Chapter 5: Evaluating a Single Project

Objective

Applying several methods to evaluate the economic profitability of a single alternative (proposed project).

Minimum attractive rate of return (MARR)

- Sometimes called the <u>hurdle rate</u>.
- Established by the organization.
- A project must provide a return that is equal to or greater than the MARR.

Methods for evaluating a single project

- Present worth (PW): most common method.
- Future worth (FW).
- Annual worth (AW).
- Internal rate of return (IRR).
- External rate of return (ERR).
- Payback period: least common method.

Present worth (PW).

- All cash inflows and outflows are discounted to the present time at an interest rate (generally MARR).
- PW $(i = \text{MARR}) \geq 0 \Rightarrow$ acceptable project (profit required by investors is satisfied or exceeded).

Example: A project has a capital investment of $50,000 and returns $18,000 per year for 4 years. At a 12% MARR, is this a good investment?

PW = -50,000 + 18,000 $(P/A, 12\%, 4)$

PW = -50,000 + 18,000 × 3.0373 = $4,671.40 → It is a good investment.

Example

A new heating system is to be purchased and installed for $110,000. This system will save approximately 300,000 kWh of electric power each year for a 6-year period with no additional O&M costs. Assume the cost of electricity is $0.10 per kWh, and company's MARR is 15% per year, and the market value of the system will be $8,000 at EOY 6. Using the PW method, is this a good idea?

Estimated annual savings = $\frac{300{,}000 \text{ kWh}}{\text{year}} \times \frac{\$0.10}{\text{kWh}} = \$30{,}000 \text{ per year.}$

PW $(i = 15\%) = -\$110{,}000 + \$30{,}000\ (P/A, 15\%, 6) + \$8{,}000\ (P/F, 15\%, 6)$

$$= -\$110{,}000 + \$30{,}000 \times 3.7845 + \$8{,}000 \times 0.4323 = \$\ 6{,}993.4$$

$\Rightarrow$ Good investment

Bond value: application of PW method

- PW is used to determine the commercial value of a bond.

$$V_N = C\ (P/F\ , i\%, N) + rZ(P/A\ , i\%, N)$$

Where:

V_N: value (price) of the bond N interest periods prior to redemption (or present worth).

Z: face, or par value of the bond.

C: redemption or disposal price (usually equal to Z).

r: bond rate (nominal interest) per interest period.

N: number of periods before redemption.

i: bond yield rate per period.

Example – bonds

A bond has a face value of \$10,000 and matures in 8 years. The bond stipulates a fixed nominal interest of 8% per year, but interest payments are made to the bondholder every 3 months. The bondholder wishes to earn 10% nominal annual interest (compounded quarterly). Assuming the redemption value is equal to the face value, how much should be paid for the bond now?

Z = face value = \$10,000.

C = redemption value = \$10,000.

i = 10% nominal per year = 2.5% per quarter.

N = 8 years = 32 quarters.

r = 8% per year = 2% per quarter.

$$V_N = \$10{,}000\ (P/F\ , 2.5\%, 32) + 0.02 \times \$10{,}000\ (P/A\ , 2.5\%, 32) = \$8{,}907.55$$

⇒ The bondholder should pay no more than \$8,907.55 for the purchase of the bond.

Example – bonds

A bond with a face value of \$5,000 pays 8% interest per year. This bond will be redeemed at par value at the end of its 20-year life and the first interest payment is due one year from now.

- How much should be paid now for this bond to receive a yield of 10% per year?

$$V_N = \$5{,}000\ (P/F\ , 10\%, 20) + 0.08 \times \$5{,}000\ (P/A\ , 10\%, 20) = \$4{,}148.44$$

- If this bond is purchased now for \$4,600, what annual yield would the buyer receive?

$V_N =$ \$4,600.

$$\$4{,}600 = \$5{,}000\ (P/F\ , i'\%, 20) + 0.08 \times \$5{,}000\ (P/A\ , i'\%, 20)$$

i'=8.9% per year.

Capitalized worth

- Capitalized worth (CW): special case of PW; where revenues or expenses occur over an infinite length of time.
- If only expenses are considered, then it is called *capitalized cost*.

$$CW = A\ (1/i)$$

Example: a bridge was constructed at a cost of \$1,900,000 and the annual upkeep cost is \$25,000. It is also estimated that maintenance will be required at a cost of \$350,000 every 8 years. What is the capitalized worth of the bridge over its life assuming MARR = 8%?

$$CW(8\%) = -\$1{,}900{,}000\ - \$350{,}000 \frac{A/F\ , 8\%, 8}{0.08} - \frac{\$25{,}000}{0.08}$$

$$= -\$2{,}623{,}815$$

Future worth (FW).

- Equivalent worth of all cash inflows and outflows at the end of the study period.
- FW is equivalent to PW $[FW = PW\ (F/P\ , i\%, N)]$.
- FW ≥ 0, project is economically justified.

<u>Example</u>: A $45,000 investment in a new conveyer system is projected to improve throughput and increase revenue by $14,000 per year for five years. The estimated market value of the conveyer at the end of five years is $4,000. Using the FW method at a MARR of 12%, is this a good investment?

$FW = -\$45{,}000\ (F/P\ , 12\%, 5) + \$14{,}000\ (F/A\ , 12\%, 5) + \$4{,}000 = \$13{,}635.7$
⇒ It is a good investment.
PW?

Annual worth (AW).

- Equal annual series equivalent to the cash inflows and outflows at a specific interest rate (normally MARR).
- AW is equivalent to PW and FW.
- AW ≥ 0, project is economically justified.

$$AW(i\%) = R - E - CR(i\%)$$

Where:

R: annual equivalent revenue.

E: annual equivalent expenses.

CR: annual capital recovery which covers the loss in value of the asset and interest (at MARR) on invested capital.

$$CR(i\%) = I\ (A/P\ , \mathrm{i}\%, N) - S\ (A/F\ , \mathrm{i}\%, N)$$

Where I is the initial cost and S is the salvage value.

Example

A project requires an initial investment of \$45,000, has a salvage value of \$12,000 after six years, incurs annual expenses of \$6,000, and provides annual revenue of \$18,000. Using a MARR of 10%, determine the AW of this project.

$$AW(i\%) = R - E - CR(i\%)$$

$$CR(10\%) = \$45{,}000\ (A/P\ , 10\%, 6) - \$12{,}000\ (A/F\ , 10\%, 6) = \$8{,}777$$

$$R = \$18{,}000$$

$$E = \$6{,}000$$

$$AW(10\%) = \$18{,}000 - \$6{,}000 - \$8{,}777 = \$3{,}223$$

⇒ It is a good investment.

Internal rate of return (IRR)

- Most widely used rate of return method in engineering economic analysis.
- Also called the investor's method, the discounted cash flow method, and the profitability index.
- IRR ≥ MARR; project is economically justified.
- IRR is the interest rate at which:

$$equivalent\ worth\ of\ cash\ inflows = equivalent\ worth\ of\ cash\ outflows$$

Using PW:

$$\sum_{k=0}^{N} R_k\ (P/F\ , i'\%, k) = \sum_{k=0}^{N} E_k\ (P/F\ , i'\%, k)$$

Where:

R_k: net revenue or savings for the k^{th} year.

E_k: net expenditures including any investment costs for the k^{th} year.

Example

An equipment requires a capital investment of \$345,000, has a salvage value of \$115,000 after six years, annual expenses of \$22,000, and provides annual revenue of \$120,000. Using a MARR of 20%, is the purchase of this equipment a good decision according to the IRR method?

Set PW = 0

$$0 = -\$345{,}000 + (\$120{,}000 - \$22{,}000)\ (P/A\,, i'\%, 6) + \$115{,}000\ (P/F\,, i'\%, 6)$$

<u>To find i' or the IRR:</u>

- Interpolation.
- Trial and error.
- Calculators with solver.
- Spreadsheets.

IRR $\approx$ 22.16% $>$ 20% $\Rightarrow$ Project is acceptable.

External rate of return (ERR)

- Interest rate (Ɛ) for reinvesting net cash flows (usually equals MARR).
- Steps to solve:
  - Discount all net cash outflows to time 0 (present) at Ɛ% per compounding period.
  - Compound all net cash inflows to period N at Ɛ% per compounding period.
  - Solve for ERR which is i' at which:

$$\sum_{k=0}^{N} R_k\ (F/P\,, \varepsilon\%, N-k) = \sum_{k=0}^{N} E_k\ (P/F\,, \varepsilon\%, k)(F/P\,, i'\%, N)$$

Where:

R_k: excess of receipts over expenses in period k.

E_k: excess of expenses over receipts in period k.

ε: external reinvestment rate per compounding period.

- ERR $\geq$ MARR; project is economically justified.

Example

For the cash flows given below, find the ERR when the external reinvestment rate (ε) = MARR = 12%.

Year	0	1	2	3	4
Cash flow	-$15,000	-$7,000	$10,000	$10,000	$10,000

Expenses: $\$15{,}000 + \$7{,}000\ (P/F\ ,12\%, 1) = \$21{,}250$

Revenue: $\$10{,}000\ (F/A\ ,12\%, 3) = \$33{,}744$

<u>Solving for ERR</u>:

$\$21{,}250\ (F/P\ , i'\%, 4) = \$33{,}744$ $\Rightarrow i' = 16.67\% > 12\%$ (acceptable project)

Payback (payout period)

- Number of years required for cash inflows to just equal cash outflows.
- Low-valued payback period is desired.
- It is a measure of <u>liquidity</u> rather than <u>profitability</u>; hence, it can be misleading (other methods are recommended).
- Two types of payback period:
  - <u>Simple payback period</u>: ignores the time value of money

$$\sum_{k=1}^{\theta}(R_k - E_k) - I \geq 0, \text{where } \theta \text{ is the simple payback period } (\theta \leq N)$$

  - <u>Discounted payback period</u>: time value of money is considered

$$\sum_{k=1}^{\theta'}(R_k - E_k)\ (P/F\ , i\%, k) - I \geq 0, \text{where } \theta' \text{ is the discounted payback period } (\theta' \leq N)$$

Examples

- An investment of \$5,000,000 yields net annual revenues of \$1,500,000. What is the simple payback period?

$$\text{Payback period} = \frac{\$5{,}000{,}000}{\$1{,}500{,}000} = 3.33 \text{ years}$$

$$\Rightarrow \text{Simple payback period } (\theta) \text{ is 4 years.}$$

- For the following cash flows, what is the simple and discounted payback periods at i = 6%?

EOY	0	1	2	3	4	5
Net cash flow	-\$42,000	\$12,000	\$11,000	\$10,000	\$10,000	\$9,000

EOY	Net cash flow	Cumulative PW at 0%	Cumulative PW at 6%
0	-\$42,000	-\$42,000	-\$42,000
1	\$12,000	-\$30,000	-\$30,479
2	\$11,000	-\$19,000	-\$20,880
3	\$10,000	-\$9,000	-\$12,493
4	\$10,000	\$1,000	-\$4,572
5	\$9,000		\$2,153

Keep calculating cumulative PW until reaching a positive value

From the table: $\theta = 4$ years and $\theta' = 5$ years

Chapter 6: Comparison and Selection Among Alternatives

Objective

Correctly evaluating investment alternatives when the time value of money is considered.

Alternatives:

- **Mutually exclusive**: selection of one alternative excludes the others.
- **Independent**: selection of one alternative does not exclude other alternatives.

Comparing different alternatives

- Acceptable alternatives: feasible alternatives + satisfy at least MARR.
- Acceptable alternative with the least capital investment is called the base alternative.
- If the incremental investment (over the base alternative) is justified by extra benefits ⇒ investment should be made.

Investment and cost alternatives

- Investment alternatives: involve initial (capital) investment and produce positive cash flows due to increased revenue or savings, reduced costs, or both.
  - PW of all acceptable alternatives ≥ 0 ⇒ Select the alternative with the largest PW.
- Cost alternatives: all negative cash flows (decision involves the most economical way of conducting an activity/project).
  - PW of all alternatives < 0 ⇒ select the alternative with the largest (smallest absolute value) PW.

Example

N = 4 years	Alternative		
	A	*B*	Δ(*B* – *A*)
Capital investment	–$60,000	–$73,000	–$13,000
Annual revenues less expenses	22,000	26,225	4,225

- MARR = 10%.

PW (10%) $_A$ = -$60,000 + $22,000 (P/A, 10%, 4) = $9,738.
PW (10%) $_B$ = -$73,000 + $26,225 (P/A, 10%, 4) = $10,131.
PW (10%) $_{B-A}$ = -$13,000 + $4,225 (P/A, 10%, 4) = $393.

⇒ Both alternatives are acceptable (PW @ MARR $\geq$ 0) … investment alternatives.
⇒ Alternative A is the *base alternative* (acceptable + lowest capital).

Select alternative B (higher PW @ MARR) ⇒ additional capital investment in B is justified.
Also, PW $_{B-A}$ is positive ⇒ additional investment is justified.

*** What if A and B were independent alternatives?

Example

End of Year	Alternative		
	C	*D*	Δ(*D* – *C*)
0	–$380,000	–$415,000	–$35,000
1	–38,100	–27,400	10,700
2	–39,100	–27,400	11,700
3	–40,100	–27,400	12,700
3[a]	0	26,000	26,000

[a] Market value.

- MARR = 10%.

⇒ Alternative C is the *base alternative* (lowest capital).

PW (10%) $_C$ = -$380,000 – $38,100 (P/A, 10%, 3) – $1,000 (P/G, 10%, 3) = -$477,077.

PW (10%) $_D$ = -$415,000 – $27,400 (P/A, 10%, 3) + $26,000 (P/F, 10%, 3) = -$463,607.

PW (10%) $_{D-C}$ = -$35,000 + $10,700 (P/A, 10%, 3) + $1,000 (P/G, 10%, 3) + $ 26,000 (P/F, 10%, 3) = +$13,470.

⇒ PW @ MARR $<$ 0 for both alternatives … cost alternatives.

Select alternative D (higher PW @ MARR) ⇒ additional capital investment in D is justified.

Study period

Study period (*or planning horizon*): selected time period over which mutually exclusive alternatives are compared.

<u>Study period cases</u>:

- Useful lives of all mutually exclusive alternatives (MEAs) are the same and equal to the study period.
  - ⇒ No cash flow adjustment.
  - ⇒ MEAs are compared using equivalent worth methods (PW, FW, or AW) or rate of return methods (IRR or ERR).
- Useful lives are unequal and at least one does not match the study period.
  - ⇒ Repeatability assumption.
  - ⇒ Coterminated assumption.

Example - equal useful lives = study period

	Alternatives			
	A	B	C	D
Capital investment	-$150,000	-$85,000	-$75,000	-$120,000
Annual revenues	$28,000	$16,000	$15,000	$22,000
Annual expenses	-$1,000	-$550	-$500	-$700
Market Value (EOL)	$20,000	$10,000	$6,000	$11,000
Life (years)	10	10	10	10

Use a <u>MARR of 12%</u> to select one alternative among the four MEAs.

PW_A = - $150,000 + $27,000 (P/A, 12%, 10) + $20,000 (P/F, 12%, 10) = $8,995.

PW_B = - $85,000 + $15,450 (P/A, 12%, 10) + $10,000 (P/F, 12%, 10) = $5,516.

PW_C = - $75,000 + $14,500 (P/A, 12%, 10) + $6,000 (P/F, 12%, 10) = $8,860.

PW_D = - $120,000 + $21,300 (P/A, 12%, 10) + $11,000 (P/F, 12%, 10) = $3,891.

All alternatives are acceptable (PW > 0) ⇒ C is the base alternative (lowest capital).

⇒ Select alternative A (highest PW)

Repeat using AW and FW methods → Should get the same conclusion (not the same numbers though).

Example

A company is planning to install a new automated plastic-molding press. Four alternatives are considered as shown in the table. Assuming that each press has the same output capacity (120,000 units per year) with no market value at the end of its useful life, and using a 10% MARR, which machine will you select?

	Alternatives			
	A	B	C	D
Capital investment	$24,000	$30,400	$49,600	$52,000
Total annual expenses	$31,200	$29,128	$25,192	$22,880
Life (years)	5	5	5	5

Same output ⇒ same revenue ⇒ minimize cost

Calculate the PW of all costs and select the one with the highest value.

	Alternatives			
	A	B	C	D
Present worth	-$142,273	-$140,818	-$145,098	-$138,734
Annual worth	-$37,531	-$37,148	-$38,276	-$36,598
Future worth	-$229,131	-$226,788	$233,689	$-223,431

Example

Three mutually exclusive design alternatives are being considered. The estimated cash flows for each alternative are given in the following table. At a MARR of 20% per year, which one will you select?

	A	B	C
Investment cost	$28,000	$55,000	$40,000
Annual expenses	$15,000	$13,000	$22,000
Annual revenues	$23,000	$28,000	$32,000
Market value	$6,000	$8,000	$10,000
Useful life	10 years	10 years	10 years
IRR	26.4%	24.7%	22.4%

PW (20%) $_A$ = -$28,000 + ($23,000 – $15,000) (P/A, 20%, 10) + $6,000 (P/F, 20%, 10) = $6,509.

PW (20%) $_B$ = -$55,000 + ($28,000 – $13,000) (P/A, 20%, 10) + $8,000 (P/F, 20%, 10) = $9,180.

PW (20%) $_C$ = -$40,000 + ($32,000 – $22,000) (P/A, 20%, 10) + $10,000 (P/F, 20%, 10) = $3,540.

⇒ Select B.

*** selection based on maximum IRR is wrong ***

Rate of return method

- DO NOT judge by just comparing the IRR of alternatives (selecting the alternative with the highest IRR can lead to incorrect decisions).
- The IRR of the increments is computed and compared against MARR.
- Steps:
  - Arrange feasible alternatives based on increasing capital investment.
  - Establish a base alternative:
    - Cost alternatives: first alternative in the ranked order is the base.
    - Investment alternatives: first acceptable alternative (IRR ≥ MARR) in the ranked order is the base.
  - Evaluate differences (incremental cash flows) between each two successive alternatives starting with the base until all have been considered. If the IRR of the incremental cash flow (2 – 1) is greater than or equal to MARR, 2 becomes the new base and 1 is eliminated.
  - Continue until all alternatives have been considered and a "winner" is found.

Example

N = 4 years	Alternative		
	A	*B*	$\Delta(B-A)$
Capital investment	−$60,000	−$73,000	−$13,000
Annual revenues less expenses	22,000	26,225	4,225

- MARR = 10%.

Calculate PW and IRR for both alternatives.

Alternative	IRR	PW (10%)
A	**17.3%**	**$9,738**
B	**16.3%**	**$10,131**

Both have positive PW, IRR > MARR … both acceptable alternatives.

- Based on PW ⇒ select B (PW method is **always correct**).
- Based on IRR ⇒ select A (misleading).
- To solve using IRR, find the IRR of the incremental cash flow (B – A)

$IRR_{B\text{-}A}$ = 11.4% > MARR … The incremental investment ($13,000) is justified.

$PW_{B\text{-}A}$ = $393 > 0 … same conclusion.

Select alternative B.

Example – IRR method

Six mutually exclusive alternatives with equal useful lives (10 years) are analyzed and compared using the IRR method. Assume MARR = 10%, which alternative will you select?

	A	B	C	D	E	F
Capital investment	$900	$1,500	$2,500	$4,000	$5,000	$7,000
Net annual income	$150	$276	$400	$925	$1,125	$1,425
IRR	10.6%	13.0%	9.6%	19.1%	18.3%	15.6%

- IRR is computed for each alternative … alternatives are ranked from lowest to highest capital.
- IRR < MARR for alternative C … C is eliminated ⇒ The rest of alternatives (A, B, D, E, and F) are all acceptable alternatives.
- Alternative A has the lowest capital among all acceptable alternatives … A is the base alternative.

B becomes the new base (A is eliminated)

	A	B – A	D – B	E – D	F – E
Δ Capital	$900	$600	$2,500	$1,000	$2,000
Δ Annual income	$150	$126	$649	$200	$300
IRR Δ	10.6%	16.4%	22.6%	15.1%	8.1%
Is increment justified?	-	yes	yes	yes	no

Select E

Example – IRR method

Four alternatives are compared at MARR = 9%.

C – A

Alternative	Capital	IRR	Δ IRR		
			A	B	C
A	$15,000	12%	-	-	-
B	$20,000	15%	15%	-	-
C	$25,000	10%	9%	10%	-
D	$30,000	20%	9.5%	12%	7%

- If all alternatives are independent, which one will you select?

All (all have IRR > MARR and the selection of one does not exclude others).

- If all alternatives are mutually exclusive, which one will you select?

Alternative C.

Unequal useful lives

Repeatability assumption:

- If the study period is infinite in length or a common multiplier of the useful lives.

- The useful lives of alternatives are repeated in subsequent cycles.

- Not common in engineering economy problems.

Example – repeatability

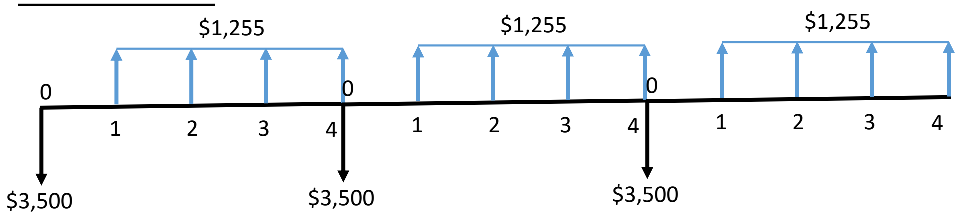
Two mutually exclusive alternatives with different useful lives. If MARR = 10% per year, and using the repeatability assumption, which alternative would you pick?

	A	B
Capital investment	\$3,500	\$5,000
Annual net cash flow	\$1,255	\$1,480
Useful lives (years)	4	6
Market value at end of useful life	0	0

- The least common multiple of useful lives = 12 years.
  - A is repeated 3 times.
  - B is repeated 2 times.

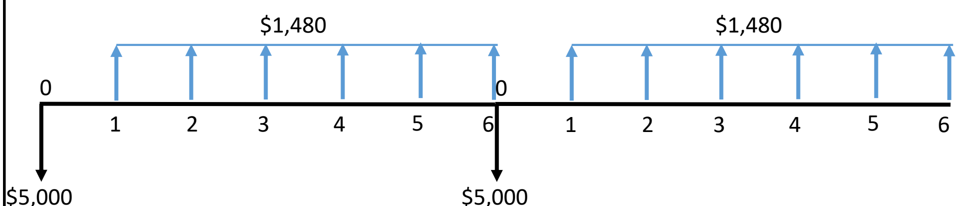
Example – repeatability

Alternative A



PW (10%)$_A$ = -\$3,500 – \$3,500 [(P/F, 10%, 4) + (P/F, 10%, 8)] + \$1,255 (P/A, 10%, 12) = \$1,028.

Alternative B



Select B

PW (10%)$_B$ = -\$5,000 – \$5,000 (P/F, 10%, 6) + \$1,480 (P/A, 10%, 12) = \$2,262.

Repeatability example – cont'd

Solve using the annual worth method

AW (10%)$_A$ = -\$3,500 (A/P, 10%, 12) – \$3,500 [(P/F, 10%, 4) + (P/F, 10%, 8)] (A/P, 10%, 12) + \$1,255 = \$151.

Or AW (10%)$_A$ = PW (10%)$_A$ × (A/P, 10%, 12) = \$151.

AW (10%)$_B$ = -\$5,000 (A/P, 10%, 12) – \$5,000 (P/F, 10%, 6) (A/P, 10%, 12) + \$1,480 = \$332.

Or AW (10%)$_A$ = PW (10%)$_B$ × (A/P, 10%, 12) = \$332.

Select B

Calculate the annual worth of each alternative over one useful life cycle:

AW (10%)$_A$ = -\$3,500 (A/P, 10%, 4) + \$1,255 = \$151.

AW (10%)$_B$ = -\$5,000 (A/P, 10%, 6) + \$1,480 = \$332.

⇒ Same result ⇒ For repeatability assumption, we can calculate the AW for each alternative over its own useful (single) life and compare directly.

Coterminated assumption

- If repeatability assumption is not applicable ⇒ <u>coterminated assumption</u>
- Coterminated assumption is more common in engineering practice.

1) Useful life < study period
   a) Cost alternatives:
   - Contracting or leasing equipment/service for the remaining years.
   - Repeat part of the useful life and truncate at the end of the study period with an estimated market value.

   b) Investment alternatives:
   - Cash flows reinvested at the MARR to the end of the study period.
   - Replace with another asset with possibly different cash flows over the remaining life.
2) Useful life > study period: truncate at the end of the study period with an estimated market value.

Example – coterminated

Two mutually exclusive alternatives with different useful lives. If MARR = 10% per year, and the <u>study period is 6 years</u>, which alternative would you pick?

	A	B
Capital investment	\$3,500	\$5,000
Annual cash flow	\$1,255	\$1,480
Useful lives (years)	4	6
Market value at end of useful life	0	0

- 6 years isn't a multiple of both lives ⇒ repeatability isn't applicable.
- Coterminated assumption:

<u>For A</u>, useful life < study period … needs cash flow adjustment.
Assume money will be reinvested at MARR until the end of the study period.
$FW\ (10\%)_A$ = [-\$3,500 (F/P, 10%, 4) + \$1,225 (F/A, 10%, 4)] × (F/P, 10%, 2) = \$847.

<u>For B</u>, useful life = study period … no cash flow adjustment is needed.
$FW\ (10\%)_B$ = -\$5,000 (F/P, 10%, 6) + \$1,480 (F/A, 10%, 6) = \$2,561.

Select B

Example

Two mutually exclusive alternatives with different useful lives. At 5% per year MARR:

	A	B
Capital investment	\$6,000	\$14,000
Annual expenses	\$2,500	\$2,400
Useful lives (years)	12	18
Market value at end of useful life	0	\$2,800

- Determine which alternative to select assuming repeatability applies.

$AW(5\%)_A$ = -\$6,000 (A/P, 5%, 12) – \$2,500 = -\$3,176.8.

$AW(5\%)_B$ = -\$14,000 (A/P, 5%, 18) – \$2,400 + \$2,800 (A/F, 5%, 18) = -\$3,497.6.

⇒ select A (lower cost).

- Determine which alternative to select if the repeatability does not apply, study period is 18 years, and a new system can be leased for \$8,000 per year after the useful life of alternative A is over.

$PW(5\%)_A$ = -\$6,000 – \$2,500 (P/A, 5%, 12) – \$8,000 (P/A, 5%, 6) (P/F, 5%, 12) = -\$50,767.45.

$PW(5\%)_B$ = -\$14,000 – \$2,400 (P/A, 5%, 18) + \$2,800 (P/F, 5%, 18) = -\$40,885.54.

⇒ select B (lower cost).

Unequal lives – rate of return methods

- IRR of the incremental investment ($i*$) is computed,
- If $i* \geq$ MARR ⇒ increment is justified.
- To solve:
  - Coterminated … straight forward application.
  - Repeatability:
    - Find AW of each alternative.
    - Equate AW of the lower capital to the AW of the higher capital and find $i*$ that satisfies the equality.
    - Compare $i*$ with MARR and make a decision.

Example – rate of return method

	A	B
Capital investment	\$3,500	\$5,000
Annual cash flow	\$1,255	\$1,480
Useful lives (years)	4	6
Market value at end of useful lives	0	0

- Study period isn't determined … repeatability.
- Repeatability means finding AW.

$AW(i^*)_A = AW(i^*)_B$

$-\$3,500\ (A/P, i^*, 4) + \$1,255 = -\$5,000\ (A/P, i^*, 6) + \$1480$

i^* = 26% … increment is justified … $\boxed{\text{Select B}}$

Chapter 7: Depreciation and Income Taxes

Objective

Explain how depreciation affects income taxes, and how income taxes affect economic decisions.

Income taxes = significant cash outflow.

Depreciation is an important element in after-tax cash flow analysis.

Depreciation

- Depreciation is the decrease in value of physical properties with the passage of time and use.
- Depreciation is an **accounting concept** (noncash or book cost) that establishes annual deduction against before-tax income.
- Depreciation begins once the property is placed in service for **business or to produce income**.

A property is depreciable if:

- It is used in business or to produce income.
- It has a determinable useful life that is longer than one year.
- It is something that wears out, decays, gets used up, or loses value from natural causes.
- It is not inventory, stock in trade, or investment property.

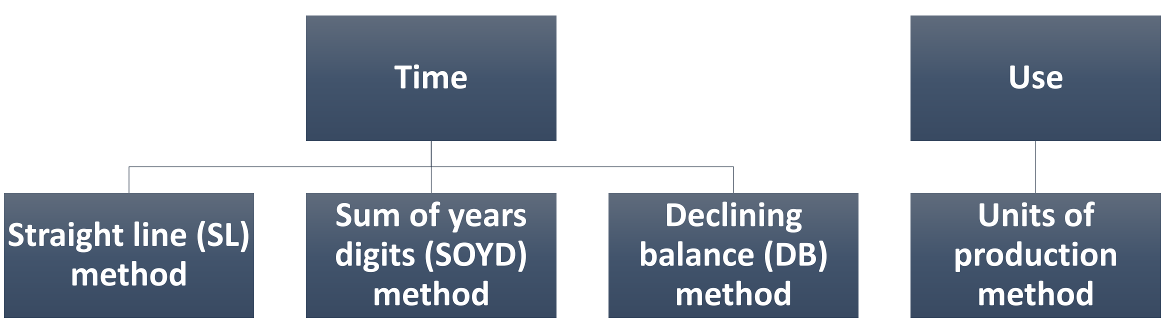
Depreciable properties classification

- Tangible property – can be seen or touched, and includes two main types:
  - Personal property: machinery, vehicles, equipment, furniture, etc.
  - Real property: land + anything on the land.

  *** land alone isn't depreciable because land doesn't have a determinable life ***

- Intangible property – copyright, patent, or franchise.

Depreciation methods



Straight line (SL) method

- Constant amount is depreciated each year over the depreciable (useful) life.

$$d_k = \frac{\mathrm{B} - SV_N}{N}$$

$$d_k^* = k \times d_k, \text{for } 1 \leq k \leq N$$

$$\mathrm{BV}_k = B - d_k^*$$

Where:

N: depreciable life of the asset in years.

B: cost basis, which is the initial cost of acquiring an asset + other associated expenses (sales tax, transportation, setup, etc.)

d_k: annual depreciation in year k.

BV_k: book value at end of year k (worth of a depreciable property on accounting records = cost basis – all allowable depreciation).

SV_k: estimated salvage value at end of year N.

d_k^*: cumulative depreciation through year k.

Example

A tool has a cost basis of \$200,000 and a five-year depreciable life. The estimated salvage value is \$20,000 at the end of five years. Determine the annual depreciation using SL method and tabulate the annual depreciation amounts and book values at the end of each year.

$\mathrm{B} = \$200{,}000$ SV_N = \$20,000 N = 5 years

$$d_1 = d_2 = d_3 = d_4 = d_5 = \frac{200{,}000 - 20{,}000}{5} = \$36{,}000$$

$d_1^* = 1 \times \$36{,}000 = \$36{,}000$
$d_2^* = 2 \times \$36{,}000 = \$72{,}000$
$d_3^* = 3 \times \$36{,}000 = \$108{,}000$
$d_4^* = 4 \times \$36{,}000 = \$144{,}000$
$d_5^* = 5 \times \$36{,}000 = \$180{,}000$

$\mathrm{BV}_1 = \$200{,}000 - \$36{,}000 = \$164{,}000$
$\mathrm{BV}_2 = \$200{,}000 - \$72{,}000 = \$128{,}000$
$\mathrm{BV}_3 = \$200{,}000 - \$108{,}000 = \$92{,}000$
$\mathrm{BV}_4 = \$200{,}000 - \$144{,}000 = \$56{,}000$
$\mathrm{BV}_5 = \$200{,}000 - \$180{,}000 = \$20{,}000$

EOY, k	d_k	BV_k
0	-	\$200,000
1	\$36,000	\$164,000
2	\$36,000	\$128,000
3	\$36,000	\$92,000
4	\$36,000	\$56,000
5	\$36,000	\$20,000

Sum of years digits (SOYD) method

- This method accelerates the recognition of depreciation (most depreciation is recognized in the first few years of the asset's life).

$$d_k = \frac{(N-k+1)}{\sum_{k=1}^{N} k}\,(\mathrm{B}-SV_N)$$

$$d_k^* = \sum_{n=1}^{k} d_k \text{ , for } 1 \le k \le N$$

$$\mathrm{BV}_k = B - d_k^*$$

Example: a property has a cost basis of \$33,000 and a salvage value of \$3,000 with a 5-year useful life. Use the SOYD method to determine the annual depreciations and book values at end of each year.

$$d_1 = \frac{(5-1+1)}{1+2+3+4+5}(\$33{,}000 - \$3{,}000) = \frac{5}{15} \times \$30{,}000 = \$10{,}000$$
$$d_2 = \frac{4}{15} \times \$30{,}000 = \$8{,}000$$
$$d_3 = \frac{3}{15} \times \$30{,}000 = \$6{,}000$$
$$d_4 = \frac{2}{15} \times \$30{,}000 = \$4{,}000$$
$$d_5 = \frac{1}{15} \times \$30{,}000 = \$2{,}000$$

EOY, k	d_k	BV_k
0	-	\$33,000
1	\$10,000	\$23,000
2	\$8,000	\$15,000
3	\$6,000	\$9,000
4	\$4,000	\$5,000
5	\$2,000	\$3,000

Declining balance (DB) method

- Also called constant-percentage method or Matheson formula.

- Annual depreciation is a fixed percentage of the BV at the beginning of the year.

$$d_k = \mathrm{B}\,(1-R)^{k-1}\,R$$

$$d_k^* = \mathrm{B}[1-(1-\mathrm{R})^k]$$

$$\mathrm{BV}_k = \mathrm{B}(1-\mathrm{R})^k$$

Where:

R: constant percentage ratio = $2/N$ when 200% DB is being used (double declining balance – DDB) and $1.5/N$ when 150% DB is used.

Example – DB method

A new cutting machine has a cost basis of \$4,000 and a 10-year depreciable life. The machine has no market value at the end of its life. Use the DB method to calculate the annual depreciation when:

(a) $R = 2/N$ (200% DB or DDB).

(b) $R = 1.5/N$ (150% DB).

$$R = \frac{2}{10} = 0.2$$

$$d_1 = \$4{,}000\ (1-0.2)^{1-1} \times 0.2 = \$800,\ \ \mathrm{BV}_1 = \$4{,}000(1-0.2)^1 = \$3{,}200.$$

$$d_2 = \$4{,}000\ (1-0.2)^{2-1} \times 0.2 = \$640,\ \ \mathrm{BV}_2 = \$4{,}000(1-0.2)^2 = \$2{,}560.$$

... and so on

$$R = \frac{1.5}{10} = 0.15$$

$$d_1 = \$4{,}000\ (1-0.15)^{1-1} \times 0.15 = \$600,\ \ \mathrm{BV}_1 = \$4{,}000(1-0.15)^1 = \$3{,}400.$$

$$d_2 = \$4{,}000\ (1-0.15)^{2-1} \times 0.15 = \$510,\ \ \mathrm{BV}_2 = \$4{,}000(1-0.15)^2 = \$2{,}890.$$

... and so on

200% DB method only		
EOY k	d_k	BV_k
0	-	\$4,000
1	\$800	\$3,200
2	\$640	\$2,560
3	\$512	\$2,048
4	\$409.6	\$1,638.4
5	\$327.68	\$1,310.72
6	\$262.14	\$1,048.58
7	\$209.72	\$838.86
8	\$167.77	\$671.09
9	\$134.22	\$536.87
10	\$107.37	\$429.50

Example – cont'd

200% DB method only		
EOY k	d_k	BV_k
0	-	\$4,000
1	\$800	\$3,200
2	\$640	\$2,560
3	\$512	\$2,048
4	\$409.6	\$1,638.4
5	\$327.68	\$1,310.72
6	\$262.14	\$1,048.58
7	\$209.72	\$838.86
8	\$167.77	\$671.09
9	\$134.22	\$536.87
10	\$107.37	\$429.50

Never reaches SV

Switchover to SL method

- Switchover occurs in the year in which the SL depreciation is greater than or equal to the DB depreciation.

Example – cont'd

200% DB method only				
Year, k	BV @ beginning of year	d_k DDB method	d_k SL method	d_k selected
1	$4,000	$800	$400	$800
2	$3,200	$640	$355.56	$640
3	$2,560	$512	$320	$512
4	$2,048	$409.6	$292.57	$409.6
5	$1,638.4	$327.68	$273.07	$327.68
6	$1,310.72	$262.14 Switch	$262.14	$262.14
7	$1,048.58	$209.72	$262.14	$262.14
8	$786.44	$167.77	$262.14	$262.14
9	$524.30	$134.22	$262.14	$262.14
10	$262.14	$107.37	$262.14	$262.14
11	0			

*** d_k SL method is calculated based on the BV @ beginning of each year, SV, and the remaining years.

For year 1, d_k SL = ($4,000 – 0)/10 = $400.

For year 2, d_k SL = ($3,200 – 0)/9 = $355.56.

Units of production method

- Decrease in value is a function of use.

$$\text{Depreciation per unit of production} = \frac{\text{B} - \text{SV}_\text{N}}{\text{estimated lifetime production units}}$$

<u>Example</u>: An equipment has a basis of $50,000 and is expected to have a $10,000 SV when replaced after 30,000 hours of use. Find the depreciation rate per hour of use and find its book value after 10,000 hours of operation.

$$\text{Depreciation per unit of production} = \frac{\$50{,}000 - \$10{,}000}{30{,}000 \text{ hours}} = \$1.33 \text{ per hour}$$

After 10,000 hours, BV $= \$50{,}000 - \frac{\$1.33}{\text{hour}} \times 10{,}000 \text{ hours} = \$36{,}700.$

Taxes

Types of taxes

- Income taxes: function of gross revenue minus allowable deductions.
- Property taxes: function of the property (e.g., land, building, equipment, etc.) value (**independent of income or profit**).
- Sales taxes: function of the value of purchased goods or services (**independent of income or profit**).
- Excise taxes: taxes imposed on the purchase of non-necessities (**independent of income or profit**).

After-tax analysis

After-tax MARR ≅ Before-tax MARR × (1 – effective income tax rate)

Taxable income = Gross income – All expenses (except capital investment) – Depreciation deductions

Example: A company generates $1,500,000 of gross income during its tax year and incurs operating expenses of $800,000. Property taxes on business assets amount to $48,000. The total depreciation deductions for the tax year equal $114,000. What is the taxable income of this firm?

Taxable income = $1,500,000 − $800,000 − $48,000 − $114,000 = $538,000.

Gain (loss) on the disposal of an asset

- When a depreciable property is sold, often market value (MV) ≠ BV.

Gain (or loss) on disposal @ EOY $_N$ = $\mathrm{MV_N} - \mathrm{BV_N}$

Gain (+ve) ⇒ tax liability

Loss (-ve) ⇒ tax credit

Example: A company sold an equipment during the current tax year for $78,600. The cost basis is $1,900,000 and the accumulated depreciation is $139,200. Assume 40% effective tax rate.

- **What is the gain (or loss) on disposal?**

BV @ the time of sale = $190,000 − $139,200 = $50,800
Gain on disposal = $78,000 − $50,800 = $27,800

- **What is the tax liability (or credit) resulting from this sale?**

Gain … tax liability = -0.4 × $27,800 = -$11,120

- **What is the tax liability (or credit) if the accumulated depreciation was $92,400 instead of $139,200?**

BV @ the time of sale = $190,000 − $92,400 = $97,600
Loss on disposal = $78,000 − $97,600 = −$19,600
Loss … tax credit = -0.4 × -$19,000 = $7,600

After-tax cash flow (ATCF)

After-tax economic analysis is the same as before-tax analysis except:

$$T_k = -t\,\underbrace{(R_k - E_k - d_k)}_{\text{Taxable income}}$$

Where:

T_k: income tax consequence during year k.
R_k: revenue (and savings) or cash inflow during year k.
E_k: cash outflows during year k.
d_k: sum of book costs during year k or accumulated depreciation.
t: effective income tax rate.

Before-tax cash flow → $\mathrm{BTCF}_k = R_k - E_k$

$$\begin{aligned}\mathrm{ATCF}_k &= \mathrm{BTCF}_k + T_k \\ &= (R_k - E_k) - t\,(R_k - E_k - d_k) \\ &= (1-t)(R_k - E_k) + t\,d_k\end{aligned}$$

Example

A new equipment is estimated to cost $180,000 and is expected to reduce net annual expenses by $36,000 for 10 years and to have a $30,000 market value at the end of the 10th year. Using the SL depreciation method, and assuming a 40% effective income tax rate, develop the ATCF and BTCF.

$d_k = \frac{\$180{,}000 - \$30{,}000}{10}$; Taxable income $= \text{BTCF} - d_k$; Income tax $= -t \times \text{taxable income}$; ATCF $= \text{BTCF} + \text{income tax}$

EOY k	Capital	R_k	E_k	BTCF	d_k	Taxable income	Income tax	ATCF
0	-$180,000	-	-	-$180,000	-	-	-	-$180,000
1	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
2	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
3	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
4	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
5	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
6	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
7	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
8	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
9	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600
10	-	$36,000	-	$36,000	$15,000	$21,000	-$8,400	$27,600

Example

A company wants to purchase a machine with an initial cost of $100,000 with additional $10,000 installation and transportation costs and a salvage value after 10 years of $10,000. If the annual revenue is $20,000 and the annual expenses are $5,000, and using the SL depreciation method and a 30% income tax rate:

- <u>What is the BTCF for the 3rd year</u>?

BTCF_3 = $20,000 – $5,000 = $15,000.

- <u>What is the ATCF for the 2nd year</u>?

BCTF_2 = $20,000 - $5,000 = $15,000.

d_k = ($100,000 + $10,000 – $10,000)/10 = $10,000 per year.

Taxable income for year 2 = BTCF - d_k = $15,000 – $10,000 = $5,000.

T_k = - 0.30 × $5,000 = -$1,500.

ATCF_2 = BTCF_2 + T_k = $15,000 + - $1,500 = $13,500.

Example

Assume the cost basis is $35,000, annual revenue is $30,000, annual expenses are $13,000 in the first year and increasing by $1,000 per year. The useful life is 4 years and the SOYD is the applicable depreciation method, develop BTCFs and ATCFs using a 15% income tax rate.

EOY k	Capital	R_k	E_k	BTCF	d_k	Taxable income	Income tax	ATCF
0	-$35,000	-	-	-$35,000	-	-	-	-$35,000
1		$30,000	-$13,000	$17,000	$14,000	$3,000	$450	$16,550
2		$30,000	-$14,000	$16,000	$10,500	$5,500	$825	$15,175
3		$30,000	-$15,000	$15,000	$7,000	$8,000	$1,200	$13,800
4		$30,000	-$16,000	$14,000	$3,500	$10,500	$1,575	$12,425

d_k
-
= 4/(1+2+3+4) × ($35,000 – $0)
= 3/(1+2+3+4) × ($35,000 – $0)
= 2/(1+2+3+4) × ($35,000 – $0)
= 1/(1+2+3+4) × ($35,000 – $0)