* partial.

1

11.1, 11.2 fourier series.

* if $f(x)$ is periodic with periode $(2L)$

$f(x \mp n(2L)) = f(x)$, then the fourier series of $f(x)$ is:

$$\to f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{nx\pi}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right).$$

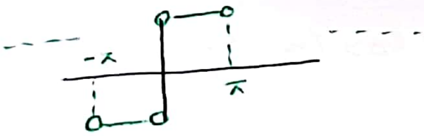
$$* \; a_0 = \frac{1}{2L} \int_{-L}^{L} f(x) \cdot dx.$$

$$* \; a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cdot \cos\left(\frac{n\pi x}{L}\right) \cdot dx.$$

$$* \; b_n = \frac{1}{L} \int_{-L}^{L} f(x) \cdot \sin\left(\frac{nx\pi}{L}\right) \cdot dx.$$

Ex. Find the Fourier series if:

$$f(x) = \begin{cases} -K, & -\pi < x < 0 \\ K, & 0 < x < \pi \end{cases}$$




The fourier series of $f(x)$ is:

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx), \; 2L = 2\pi \to \boxed{L = \pi}.$$

$$\to a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \cdot dx = \frac{1}{2\pi} \left[ \int_{-\pi}^{0} -K\, dx + \int_{0}^{\pi} K \cdot dx \right] = \frac{1}{2\pi} \left[ -K\pi + K\pi \right] = \text{Zero}$$

(odd fun...)

$$\to a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{f(x)}_{\text{odd}} \underbrace{\cos(nx)}_{\times \text{ even}} \cdot dx = \frac{1}{\pi} \left[ \int_{-\pi}^{0} -K\cos nx \cdot dx + \int_{0}^{\pi} K\cos(nx) \cdot dx \right] = \text{Zero}$$

$= \text{odd}.$

$$\hookrightarrow \int_{-L}^{L} \text{odd} = \text{zero}.$$

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$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,.\,\sin(\pi x)\,.\,dx = \frac{2}{\pi}\int_0^L f(x)\sin(nx)\,dx$$

odd × odd = even.

$$= \frac{2}{\pi}\int_0^{\pi} K\sin(nx)\,.\,dx = \frac{-2}{\pi}\,\frac{\cos(nx)}{n} * K\Big|_0^{\pi} = \frac{2K}{\pi}\left[\frac{1-\cos(n\pi)}{n}\right]$$

$\cos(n\pi) \to (-1)^n$

$$= \frac{2K}{\pi}\left[\frac{1-(-1)^n}{n}\right]$$

$$\equiv \begin{cases} \dfrac{2K}{\pi} * \dfrac{2}{n}, & n:\text{odd.} \\ \text{zero}, & n:\text{even.} \end{cases} \longrightarrow \frac{4K}{\pi} * \frac{1}{(2n-1)}$$

بتعطيني الأرقام الفردية فقط.

* So, the fourier series of $f(x)$:

$$f(x) \sim \sum_{n=1}^{\infty} \frac{4K}{\pi(2n-1)} \sin((2n-1)x)$$

---

*** Remark :-**

① if $f$ is an even function then the fourier series of $f(x)$ is:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right).$$

$$\# \; a_0 = \frac{2}{2L}\int_0^L f(x)\,.\,dx.$$

$$\# \; a_n = \frac{2}{L}\int_0^L f(x)\,.\,\cos\left(\frac{n\pi x}{L}\right).$$

② if $f$ is an odd function then the fourier series is:

$$f(x) \sim \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

$$\# \; b_n = \frac{2}{L}\int_0^L f(x)\,.\,\sin\left(\frac{n\pi x}{L}\right).\,dx.$$

(3) let $f(x)$ be periodic with period $(2L)$ & piecewise cont. on $-L \leq x \leq L$ moreover $f(x)$. have left hand derivative & right hand derivative at any $x$, then the fourier series of $f$, converge. 

→ if $f$ is cont. at $x_0$, then the series at $x_0$ converge to $f(x_0)$.

→ if $f$ is discont. at $x_0$, then the series at $x_0$ converge to

$$\frac{f(x_0^+) + f(x_0^-)}{2}$$ (average)

---

**Ex.**

$$f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$$

$$f(x+2\pi) = f(x).$$

* we find the fourier series of $f(x)$:

$$f(x) \sim \sum_{n=1}^{\infty} \frac{4K}{\pi(2n-1)} \sin((2n-1)x)$$

→ AT $t = \pi/2$. then $f(\pi/2) = \sum_{n=1}^{\infty} \frac{4K}{\pi(2n-1)} \sin\left(\frac{\pi}{2}(2n-1)\right)$ → $(-1)^{n+1}$

$0 < \pi/2 < \pi$

$$\frac{\pi}{4\cancel{K}} * \cancel{K} = \sum_{n=1}^{\infty} \frac{4K(-1)^{n+1}}{\pi(2n-1)} \quad * \frac{\pi}{4K}$$

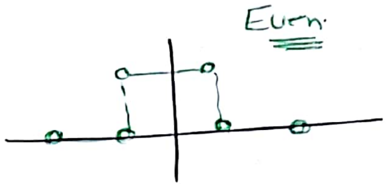
$$\frac{\pi}{4} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)}$$

#

**Example**

find the fourier series of.

$$f(x) = \begin{cases} 0 & , -2 < x < -1 \\ k & , -1 < x < 1 \\ 0 & , 1 < x < 2 . \end{cases}$$




$$f(x) = f(x+4)$$

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2}\right)$$

$$\# \; a_0 = \frac{2}{2(2)} \int_0^2 k\, f(x) \, dx = \frac{1}{2} \left[ \int_0^1 k \, dx + \int 0 \right] = k/2 .$$

$$\# \; a_n = \int_0^1 k \cos\left(\frac{n\pi x}{2}\right) . dx = \sin\left(\frac{n\pi x}{2}\right) * \frac{2k}{n\pi} \Big|_0^1 = \frac{2k}{n\pi} \sin\left(\frac{n\pi}{2}\right) .$$

$$* \; \frac{2k}{n\pi} \sin\left(\frac{n\pi}{2}\right) = \begin{cases} \dfrac{2k}{n\pi} & , n = 1, 5, 9, \ldots \\ \dfrac{-2k}{n\pi} & , n = 3, 7, 11, \ldots \end{cases}$$

$$= \frac{k}{2} + \frac{2k}{\pi(1)} \cos\left(\frac{\pi x}{2}\right) + \frac{2k}{\pi(3)} (-1) \cos\left(\frac{3\pi x}{2}\right) + \frac{2k}{\pi(5)} (1) \cos\left(\frac{5\pi x}{2}\right)$$

$$= \frac{k}{2} + \sum_{n=1}^{\infty} \frac{2k(-1)^{n+1}}{\pi(2n-1)} \cos\left(\frac{(2n-1)\pi x}{2}\right) \quad \#$$

→ Show that.

① show that $\displaystyle \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = \frac{\pi}{4}$ .

at $x=0$ (لأن ال cos ثابت)

the series convergent.

$f(0) = k$

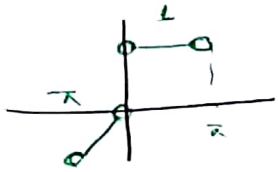
$$k = k/2 + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2k}{n} \cos\left((2n-1)\frac{\pi x}{2}\right) = \frac{k}{2} + \sum_{n=1}^{\infty} \frac{2k(-1)^{n+1}}{\pi(2n-1)} \Rightarrow \sum_{n=1}^{\infty} \frac{2k(-1)^{n+1}}{\pi(2n-1)} = k/2$$

$$\rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\pi(2n-1)} = \frac{\pi}{4} \quad \#$$

## Example 6.

find the fourier series of :

$$f(x) = \begin{cases} x, & -\pi < x < 0. \\ 1, & 0 < x < \pi. \end{cases}$$




$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

$$\# \; a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,.dx = \frac{1}{2\pi}\left[\int_{-\pi}^{0} x\,.dx + \int_{0}^{\pi} 1\,.dx\right] = \frac{1}{2\pi}\left[\frac{x^2}{2}\Big|_{-\pi}^{0} + \pi\right]$$

$$= \boxed{\frac{1}{2} - \frac{\pi}{4}}$$

$$\# \; a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,.dx$$

$$= \frac{1}{\pi}\Big[\underbrace{\int_{-\pi}^{0} x\cos(nx)\,.dx}_{\text{By parts.}} + \int_{0}^{\pi}\cos(nx)\,.dx. \quad \to \frac{\sin(nx)}{n}$$

$$\begin{array}{lcl} x & \oplus & \cos(nx) \\ 1 & \ominus & \frac{\sin(nx)}{n} \\ 0 & & -\frac{\cos(nx)}{n^2} \end{array}$$

$$a_n = \frac{1}{\pi}\left[\underbrace{\cancel{\frac{x\sin(nx)}{n}}}_{=\text{zero}} + \frac{\cos nx}{n^2}\right]\Bigg|_{-\pi}^{0} + \underbrace{\cancel{\frac{\sin(nx)}{n}}\Bigg|_{0}^{\pi}}_{=\text{zero}} = \frac{1}{\pi}\frac{\cos(nx)}{n^2}\Bigg|_{-\pi}^{0}$$

$$= \frac{1}{\pi n^2} - \frac{\cos(-nx)}{\pi n^2} = \left(\frac{1}{n^2} - \frac{\overbrace{\cos(nx)}^{\to (-1)^n}}{n^2}\right)1/\pi$$

$$= \cancel{\frac{1}{\pi n^2}} - \frac{1}{\pi}\left(\frac{1}{n^2} - \frac{(-1)^n}{n^2}\right) = \frac{1-(-1)^n}{\pi n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx).dx.$$

$$= \frac{1}{\pi}\left(\int_{-\pi}^{0} x\sin(nx).dx + \int_{0}^{\pi}\sin(nx).dx\right).$$

$x$ $\quad$ $\sin(nx)$

$1$ $\quad$ $-\frac{\cos(nx)}{n}$

$0$ $\quad$ $-\frac{\sin(nx)}{n^2}.$

$$= \frac{1}{\pi}\left(\left(-\frac{x\cos(nx)}{n} + \frac{\sin(nx)}{n^2}\right) + -\frac{\cos(nx)}{n}\right)\Bigg|_0^{\pi}.$$

$$= \frac{1}{\pi}\left[\left(0 - \frac{\pi\cos(n(-\pi))}{n}\right) + \frac{1}{n} - \frac{\cos(n\pi)}{n}\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi(-1)^n}{n} + \frac{1-(-1)^n}{n}\right]$$

$$= \frac{(-1)^{n+1}}{n} + \frac{1-(-1)^n}{\pi n}.$$

The Fourier Series of $f$ is:

$$f(x) \sim \frac{1}{2} - \frac{\pi}{4} + \sum_{n=1}^{\infty}\frac{2}{\pi(2n-1)^2}\cos((2n-1)x) + \sum_{n=1}^{\infty}\left(\frac{(-1)^{n+1}}{n} + \frac{(1-(-1)^n)}{\pi n}\right)\sin(nx)$$

* show that:-

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = \frac{\pi^2}{8}.$$

At $x=0$, the series converges to

$$\frac{f(0^+)+f(0^-)}{2} = \frac{1}{2} - \frac{\pi}{4} + \sum_{n=1}^{\infty}\frac{2\cos((2n-1)(0))}{\pi(2n-1)^2}$$

$$= \cancel{\frac{1}{2}} = \cancel{\frac{1}{2}} - \frac{\pi}{4} + \sum_{n=1}^{\infty}\frac{2}{\pi(2n-1)^2}$$

$$\frac{\pi}{2} * \frac{\pi}{4} = \sum_{n=1}^{\infty}\frac{\cancel{2}}{\cancel{\pi}(2n-1)^2} * \cancel{\frac{\pi}{2}}$$

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} \quad \#$$

→ At $x = \frac{21\pi}{4}$

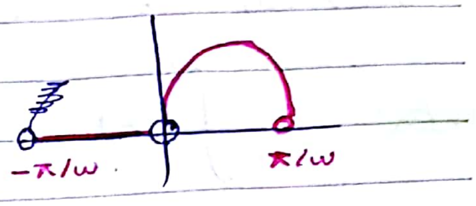
$\frac{21\pi}{4} \rightarrow \frac{20\pi}{4} + \frac{\pi}{4} \rightarrow 4\pi + \pi + \frac{\pi}{4} \rightarrow 4\pi + \frac{5\pi}{4}$

the series converge to $\frac{-3\pi}{4}$.

Ex.

Find the fourier series.

$$f(x) = \begin{cases} 0 & , \frac{-\pi}{w} < x < 0 \\ E\sin(wx) & , 0 < x < \frac{\pi}{w} \end{cases}$$




$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos(nwx) + b_n \sin(nwx)$$

* $a_0 = \frac{1}{2(\frac{\pi}{w})} \int_{-\pi/w}^{\pi/w} f(x).\, dx = \frac{w}{2\pi} \int_0^{\pi/w} E\sin(wx).\, dx = \frac{w}{2\pi} \left(\frac{-E\cos(wx)}{w}\right)\Big|_0^{\pi/w}$

$$\boxed{a_0 = E/\pi}$$

* $a_n = \frac{w}{\pi} \int_{-\pi/w}^{\pi/w} f(x)\cos(nwx).\, dx$

$= \frac{w}{\pi} \int_0^{\pi/w} E\sin(wx)\cos(nwx)\, dx.$

~~[illegible]~~

$\sin(a \pm b) = \sin a\cos b \pm \cos a \sin b$

$2\sin(a \pm b) = \sin(a+b) + \sin(a-b)$.

$= \frac{wE}{2\pi} \int_0^{\pi/w} \sin(wx).\cos(nwx).\, dx.$

$= \frac{wE}{2\pi} \left( \int_0^{\pi/w} \sin((1+n)wx) + \sin((1-n)wx) \right).dx.$

( sin sin ui )

$= \frac{wE}{2\pi} \left[ \frac{1}{w(1+n)} + \frac{1}{w(1-n)} - \frac{(-1)^{n+1}}{w(1+n)} - \frac{(-1)^{n+1}}{w(1-n)} \right]$

$= \frac{wE}{2\pi} \left[ \frac{2}{w(1+n)} + \frac{2}{w(1-n)} \right]$

$\frac{\cancel{w}E\cancel{*2}}{\cancel{2}\pi\cancel{w}} \left[ \frac{1 + \cancel{2n} + 1 \cancel{- 2n}}{(1+2n)(1-2n)} \right] = \frac{2E}{\pi(1-4n^2)}$

$\underline{\underline{n=1}}$

$$\frac{wE}{\pi}\int_0^{\pi/w} \sin(wx)\cos(wx).dx.$$

$$= \frac{wE}{\pi} \left.\frac{\sin^2 wx}{2}\right|_0^{\pi/w} = 0$$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b.$

$2\sin(a)\sin b = \cos(a-b) - \cos(a+b).$

$$*b_n = \frac{w}{\pi}\int_{-\pi/w}^{\pi/w} f(x) \sin(nwx).dx.$$

$$= \frac{w}{\pi}\int_0^{\pi/w} E \sin(wx)\sin(nwx).dx$$

$$= \frac{wE}{\pi(2)}\int_0^{\pi/w} (\cos((1+n)wx) - \cos((1-n)wx)) = 0, n \neq 1$$

$\underline{\underline{n=1}}$

$$\frac{wE}{\pi}\int_0^{\pi/w} \sin(wx)\sin(wx).dx = \frac{wE}{\pi} * \frac{\pi}{2} = \boxed{\frac{wE}{2}}$$

* The fourier Series of $f(x)$

$$f(x) = \frac{E}{\pi} + \frac{E}{2} * \sin(wx) + \sum_{n=2}^{\infty} \frac{2E}{\pi} \frac{\cos(2nwx)}{(1-4n^2)}$$

* Show that : $\sum_{n=2}^{\infty} \frac{1}{4n^2-1} = \frac{1}{2}$

At $x=0$ the series conv. to $0$

$$0 = \frac{E}{\pi} + \frac{E}{2}\sin(w(0)) + \sum_{n=2}^{\infty} \frac{2E\cos(2nw(0))}{1-4n^2}$$

$$= \sum_{n=2}^{\infty} \frac{2E}{\pi} * \frac{1}{1-4n^2} = -\frac{E}{\pi}$$

$$\sum_{n=2}^{\infty} \frac{1}{1-4n^2} = -\frac{1}{2} \quad \#$$

* Suppose $f(x)$, $0 < x < L$.

Then the fourier cosine of $f$, is given:

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x/L).$$

$$a_0 = \frac{2}{2L}\int_0^L f(x)\,dx, \quad a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx.$$

* The Fourier sine of $f(x)$.

$$\rightarrow f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right).$$

$$\rightarrow b_n = \frac{2}{L}\int_0^L f(x)\,\sin\left(\frac{n\pi x}{L}\right)dx.$$

Ex.

Find the fourier cosine for the fourier sine Series of $f(x) = x$, $0 < x < \pi$.

$\rightarrow$ fourier cosine:

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

$$\sim a_0 + \sum_{n=1}^{\infty} a_n \cos(nx), \quad L = \pi.$$

$$\rightarrow a_0 = \frac{2}{2\pi}\int_0^\pi x\,dx = \frac{1}{\pi} * \frac{x^2}{2}\Big|_0^\pi = \frac{1}{\pi} * \frac{\pi^2}{2} = \frac{\pi}{2}.$$

$$\rightarrow a_n = \frac{2}{\pi}\int_0^\pi x\cos(nx)\,dx = \frac{2}{\pi}\left(\underbrace{\cancel{\frac{x\sin(nx)}{n}}}_{=0} + \frac{\cos(nx)}{n^2}\right)\Big|_0^\pi$$

$$= \frac{2}{\pi}\left(\frac{(-1)^n - 1}{n^2}\right) = \begin{cases} \frac{-2}{n^2}, & n \in \text{odd}. \\ 0, & n \in \text{even}. \end{cases}$$

$$f(x) \sim \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{-4}{\pi(2n-1)^2}\cos((2n-1)x).$$

* The Fourier sine of $f(x)$.

$$f(x) \sim \sum_{n=1}^{\infty} b_n \sin(nx).$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx) = \frac{2}{\pi}\int_0^{\pi} x\sin(nx)\,.dx \quad \text{(by parts)}$$

$$b_n = \frac{2}{\pi}\left[\frac{-x\cos(nx)}{n} + \frac{\sin(nx)}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\pi\cos(n\pi)}{n}\right] = \frac{-2(-1)^n}{n}$$

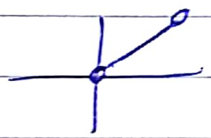
$$f(x) \sim \sum_{n=1}^{\infty} \frac{-2(-1)^n}{n}\sin(nx).$$

At $x=-2$, the Fourier cosine converge to $\underline{\underline{2}}$

$x=-2$, the Fourier sine " " $-2$.

$x=2$, the " cosine " " $2$.

$x=2$, " " sine " " $2$.



Cos & sin converge on $x$ only for $0<x<\pi$

[cos converge to even extension]

[sin converge to odd extension]

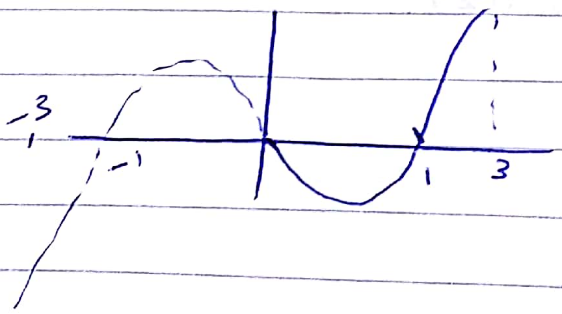
cos → better than sin.

(because of $(2n-1)$)

[illegible]

**Ex:**

Find the Fourier series of $f(x)$ $x^2 - x$, $1<x<3$.




(odd extension)

so → only sin.

* The fourier sine series of $f(x)$

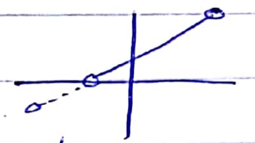
$$f(x) \sim \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{3}\right)$$

$$b_n = \frac{2}{3}\int_0^3 f(x)\sin\left(\frac{n\pi x}{3}\right).dx = \frac{2}{3}\int_0^3 \underbrace{(x^2-x)\sin\left(\frac{n\pi x}{3}\right)}_{\text{by parts.}}.dx$$

$$= \frac{2}{3}\Big[(x^2-x)(-3/n\pi)\cos(n\pi x/3) + (2x-1)(3/n\pi)^2\sin(n\pi x/3) + 2(3/n\pi)^3\cos(n\pi x/3)\Big]\Big|_0^3$$

| | | |
|---|---|---|
| $x^2-x$ | $+$ | $\sin(n\pi x/3)$ |
| $2x-1$ | $-$ | $-3/n\pi \cos(n\pi x/3)$ |
| $2$ | $+$ | $-(3/n\pi)^2\sin(n\pi x/3)$ |
| $0$ | | $+(3/n\pi)^3\cos(n\pi x/3)$ |

**Ex:** find fourier series of $f(x) = x+1, \ -1 < x < 2$.



↳ not even or odd extension.

So, the fourier series of $f(x)$ is:

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{2}\right) + b_n\sin\left(\frac{n\pi x}{2}\right).$$

$$\rightarrow a_0 = \frac{1}{2(2)}\int_{-2}^{2}(x+1).dx = 1.$$

$$\rightarrow a_n = \frac{1}{2}\int_{-2}^{2}\underbrace{(x+1)\cos(n\pi x/2)}_{\text{By parts.}}.dx$$

| | |
|---|---|
| $x+1$ | $\cos(n\pi x/2)$ |
| $1$ | $(2/n\pi)\sin(n\pi x/2)$ |
| $0$ | $-(2/n\pi)^2\cos(n\pi x/2)$ |

$$= \frac{1}{2}\left[\frac{(x+1)^2}{n\pi}\sin\left(\frac{n\pi x}{2}\right) + \frac{4}{n^2\pi^2}\overbrace{\cos\left(\frac{n\pi x}{2}\right)}^{\text{even.}}\right]\Bigg|_{-2}^{2} = \text{Zero}$$

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$$b_n = \frac{1}{2}\int_{-2}^{2} (x+1)\sin\left(\frac{n\pi x}{2}\right) . dx$$

by parts.

$x+1$ (+) $\sin(n\pi x/2)$
$1$ (−) $-(2/n\pi)\cos(n\pi x/2)$
$0$ $-(2/n\pi)^2\sin(n\pi x/2)$

$$= \frac{1}{2}\left[(x+1)(-2/n\pi)\cos(n\pi x/2) + (2/n\pi)^2\sin(n\pi x/2)\right]\Big|_{-2}^{2}$$

$$= \frac{1}{2}\left[-3\,(2/n\pi)\cos(n\pi) - (+1)(2/n\pi)\cos(-n\pi)\right]$$

## * Remark.

(Sum of scalar multiple):–

the fourier ~~series~~ coefficient of sum $f_1 + f_2$ are the sum of the corresponding fourier coefficient of $f_1$ & $f_2$.

The fourier coeff. of of are $c$ times the fourier coeff(s) of $f$.

**Ex.**

Find the fourier series of $f(x) = x+1$
$-2 \leq x \leq 1$

* $f_1(x) = 1$, the fourier series of $f(x) = 1$, is $f_1(x) = 1$.

* $f_2(x) = x \to$ the fourier series of $f_2(x) = x$, is : odd.

$f_2(x) \sim \sum b_n \sin(n\pi x/2)$

$$b_n = \frac{2}{2}\int_0^2 x \sin\left(\frac{n\pi x}{2}\right) . dx \quad \text{(by parts)}.$$

$$= -\frac{2x}{n\pi}\cos(n\pi x/2) + \left(\frac{2}{n\pi}\right)^2 \sin(n\pi x/2)\Big|_0^2 = \frac{-4(-1)^n}{4\pi}.$$

the fourier series of $f_2(x) = x$.

$$= \sum \frac{-4(-1)^n}{4\pi}\sin\left(\frac{n\pi x}{2}\right).$$

the fourier series of $(x+1)$ $= 1 + \sum \frac{-4(+1)^n}{4\pi}\sin(n\pi x/2)$.

**Ex.** find the fourier ~~Sri~~ Series of.

$$f(x) = x + \cos(4x) + \sin(3x) + \cos^2(x), \quad x \in (-\pi, \pi).$$

$$\rightarrow a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

$$f_1(x) = x.$$

$$f_2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x) + \cos 4x + \sin 3x.$$

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