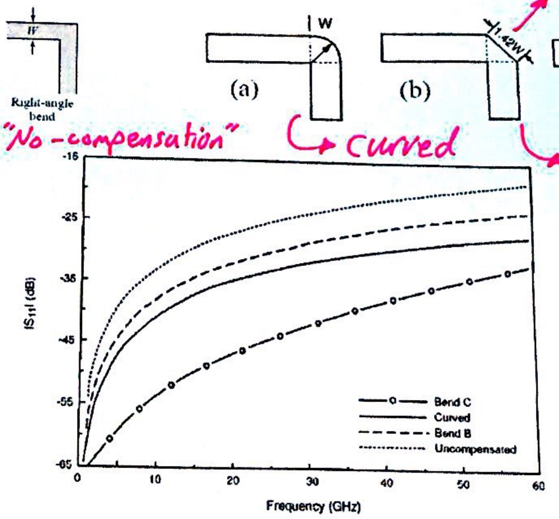
S_{11} should be small as could as it possible.

Microstrip Discontinuity Compensation $\sqrt{2}\,w$.

In practical circuits, microstrip **bends** are chamfered to compensate for the excess capacitance.
In figure (a), minimum reflection occurs if the radius of the curvature (of conductor center) is greater than three times of the width of the microstrip line.
In the graph below, S_{11} (in dB) is plotted for the uncompensated and the compensated lines.
Observe the reduction if in reflection coefficient



The better one

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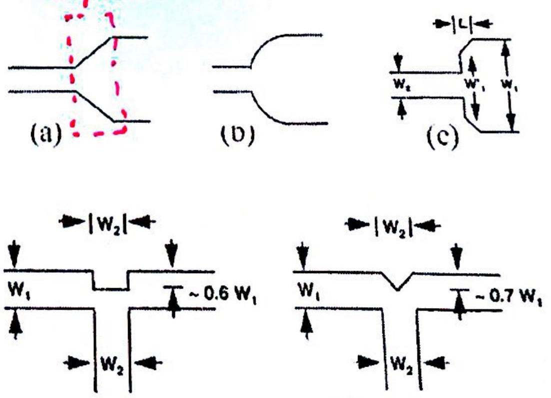
Compensation of a Step and T-junction discontinuity

- The compensation of a **step discontinuity** using appropriate tapers has been reported. In this case the effect of discontinuity reactance is reduced by chamfering the large width. The taper length depends upon the value of step ratio, dielectric constant, and the substrate thickness (h).
- Figures (a, b, c) shows **T-junction** compensation configurations using rectangular and triangular notches and their approximate dimensions for $h/\lambda \ll 1$. However, accurate dimensions of the compensated configuration depend upon the **line widths, dielectric constant**, and the **substrate thickness.**

Tapered.



Better than

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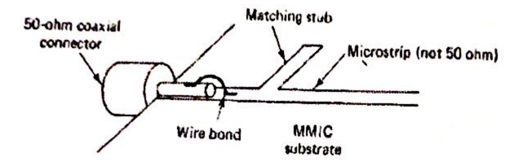
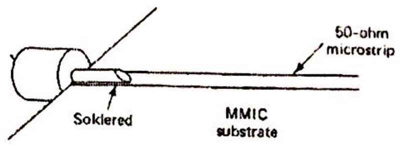
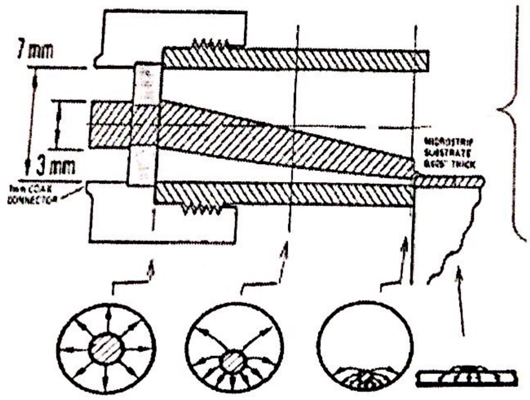
Coaxial to Microstrip transition

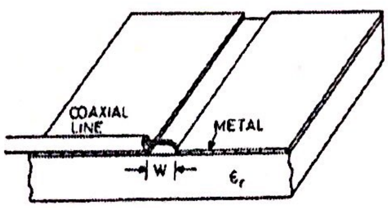




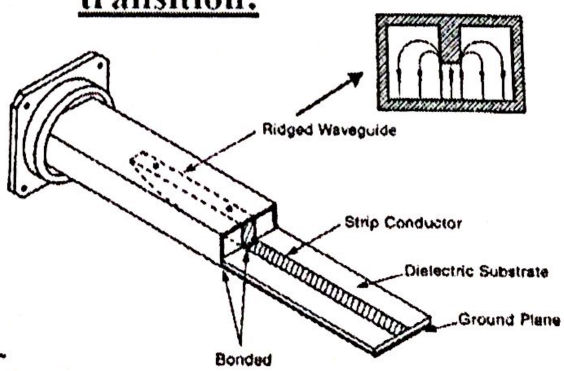


Coaxial Cable to slotline:

~~Slotline to Microstrip~~ transition:



Waveguide to Microstrip transition:



Topics in Communications.
"Microwaves"

Spring 2017/2018

Dr. Yanal Al-Faouri

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* * *

Microwave Engineering

Chapter 5

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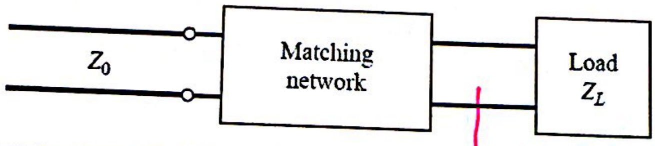
IMPEDANCE MATCHING AND TUNING

- Matching with Lumped Elements (L Networks)
- *Single-Stub Tuning*
- Double-Stub Tuning
- The Quarter-Wave Transformer
- The Theory of Small Reflections

* every 2-port network will need two matching networks Before & After the 2-port network will be called: input matching Network & output matching Network.

Impedance Matching (1)

- Impedance matching is often an important part of a larger design process for a microwave component or system.
- The matching network is ideally lossless, to avoid unnecessary loss of power, and is usually designed so that the impedance seen looking into the matching network is Z_0. Then reflections will be eliminated on the transmission line to the left of the matching network, although there will usually be multiple reflections between the matching network and the load. This procedure is sometimes referred to as tuning.



here multiple reflection will occur.

Impedance Matching (2)

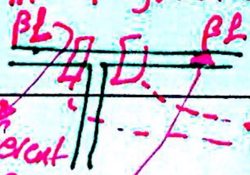
- Impedance matching or tuning is important for the following reasons:

1. Maximum power is delivered when the load is matched to the line (assuming the generator is matched), and power loss in the feed line is minimized.

2. Impedance matching sensitive receiver components (antenna, low-noise amplifier, etc.) may improve the signal-to-noise ratio of the system.

3. Impedance matching in a power distribution network (such as an antenna array feed network) may reduce amplitude and phase errors.

in T-junction.



different phase with

these matching Networks will match the phase shift between the two sides.

Matching Network Selection

* All matching Networks are Lossless.

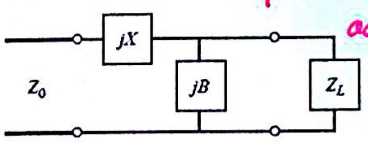
- **Complexity**: the simplest design that satisfies the required specifications is generally preferable. A simpler matching network is usually cheaper, smaller, more reliable, and less lossy than a more complex design.
- **Bandwidth**: Any type of matching network can ideally give a perfect match (zero reflection) at a single frequency. In many applications, however, it is desirable to match a load over a band of frequencies. There are several ways of doing this, with, of course, a corresponding increase in complexity.
- **Implementation**: Depending on the type of transmission line or waveguide being used, one type of matching network may be preferable to another. For example, tuning stubs are much easier to implement in waveguide than are multi-section quarter-wave transformers.
- **Adjustability**: In some applications the matching network may require adjustment to match a variable load impedance. Some types of matching networks are more amenable than others in this regard.

"immune"

Matching with Lumped Elements (L Networks)

- The simplest type of matching network which uses two reactive elements to match an arbitrary load impedance to a transmission line
- There are two possible configuration depending on the normalized load impedance z_L:

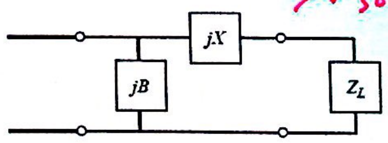
solve using admittance.



If z_L is inside the $1 + jx$ circle

Normalized.

solve using impedance.



If z_L is outside the $1 + jx$ circle

The $1 + jx$ circle is the resistance circle on the impedance Smith chart for which $r = 1$

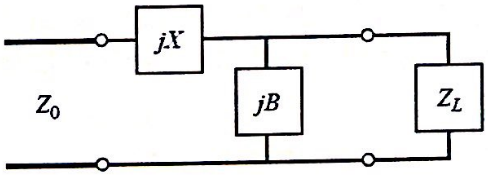
Analytic Solutions for L Matching Network (1)

$$Z_0 = jX + \frac{1}{jB + 1/(R_L + jX_L)}$$

$$B(XR_L - X_L Z_0) = R_L - Z_0,$$
$$X(1 - BX_L) = BZ_0R_L - X_L$$

$$B = \frac{X_L \pm \sqrt{R_L/Z_0}\sqrt{R_L^2 + X_L^2 - Z_0R_L}}{R_L^2 + X_L^2}$$

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{BR_L}$$



Two solutions:
Take +ve B with +ve X
, -ve B with -ve X.

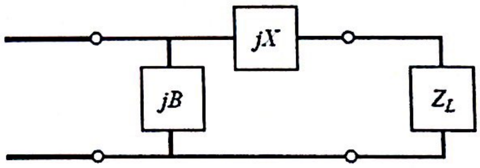
Analytic Solutions for L Matching Network (2)

$$\frac{1}{Z_0} = jB + \frac{1}{R_L + j(X + X_L)}$$

$$BZ_0(X + X_L) = Z_0 - R_L \qquad (X + X_L) = BZ_0R_L$$

$$X = \pm\sqrt{R_L(Z_0 - R_L)} - X_L$$

$$B = \pm\frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}$$



Because $R_L < Z_0$, the arguments of the square roots are always positive and the two solutions are possible.

$B = \omega C$

$B = 2\pi f C$

$\Rightarrow C = \dfrac{B}{2\pi f} = \dfrac{b}{2\pi f Z_0}$

$X_L = \omega L$

$X = 2\pi f L$

$\Rightarrow L = \dfrac{X}{2\pi f} = \dfrac{x Z_0}{2\pi f}$

Smith Chart Solutions

- **Example: L-Section Impedance Matching**

Design an L-section matching network to match a series RC load with an impedance $Z_L = 200 - j100\ \Omega$ to a 100 Ω line at a frequency of 500 MHz.

$z_L = 2 - j1$ ➔ inside the $1 + jx$ circle

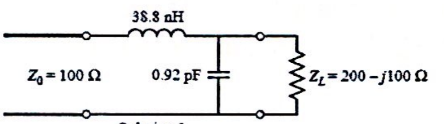
Analytical: First Solution: b = 0.29, x = 1.22 Second Solution: b = −0.69, x = −1.22

Smith: b = 0.3, x = 1.2

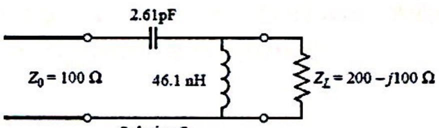
Smith: b = -0.7, x = -1.2

$$C = \frac{b}{2\pi f Z_0} = 0.92\ \text{pF} \qquad C = \frac{-1}{2\pi f x Z_0} = 2.61\ \text{pF}$$

$$L = \frac{x Z_0}{2\pi f} = 38.8\ \text{nH} \qquad L = \frac{-Z_0}{2\pi f b} = 46.1\ \text{nH}$$



Solution 1



Solution 2

Solution:

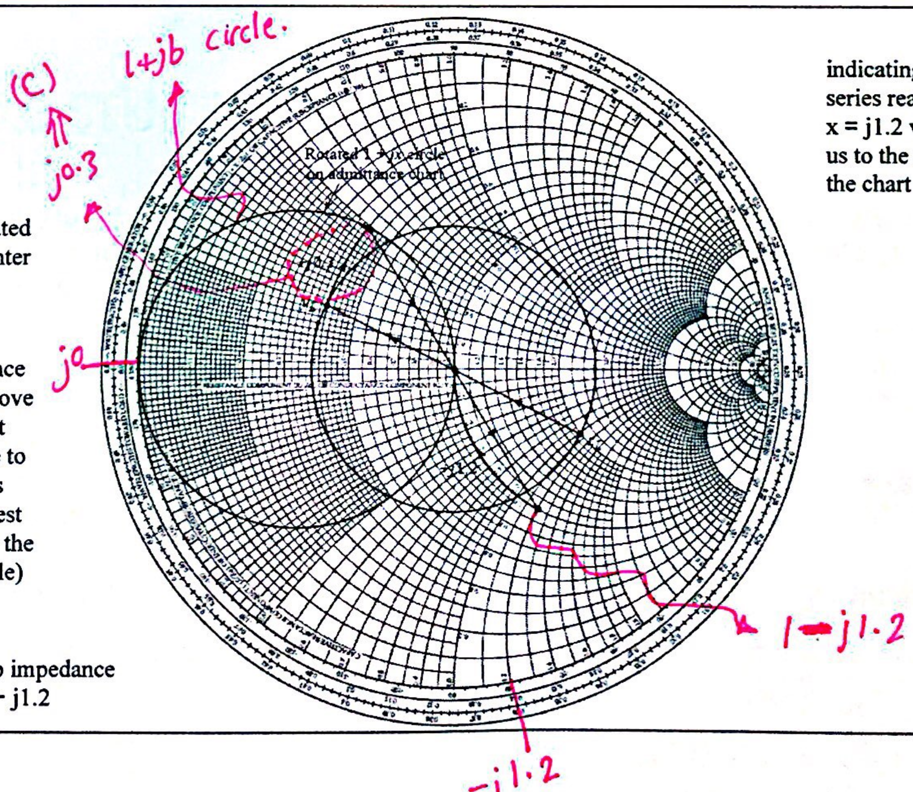
$z_L = 2 - j\,1$

$y_L = 0.4 + j0.2$

construct the rotated $1 + j\,x$ circle (center at r = 0.333)

Then we see that adding a susceptance of jb = j0.3 will move us along a constant conductance circle to y = 0.4 + j0.5 (this choice is the shortest distance from y_L to the shifted 1 + j x circle)

Converting back to impedance leaves us at z = 1 − j1.2



indicating that a series reactance of x = j1.2 will bring us to the center of the chart

$\Rightarrow j0 - (-j1.2) = \underline{j1.2}$

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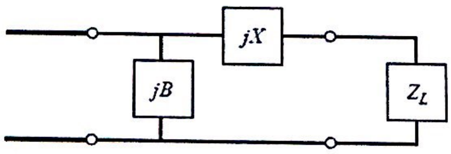
Smith Chart Solutions

- **Example: L-Section Impedance Matching (outside the 1 + jx circle)**

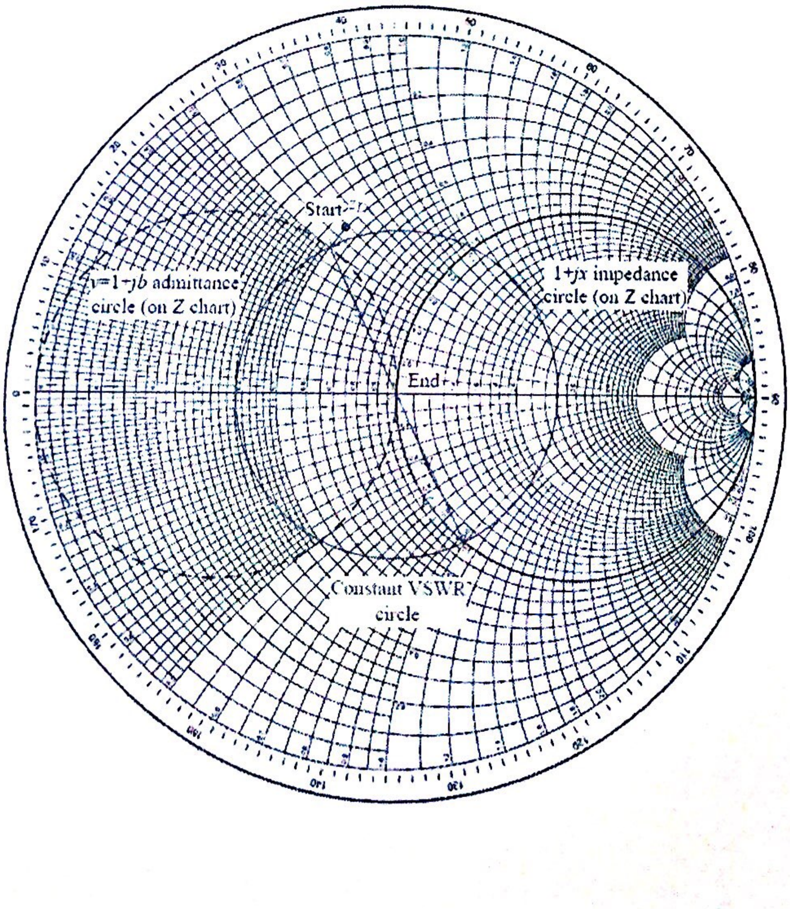
Design an L-section matching network to match a series RL load with an impedance $Z_L = 25 + j30\ \Omega$ to a 50 Ω line at a frequency of 1 GHz.

- Solution:
- $z_L = 0.25 + j0.3 \Rightarrow r_L < 1 \Rightarrow$



- Analytical Solution:

$$jX = \pm\sqrt{R_L(Z_0 - R_L)} - X_L = \pm 25 - 30 = \begin{cases} -5\ \Omega \\ -55\ \Omega \end{cases} \qquad B = \pm\frac{1}{Z_0}\sqrt{\frac{Z_0 - R_L}{R_L}} = \begin{cases} 0.02\ \text{S} \\ -0.02\ \text{S} \end{cases}$$

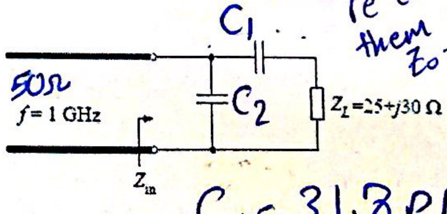


$$jX = jx\cdot Z_0 = -j0.1\cdot 50 = -j5\ \Omega$$

$$jB = jb\cdot Y_0 = j1.0\cdot\frac{1}{50} = j0.02\ \text{S}$$

re calculate them for $Z_0 = 100\ \Omega$.

...



$C_1 = 31.8\ pF$

$C_2 = 3.18\ pF$.

SINGLE-STUB TUNING (1)

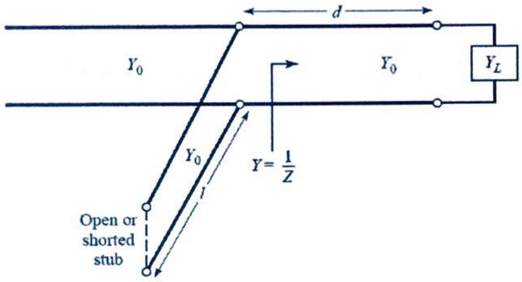
- At microwave frequencies, lumped inductors/capacitors are difficult to make.
- Another popular matching technique uses a single open-circuited or short-circuited length of transmission line (a stub) connected either in parallel or in series with the transmission feed line at a certain distance from the load.
- Such a single-stub tuning circuit is often very convenient because the stub can be fabricated as part of the transmission line media of the circuit, and lumped elements are avoided.
- Shunt stubs are preferred for microstrip line or stripline, while series stubs are preferred for slotline or coplanar waveguide.

SINGLE-STUB TUNING (2)

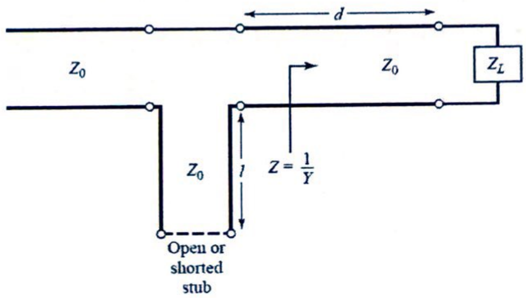
- In single-stub tuning the two adjustable parameters are the **distance (d) from the load to the stub position**, and the **value of susceptance or reactance provided by the stub**.
- For the shunt-stub case, the basic idea is to select d so that the admittance (Y) seen looking into the line at distance d from the load is of the form (Y_0 + jB). Then the stub susceptance is chosen as (−jB), resulting in a matched condition.
- For the series-stub case, the distance (d) is selected so that the impedance (Z) seen looking into the line at a distance d from the load is of the form (Z_0 + jX).
- Then the stub reactance is chosen as (−jX), resulting in a matched condition.

SINGLE-STUB TUNING (3)

The proper length of an open or shorted transmission line section can provide any desired value of reactance or susceptance.
For a given susceptance or reactance, the difference in lengths of an open- or short-circuited stub is $\lambda/4$



Shunt single-stub tuning circuits



Series single-stub tuning circuits

$\lambda \equiv 720^\circ$, $\frac{\lambda}{2} = 360^\circ$ "one revolution"

SINGLE-STUB TUNING (4)

- For transmission line media such as microstrip or stripline, open-circuited stubs are easier to fabricate since a via hole through the substrate to the ground plane is not needed.
- For lines like coax or waveguide, however, short-circuited stubs are usually preferred because the cross-sectional area of such an open-circuited line may be large enough (electrically) to radiate.
- In which case the stub is no longer purely reactive.

Shunt Stubs

Example: Single-Stub Shunt Tuning

- For a load impedance Z_L = 60 − j80 Ω, design two single-stub (short circuit) shunt tuning networks to match this load to a 50 Ω line. Assuming that the load is matched at 2 GHz and that the load consists of a resistor and capacitor in series, plot the reflection coefficient magnitude from 1 to 3 GHz for each solution.

d ⇒ matching the real part
ℓ ⇒ matching the imaginary part.

Solution:

$z_L = 1.2 - j1.6$

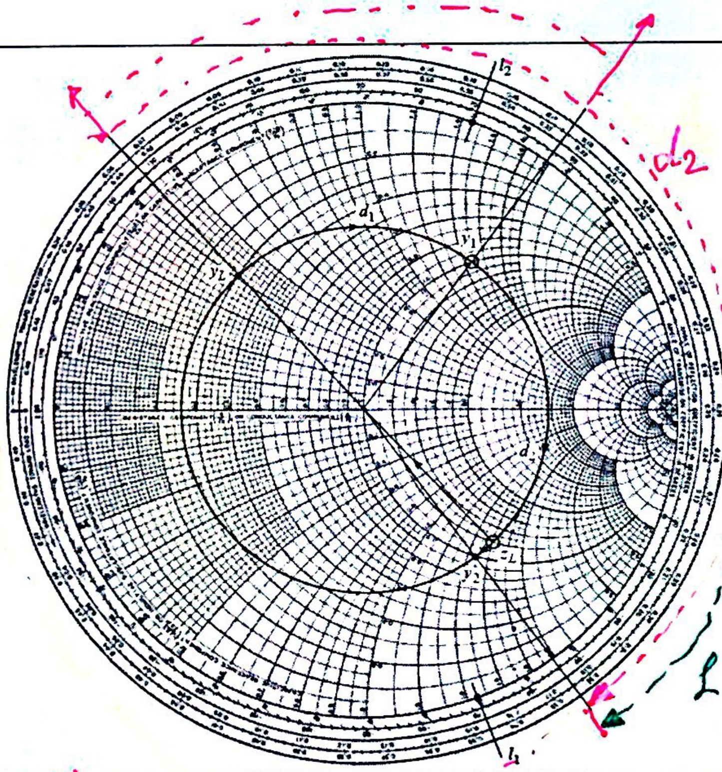
The SWR circle intersects the 1 + jb circle at two points, denoted as y_1 and y_2

$d_1 = 0.176 - 0.065 = 0.110\lambda$,
$d_2 = 0.325 - 0.065 = 0.260\lambda$

always found (toward the gen.)

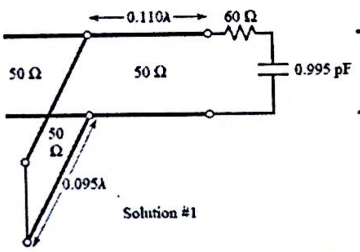
$y_1 = 1.00 + j1.47$
$y_2 = 1.00 - j1.47$

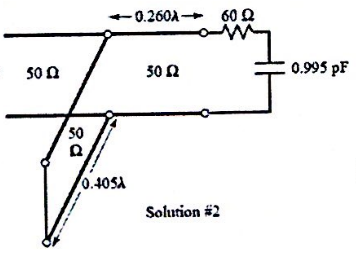
$\ell_1 = 0.095\lambda$
$\ell_2 = 0.405\lambda$

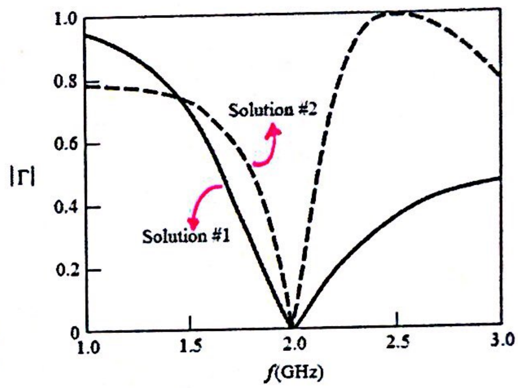


Actually, there is an infinite number of distances d around the SWR circle that intersect the 1 + jb circle.
Usually it is desired to keep the matching stub as close as possible to the load to **improve the bandwidth of the match** and to **reduce losses** caused by a possibly large standing wave ratio on the line between the stub and the load.

Solution:







Solution 1 has a significantly better bandwidth than solution 2; this is because both d and ℓ are shorter for solution 1, which reduces the frequency variation of the match.

Series Stubs

Example: Single-Stub Series Tuning

- Match a load impedance of Z_L = 100 + j80 Ω to a 50 Ω line using a single series open-circuit stub. Assuming that the load is matched at 2 GHz and that the load consists of a resistor and inductor in series, plot the reflection coefficient magnitude from 1 to 3 GHz.

series short
Z=0
y=∞

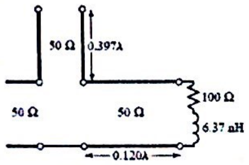
shunt short
y=0
Z=∞

Solution:

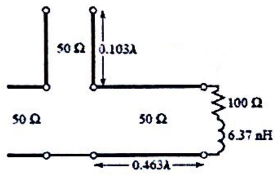
$z_L = 2 + j1.6$

$d_1 = 0.328 - 0.208 = 0.120\lambda$,
$d_2 = (0.5 - 0.208) + 0.172 = 0.463\lambda$

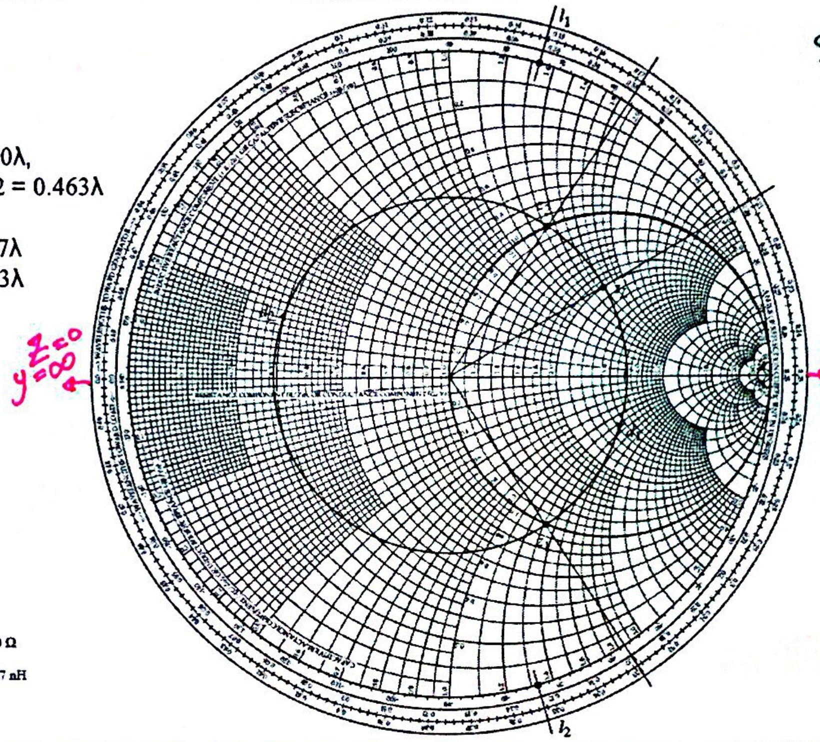
$z_1 = 1 - j1.33 \quad \ell_1 = 0.397\lambda$
$z_2 = 1 + j1.33 \quad \ell_2 = 0.103\lambda$



Solution 1



Solution 2

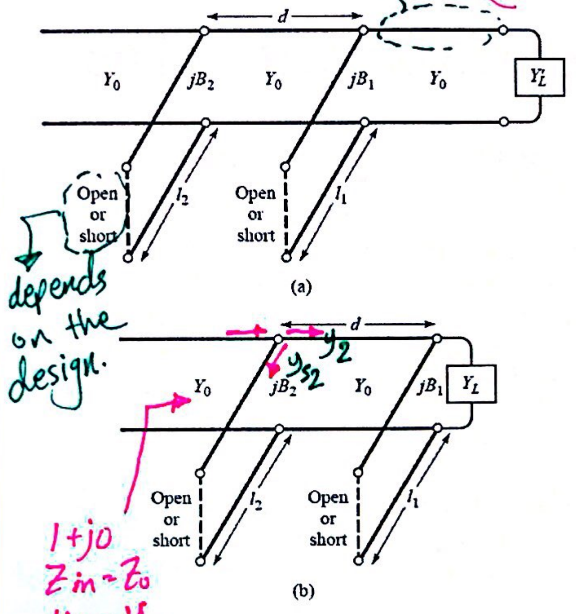


Double-Stub Tuning

- The single-stub tuner of the previous section is able to match any load impedance (having a positive real part) to a transmission line, but suffers from the disadvantage of requiring a variable length of line between the load and the stub.
- This may not be a problem for a fixed matching circuit, but would probably pose some difficulty if an adjustable tuner was desired.
- In this case, the double-stub tuner, **which uses two tuning stubs in fixed positions,** can be used.
- Such tuners are often fabricated in coaxial line with adjustable stubs connected in shunt to the main coaxial line.
- Note that a double-stub tuner cannot match all load impedances.

we add an additional distance if the load was in the forbidden region

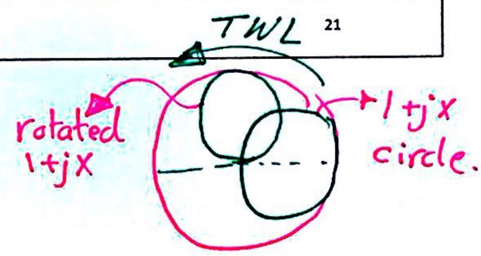
we consider the worst case when design.



- In the double-stub tuner circuit, the load may be an arbitrary distance from the first stub.
- Although this is more representative of a practical situation, the circuit of figure (b), where the load Y_L has been transformed back to the position of the first stub, is easier to deal with and does not lose any generality.
- The shunt stubs can be conveniently implemented for some types of transmission lines, while series stubs are more appropriate for other types of lines.
- In either case, the stubs can be open-circuited or short-circuited.

$y_{in} = y_2 + y_{s2}$

$1+j0 = (1+jx) - (jx)$

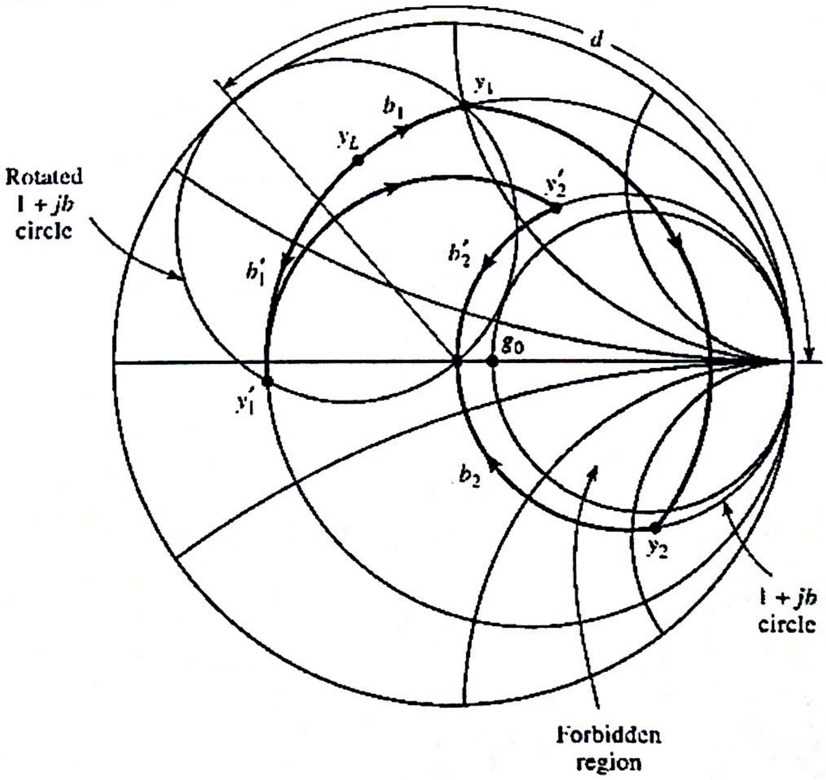


Smith Chart Solution (1)

- As in the case of the single-stub tuner, two solutions are possible.
- The susceptance of the first stub b_1 (or b'_1 for the second solution), moves the load admittance to y_1 (or y'_1).
- These points lie on the rotated 1 + jb circle; the amount of rotation is d wavelengths toward the load, where d is the electrical distance between the two stubs.
- Then transforming y_1 (or y'_1) toward the generator through a length d of line leaves us at the point y_2 (or y'_2), which must be on the 1 + jb circle.
- The second stub then adds a susceptance b_2 (or b'_2), which brings us to the center of the chart and completes the match.

Smith Chart Solution (2)

- Notice that; if the load admittance y_L were inside the shaded region of the $g_0 + jb$ circle, no value of stub susceptance b_1 could ever bring the load point to intersect the rotated $1 + jb$ circle.
- This shaded region thus forms a forbidden range of load admittances that cannot be matched with this particular double-stub tuner.
- A simple way of reducing the forbidden range is to reduce the distance d between the stubs. This has the effect of swinging the rotated $1 + jb$ circle back toward the $y = \infty$ point, but d must be kept large enough for the practical purpose of fabricating the two separate stubs.
- In addition, stub spacings near 0 or $\lambda/2$ lead to matching networks that are very frequency sensitive.
- In practice, stub spacings are usually chosen as $\lambda/8$ or $3\lambda/8$.
- If the length of line between the load and the first stub can be adjusted, then the load admittance y_L can always be moved out of the forbidden region.



Example: Double-Stub Tuning

- Design a double-stub shunt tuner to match a load impedance $Z_L = 60 - j80\ \Omega$ to a 50 Ω line. The stubs are to be open-circuited stubs and are spaced $\lambda/8$ apart.
  90°
- Assuming that this load consists of a series resistor and capacitor and that the match frequency is 2 GHz, plot the reflection coefficient magnitude versus frequency from 1 to 3 GHz.

Solution:

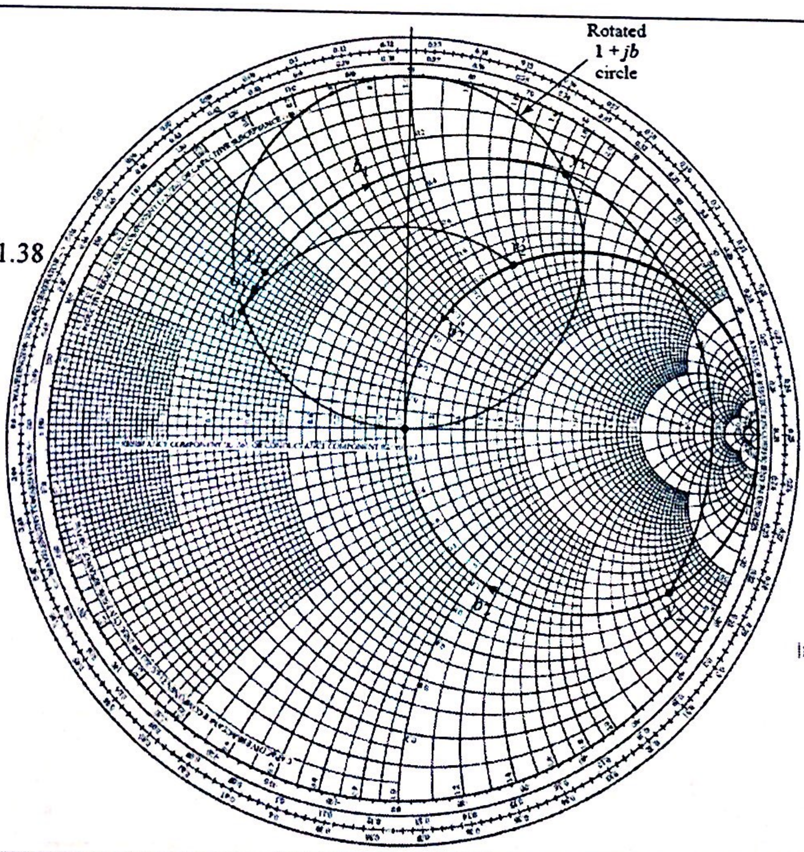
$y_L = 0.3 + j0.4$

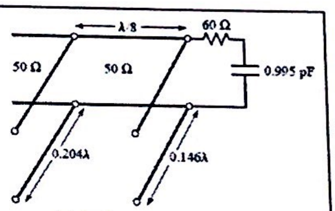
$b_1 = 1.314$ or $b_1 = -0.114$

$y_2 = 1 - j3.38$ or $y_2 = 1 + j1.38$

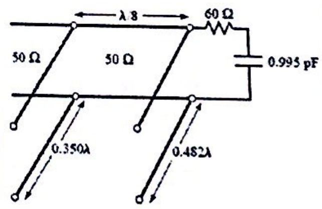
Then the susceptance of the second stub should be $b_2 = 3.38$ or $b_2 = -1.38$

$\ell_1 = 0.146\lambda$, $\ell_2 = 0.204\lambda$ or $\ell_1 = 0.482\lambda$, $\ell_2 = 0.350\lambda$

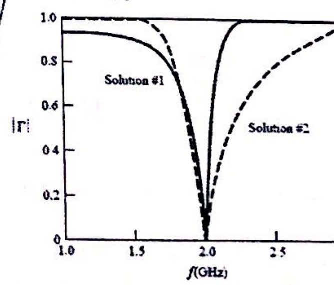




Solution 1



Solution 2



Quarter-Wave Transformer (1)

- The quarter-wave transformer is a simple and useful circuit for matching a real load impedance to a transmission line.
- An additional feature of the quarter-wave transformer is that it can be extended to multi-section designs in a methodical manner to provide broader bandwidth.
- If only a narrow band impedance match is required, a single-section transformer may suffice.
- However, multi-section quarter-wave transformer designs can be synthesized to yield optimum matching characteristics over a desired frequency band; such networks are closely related to bandpass filters.

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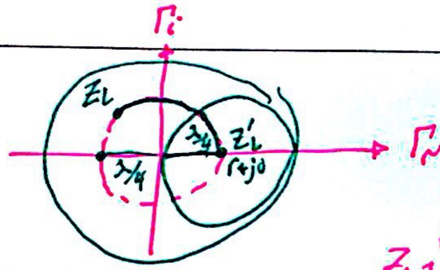


$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$

$Z_{in} = \frac{Z_0 \cdot Z_0}{Z_L}$

$Z_0 = \sqrt{Z_{in} Z_L}$

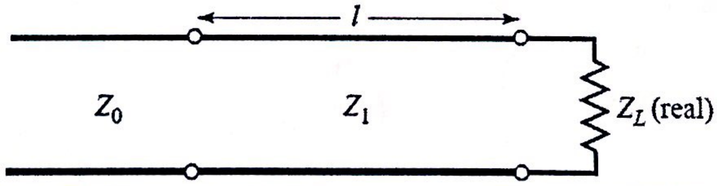
$Z_0 \frac{\lambda}{4}$

Z_0 T.L.

Quarter-Wave Transformer (2)

- One drawback of the quarter-wave transformer is that it can only match a real load impedance.
- A complex load impedance can always be transformed into a real impedance, however, by using an appropriate length of transmission line between the load and the transformer, or an appropriate series or shunt reactive element.
- These techniques will usually alter the frequency dependence of the load, and this often has the effect of reducing the bandwidth of the match.

$$Z_1 = \sqrt{Z_0 Z_L}$$



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Quarter-Wave Transformer (3)

$$Z_{in} = Z_1 \frac{Z_L + jZ_1 t}{Z_1 + jZ_L t}$$

$t = \tan\beta\ell = \tan\theta$, and $\beta\ell = \theta = \pi/2$

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_1(Z_L - Z_0) + jt\,(Z_1^2 - Z_0 Z_L)}{Z_1(Z_L + Z_0) + jt\,(Z_1^2 + Z_0 Z_L)}$$

Because $Z_1^2 = Z_0 Z_L$, this reduces to

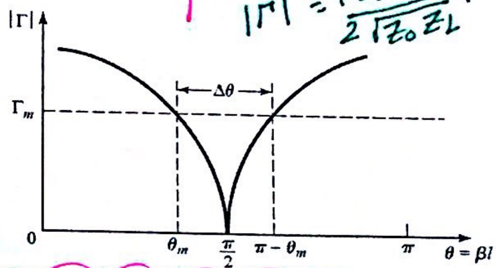
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0 + j2t\sqrt{Z_0 Z_L}}$$

$$|\Gamma| = \frac{|Z_L - Z_0|}{\left[(Z_L + Z_0)^2 + 4t^2 Z_0 Z_L\right]^{1/2}}$$

$$|\Gamma| \simeq \frac{|Z_L - Z_0|}{2\sqrt{Z_0 Z_L}}|\cos\theta| \quad \text{for } \theta \text{ near } \pi/2$$

$\hookrightarrow |\Gamma| = \text{zero}.$

this graph for $|\Gamma| = \frac{|Z_L - Z_0|}{2\sqrt{Z_0 Z_L}}|\cos\theta|$



* * $\cos\theta_m = \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}$ * very important.

* $f_m = \frac{2\theta_m f_0}{\pi}$ $\qquad \Delta\theta = 2\left(\frac{\pi}{2} - \theta_m\right)$ *

If we assume TEM lines, then $\theta = \beta\ell = \frac{2\pi f}{v_p}\frac{v_p}{4 f_0} = \frac{\pi f}{2 f_0}$

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{2 f_m}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right] \equiv \text{Bandwidth (\%) "Normalized"}$$

f_0 is the operating frequancy.

maximum reflection coeficient you can handle. (@ θ_m).

Example: Quarter-Wave Transformer Bandwidth

- Design a single-section quarter-wave matching transformer to match a 10 Ω load to a 50 Ω transmission line at f_0 = 3 GHz. Determine the percent bandwidth for which the SWR ≤ 1.5

$\hookrightarrow$ Maximum.

$$Z_1 = \sqrt{Z_0 Z_L} = \sqrt{(50)(10)} = 22.36\ \Omega$$

$$\Gamma_m = \frac{\text{SWR} - 1}{\text{SWR} + 1} = \frac{1.5 - 1}{1.5 + 1} = 0.2$$

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right] = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{0.2}{\sqrt{1 - (0.2)^2}} \frac{2\sqrt{(50)(10)}}{|10 - 50|}\right] = 0.29, \text{ or } 29\%$$

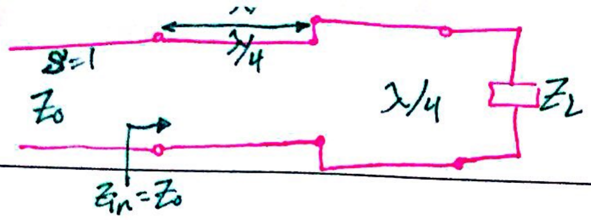
$3G - \frac{0.29}{2} \times 3G$ $\qquad$ $3G + \frac{0.29}{2} \times 3G$

f_0 $\qquad$ f (HZ)

3GHZ

Δf

$\frac{\Delta f}{2}$



The Theory of Small Reflections

- The quarter-wave transformer provides a simple means of matching any real load impedance to any transmission line impedance.
- For applications requiring more bandwidth than a single quarter-wave section can provide, multi-section transformers can be used.
- To design such transformers; approximate results for the total reflection coefficient caused by the partial reflections from several small discontinuities. This topic is generally referred to as the theory of small reflections.

Single-Section Transformer

$= 90^\circ$ if $\ell = \lambda/4$

- The partial reflection and transmission coefficients for single section transformer are:

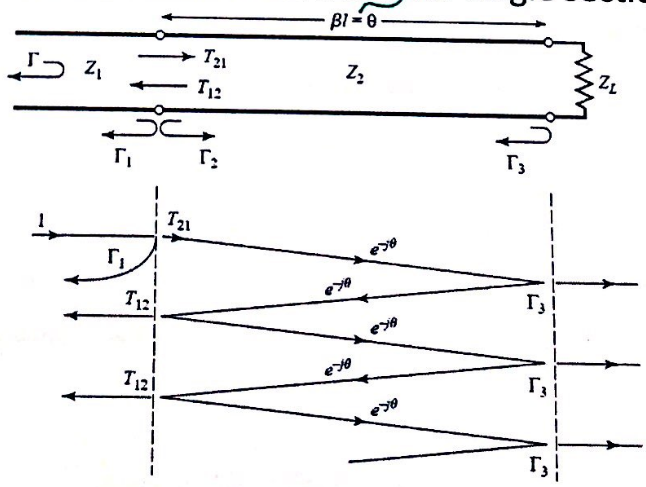
$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\Gamma_2 = -\Gamma_1$$

$$\Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2}$$

$$T_{21} = 1 + \Gamma_1 = \frac{2Z_2}{Z_1 + Z_2}$$

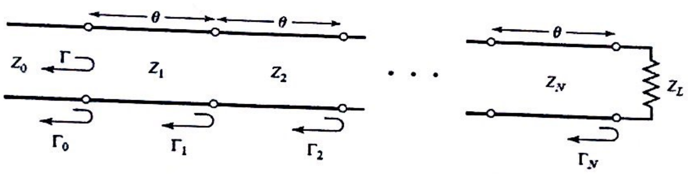
$$T_{12} = 1 + \Gamma_2 = \frac{2Z_1}{Z_1 + Z_2}$$



sheilding effectiveness $\equiv 20 \log \tau_{\text{over all}}$

Multi-section Transformer

$$\Gamma_0 = \frac{Z_1 - Z_0}{Z_1 + Z_0},$$
$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n},$$
$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}.$$

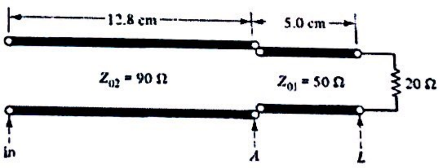


$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 e^{-2j\theta} + \Gamma_2 e^{-4j\theta} + \cdots + \Gamma_N e^{-2jN\theta}$$ usually we take first 3 terms

- The importance of these results lies in the fact that we can synthesize any desired reflection coefficient response as a function of frequency (Θ) by properly choosing the Γ_n and using enough sections (N).
- This should be clear from the realization that a Fourier series can approximate an arbitrary smooth function if enough terms are used.
- Two of the most commonly used passband responses: the binomial (maximally flat) response, and the Chebyshev (equal-ripple) response can be used to design multisection transformers.

Example:

For the lossless T.L of $\lambda = 20$ cm, find Z_{in}



$Z_{LN} = Z_L/Z_{01} = 0.4$

Plot $(Z_L)_N$; Draw SWR circle (~2.5)
Move in TWG axis by, $\ell = 5\text{cm} = 0.25\lambda$
read $Z_{AN} = 2.5 \rightarrow Z_A = 125\ \Omega$
Normalizing with respect to Z_{02};
$Z_{AN2} = Z_A/Z_{02} = 1.39$
Plot $(Z_A)_{N2}$; Draw SWR circle
The new value of SWR=1.39
Move using TWG. axis by, $\ell = 12.8\text{cm} = 12.8/20 = 0.64\lambda$,
read from chart, $(Z_{in})_N = 0.9 - j0.3$

$Z_{in} = 81 - j27\ \Omega$

Topics in Communications.

"Microwaves"

Spring 2017/2018

Dr. Yanal Al-Faouri

By: Mohammad Abu Hashya.

* * *

Microwave Engineering

Chapter 6

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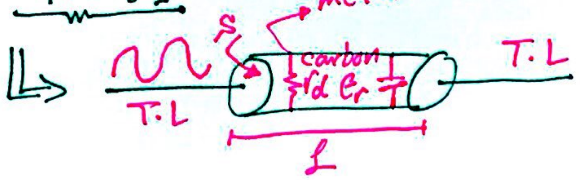
Passive Circuit Elements Used in MIC's

- Radio Frequency/Microwave Resistors
- Radio Frequency/Microwave Capacitors
- Radio Frequency/Microwave Inductors
- Capacitors and Inductors using Microstrip Lines
- Series and Parallel Resonant Circuits
- Transmission Line Resonators

Lumped Elements Used in MIC's

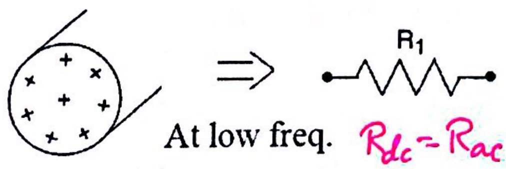
- The lumped-element form of MICs consists of capacitors, inductors and resistors, that are a fraction of a wavelength in size.
- Lumped means the values of the components are independent of frequency.
- In the past, this type of circuit was not feasible at microwave frequencies because conventional fabrication techniques could not provide coils and capacitors small enough to behave as true lumped elements.
- Recently, with the advent of new photolithographic techniques, fabrication of lumped element, that was limited to X-band, can be extended to about 60 GHz
- To analyze the lumped form of capacitors, inductors and resistors, comprehensive mathematical model is needed that cater the affects of fringing field, proximity effect, parasitic and ground plane.

$R = \frac{l}{\sigma S}$



RF/MW Resistors

- At high frequencies, due to skin-effect and straight-wire-conductor:

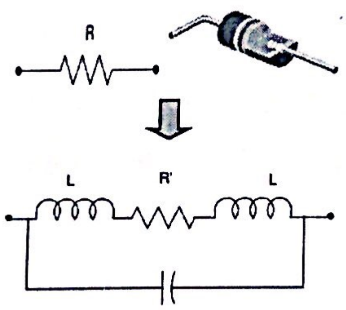


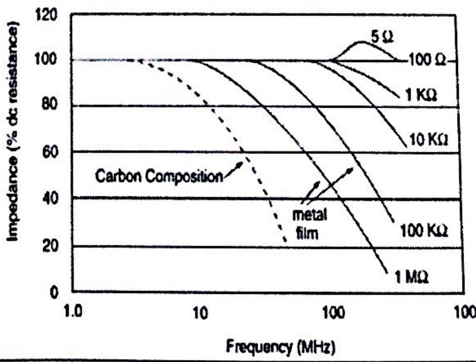
$R_{dc} = R_{ac}$

$R_{ac} > R_{dc}$

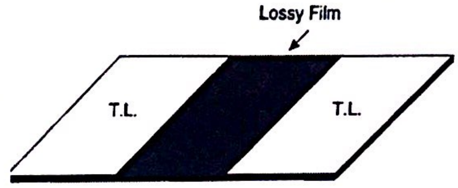
$R_2 > R_1$

$f\uparrow \Rightarrow R_{ac}\uparrow$





Types of Lumped Resistors (1)



(1) Thin film resistors:

- can be realized by depositing thin films of lossy material (Nichrome or Tantalum-nitride) on a dielectric base.
- Design of these resistors require knowledge of :

(i) sheet resistance,

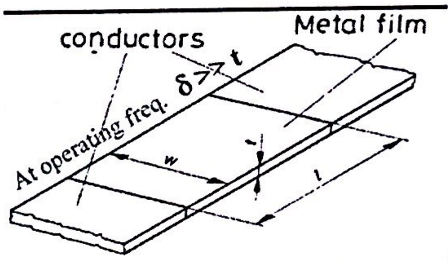
(ii) thermal resistance,

(iii) current-handling capacity,

(iv) nominal tolerance and

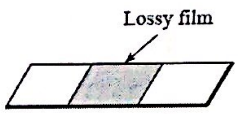
(v) temperature coefficient of the film.

$R = \ell/(\sigma w t)$



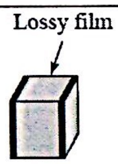
- To satisfy lumped condition, 'w << λ' & 'ℓ << λ', means uniform current distribution along resistor cross section and thus resistance is independent of frequency.

Temp ↑ ⇒ R ↓



Planar resistor

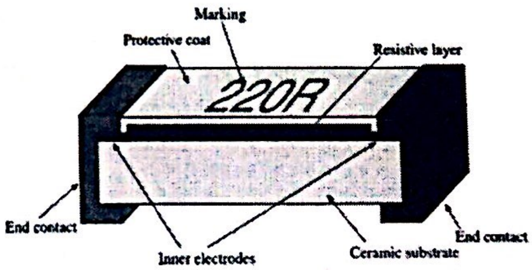
Types of Lumped Resistors (2)



Chip resistor

(2) Chip resistors: are realized by depositing a metal film (nichrome) layer on a ceramic body (aluminum oxide). Later resistive film is coated for protection.

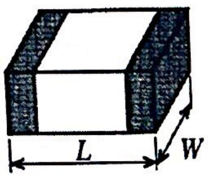
- Size of chip resistors can be as small as 40x20 mils (1 mil=0.001 inch) for 0.5W power ratings and up to 1x1 inch for 1KW ratings.



- Resistance value can range from 0.1Ω up to several MΩ.

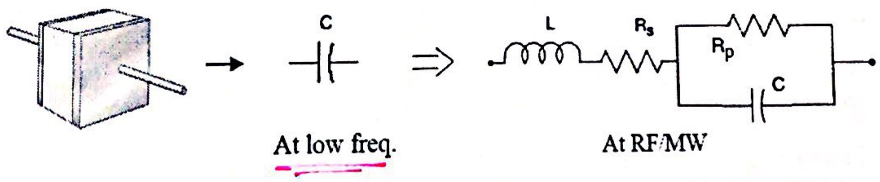
Types of Lumped Resistors (3)

To determine the size code, note that first two digit represent the length (in mil/10) and last two digit represent the width (in mil/10) of the component. These resistors are surface mounted to the circuit.

Geometry	Size Code	Length L, mils	Width W, mils
	0402	40	20
	0603	60	30
	0805	80	50
	1206	120	60
	1218	120	180

RF/MW Capacitors (1)

- At RF/MW frequencies, the parasitic elements of the capacitor become important.
- In the equivalent circuit, 'C' is actual capacitance, R_p is insulating resistance, R_s is series resistance (~δ_s) and 'L' is lead inductance.



RF/MW Capacitors (2)

- For **high frequency** and **high power** cases, capacitors are characterized by their dielectric loss {tan δ = σ/(ωε)}.
- Thus, higher losses yields high R_{eq} seen by microwave signal and also causes larger heat generation within capacitor (undesirable).
- Quality factor (Q) describes the ability of an element to store energy.
- Thus, high Q is desirable.
- The relation of Q and loss tangent is given by 'Q = 1/tan δ'.

$$Q = \omega \frac{\text{average energy stored}}{\text{energy loss/second}} = \omega \frac{W_m + W_e}{P_{\text{loss}}}$$

Capacitor power loss/dissipation at 500 MHz

Parameters	For Dielectric Substrate	
	Glass Epoxy	Alumina
tan δ	0.05	0.001
Quality factor (Q=1/tan δ)	20	1000
Capacitance (C)	10 pF	10 pF
Reactance (X_c=1/ωc) at 500 MHz	32 Ω	32 Ω
$R_{dissipative}=X_c/Q$	1.6 Ω	0.032 Ω
Current (I)	10 mA (*1A*)	10 mA (*1A*)
Power dissipated ($P=I^2 R_{diss}$)	0.16 mW (*1.6W*)	3.2 μW (*32mW*)

name of the material.

Types of Lumped Capacitors (1)

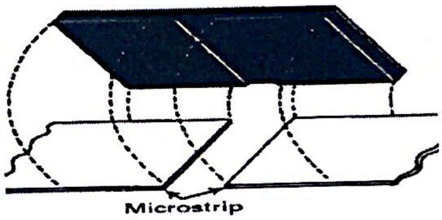
- There are two groups of planar capacitors suitable for RF/MW MIC's:

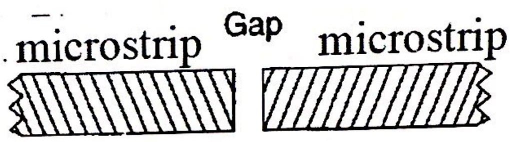
(1) Capacitors achieved with single metalization scheme, that are formed by fringing fields of the gap via the substrate.

Typically, **C < 1.0 pF** and **Q < 50**

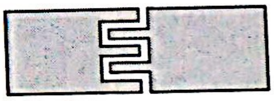
(a) GAP capacitor:

- mainly used as coupling capacitors (DC block)





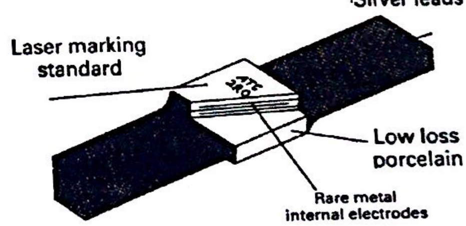
Types of Lumped Capacitors (2)

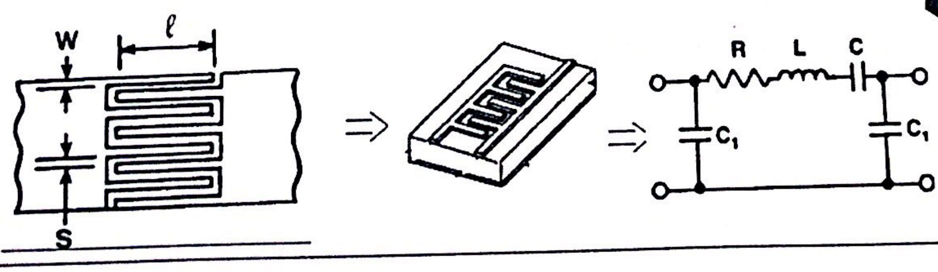


Interdigital gap capacitor

(b) Interdigitated Capacitors

- Used in MMIC due to easy way of construction and repeatability.
- Series capacitances is function of <u>number</u> and <u>length of fingers</u>.





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Analysis of Interdigitated Capacitors Based on TL Theory

- Capacitor dimensions are set to be much less than wavelength.
- Two terminal microstrip's can be represent as TL's and fingers can be considered as effective distributed shunt admittance across the terminal.
- Each pair of fingers can be analyzed as coupled microstrip with length 'l' and open-circuited at opposite ends that has even and odd mode "α's".
- To obtain maximum Q and minimum parasitic effect, **finger width** and **gap spacing** are utilized.
- Typically, for an interdigitated capacitors on 0.202 mm GaAs substrate:

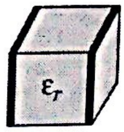
- To minimize area, low capacitances value are realized.
- To minimize losses, finger-width ≥ 10 µm and finger-gaps < 5 µm are used.
- Based on this values, the constructed capacitor behaved as lumped element up to 18 GHz.

- For high capacitance value (decoupling cap's), use **overlay** structures.

Interdigital Capacitors Dimensions and Associated Values

Physical Dimensions of Finger	INDIG<u>80</u>	INDIG180	INDIG300	INDIG400	Units
length, l (#)	<u>80</u>	180	300	400	µm
width, w	12	12	12	12	µm
spacing (side), S	8	8	8	8	µm
spacing (end), SE	12	12	12	12	µm
Thickness, t	5	5	5	5	µm
Number, n	20	20	20	20	~~µm~~
Sub. thickness, h	125	125	125	125	µm
Capacitances, C (⇑)	0.126	0.252	0.405	0.527	pF
Inductance, L (⇑)	0.000	0.025	0.064	0.101	nH
Resistance, R (⇓)	1.89	0.850	0.500	0.441	Ω
Shunt Capaci., C_S	0.028	0.052	0.080	0.104	pF

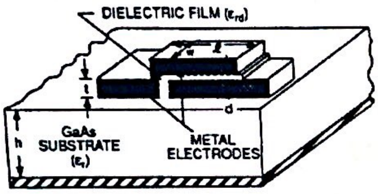
Types of Lumped Capacitors (3)

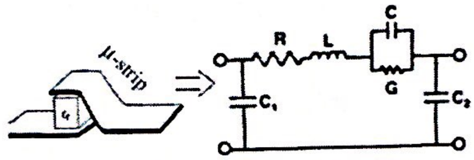


Chip capacitor

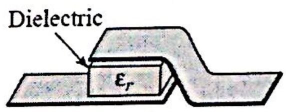
(2) Capacitors achieved with **overlay structures**, which use low loss dielectric films {Si_3N_4 (ε_r=6-8), Ta_5O_5 (ε_r=20-25)} between two metal plates, where top plate ('t' < δ_S R_0 is conductor loss) is connected via air-bridge (wire bond).

- **MIM capacitor**: for large capacitance value (**C < 25 pF**) and high Q **(50<Q<100)** "Metal Insulator Metal"





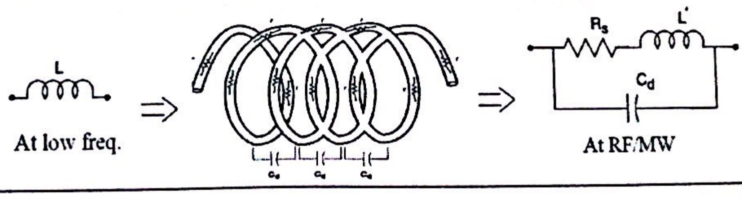
where, 'G' is the dielectric film loss
'C_1' & 'C_2' fringing cap. of top & bottom μ-strip



Metal-insulator-metal capacitor

RF/MW Inductors

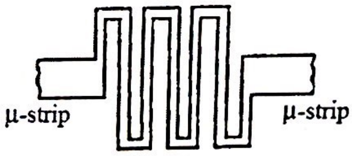
- At RF/MW frequencies, the parasitic elements of the inductor changes its behavior.
- At low frequencies, Q is large, thus R_S is small. But as frequency increases, Q is degraded by increasing value of R_S (due to δ_s) and thus C_d.
- Planar RF/MW inductors are made of single layer microstrip spirals or minder line, where mutual coupling between segments are exploited to achieve high inductance in a small area. Typically (lumped 'L'): **10 nH > L > 0.5nH** and **Q ≈ 50**
- To be lumped, total line segment should be small fraction of a wavelength.



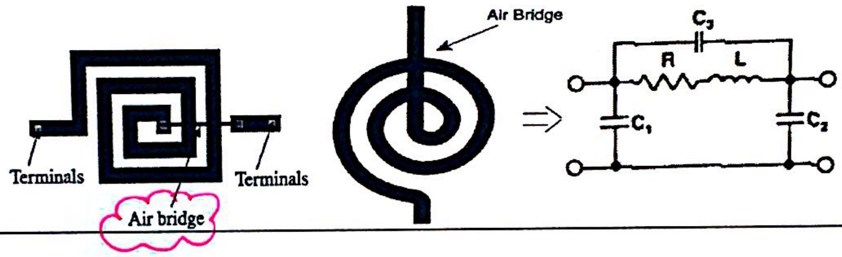
Types of Lumped Inductors (1)

- There are two major types of planar suitable for RF/MW MIC's:

(1) Meander line Inductance:

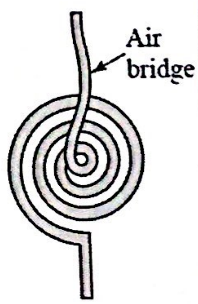


(2) Microstrip Coil inductors: are surface mounted flat inductors, useful in circuits with thickness constraints. Can easily integrate with microstrip TL's.



Types of Lumped Inductors (2)

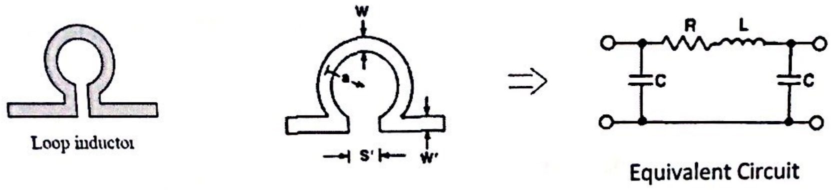
- Fabricated using standard Integrated Circuit (IC) process.
- Inner most turn is connected to external circuitry using air-bridge (in MMIC) and wire bond (in HMIC).
- The width and thickness of the conductor determines current carrying capacity of the inductor.
- Also if substrate height is reduced (GND plane come closer) then the inductance is reduced.
- For high Q, conductor thickness must be ≈ four times the skin depth.



Spiral inductor

Types of Lumped Inductors (3)

(3) Loop Inductor



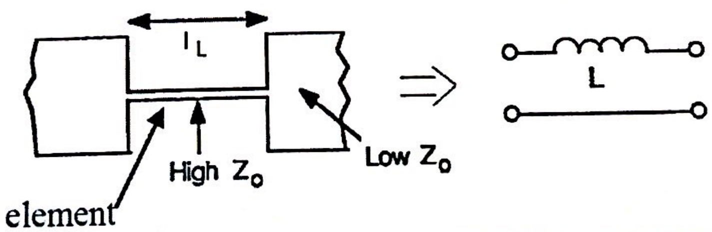
Step in Width Inductors for RF/MW Circuits

- Series inductance are also synthesized using short lengths of high impedance microstrip lines terminated in low impedance microstrip line.
- Characteristic impedance of a microstrip line is a function of its width 'w'. ('w'⇓, Z_0 ⇑).
- Inductance (L) of the circuit is expressed as;

$$L = \frac{l_L Z_{0L}}{f \lambda_g} \text{ Henry}$$

not a current

f = operating frequency
l_L = element length
Z_{0L} = element impedance
λ_g = guide wavelength



$\lambda_{eff.}$

$\lambda_g = \frac{c}{f\sqrt{\epsilon_r}}$

$\lambda_{g_e} = \frac{c}{f\sqrt{\epsilon_{r_e}}}$

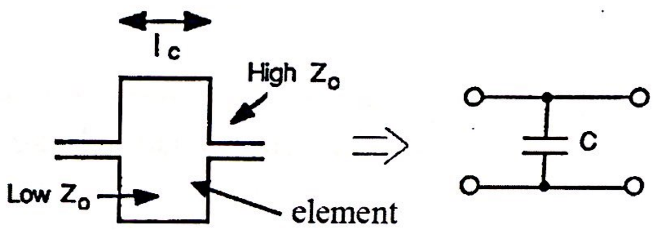
$W \propto \frac{1}{Z_0}$

Step in Width Capacitor for RF/MW Circuits

- shunt capacitance are also synthesized by terminating short lengths of low impedance microstrip lines by a high impedance lines.
- Capacitance value are;

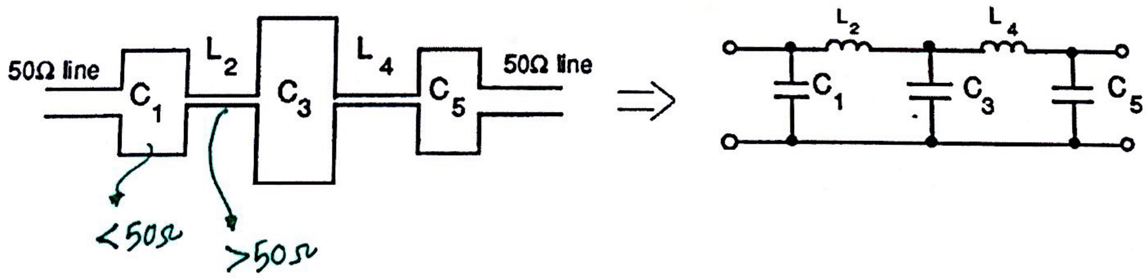
$$C = \frac{l_C}{Z_{0C}\,\lambda_g f} \quad \text{Farads}$$

f = operating frequency
l_C = element length
Z_{0C} = element impedance
λ_g = guide wavelength



Low Pass Filter Example

- Thus using these inductive and capacitive elements we can construct a Low Pass Filter as shown in the figure below:



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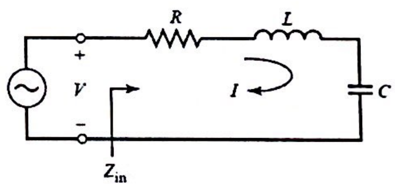
Microwave Resonators

- Microwave resonators are used in a variety of applications, including filters, oscillators, frequency meters, and tuned amplifiers.

- The operation of microwave resonators is very similar to that of lumped-element resonators of circuit theory.

- At frequencies near resonance, a microwave resonator can usually be modeled by either a series or parallel RLC lumped-element equivalent circuit.

Series Resonant Circuit (Series RLC)



$$Z_{in} = R + j\omega L - j\frac{1}{\omega C}$$

The complex power delivered to the resonator is:

$$P_{in} = \frac{1}{2}VI^* = \frac{1}{2}Z_{in}|I|^2 = \frac{1}{2}Z_{in}\left|\frac{V}{Z_{in}}\right|^2 = \frac{1}{2}|I|^2\left(R + j\omega L - j\frac{1}{\omega C}\right)$$

The power dissipated by the resistor R is: $P_{loss} = \frac{1}{2}|I|^2 R$

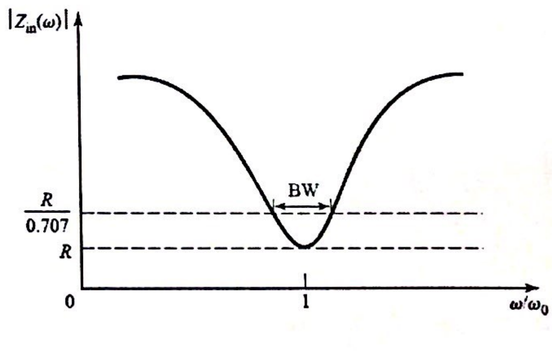
The average magnetic energy stored in the inductor L is: $W_m = \frac{1}{4}|I|^2 L$

The average electric energy stored in the capacitor C is:

$$W_e = \frac{1}{4}|V_c|^2 C = \frac{1}{4}|I|^2\frac{1}{\omega^2 C}$$

Then; $P_{in} = P_{loss} + 2j\omega(W_m - W_e)$

$$Z_{in} = \frac{2P_{in}}{|I|^2} = \frac{P_{loss} + 2j\omega(W_m - W_e)}{\frac{1}{2}|I|^2}$$



- Resonance occurs when the average stored magnetic and electric energies are equal ($W_m = W_e$). Then the input impedance at resonance is:

$$Z_{in} = \frac{P_{loss}}{\frac{1}{2}|I|^2} = R$$

$W_m = W_e$ ➔ implies that the resonant frequency, ω_0, can be defined as; $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Q = \omega \frac{\text{average energy stored}}{\text{energy loss/second}} = \omega \frac{W_m + W_e}{P_{loss}}$$

- Thus Q is a measure of the loss of a resonant circuit; lower loss implies a higher Q.
- Resonator losses may be due to **conductor loss, dielectric loss**, or **radiation loss**, and are represented by the resistance, R, of the equivalent circuit.
- An external connecting network may introduce additional loss. Each of these loss mechanisms will have the effect of lowering the Q.
- The Q of the resonator itself, disregarding external loading effects, is called the unloaded Q, denoted as Q_0.

$Q_0 = \omega_0 \frac{2W_m}{P_{loss}} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$ ➔ which shows that Q increases as R decreases. $\text{BW} = \frac{1}{Q_0}$

Parallel Resonant Circuit (Parallel RLC)

$$Z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)^{-1}$$

$$P_{in} = \frac{1}{2}VI^* = \frac{1}{2}Z_{in}|I|^2 = \frac{1}{2}|V|^2\frac{1}{Z_{in}^*} = \frac{1}{2}|V|^2\left(\frac{1}{R} + \frac{j}{\omega L} - j\omega C\right)$$

$$P_{loss} = \frac{1}{2}\frac{|V|^2}{R}$$

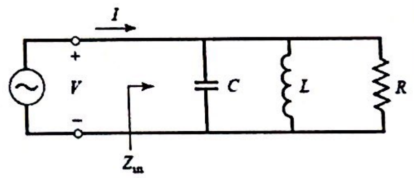
$$W_e = \frac{1}{4}|V|^2 C$$

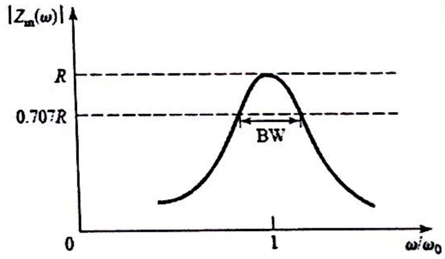
$$W_m = \frac{1}{4}|I_L|^2 L = \frac{1}{4}|V|^2\frac{1}{\omega^2 L}$$

$$P_{in} = P_{loss} + 2j\omega(W_m - W_e)$$

$$Z_{in} = \frac{2P_{in}}{|I|^2} = \frac{P_{loss} + 2j\omega(W_m - W_e)}{\frac{1}{2}|I|^2}$$

when $W_m = W_e$ $\quad Z_{in} = \frac{P_{loss}}{\frac{1}{2}|I|^2} = R$





$$BW = \frac{1}{Q_0}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

which is identical to the series resonant circuit case. Resonance in the case of a parallel RLC circuit is sometimes referred to as an anti-resonance

The unloaded Q of the parallel resonant circuit can be expressed as;

$$Q_0 = \omega_0 \frac{2W_m}{P_{loss}} = \frac{R}{\omega_0 L} = \omega_0 RC$$

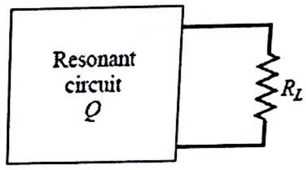
This result shows that the Q of the parallel resonant circuit increases as R increases.

Summary of Results for Series and Parallel Resonators

Quantity	Series Resonator	Parallel Resonator
Input impedance/admittance	$Z_{in} = R + j\omega L - j\frac{1}{\omega C}$	$Y_{in} = \frac{1}{R} + j\omega C - j\frac{1}{\omega L}$
	$\simeq R + j\frac{2RQ_0\Delta\omega}{\omega_0}$	$\simeq \frac{1}{R} + j\frac{2Q_0\Delta\omega}{R\omega_0}$
Power loss	$P_{loss} = \frac{1}{2}\lvert I\rvert^2 R$	$P_{loss} = \frac{1}{2}\frac{\lvert V\rvert^2}{R}$
Stored magnetic energy	$W_m = \frac{1}{4}\lvert I\rvert^2 L$	$W_m = \frac{1}{4}\lvert V\rvert^2 \frac{1}{\omega^2 L}$
Stored electric energy	$W_e = \frac{1}{4}\lvert I\rvert^2 \frac{1}{\omega^2 C}$	$W_e = \frac{1}{4}\lvert V\rvert^2 C$
Resonant frequency	$\omega_0 = \frac{1}{\sqrt{LC}}$	$\omega_0 = \frac{1}{\sqrt{LC}}$
Unloaded Q	$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$	$Q_0 = \omega_0 RC = \frac{R}{\omega_0 L}$
External Q	$Q_e = \frac{\omega_0 L}{R_L}$	$Q_e = \frac{R_L}{\omega_0 L}$

Loaded and Unloaded Q

- The unloaded Q (Q_0) is a characteristic of the resonator itself, in the absence of any loading effects caused by external circuitry.
- In practice, however, a resonator is invariably coupled to other circuitry, which will have the effect of lowering the overall, or loaded Q (Q_L) of the circuit.
- Define Q_e as an external Q



A resonant circuit connected to an external load R_L

$$Q_e = \begin{cases} \dfrac{\omega_0 L}{R_L} & \text{for series circuits} \\ \dfrac{R_L}{\omega_0 L} & \text{for parallel circuits} \end{cases}$$

$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_0}$$

Transmission Line Resonators

- Ideal lumped circuit elements are often unattainable at microwave frequencies, so distributed elements are frequently used.
- The use of transmission line sections with various lengths and terminations (usually open- or short circuited) to form resonators are introduced.
- Because we are interested in the Q of these resonators, we must consider transmission lines with losses.

write it on formula sheet.

$Zin_{sc} = jZ_0 \tan\beta l \Rightarrow j\tan\beta l \equiv \tanh\gamma l.$

A. Short-Circuited λ/2 Line

$$Z_{in} = Z_0 \tanh(\alpha + j\beta)\ell$$

$$Z_{in} = Z_0 \frac{\tanh\alpha\ell + j\tan\beta\ell}{1 + j\tan\beta\ell\tanh\alpha\ell}$$

$Z_{in} = jZ_0\tan\beta\ell$ if $\alpha = 0$ (a lossless line)

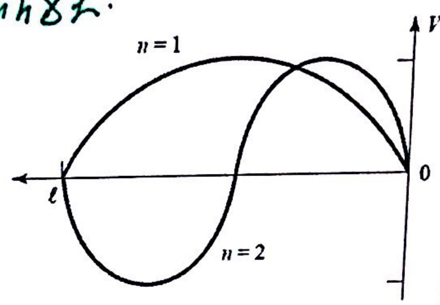
$$R = Z_0\alpha\ell$$

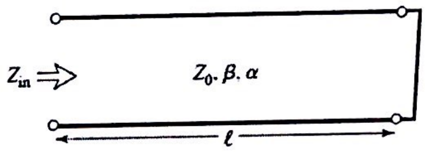
$$L = \frac{Z_0\pi}{2\omega_0}$$

$$C = \frac{1}{\omega_0^2 L}$$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{\pi}{2\alpha\ell} = \frac{\beta}{2\alpha} \quad ; \ l = \frac{\lambda}{2}$$

$\beta\ell = \pi$ at the first resonance. This result shows that the Q decreases as the attenuation of the line increases.





A short-circuited length of lossy transmission line, and the voltage distributions for n = 1 ($\ell = \lambda/2$) and n = 2 ($\ell = \lambda$) resonators.

Example: Q of Half-Wave Coaxial Line Resonator

- A λ/2 resonator is made from a piece of copper coaxial line having an inner conductor radius of 1 mm and an outer conductor radius of 4 mm. If the resonant frequency is 5 GHz, compare the unloaded Q of an air-filled coaxial line resonator to that of a Teflon-filled coaxial line resonator. (conductivity of copper is $\sigma = 5.813 \times 10^7$ S/m)

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Solution:

$$R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 1.84 \times 10^{-2}\ \Omega$$

α_d = Zero. Since σ_{air} = Zero.

The attenuation due to conductor loss for the air-filled line is:

$$\alpha_c = \frac{R_s}{2\eta \ln b/a}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{1.84 \times 10^{-2}}{2(377)\ln(0.004/0.001)}\left(\frac{1}{0.001}+\frac{1}{0.004}\right) = 0.022\ \text{Np/m}$$

For Teflon, ε_r = 2.08 and tan δ = 0.0004, so the attenuation due to conductor loss for the Teflon-filled line is:

$$\alpha_c = \frac{1.84 \times 10^{-2}\sqrt{2.08}}{2(377)\ln(0.004/0.001)}\left(\frac{1}{0.001}+\frac{1}{0.004}\right) = 0.032\ \text{Np/m}$$

The dielectric loss of the air-filled line is zero, but the dielectric loss of the Teflon filled line is:

$$\alpha_d = k_0\frac{\sqrt{\epsilon_r}}{2}\tan\delta = \frac{(104.7)\sqrt{2.08}(0.0004)}{2} = 0.030\ \text{Np/m}$$

$$Q_{\text{air}} = \frac{\beta}{2\alpha} = \frac{104.7}{2(0.022)} = 2380$$

$$Q_{\text{Teflon}} = \frac{\beta}{2\alpha} = \frac{104.7\sqrt{2.08}}{2(0.032+0.030)} = 1218$$

Thus it is seen that the Q of the air-filled line is almost twice that of the Teflon filled line. The Q can be further increased by using silver-plated conductors.

$Q_{Teflon} << Q_{air}$ $Q\uparrow \Rightarrow$ losses$\downarrow$

write it on formula sheet.

B. Short-Circuited λ/4 Line

$$Z_{\text{in}} = Z_0 \tanh(\alpha + j\beta)\ell = Z_0\frac{\tanh\alpha\ell + j\tan\beta\ell}{1 + j\tan\beta\ell\tanh\alpha\ell} = Z_0\frac{1 - j\tanh\alpha\ell\cot\beta\ell}{\tanh\alpha\ell - j\cot\beta\ell}$$

Assume that $\ell = \lambda/4$ at $\omega = \omega_0$, and let $\omega = \omega_0 + \omega$. Then, for a TEM line,

$$\beta\ell = \frac{\omega_0\ell}{v_p} + \frac{\Delta\omega\ell}{v_p} = \frac{\pi}{2} + \frac{\pi\Delta\omega}{2\omega_0}$$

$$R = \frac{Z_0}{\alpha\ell}$$

$$C = \frac{\pi}{4\omega_0 Z_0}$$

$$L = \frac{1}{\omega_0^2 C}$$

$$Q_0 = \omega_0 RC = \frac{\pi}{4\alpha\ell} = \frac{\beta}{2\alpha}$$

C. Open-Circuited $\lambda/2$ Line

$$Z_{in} = Z_0 \coth(\alpha + j\beta)\ell = Z_0 \frac{1 + j \tan \beta\ell \tanh \alpha\ell}{\tanh \alpha\ell + j \tan \beta\ell}$$

Assume that $\ell = \lambda/2$ at $\omega = \omega_0$, and let $\omega = \omega_0 + \omega$. Then, for a TEM line,

$$\beta\ell = \pi + \frac{\pi \Delta\omega}{\omega_0}$$

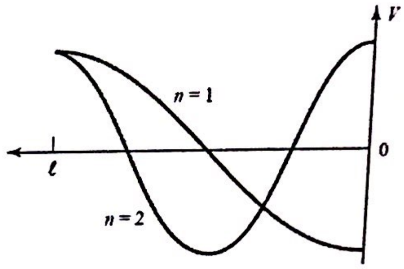
$$R = \frac{Z_0}{\alpha\ell}$$

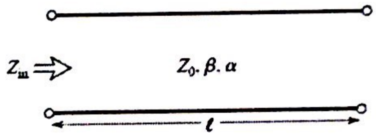
$$C = \frac{\pi}{2\omega_0 Z_0}$$

$$L = \frac{1}{\omega_0^2 C}$$

$$Q_0 = \omega_0 RC = \frac{\pi}{2\alpha\ell} = \frac{\beta}{2\alpha}$$

since $\ell = \pi/\beta$ at resonance





Example: A Half-Wave Microstrip Resonator

- Consider a microstrip resonator constructed from a $\lambda/2$ length of 50 Ω open circuited microstrip line. The substrate is Teflon ($\varepsilon_r = 2.08$, tan δ = 0.0004), with a thickness of 0.159 cm, and the conductors are copper. Compute the required length of the line for resonance at 5 GHz, and the unloaded Q of the resonator. Ignore fringing fields at the end of the line.

Use the equations in CH3.

Solution:

$$\ell = \frac{\lambda}{2} = \frac{v_p}{2f} = \frac{c}{2f\sqrt{\epsilon_e}} = \frac{3 \times 10^8}{2(5 \times 10^9)\sqrt{1.80}} = 2.24 \text{ cm}$$

$$\beta = \frac{2\pi f}{v_p} = \frac{2\pi f \sqrt{\epsilon_e}}{c} = \frac{2\pi(5 \times 10^9)\sqrt{1.80}}{3 \times 10^8} = 151.0 \text{ rad/m}$$

$$\alpha_c = \frac{R_s}{Z_0 W} = \frac{1.84 \times 10^{-2}}{50(0.00508)} = 0.0724 \text{ Np/m}$$

$$\alpha_d = \frac{k_0 \epsilon_r (\epsilon_e - 1)\tan\delta}{2\sqrt{\epsilon_e}(\epsilon_r - 1)} = \frac{(104.7)(2.08)(0.80)(0.0004)}{2\sqrt{1.80}(1.08)} = 0.024 \text{ Np/m}$$

$$Q_0 = \frac{\beta}{2\alpha} = \frac{151.0}{2(0.0724 + 0.024)} = 783$$

solve this question for $\lambda/2$ short circuit

& compare the answer of Q_0 with 783.