

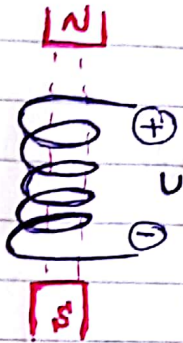
* Poly phase system *

Q → how do we generate electricity?

Induced voltage. $V = -N \frac{d\phi}{dt}$ magnetic flux

Coil \rightarrow Faraday

#no. of turns.

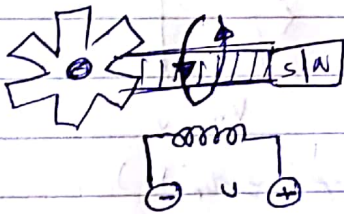


OR
Moving coil
Static field.

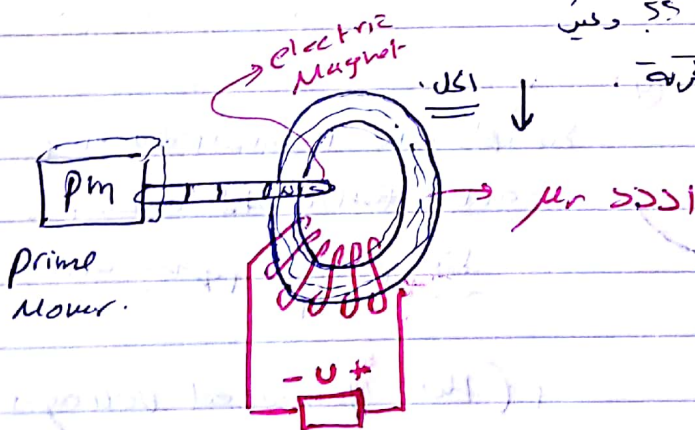


static
coil moving
field.

* (synchronous generator) *

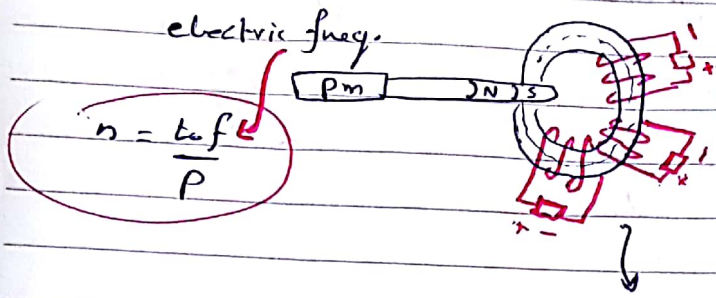


اكد للتكامل في عملية
حينه التوليد في
وصفها التوليد.



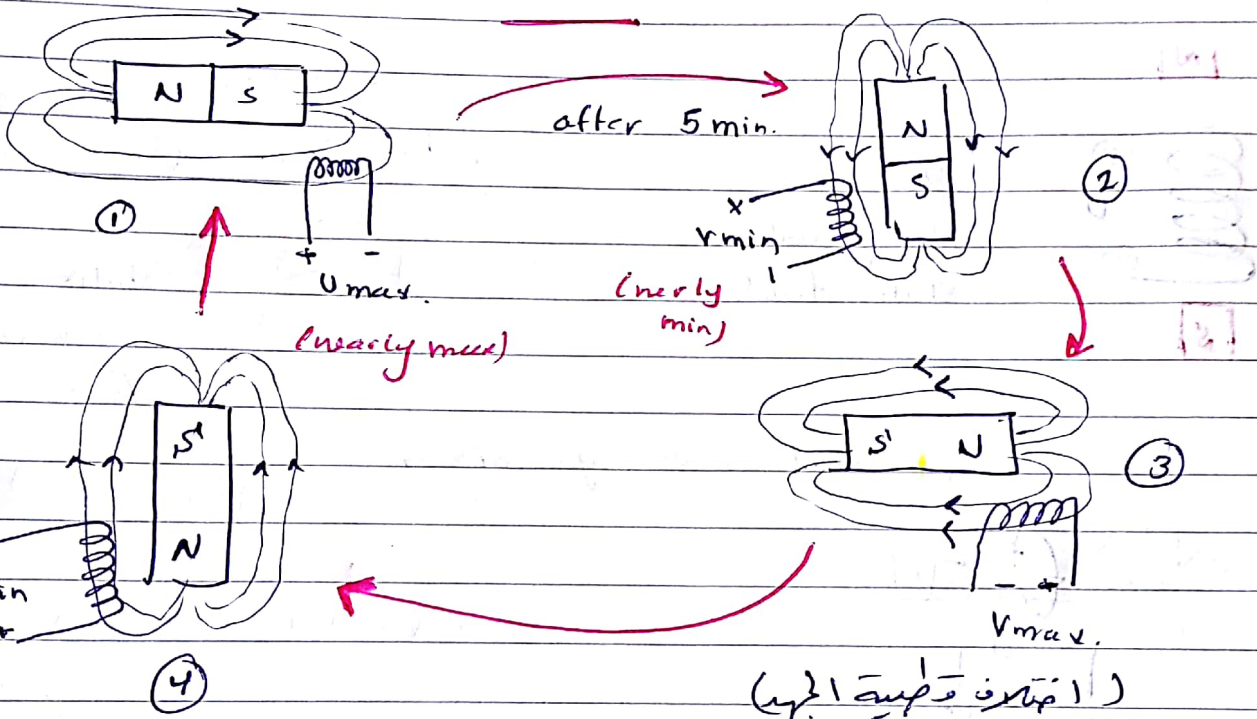
the whole
structure named
as (generator)

* عنوان تيار الكهارة # اولى ان تيار بلاتن احدث *

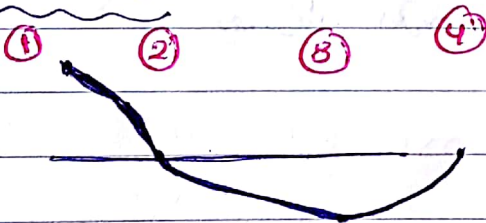


* اقول تيار تم اكتشاه
 على وجه
 بلاتن
 (اكتشاه ريبا وسكايكتيا)

* 3-phase shift *



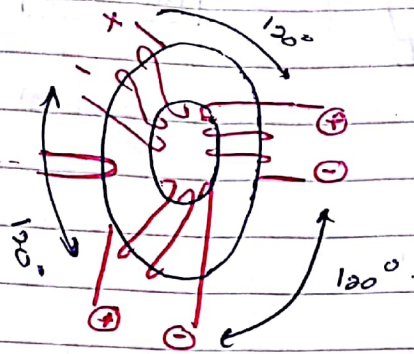
$$U = -N \frac{d\phi}{dt}$$



* اقول ان السبب كان على واحد فقط
 * بالسيه للبلاتن احدث
 * تيار تيار phase shift

+ (the generated voltages are AC voltages).

* To get less vibration *
 we need a displacement between the inductors has (120°)
 value.

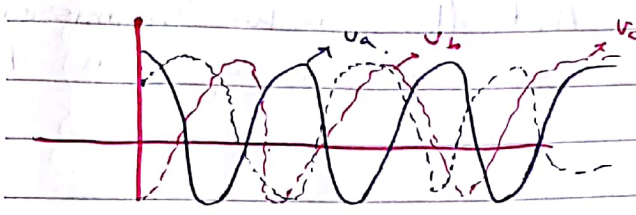


$$(U_a) U_{1 \text{ max}} = U_{\text{max}} \cos(\omega t)$$

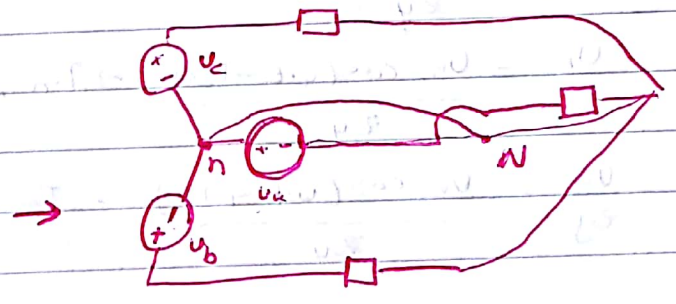
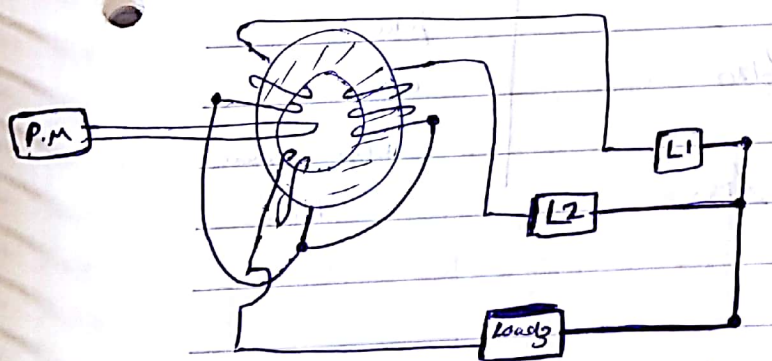
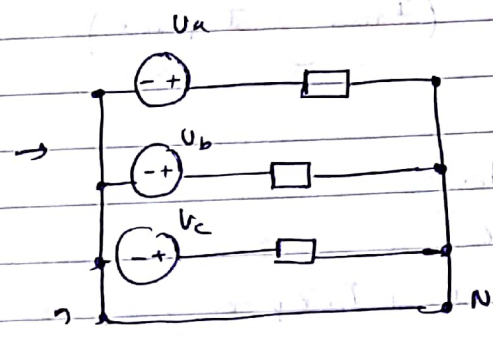
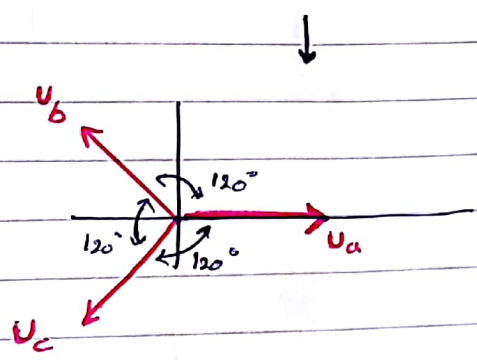
$$(U_b) U_2 = U_{\text{max}} \cos(\omega t - 120^\circ)$$

$$(U_c) U_3 = U_{\text{max}} \cos(\omega t + 120^\circ)$$

(All of them have the same max value).



$$U = -N \frac{d\phi}{dt}$$

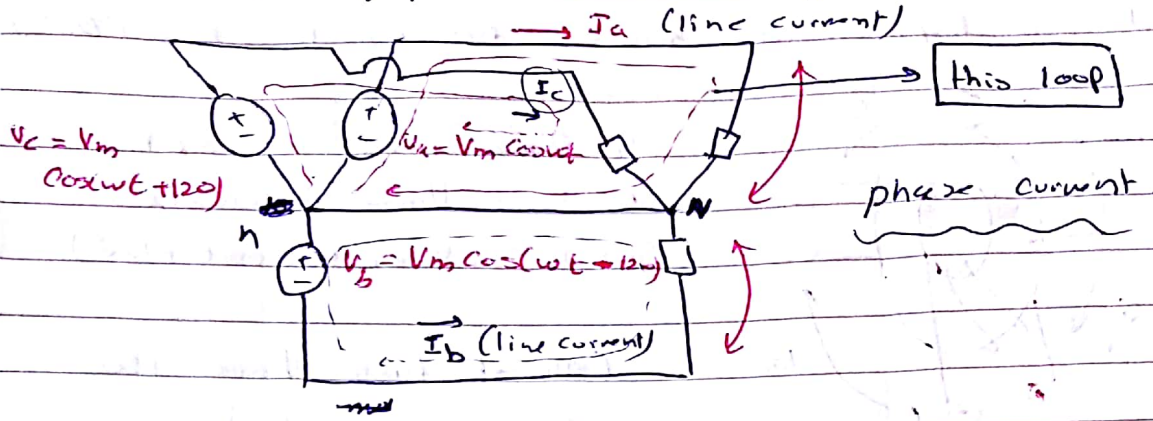


3-phase source.

3-phase generator.

Y-connected load generator
 → the neutral points of the
 3 sources are connected
 together.

* Balanced y - y connected 3-phase system



- * **line current**: the current that flows in the Transmission line.
- * **phase current**: " " " " " " " " " " " load.

In y connected loads
 $(I_{min} = I_{phase})$

Q this loop :-
KVL

$$-V_m \cos \omega t + I_a Z_Y = 0$$

$$I_a = \frac{V_a}{Z_Y} = \frac{V_m \cos(\omega t)}{Z_Y} = I_m \angle 0$$

$$I_b = \frac{V_b}{Z_Y} = \frac{V_m \cos(\omega t - 120^\circ)}{Z_Y} = I_m \angle -120$$

$$I_c = \frac{V_c}{Z_Y} = \frac{V_m \cos(\omega t + 120^\circ)}{Z_Y} = I_m \angle 120$$

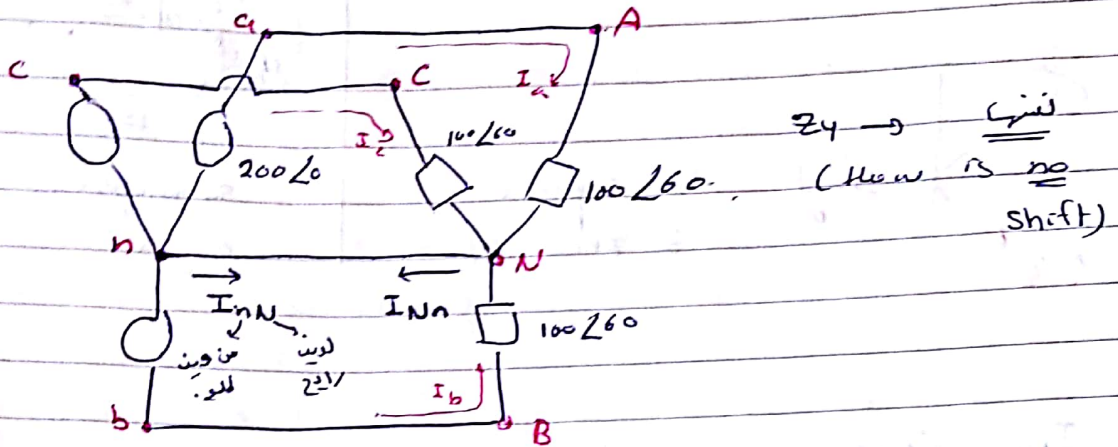
2
 line 2
 phase
 current
 ?
 the same
 Here.

A Balance load will produced balanced current.

The voltage across the loads is called phase voltage.

Ex.

find the phase & line current & the total power absorbed by the load for this balanced Y-Y connected 3-phase sys.



$$V_{an} = 200 \angle 0^\circ$$

$$V_{bn} = 200 \angle -120^\circ$$

$$V_{cn} = 200 \angle 120^\circ$$

$$I_a = I_{aA} = \frac{V_{an}}{Z_Y} = \frac{200 \angle 0^\circ}{100 \angle 60^\circ} = 2 \angle -60^\circ \text{ A}$$

$$I_b = 2 \angle -180^\circ = 2 \angle -60^\circ - 120^\circ \quad (= V_{bn} / Z_Y = \frac{200 \angle -120^\circ}{100 \angle 60^\circ} = 2 \angle -180^\circ \text{ A})$$

(voltage shift)

$$I_c = 2 \angle -60^\circ + 120^\circ = 2 \angle 60^\circ \text{ or } (I_c = V_{cn} / Z_Y = \frac{200 \angle 120^\circ}{100 \angle 60^\circ} = 2 \angle 60^\circ)$$

* Real power in phase A.

$$P_A = V_{rms} I_{rms} \cos(\theta - \phi) = V_{AN} I_a \cos(\theta_{V_{AN}} - \phi_{I_a})$$

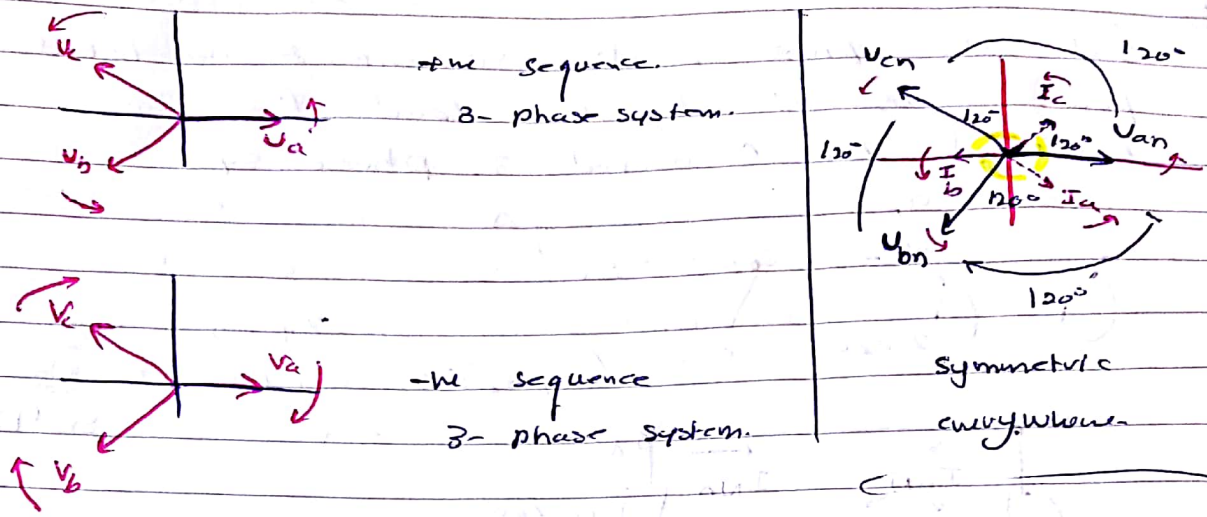
$$= 200 * 2 * \cos(0 - 60) = 200 \text{ W}$$

$$P_B = V_{BN} I_B \cos(\theta_{V_{BN}} - \phi_{I_B}) = 200 * 2 * \cos(-120 - (-180)) = 400 \cos 60 = 200 \text{ W}$$

$$P_C = V_{CN} I_C \cos(\theta_{V_{CN}} - \phi_{I_C}) = 200 * 2 * \cos(120 - 60) = 200 \text{ W}$$

$$P_{3\phi} = P_A + P_B + P_C = 600 \text{ W}$$

↳ 3-phase



I_{Nn} (Neutral current)

$$\begin{aligned}
 I_{Nn} &= I_a + I_b + I_c \\
 &= 2 \angle 60^\circ + 2 \angle -120^\circ + 2 \angle 60^\circ \\
 &= 2 \cos 60^\circ + 2j \sin 60^\circ + 2 \cos -120^\circ + 2j \sin -120^\circ \\
 &\quad + 2 \cos 60^\circ + 2j \sin 60^\circ \\
 &= 2 - \sqrt{3} - 2 + 1 + \sqrt{3} \\
 &= 2 - 2 = \underline{\underline{Zero}}
 \end{aligned}$$

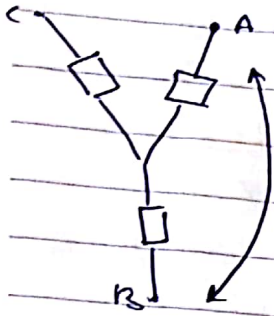
Balanced 3-phase y-y connected system :-

$$I_a = I_m \angle \theta, \quad I_b = I_m \angle \theta - 120^\circ, \quad I_c = I_m \angle \theta + 120^\circ$$

$$\begin{aligned}
 I_{Nn} &= I_m \angle \theta + I_m \angle \theta - 120^\circ + I_m \angle \theta + 120^\circ \\
 &= I_m \cos \theta + j I_m \sin \theta + I_m \cos(\theta - 120^\circ) + j I_m \sin(\theta - 120^\circ) \\
 &\quad + I_m \cos(\theta + 120^\circ) + j I_m \sin(\theta + 120^\circ) \\
 &= \underline{\underline{Zero}}
 \end{aligned}$$

The current flowing in a balanced 3 phase y-y connected is equal to zero.

$U_{AB} \Rightarrow$ line to line voltage.
 (line voltage) \searrow phase to phase voltage.



$U_{AB} = \overbrace{U_{AN}}^{\text{phase voltage}} - \overbrace{U_{BN}}^{\text{phase to neutral voltage}}$

$$\begin{aligned}
 &= V_m \angle 0^\circ - V_m \angle -120^\circ \\
 &= V_m - V_m \cos(-120^\circ) - j V_m \sin(-120^\circ) \\
 &= V_m + \frac{1}{2} V_m + \frac{\sqrt{3}}{2} V_m
 \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{3} V_m \left(\frac{\sqrt{3}}{2} + \frac{1}{2} j \right) \\
 &= \sqrt{3} V_m \angle +30^\circ
 \end{aligned}$$

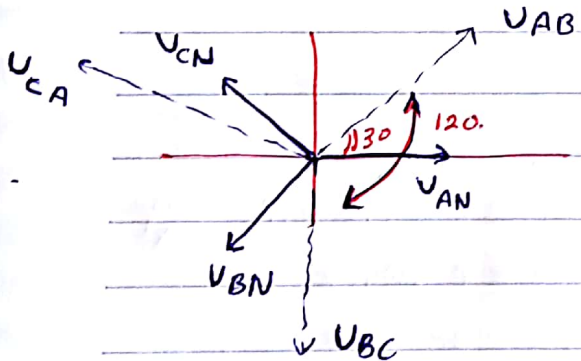
cos 30 + j sin 30

$V_{BC} \rightarrow$ line to line voltage. (line voltage)

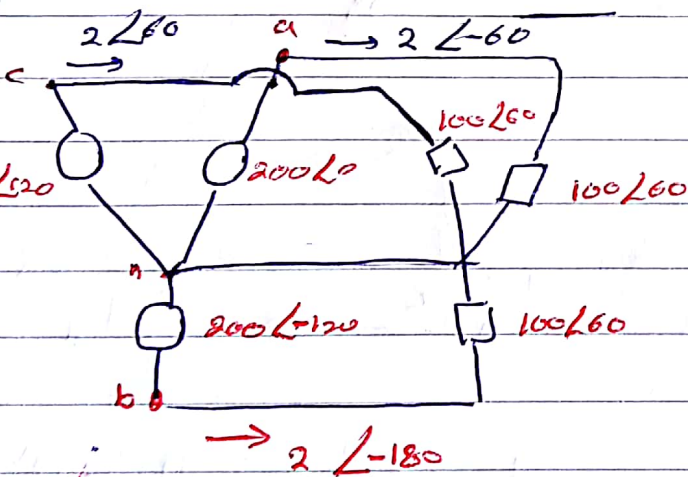
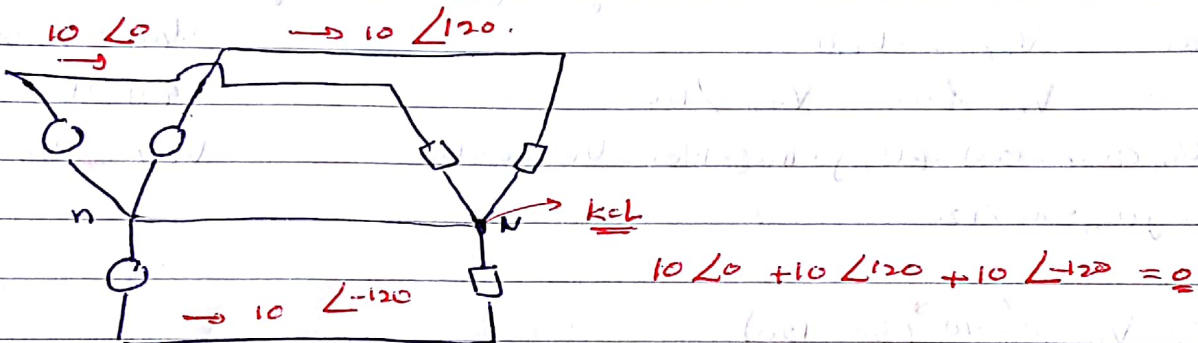
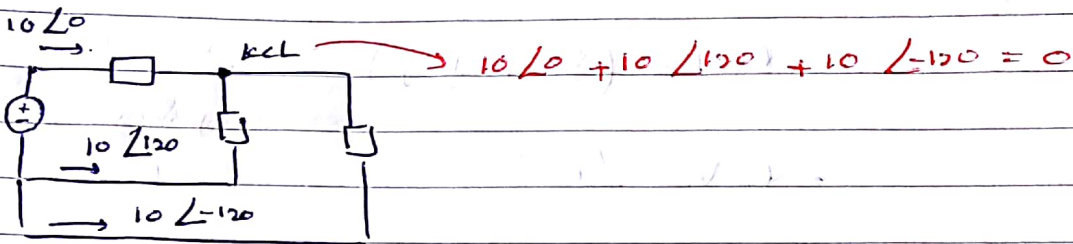
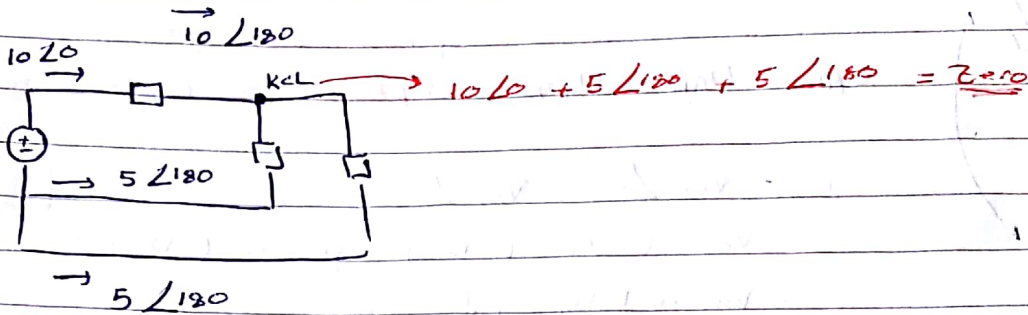
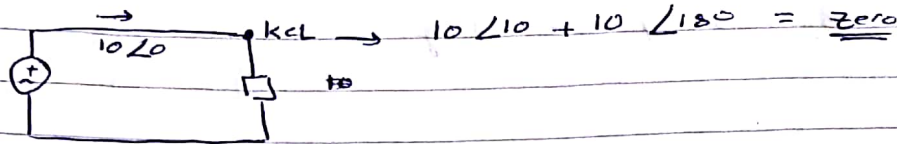
$$\begin{aligned}
 V_{BC} &= V_{BN} - V_{CN} \\
 &= V_m \angle -120^\circ - V_m \angle 120^\circ \\
 &= V_m \cos(-120^\circ) + V_m j \sin(-120^\circ) - V_m \cos 120^\circ \\
 &\quad - j V_m \sin 120^\circ
 \end{aligned}$$

$$\begin{aligned}
 U_{AN} &= V_m \angle 0^\circ \\
 V_{BN} &= V_m \angle -120^\circ \\
 V_{CN} &= V_m \angle 120^\circ
 \end{aligned}$$

$$= \sqrt{3} V_m \angle -90^\circ \quad (30 - 120)$$



V-Y Connected loads.



$$V_{AN}(t) = 200\sqrt{2} \cos \omega t$$

$$i_A(t) = 2\sqrt{2} \cos(\omega t - 60)$$

$$P_A(t) = 800 [\cos \omega t * \cos \omega t - 60]$$

$$= 200 + 400 \cos(2\omega t - 60) \text{ W}$$

$$V_{BN}(t) = 200\sqrt{2} \cos(\omega t - 120)$$

$$i_B(t) = 2\sqrt{2} \cos(\omega t - 180)$$

$$P_B(t) = V_{BN}(t) \cdot i_B(t)$$

$$= 200 + 400 \cos(2\omega t + 60) \text{ W}$$

→ for **single phase load** ($p \rightarrow \text{func}(t)$)

$$P(t) = \frac{V_m I_m}{2} \cos(\theta - \phi) + \frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)$$

$$V_{CN}(t) = 200\sqrt{2} \cos(\omega t + 120)$$

$$i_C(t) = 2\sqrt{2} \cos(\omega t + 60)$$

$$P_C(t) = V_{CN}(t) \cdot i_C(t)$$

$$= 200 + 400 \cos(2\omega t + 180)$$

$$P_{3\phi} = P_C(t) + P_B(t) + P_A(t)$$

$$P_{3\phi} = \underline{600 + 400 \cos(2\omega t - 60) + 400 \cos(2\omega t + 60) + 400 \cos(2\omega t + 180)}$$

$$P_{3\phi} = 200 + 400 \angle -60 + 200 + 400 \angle 60 + 200 + 400 \angle 180$$

$$= 200 + 400 \cos -60 + 400j \sin(-60) + 200 + 400 \cos 60 + j400 \sin 60$$

$$+ 200 + 400 \cos 180 + j400 \sin 180$$

$$= 600 \text{ W}$$

$$P_{3\phi} = \underline{600} \text{ W (not function of } t)$$

Phase shift 120° equal

Zero = $\cancel{j400 \sin}$

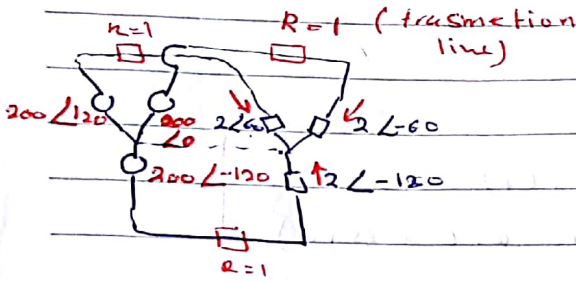
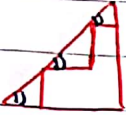
3φ
(600 W)

PF = 0.5

$PF_A = 0.5 = \cos(0 + 60)$

$PF_B = 0.5 = \cos(-120 + 120)$

$PF_C = 0.5 = \cos(120 - 60)$



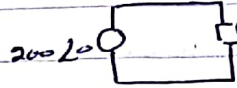
power losses by T.L., $R=1\Omega$

$= I_a^2 R + I_b^2 R + I_c^2 R$

$= 4 + 4 + 4 = 12 \text{ W}$

1φ

(600 W), PF = 0.5



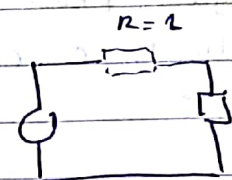
$P = V_{rms} I_{rms} \cos(\theta - \phi)$
 $600 = 200 I_{rms} \cos(\theta - \phi)$

$I_{rms} = 6 \text{ A}$

التيار الي ص ح

مسار انتقال تسي الالة

تير ازم الالتيار ال 1φ

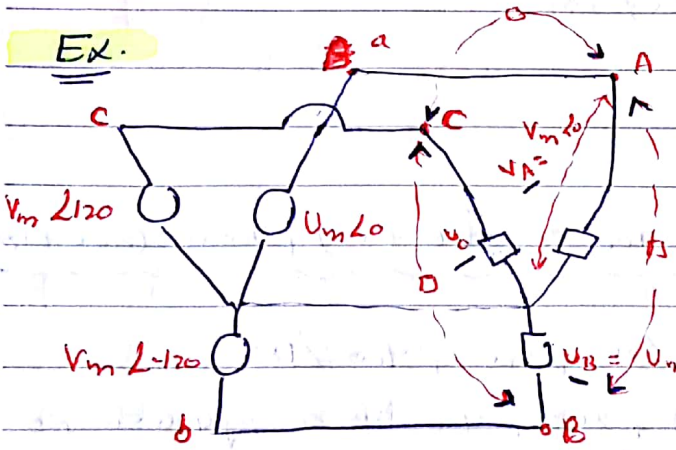


T.L losses.

$P = I^2 R = 36 \text{ W}$

3φ الالوس ال الال

Ex.



* In Y connected load

$I_{load} = I_{phase}$

* Voltage across load is phase to neutral voltage

(phase voltage)

↳ from phase to neutral.

* line to line voltage.

↳ from phase to phase.

$V_{BC} = V_{BN} - V_{CN} = \sqrt{3} U_m \angle -90$

$V_{AB} = V_{AN} - V_{BN} = \sqrt{3} U_m \angle 30$

line to line voltages.

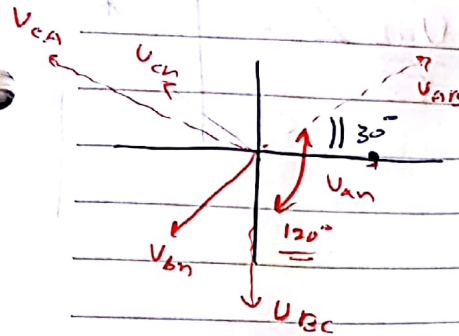
$V_A \angle V_B \angle V_C$

phasor voltages.

$$\begin{aligned}
 V_{CA} &= V_{CN} - V_{AN} = V_m \angle 120 - V_m \angle 0 \\
 &= V_m \cos 120 + j V_m \sin 120 - V_m \\
 &= -\frac{V_m}{2} + j \frac{\sqrt{3} V_m}{2} - V_m \\
 &= V_m \frac{\sqrt{3}}{2} j - \frac{3 V_m}{2} \\
 &= \sqrt{3} V_m \left[\frac{1}{2} j - \frac{\sqrt{3}}{2} \right] = \sqrt{3} V_m \angle 150^\circ
 \end{aligned}$$

\swarrow $\sin 150$ \searrow $\cos 150$

$V_{CA}, V_{AB}, V_{BC} \rightarrow$ 20/11/19/10
120°



$$V_{AN} = V_{AB} \cos 30^\circ$$

$$|V_{AB}| > |V_A|$$

$\sqrt{3}$ times

Example:-

Given that $V_{an} = 50 \angle 15^\circ$, $Z_Y = 5 \angle 60$

→ find phase voltages, line voltages, line currents, phase currents, given that its a balanced Y-Y connected load.

Solution

$$V_{an} = 50 \angle 15^\circ$$

$$V_{bn} = 50 \angle -105^\circ$$

$$V_{cn} = 50 \angle 135^\circ$$

Phase voltages.

$$V_{AB} = 86.6 \angle 45^\circ \quad (30 + 15)$$

$$V_{BC} = 86.6 \angle -75^\circ \quad (105 + 30)$$

$$V_{CA} = 86.6 \angle 165^\circ \quad (135 + 30)$$

Line voltages.

$$I_a = \frac{V_{an}}{Z_Y} = \frac{50 \angle 15}{5 \angle 60} = 10 \angle -45^\circ \text{ A}$$

$$I_b = 10 \angle -45 - 120 = 10 \angle -165^\circ \text{ A}$$

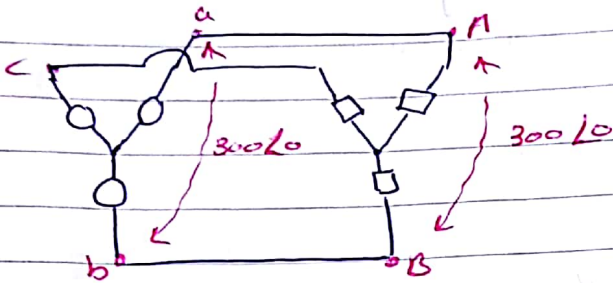
$$I_c = 10 \angle -45 + 120 = 10 \angle 75^\circ \text{ A}$$

line currents = phase currents.

in Balanced Y connected load.

Example.

A Balanced 3φ y-y connected system with line voltage (line to line) $V_{ab} = 300 \angle 0$ Volts supplying a load with 1200 w & a leading PF of .8, find the line current & per-phase load impedance.



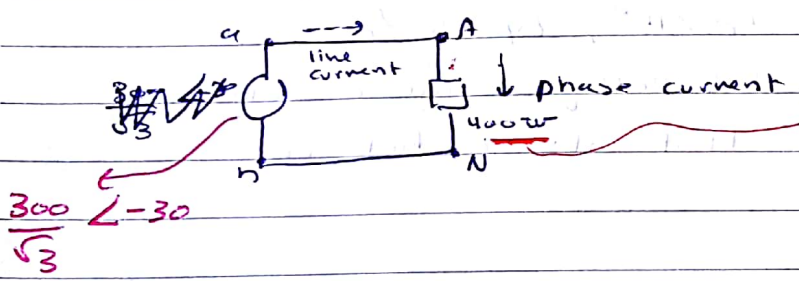
$$V_{ab} = \sqrt{3} V_{an} \angle 30$$

$$V_{an} = \frac{V_{ab}}{\sqrt{3}} \angle -30$$

$$V_{an} = \frac{V_{ab}}{\sqrt{3}} \angle -30 = \frac{300}{\sqrt{3}} \angle -30$$

per-phase equivalent ckt.

⇒ Take phase A.



1200 w (3φ)
 $1\phi = \frac{1200}{3} = 400w$
 usually power given in question it is 3φ power.

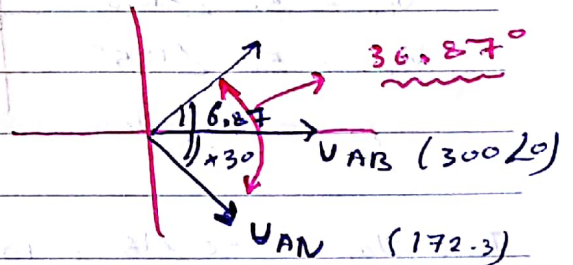
line current = phase current

$$P = V_{rms} I_{rms} \cos \theta - \phi \rightarrow PF$$

$$P = V_{AN} I_a \cos \theta - \phi$$

$$P = V_{AN} I_a * PF$$

$$400 = \frac{200}{\sqrt{3}} * I_a * 0.8$$



$$I_A = 2.89 \text{ A}$$

Current leading voltage by $\cos^{-1}(0.8) = 36.87^\circ$ So

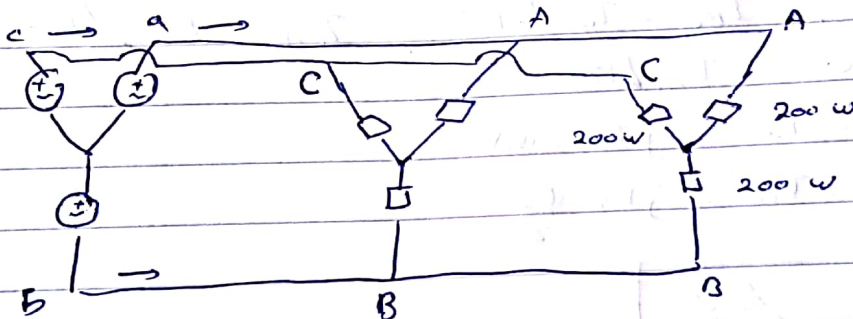
(PF = .8 leading)

$$I_a = 2.89 \angle 16.87$$

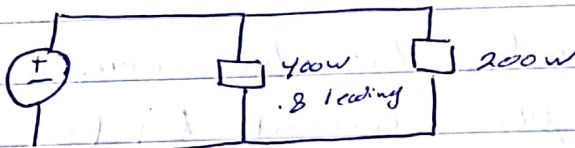


Ex.

if a balanced 600-w load is added to the previous system (in parallel),
Find the new line current.



$I = 2.89$ ← PF in the 1 to 1.5

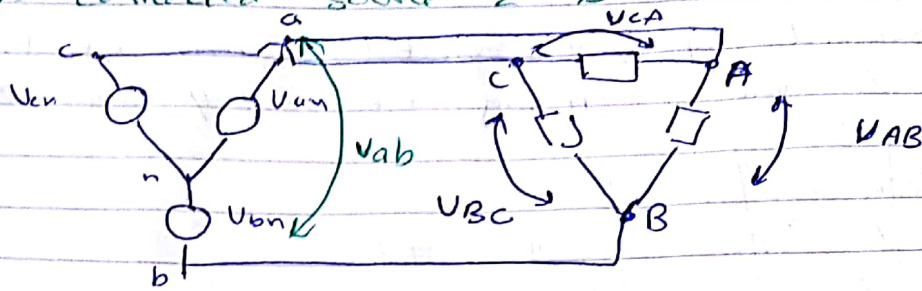


$$P = |V||I| \cos(\theta - \phi) \Rightarrow 200 = 173.2 * |I|$$

$$|I| = 1.15, \quad I = 1.15 \angle -30^\circ$$

$$* I_{line} = 1.15 \angle -30^\circ + 2.89 \angle 6.87^\circ = 3.87 \angle -3.4^\circ A$$

* Y-connected source & Δ connected load.



- The neutral point is not accessible.
- increasing the load voltage.

* The voltage across the load is line to line voltage.

$$V_{AB} = \sqrt{3} V_{an} \angle +30^\circ$$

* In Δ connection: $I_{line} \neq I_{phase}$

$$\text{at CA: } I_a + I_{CA} = I_{AB}$$

$$\rightarrow I_a = I_{AB} - I_{CA}$$

* let's suppose that:

$$I_{AB} = I_m \angle 0^\circ, \quad I_{BC} = I_m \angle -120^\circ, \quad I_{CA} = I_m \angle 120^\circ$$

$$\text{So, } I_a = I_m \angle 0^\circ - I_m \angle 120^\circ$$

$$I_a = I_m - I_m \cos 120^\circ - j I_m \sin 120^\circ$$

$$I_a = I_m + 0.5 I_m - j I_m \frac{\sqrt{3}}{2}$$

$$I_a = \sqrt{3} I_m \angle -30^\circ$$

* for a Δ connected load, the line current is greater than the phase current by $\sqrt{3}$ and lags it by 30°

Ex. Find the line currents for 3 phase y-connected balanced system with a line voltage of $V_{ab} = 300 \text{ V}$ and supplies 1200 W to a Δ -connected load, $\text{PF} = 0.8$ lagging, then find the phase impedance.

* Since $V_{ab} = 300 \angle 0^\circ$ so, $U_{AB} = 300 \angle 0$

do find the currents:-

$$P_{AB} = |U_{AB}| |I_{AB}| \cos(\theta - \phi)$$

$$|I_{AB}| = \frac{400}{300 \times 0.8} = 1.667 \text{ A}$$

$$\phi = -\cos^{-1}(\text{PF}) = -36.87$$

$$\text{so, } I_{AB} = 1.667 \angle -36.87$$

phase currents:

$$I_{AB} = 1.667 \angle -36.87^\circ, I_{BC} = 1.667 \angle -126.87^\circ, I_{CA} = 1.667 \angle 83.13^\circ$$

→ to find the line currents:

$$I_{\text{line}} = \sqrt{3} I_{\text{phase}} \angle -30^\circ$$

$$I_A = \sqrt{3} \times 1.667 \angle -36.87^\circ \times \angle -30^\circ = 2.89 \angle -66.87^\circ$$

$$I_B = \sqrt{3} \times 1.667 \angle -126.87^\circ \times \angle -30^\circ = 2.89 \angle -156.87^\circ$$

$$I_C = 2.89 \angle 53.13^\circ$$

$$I_C = 2.89 \angle 53.13^\circ$$

V_{ab} is $\sqrt{3}$ times V_{an} $V_{ab} = \sqrt{3} V_{an} \angle +30^\circ$

$$V_{ab} = \sqrt{3} V_{an} \angle +30^\circ$$

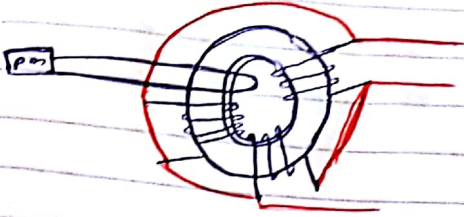
→ To find Z

$$Z_A = \frac{U_{AB}}{I_{AB}} = 180 \angle 36.87^\circ = Z_B = Z_C$$

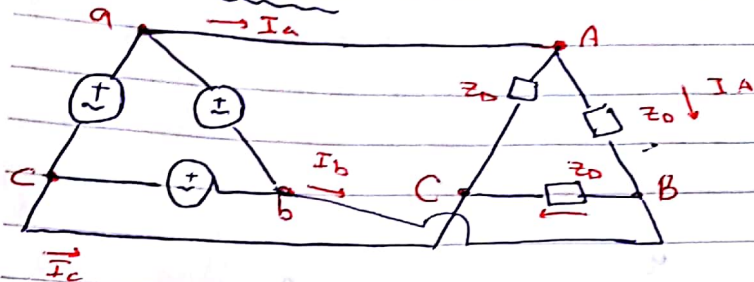
* we have 4 connections.

- ① Y-Y ② Y- Δ ③ Δ -Y ④ Δ - Δ

* Can we connect the source as a Δ ?



Δ - Δ Connected

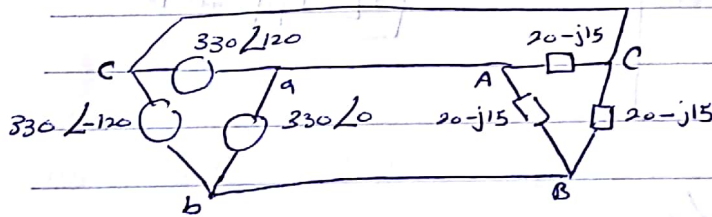


- * The voltage across the load is (line to line voltage)
- * $I_{\text{line}} \neq I_{\text{phase}}$
- * $I_{\text{line}} = \sqrt{3} * I_{\text{phase}} \angle -30^\circ$

Ex:

a Balanced Δ connected load of $Z_D = 20 - j15 \Omega$ is connected to Δ source with $V_{ab} = 330 \angle 0^\circ$, find phase & line current.

خطا الصواب ان 36.87° لا 120°
 ال I_{phase} ل I_{line}



$$I_{AB} = \frac{U_{AB}}{Z_D} = \frac{V_{ab}}{Z_D} = \frac{330 \angle 0^\circ}{20 - j15} = 13.2 \angle 36.87^\circ$$

$$I_{BC} = 13.2 \angle 36.87^\circ - 120^\circ$$

$$I_{CA} = 13.2 \angle 36.87^\circ + 120^\circ$$

$$I_{Line} = \sqrt{3} I_{Phase} \angle -30$$

$$I_a = \sqrt{3} I_{AB} \angle -30$$

$$I_b = \sqrt{3} I_{BC} \angle -30$$

$$I_c = \sqrt{3} I_{CA} \angle -30$$

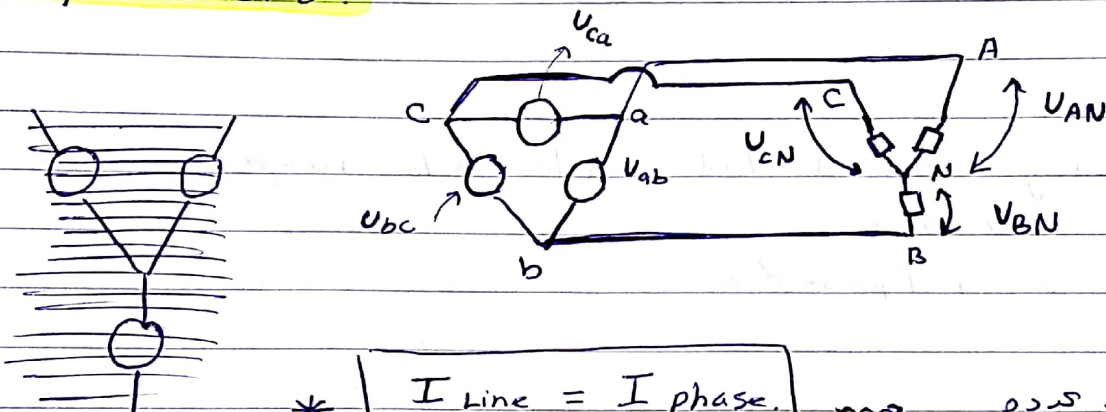
$$I_a = \sqrt{3} \times 13.2 \angle 36.87 - 30$$

$$= 22.86 \angle 6.87 \text{ A}$$

$$I_b = 22.86 \angle 6.87 - 120 \text{ A}$$

$$I_c = 22.86 \angle 6.87 + 120 \text{ A}$$

D-Y connected



كان الحثي جوده
طريقة تعلق ال load

ان كان ال تعلق
Y connected

$$I_L = I_p \rightarrow \bar{N}$$

* The voltage across the load is line to neutral voltage

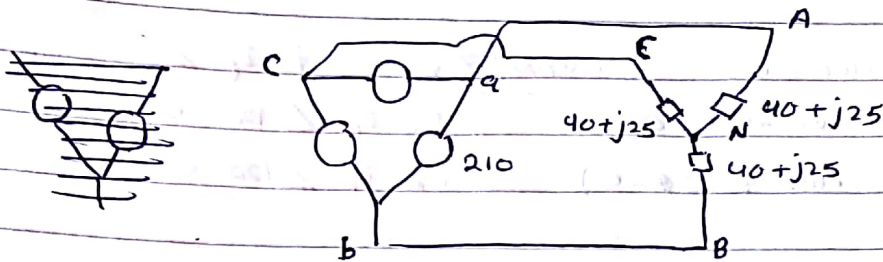
$$V_{LL} \text{ (line to line)} = \sqrt{3} V_{LN} \text{ (line to neutral)} \angle 30$$

$$V_{LN} = \frac{V_{LL}}{\sqrt{3}} \angle -30^\circ$$

Example.

A Balanced Y -connected load with $Z_Y = 40 + j25$ is supplied by a Δ -connected source with a line voltage of 210 Volt.

Calculate the phasor & line currents, Take V_{ab} as your reference.



$$I_a = \frac{U_{AN}}{Z_Y}$$

$$I_a = \frac{121.2 \angle -30^\circ}{40 + j25} = 2.57 \angle -62^\circ \text{ A}$$

$$|U_{ab}| = 210$$

$$U_{ab} = 210 \angle 0^\circ$$

$$U_{AN} = \frac{210 \angle -30^\circ}{\sqrt{3}} = 121.2 \angle -30^\circ \text{ volt}$$

$$I_b = 2.57 \angle -62 - 120^\circ \text{ A}$$

$$I_c = 2.57 \angle -62 + 120^\circ \text{ A}$$

~~$V_{bc} = 210 \angle 120^\circ$~~

~~$V_{ca} = 210 \angle 240^\circ$~~

my reference.

phase currents. (= line cur

$$V_{bc} = 210 \angle 0^\circ$$

$$V_{ab} = 210 \angle -120^\circ$$

$$V_{ca} = 210 \angle 120^\circ \text{ or } \angle -240^\circ$$

$$U_{an} = \frac{210 \angle 90^\circ}{\sqrt{3}}$$

* power in 3 ϕ system.

$$V_{AN} = \sqrt{2} V_p \cos(\omega t) \rightarrow \sqrt{2} V_p \angle 0$$

$$V_{BN} = \sqrt{2} V_p \cos(\omega t - 120) \rightarrow \sqrt{2} V_p \angle -120$$

$$V_{CN} = \sqrt{2} V_p \cos(\omega t + 120) \rightarrow \sqrt{2} V_p \angle 120$$

→ Take $Z_Y = Z \angle \theta$

$$i_a = \sqrt{2} I_p \cos(\omega t - \theta) \equiv (V_{AN}/Z_Y) \rightarrow \sqrt{2} I_p \angle \theta$$

$$i_b = \sqrt{2} I_p \cos(\omega t - 120 - \theta) \rightarrow \sqrt{2} I_p \angle -120 - \theta$$

$$i_c = \sqrt{2} I_p \cos(\omega t + 120 - \theta) \rightarrow \sqrt{2} I_p \angle 120 - \theta$$

$$P_A = |I_p| |V_p| \cos \theta \rightarrow (\theta - \phi)$$

$$P_B = |I_p| |V_p| \cos \theta$$

$$P_C = |I_p| |V_p| \cos \theta$$

$$P_{total} = P_{3\phi} = 3 |V_p| |I_p| \cos \theta$$

↙ phase voltage (RMS)
↘ phase current (RMS value)
↗ angle between phase to neutral voltage & phase current.

$$P_{3\phi} = 3 * P_{1\phi}$$

$$Q_{total} = Q_{3\phi} = 3 |V_p| |I_p| \sin \theta$$

$$= 3 * Q_{1\phi}$$

$$S'_{total} = S'_{3\phi} = 3 V_p I_p^* = 3 * S'_{1\phi}$$

$$S_{3\phi} = P_{3\phi} + j Q_{3\phi}$$

$$|S'_{total}| = 3 |V_p| |I_p| = 3 * |S'_{1\phi}|$$

↳ Apparent power.

* How to represent power quantities using line voltage & line current.

$$P_{3\phi} = 3 |V_p| |I_p| \cos \theta$$

Y-connected loads

$$I_p = I_L$$

$$|V_p| = \frac{|V_L|}{\sqrt{3}}$$

Δ-connected

$$V_p = V_L$$

$$|I_L| = \sqrt{3} |I_p|$$

line to line. angle between phase & neutral voltage

$$P_{3\phi} = 3 * \frac{|V_L|}{\sqrt{3}} * |I_L| \cos \theta$$

↳ line current & phase current.

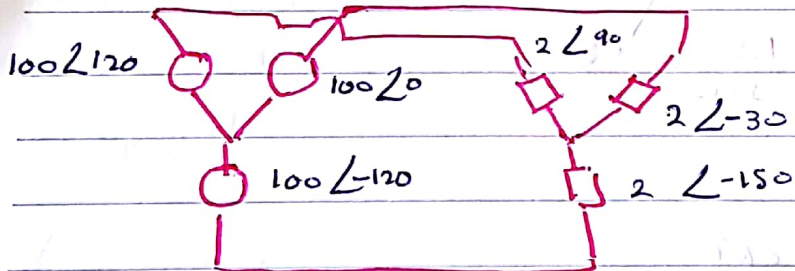
$$P_{3\phi} = \frac{3 |V_L| |I_L| \cos \theta}{\sqrt{3}}$$

$$P_{3\phi} = \sqrt{3} |V_L| |I_L| \cos \theta$$

$$* P_{3\phi} = 3 |V_p| |I_p| \cos \theta$$

$$* P_{3\phi} = \sqrt{3} |V_L| |I_L| \cos \theta$$

↳ $V_p \approx I_p$



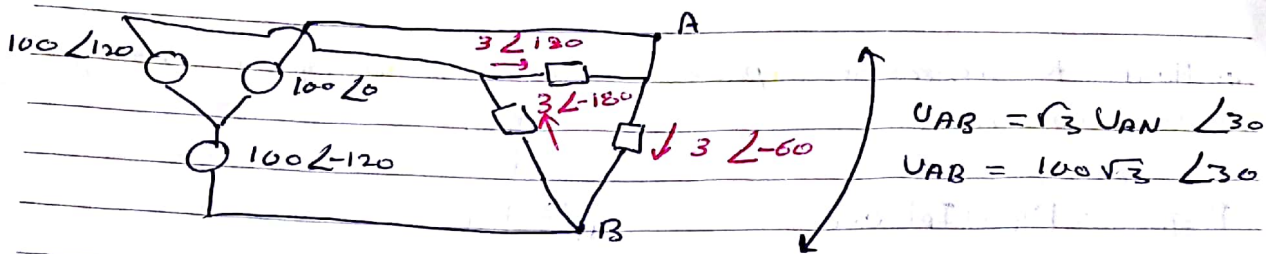
$$P_{3\phi} = 3 * 100 * 2 * \cos(0 - -30)$$

$$= \sqrt{3} * 300 \text{ w}$$

Using line values :-

$$P_{3\phi} = \sqrt{3} * \sqrt{3} * 100 * 2 \cos(30)$$

$$= 300 \sqrt{3} \text{ w}$$



$$P_{3\phi} = 3 |U_p| |I_p| \cos \theta$$

$$= 3 * [100 * \sqrt{3}] * 3 * \cos(30 - -60) = \underline{\underline{Zero}}$$

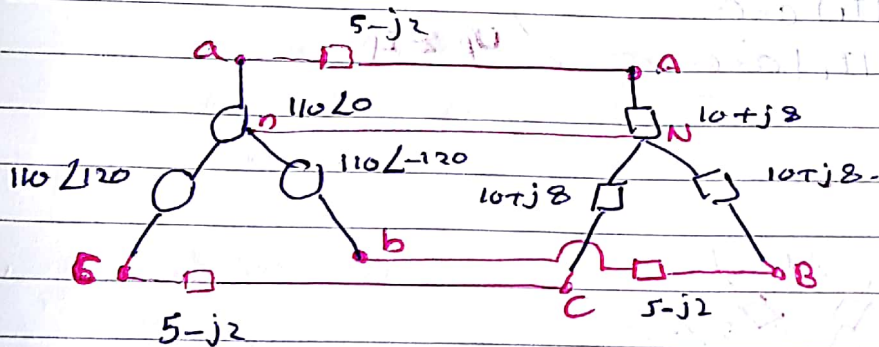
Using line:-

$$P_{3\phi} = \sqrt{3} |U_L| |I_L| \cos \theta$$

$$= \sqrt{3} * 100 * \sqrt{3} * 3 * \cos(30 + 60) = \underline{\underline{Zero}}$$

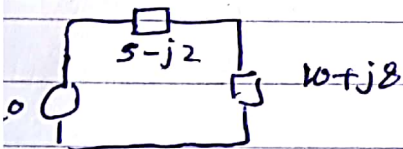
Ex.

Given the Ckt shown find the source complex power & the load complex power & the T.L losses.



* Single phase equivalent ckt

→ Take phase A



$$I_a = \frac{110 \angle 0}{15 + j6} = 6.8 \angle -21.8^\circ$$

$$S_{3\phi} (\text{source}) = 3 * U_p * I_p^*$$

$$= 3 * 110 \angle 0 * 6.8 \angle 21.8$$

$$= 2247 \angle 21.8$$

$$= 2087 + j 834.6$$

$P_{3\phi}$ ← $Q_{3\phi}$

$S(\text{load})$

$$U_{AN} = (\text{voltage division}) = \frac{110 \angle 0^\circ * (10 + j8)}{15 + 6j}$$

$$S_{\text{load}} = 3 U_p I_p^*$$

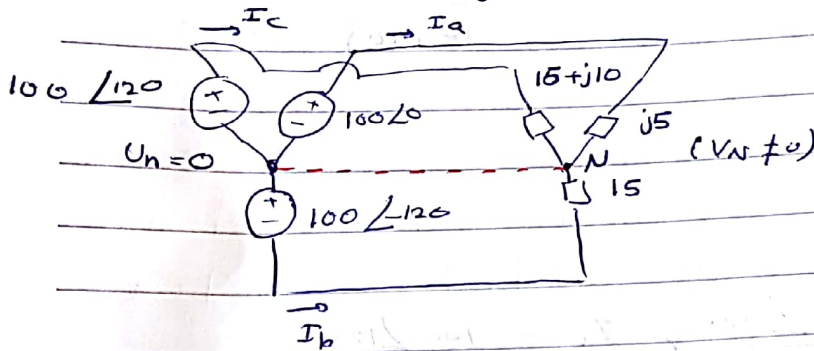
$$= 3 Z_p * I_p * I_p^*$$

$$= 3 Z_p |I_p|^2$$

$$= 3 * (10 + j8) * (6.8)^2$$

$$= 1392 + j1113 = 1782 \angle 38.66^\circ$$

* unbalanced system:-



KVL. (First way).

$$\begin{aligned} 1 & -100 \angle 0 + j5 - 15 I_b + 100 \angle 120 = 0 \\ 2 & -100 \angle 120 + I_c (10 + j10) + 15 I_b + 100 \angle -120 = 0 \\ 3 & \dots \end{aligned}$$

* ~~first way~~ second way:

KCL.

$$I_a + I_b + I_c = 0, \quad U_N \neq 0$$

nN $\sum \vec{I} = 0$ *
 $I_a + I_b + I_c = I_N$
 $U_N = \underline{\underline{Zero}}$

$\therefore \vec{I}_a + \vec{I}_b + \vec{I}_c = 0$ *

$$I_a + I_b + I_c = 0.$$

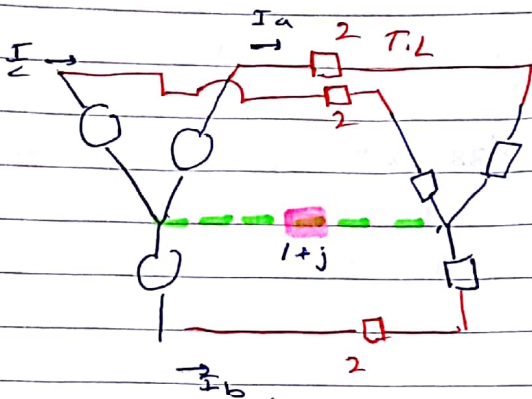
$$* I_a = \frac{100 \angle 0 - U_N}{j5}$$

$$* I_b = \frac{100 \angle -120 - U_N}{15}$$

$$* I_c = \frac{100 \angle 120 - U_N}{10 - j10}$$

$$\frac{100 \angle 0 - U_N}{j5} + \frac{100 \angle -120 - U_N}{15} + \frac{100 \angle 120 - U_N}{10 + j10} = 0$$

* find U_N then I_a, I_b, I_c



$$I_a = \frac{100 \angle 0 - U_N}{2 + j5}$$

$$I_b = \frac{100 \angle -120 - U_N}{12}$$

$$I_c = \frac{100 \angle 120 - U_N}{17 + j10}$$

(لو رصنا الممر)

$$I_a = \frac{100 \angle 0}{2 + j5}, \quad I_b = \frac{100 \angle -120}{12}, \quad I_c = \frac{100 \angle 120}{17 + j10}$$

(لو رصنا نقطة الصفر)

$$U_N \neq 0$$

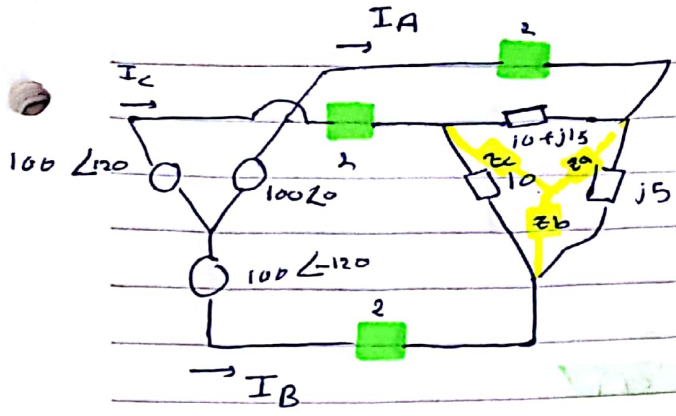
$$I_{Nn} = \frac{U_N}{1 + j} = I_a + I_b + I_c$$

$$I_a = \frac{100 \angle 0 - U_N}{2 + j5}$$

$$I_b = \frac{100 \angle -120 - U_N}{12}$$

$$I_c = \frac{100 \angle 120 - U_N}{17 - j10}$$

$$\frac{100 - U_N}{2 + j5} + \frac{100 \angle -120 - U_N}{12} + \frac{100 \angle 120 - U_N}{17 + j10} = \frac{U_N}{1 + j}$$



* Without T.L impedance:-

$$I_{AB} = \frac{U_{AB}}{j5} \quad I_{BC} = \frac{U_{BC}}{10}$$

$$I_{AB} = \frac{\sqrt{3} \cdot 100 \angle 30}{j5}, \quad I_{BC} = \frac{\sqrt{3} \cdot 100 \angle -40}{10}$$

$$I_{AB} = \frac{U_{AB}}{j5} = \frac{\sqrt{3} \cdot 100 \angle 30}{j5}$$

$$I_{BC} = \frac{U_{BC}}{10} = \frac{\sqrt{3} \cdot 100 \angle -90}{10}$$

$$I_{CA} = \frac{U_{CA}}{10+j15} = \frac{\sqrt{3} \cdot 100 \angle 150}{j15+10}$$

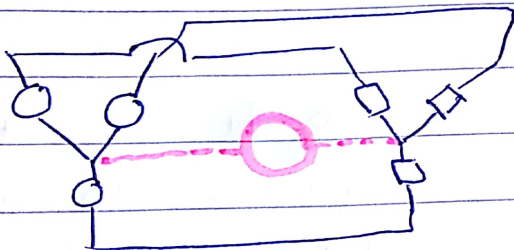
* with T.L

$U_{ab} \neq U_{AB}$

Δ-Y transformation.

$$Z_a = \frac{j5(10+j5)}{j5+10+j5+10}$$

$$I_a = \frac{100 \angle 0 - U_n}{2 + Z_a}$$



(A)

short circuit

(there is N_p line)

(V)

open ckt

(there is no N_p line)

* power measurement.

→ power is measured in the lab using a device called wattmeter.

How does wattmeter work:

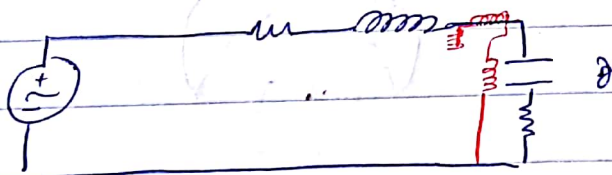
(displacement of pointer)
Deflection.

$P_{\text{measured}} \propto V, I, \cos\theta.$

$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta - \phi)$

Ex.

find the wattmeter reading (find the real power absorbed by the load)

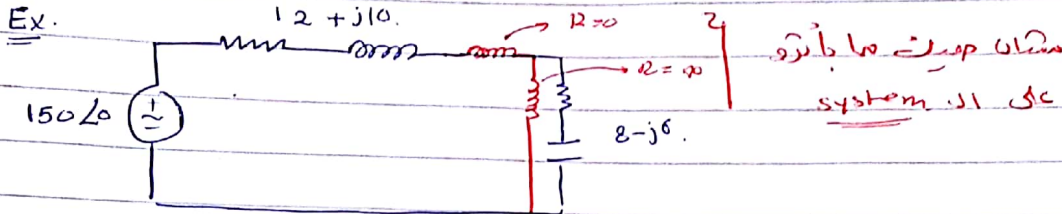
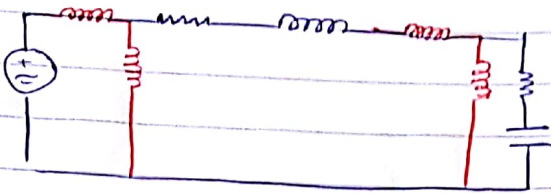


2 coils → parallel (voltage coil)
↳ series (current coil)

current coil → load current.

voltage coil → load voltage.

load power is measured by wattmeter
 load source is power is measured by
 coil



$$P = V_{rms} I_{rms} \cos(\theta - \phi)$$

$$V_{load} \rightarrow \text{voltage division} = \frac{150 \angle 0^\circ * (8 - j6)}{(20 + j4)} = 73.5 \angle -84^\circ$$

$$I_{load} \rightarrow \frac{150 \angle 0^\circ}{20 + j4} = 7.35 \angle -11.3^\circ$$

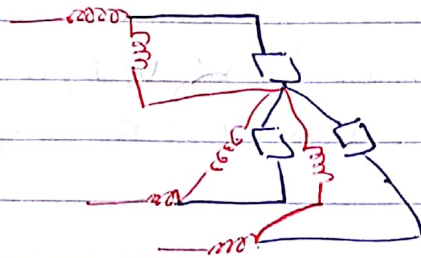
$$\begin{aligned} W_{reading} &= P(\text{load}) = V_{rms} I_{rms} \cos(\theta - \phi) \\ &= 73.5 * 7.32 * \cos(-84 + 11.3) \\ &= 432.1 \text{ W} \end{aligned}$$

* Power measurement in 3 phase system

$$P_{3\phi} = W_1 + W_2 + W_3$$

if the load is Balanced

$$P_{3\phi} = 3W_1$$

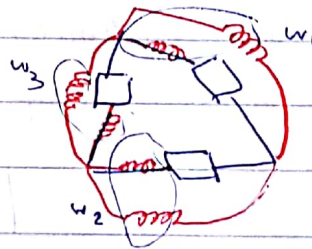


* for a Δ connected load.

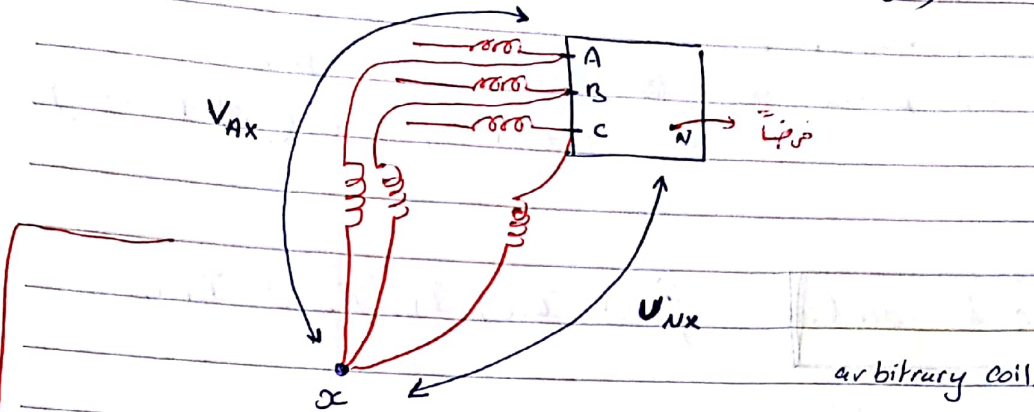
$$P_{3\phi} = W_1 + W_2 + W_3$$

if the load is Balanced

$$P_{3\phi} = 3W$$



(the neutral point is not accessible)

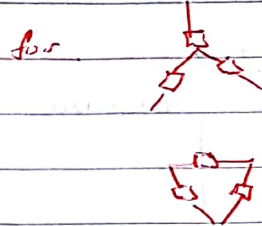


$$P_{3\phi} \stackrel{??}{=} w_1 + w_2 + w_3$$

$$w_1 = \frac{1}{T} \int_0^T U_{AN}(t) \cdot i_a(t) \cdot dt$$

$$w_2 = \frac{1}{T} \int_0^T U_{BN}(t) \cdot i_b(t) \cdot dt$$

$$w_3 = \frac{1}{T} \int_0^T U_{CN}(t) \cdot i_c(t) \cdot dt$$



$$w_1 = \frac{1}{T} \int_0^T U_{AX}(t) \cdot i_a(t) \cdot dt$$

$$w_2 = \frac{1}{T} \int_0^T U_{BX}(t) \cdot i_b(t) \cdot dt$$

$$w_3 = \frac{1}{T} \int_0^T U_{CX}(t) \cdot i_c(t) \cdot dt$$

$$\begin{aligned} U_{AX} &= U_{AN} + U_{NX} \\ U_{BX} &= U_{BN} + U_{NX} \\ U_{CX} &= U_{CN} + U_{NX} \end{aligned}$$

$$P_{3\phi} = \frac{1}{T} \int_0^T (U_{AN} + U_{NX}) \cdot i_a(t) \cdot dt + \frac{1}{T} \int_0^T (U_{BN} + U_{NX}) \cdot i_b(t) \cdot dt$$

$$+ \frac{1}{T} \int_0^T (U_{CN} + U_{NX}) \cdot i_c(t) \cdot dt$$

$$= \frac{1}{T} \left[U_{AN}(t) \cdot i_a(t) + U_{BN}(t) \cdot i_b(t) + U_{CN}(t) \cdot i_c(t) + U_{NX} (i_a + i_b + i_c) \right]$$

$$= \frac{1}{T} \int_0^T U_{AN}(t) \cdot i_a(t) \cdot dt + \frac{1}{T} \int_0^T U_{BN}(t) \cdot i_b(t) \cdot dt + \frac{1}{T} \int_0^T U_{CN}(t) \cdot i_c(t) \cdot dt$$

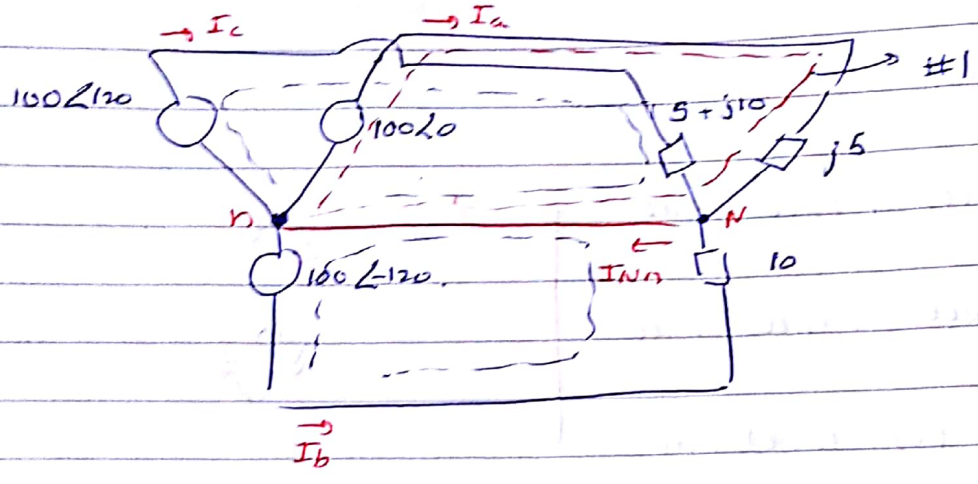
= zero (in Δ & y without neutral)

so

$P_{3\phi} = w_1 + w_2 + w_3$
 the ~~center~~ choice of point Σ will not affect
 the total power (3 ϕ power)

Unbalanced load

find I_a, I_b, I_c & I_{nn}



KVL at #1

$$-100 \angle 0 + j5 I_a = 0$$

$$I_a = \frac{100 \angle 0}{j5} = 20 \angle -90^\circ \text{ A}$$

KVL

$$I_b = \frac{100 \angle -120}{10} = 10 \angle -120 \text{ A}$$

KVL

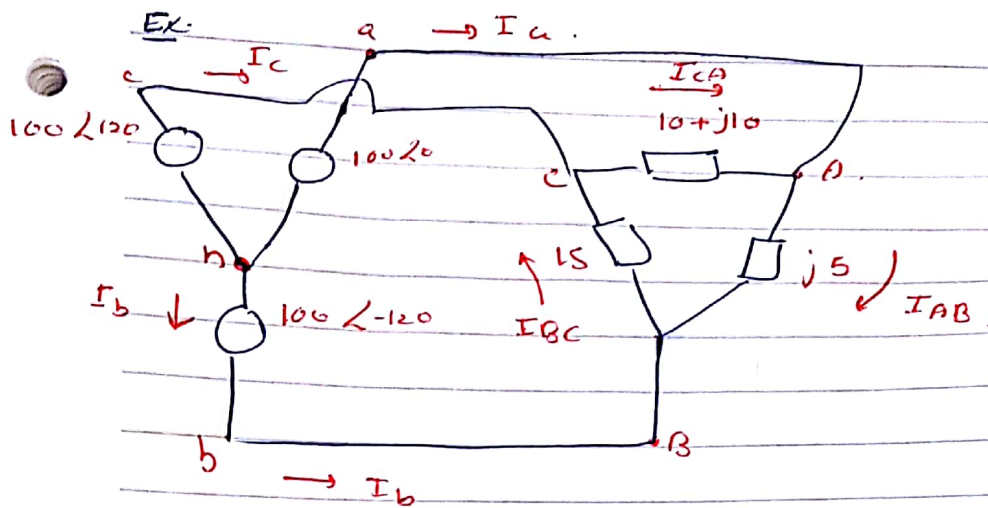
$$I_c = \frac{100 \angle +120}{5 + 10j} \neq$$

$$I_{nn} = I_a + I_b + I_c = 20 \angle -90^\circ + 10 \angle -120^\circ + \frac{100 \angle 120}{5 + 10j} \neq \underline{\underline{zero}}$$

$$V_N = V_n = 0$$

always = zero

تمام نيزک غير ذلك



find phase
2 line
currents.

$$I_{AB} = \frac{U_{AB}}{j5} = \frac{U_{ab}}{j5} = \frac{100\sqrt{3} \angle 30}{j5} = 20\sqrt{3} \angle -60 \text{ A.}$$

$$I_{BC} = \frac{U_{BC}}{15} = \frac{U_{bc}}{15} = \frac{100\sqrt{3} \angle -90}{15} = 6.66\sqrt{3} \angle -90 \text{ A.}$$

$$I_{CA} = \frac{U_{CA}}{j10} = \frac{U_{ca}}{10+j10} = \frac{\sqrt{3} * 100 \angle 150}{10+j10} \#$$

120°

→ for balanced load

$$I_{line} = \sqrt{3} I_{phase} \angle -30$$

→ for here ~~we~~ (non balanced load) we applied **KCL**

$$I_a + I_{CA} = I_{AB}$$

$$I_a = I_{AB} - I_{CA}$$

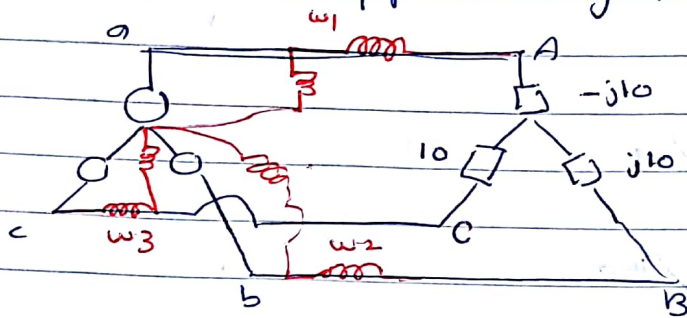
$$I_b + I_{AB} = I_{BC}$$

$$I_b = I_{BC} - I_{AB}$$

$$I_c + I_{BC} = I_{CA}$$

$$I_c = I_{CA} - I_{BC}$$

Find the reading of the 3 wattmeters & find the total power supplied by the source.



$$U_{ab} = 100 \angle 0^\circ$$

$$W_1 \Rightarrow U_{an} \text{ \& \ } I_a, \quad U_{an} = (100/\sqrt{3}) \angle -30^\circ$$

$$W_2 \Rightarrow U_{bn} \text{ \& \ } I_b, \quad U_{bn} = (100/\sqrt{3}) \angle -150^\circ$$

$$W_3 \Rightarrow U_{cn} \text{ \& \ } I_c, \quad U_{cn} = (100/\sqrt{3}) \angle 90^\circ$$

$$\frac{100}{\sqrt{3}} \angle -30^\circ - U_N + \frac{100}{\sqrt{3}} \angle -150^\circ - U_N + \frac{100}{\sqrt{3}} \angle 90^\circ - U_N = 0$$

$$\frac{100}{\sqrt{3}} \angle -30^\circ + \frac{100}{\sqrt{3}} \angle -150^\circ + \frac{100}{\sqrt{3}} \angle 90^\circ = 3U_N$$

→ find U_N then find I_a, I_b, I_c

$$* I_a = 19.32 \angle 15^\circ$$

$$* I_b = 19.32 \angle 16.2^\circ$$

$$* I_c = 10 \angle -90^\circ$$

$$w_1 \cdot P_1 = U_{an} \cdot I_a \cdot \cos(\theta_{U_{an}} - \phi_{I_a})$$

$$= \frac{100}{\sqrt{3}} \cdot 19.32 \cdot \cos(-30 - 15) = 788.7 \text{ W}$$

$$w_2 \cdot P_2 = U_{bn} \cdot I_b \cdot \cos(\theta - \phi)$$

$$= \frac{100}{\sqrt{3}} \cdot 19.32 \cdot \cos(-150 - 165)$$

$$= 788.7 \text{ W}$$

$$w_3 \cdot P_3 = U_{cn} \cdot I_c \cdot \cos(\theta - \phi)$$

$$= \frac{100}{\sqrt{3}} \cdot 10 \cdot \cos(90 + 90)$$

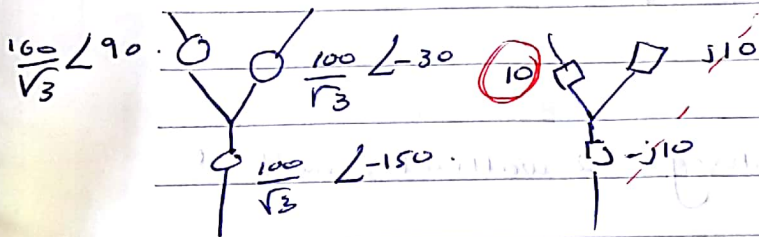
$$= -577.4 \text{ W}$$

total power supplied by the source is:

$$P_{3\phi} = P_1 + P_2 + P_3$$

$$= w_1 + w_2 + w_3$$

$$= 788.7 + 788.7 - 577.4 = 1000 \text{ W}$$



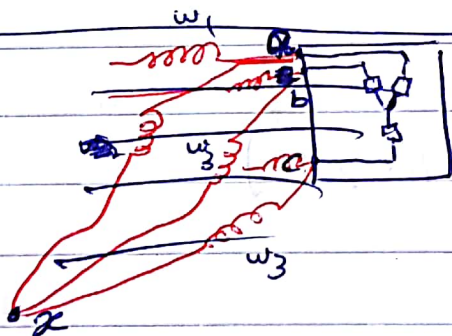
Real power only on 10Ω

$$P_{3\phi} = (I_c)^2 \cdot R$$

$$= (10)^2 \cdot 10$$

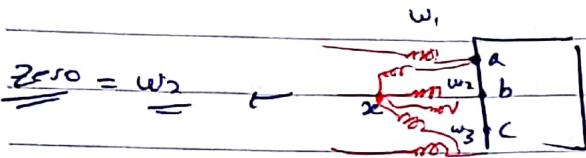
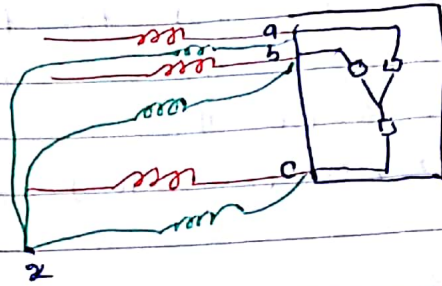
$$= 1000 \text{ W} \quad \#$$

The two wattmeter method :-



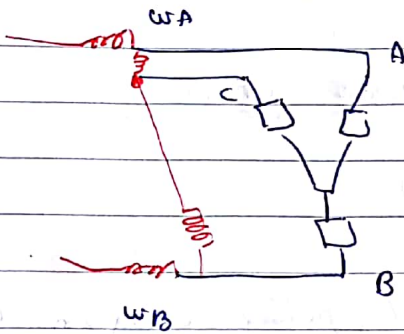
The Two wattmeter method:

α : what if α is chosen such that α is put on one of phases.



Two wattmeters.

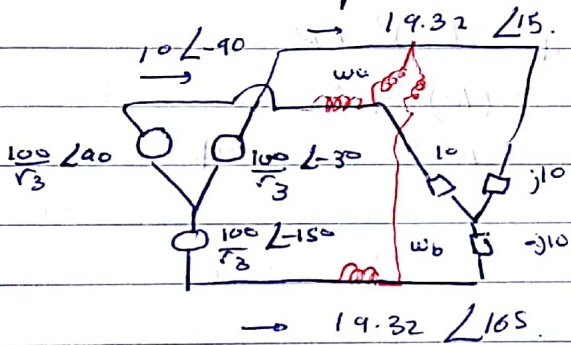
Two wattmeters.



$w_A \Rightarrow U_{AC}, I_a$

$w_B \Rightarrow U_{BC}, I_b$

* Solve the previous exp. using 2 wattmeter method



$w_a : U_{ca}, I_c$

$w_b \Rightarrow U_{ba}, I_b$

$U_{AB} = \sqrt{3} V_{an} \angle 30$

$V_{ab} = 100 \angle 0 \quad (V_{ba} = -V_{ab} = 100 \angle 180)$

$V_{bc} = 100 \angle -120$

$V_{ca} = 100 \angle 120$

$w_a : P_a : |V_{ca}| |I_c| \cos(\theta_{V_{ca}} - \phi_{I_c}) = 100 * 10 * \cos(120 + 90) = -886$

$w_b : P_b : |V_{ba}| |I_b| \cos(\theta_{V_{ba}} - \phi_{I_b}) = 100 * 19.32 \cos(180 - 165) = 1886 \text{ W}$

$1886 - 886 = 1000 \text{ W} \#$

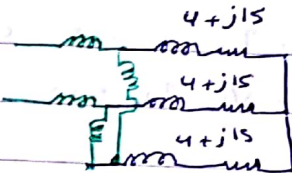
* Given that $V_{AB} = 230 \angle 0$

→ find the reading of each wattmeter.

∑ the total power supplied to the load.

$W_A : U_{AC} : I_A$

$W_B : U_{BC} : I_B$



$V_{AB} = 230 \angle 0$

$U_{BC} = 230 \angle -120$

$U_{CA} = 230 \angle 120, U_{AC} = 230 \angle 120 - 180$

$$I_a = \frac{U_{AN}}{4 + j15} = \frac{(U_{AB} / \sqrt{3}) \angle -30}{4 + j15} = \frac{230 \angle -30}{\sqrt{3} (4 + j15)} = 8.554 \angle -105.1^\circ$$

$I_b = 8.554 \angle -106 - 120$

$I_c = 8.554 \angle -105.1 + 120$

$W_1 : P_1 = 230 * 8.554 \cos(-60 + 105.1) = 1389 \text{ w}$

$W_2 : P_2 = 230 * 8.554 \cos(-120 + 125.1) = -512 \text{ w}$

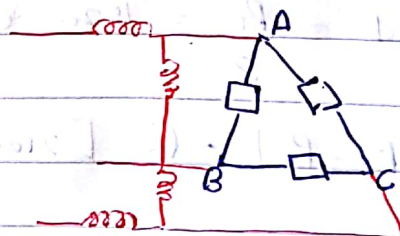
$P_T = P_1 + P_2 = 876.5 \text{ w}$

$P_T = 3 |I|^2 R = 3 * (8.554)^2 * 4 = 876.5 \text{ w}$

How to calculate the pF from the reading of wattmeter.

$$W_A = |U_{AB}| |I_A| \cos(\theta_{U_{AB}} - \phi_{I_A})$$

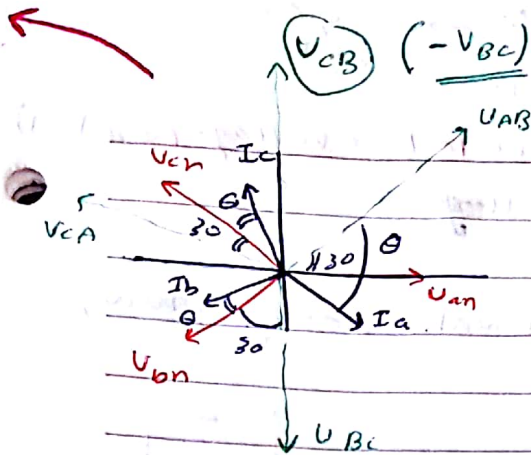
$$= |U_{LL}| |I_{LL}| \cos(\theta_{U_{AB}} - \phi_{I_A})$$



$$W_B = |U_{CB}| |I_C| \cos(\theta_{U_{CB}} - \phi_{I_C})$$

$$= |U_{LL}| |I_{LL}| \cos(\theta_{U_{CB}} - \phi_{I_C})$$

⇒ consider inductive load (voltage leads current)



$$P_A = V_{LL} I_L \cos(30 + \theta)$$

$$P_B = V_{LL} I_L \cos(30 - \theta)$$

$$\frac{P_A}{P_B} = \frac{V_{LL} I_L \cos(30 + \theta)}{V_{LL} I_L \cos(30 - \theta)}$$

$$\frac{P_A}{P_B} = \frac{\cos 30 \cos \theta - \sin 30 \sin \theta}{\cos 30 \cos \theta + \sin 30 \sin \theta}$$

$$\frac{P_A}{P_B} = \frac{\cos 30 \cos \theta - \sin 30 \sin \theta}{\cos 30 \cos \theta + \sin 30 \sin \theta}$$

$$= \frac{\left(\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right)}{\left(\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta \right)} \cdot \frac{2}{2} \cdot \frac{\cos \theta}{\cos \theta}$$

$$\frac{P_A}{P_B} = \frac{\sqrt{3} - \tan \theta}{\sqrt{3} + \tan \theta}$$

$$P_A (\sqrt{3} + \tan \theta) = P_B (\sqrt{3} - \tan \theta)$$

$$P_A \sqrt{3} + P_A \tan \theta = P_B \sqrt{3} - P_B \tan \theta$$

$$\tan \theta [P_A + P_B] = \sqrt{3} [P_B - P_A]$$

$$\tan \theta = \frac{\sqrt{3} [P_B - P_A]}{[P_A + P_B]}$$

$$\rightarrow \tan \theta = \frac{\sqrt{3} [P_2 - P_1]}{[P_1 + P_2]}$$

$P_2 = P_1$
(Resistive load)

$P_2 > P_1$
(Inductive load)

$P_2 < P_1$
(Capacitive load)

* ولقد صحت العلاقات
بكونه على وقت
معيّن.

P_1 ← الفولت
الفولت

P_2 ← الفولت
الفولت

$$P_T = P_1 + P_2 = V_{LL} I_L \cos(30 + \theta) + V_{LL} I_L \cos(30 - \theta)$$

$$= V_{LL} I_L \cos 30 \cos \theta - V_{LL} I_L \sin 30 \sin \theta + V_{LL} I_L \cos 30 \cos \theta + V_{LL} I_L \sin 30 \sin \theta$$

$$= 2 V_{LL} I_L \cos 30 \cos \theta = \boxed{\sqrt{3} V_{LL} I_L \cos \theta} = P_T$$

$$\rightarrow \sqrt{3} [P_2 - P_1] = \sqrt{3} V_{LL} I_L \cos(30 - \theta) - \sqrt{3} V_{LL} I_L \cos(30 + \theta)$$

$$= \sqrt{3} V_{LL} I_L \cos 30 \cos \theta + \sqrt{3} V_{LL} I_L \sin 30 \sin \theta$$

$$- \sqrt{3} V_{LL} I_L \cos 30 \cos \theta + \sqrt{3} V_{LL} I_L \sin 30 \sin \theta$$

$$= \sqrt{3} V_{LL} I_L \sin \theta = Q_T$$

$$P_T = P_2 + P_1$$

$$Q_T = \sqrt{3} [P_2 - P_1]$$

$$|S| = \sqrt{P_2^2 + P_1^2}$$

$$\tan \theta = \frac{Q_T}{P_T} = \frac{\sqrt{3} [P_2 - P_1]}{[P_2 + P_1]}$$

Ex. Two wattmeters produce readings $P_1 = 1560$, $P_2 = 2100$ w when connected to 3 ϕ load, find:

$$[1] P_T = P_1 + P_2 = 1560 + 2100 = 3660 \text{ w}$$

if the load is balanced,

$$P_{1\phi} = \frac{P_T}{3} = 1220 \text{ w}$$

$$[2] Q_T = \sqrt{3} [P_2 - P_1] = \sqrt{3} [2100 - 1560] = 935 \text{ VAR}$$

if the load is balanced,

$$Q_{1\phi} = \frac{Q_T}{3} = 311.67 \text{ VAR}$$

$$[3] PF = \cos \left[\tan^{-1} \left[\frac{\sqrt{3} [2100 - 1560]}{2100 + 1560} \right] \right]$$

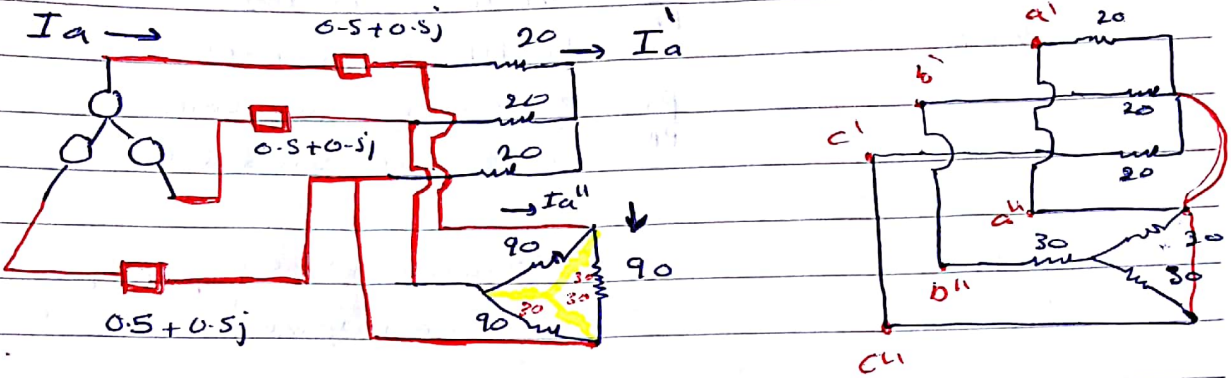
$$= 0.9689 \text{ lagging } (P_2 > P_1)$$

$\angle 0 \rightarrow$ $\frac{V_{rms}}{\sqrt{3}}$
 RMS.
 line to line voltage (line to line)

Example:-

Voltage at generator = 208 voltage

Find the line currents & phase currents (away from)



"parallel"

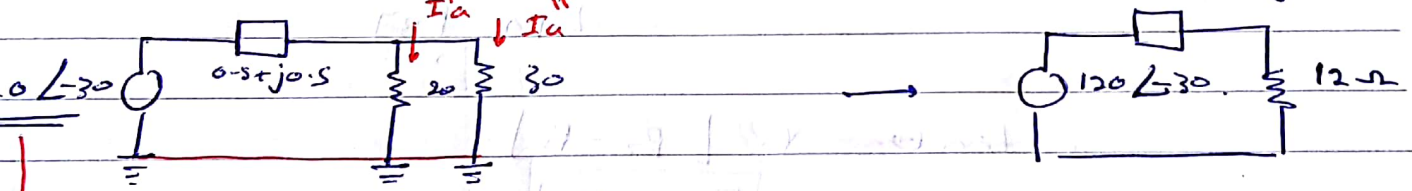
* Convert Δ to Y .

$$Z_Y = \frac{1}{3} Z_{\Delta} \text{ (balanced load)}$$

$$= 30 \Omega$$

* then single phase equivalent circuit.

(take phase A)



Take V_{AB} as our reference

$$V_{AB} = 208 \angle 0$$

$$V_{AN} = \frac{V_{AB}}{\sqrt{3}} \angle -30 = 120 \angle -30$$

$$I_a = \frac{120 \angle -30}{12 + 0.5 + j0.5}$$

$$12 + 0.5 + j0.5$$

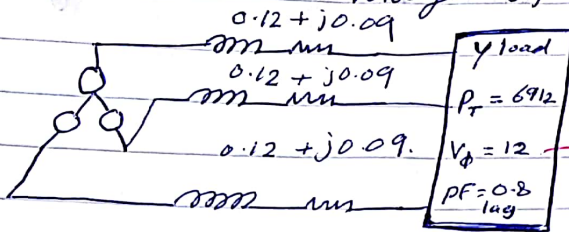
$$I_a' = \frac{30}{50} I_a$$

$$I_a'' = \frac{20}{50} I_a$$

Current division

Ex.

Find the line voltage of the source side.

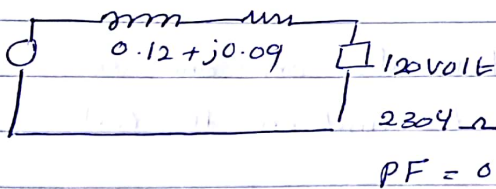


Y load
 $P_T = 6912$
 $V_\phi = 12$
 $PF = 0.8$
 lag

→ phase to neutral voltage.

→ single phase eq. ckt.

* Take V_{AN} as our reference.



* $P = U \cdot I \cdot PF$

$2304 = |120| |I| \cdot 0.8$

$|I| = 24 \text{ A}$

$V_{an} = 120 \angle 0^\circ + (0.12 + j0.09) * (24 \angle -36.84^\circ)$

$= 123.6 \angle 0^\circ$

(lagging)

$V_{ab} = \sqrt{3} * 123.6 \angle +30^\circ = 214.1 \angle 30^\circ \text{ volt}$