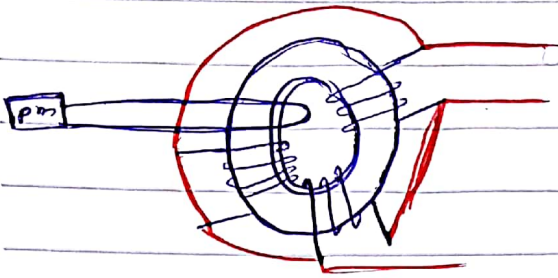
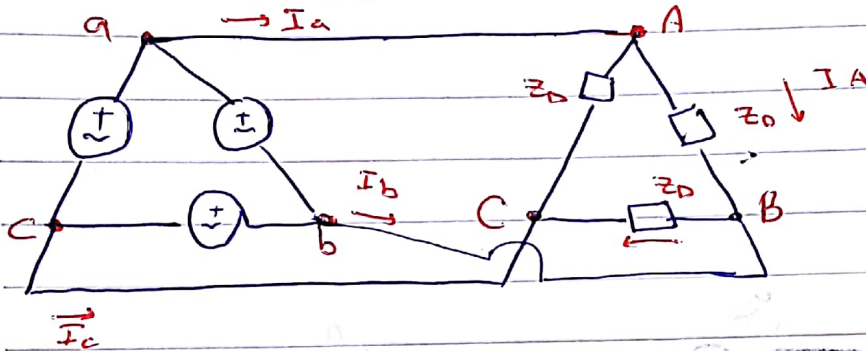


* Can we connect the source as a D?



D-D Connected

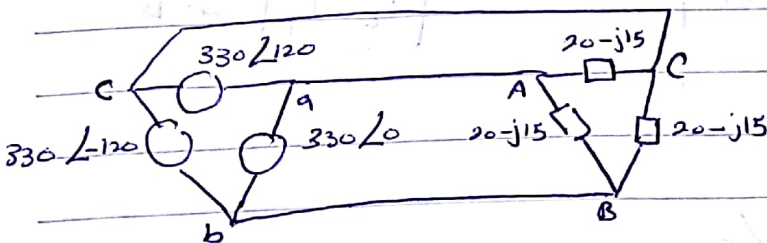


- * The voltage across the load is (line to line voltage)
- * $I_{line} \neq I_{phase}$
- * $I_{line} = \sqrt{3} * I_{phase} \angle -30^\circ$

Ex

a Balanced D connected load of $Z_D = 20 - j15 \Omega$ is connect to D source with $V_{ab} = 330 \angle 0$, find phase & line current.

المطلوب هو التيار الخطي I_{line} و التيار الطور I_{phase}



$$I_{AB} = \frac{U_{AB}}{Z_D} = \frac{V_{ab}}{Z_D} = \frac{330 \angle 0}{20 - j15} = 13.2 \angle 36.87^\circ$$

$$I_{BC} = 13.2 \angle 36.87^\circ - 120^\circ$$

$$I_{CA} = 13.2 \angle 36.87^\circ + 120^\circ$$

$$* I_{Line} = \sqrt{3} I_{Phase} \angle -30$$

$$I_a = \sqrt{3} I_{AB} \angle -30$$

$$I_b = \sqrt{3} I_{BC} \angle -30$$

$$I_c = \sqrt{3} I_{CA} \angle -30$$

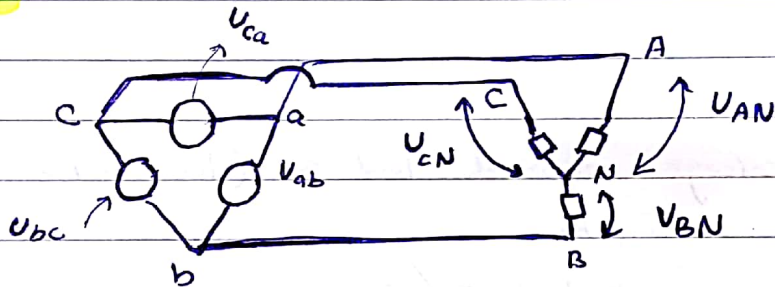
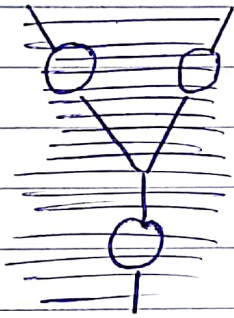
$$I_a = \sqrt{3} * 13.2 \angle 36.87 - 30$$

$$= 22.86 \angle 6.87 \text{ A}$$

$$I_b = 22.86 \angle 6.87 - 120 \text{ A}$$

$$I_c = 22.86 \angle 6.87 + 120 \text{ A}$$

D-Y connected.



$$* I_{Line} = I_{Phase}$$

لما كان الحمل متصلاً بالنول
فإن التيار في الخط يساوي تيار الحمل

Y connected

$$I_L = I_p$$

* The voltage across the load is line to neutral voltage

$$V_{LL} = \sqrt{3} V_{LN} \angle 30$$

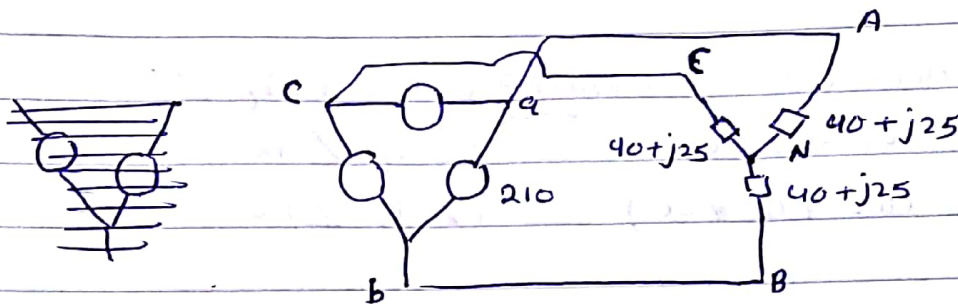
(line to line) (line to neutral)

$$V_{LN} = \frac{V_{LL}}{\sqrt{3}} \angle -30$$

Example.

A Balanced γ -connected load with $Z_Y = 40 + j25$ is supplied by a Δ -connected source with a line voltage of 210 Volt.

Calculate the phasor & line currents, Take V_{ab} as your reference.



$$I_a = \frac{U_{AN}}{Z_Y}$$

$$I_a = \frac{121.2 \angle -30^\circ}{40 + j25} = 2.57 \angle -62^\circ \text{ A}$$

$$|U_{ab}| = 210$$

$$U_{ab} = 210 \angle 0^\circ$$

$$U_{AN} = \frac{210}{\sqrt{3}} \angle -30^\circ = 121.2 \angle -30^\circ \text{ volt}$$

$$I_b = 2.57 \angle -62^\circ - 120^\circ \text{ A}$$

$$I_c = 2.57 \angle -62^\circ + 120^\circ \text{ A}$$

~~V_{bc} = 210 ∠ 0°~~

~~V_{bc} is used *~~

my reference.

phase currents, (= line current)

$$V_{bc} = 210 \angle 0^\circ$$

$$V_{ab} = 210 \angle -120^\circ$$

$$V_{ca} = 210 \angle 120^\circ \text{ or } \angle -240^\circ$$

$$U_{an} = \frac{210}{\sqrt{3}} \angle 90^\circ$$

* Power in 3 ϕ system.

$$V_{AN} = \sqrt{2} V_p \cos(\omega t) \rightarrow \sqrt{2} V_p \angle 0$$

$$V_{BN} = \sqrt{2} V_p \cos(\omega t - 120) \rightarrow \sqrt{2} V_p \angle -120$$

$$V_{CN} = \sqrt{2} V_p \cos(\omega t + 120) \rightarrow \sqrt{2} V_p \angle 120$$

$$\rightarrow \text{Take } Z_Y = Z \angle \theta$$

$$i_a = \sqrt{2} I_p \cos(\omega t - \theta) \equiv \frac{(V_{AN}/Z_Y)}{\sqrt{2}} \rightarrow \sqrt{2} I_p \angle \theta$$

$$i_b = \sqrt{2} I_p \cos(\omega t - 120 - \theta) \rightarrow \sqrt{2} I_p \angle -120 - \theta$$

$$i_c = \sqrt{2} I_p \cos(\omega t + 120 - \theta) \rightarrow \sqrt{2} I_p \angle 120 - \theta$$

$$P_A = |I_p| |V_p| \cos \theta \rightarrow (\theta - \phi)$$

$$P_B = |I_p| |V_p| \cos \theta$$

$$P_C = |I_p| |V_p| \cos \theta$$

$$P_{\text{total}} = P_{3\phi} = 3 |V_p| |I_p| \cos \theta$$

\swarrow phase current (RMS value)
 \searrow phase voltage (RMS)
 \nearrow neutral voltage
 \nwarrow angle between phase to neutral voltage & phase current.

$$P_{3\phi} = 3 * P_{1\phi}$$

$$Q_{\text{total}} = Q_{3\phi} = 3 |V_p| |I_p| \sin \theta$$

$$= 3 * Q_{1\phi}$$

$$S'_{\text{total}} = S'_{3\phi} = 3 V_p I_p^* = 3 * S'_{1\phi}$$

$$S'_{3\phi} = P_{3\phi} + j Q_{3\phi}$$

$$|S'_{\text{total}}| = 3 |V_p| |I_p| = 3 * |S'_{1\phi}|$$

Apparent power.

* How to represent power quantities using line voltage & line current.

$$P_{3\phi} = 3 |V_p| |I_p| \cos \theta$$

Y-connected loads

$$I_p = I_L$$

$$|V_p| = \frac{|V_L|}{\sqrt{3}}$$

Δ-connected

$$V_p = V_L$$

$$|I_L| = \sqrt{3} |I_p|$$

$P_{3\phi} = 3 * \frac{|V_L|}{\sqrt{3}} * |I_L| \cos \theta$
 ↳ line to line. angle between phase & neutral voltage
 ↳ line current & phase current.

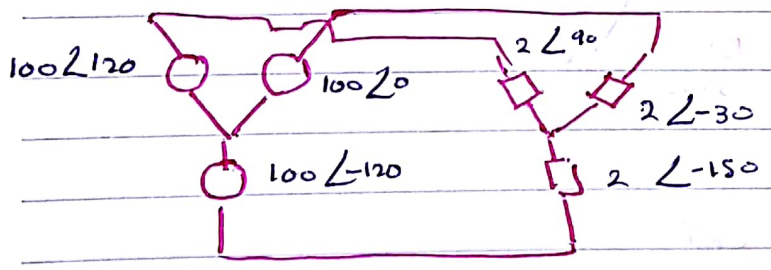
$$P_{3\phi} = 3 |V_L| |I_L| \cos \theta$$

$$P_{3\phi} = \sqrt{3} |V_L| |I_L| \cos \theta$$

$$* P_{3\phi} = 3 |V_p| |I_p| \cos \theta$$

$$* P_{3\phi} = \sqrt{3} |V_L| |I_L| \cos \theta$$

$V_p \neq I_p$



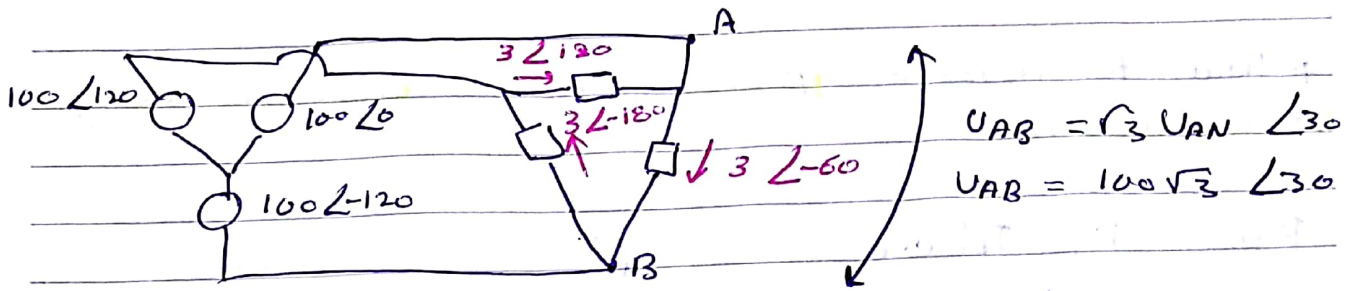
$$P_{3\phi} = 3 * 100 * 2 * \cos(0 - -30)$$

$$= \sqrt{3} * 800 \text{ w}$$

Using line values :-

$$P_{3\phi} = \sqrt{3} * \sqrt{3} * 100 * 2 \cos(30)$$

$$= 300 \sqrt{3} \text{ w}$$



$$P_{3\phi} = 3 |U_p| |I_p| \cos \theta$$

$$= 3 * [100 * \sqrt{3}] * 3 * \cos (30 - -60) = \underline{\underline{Zero}}$$

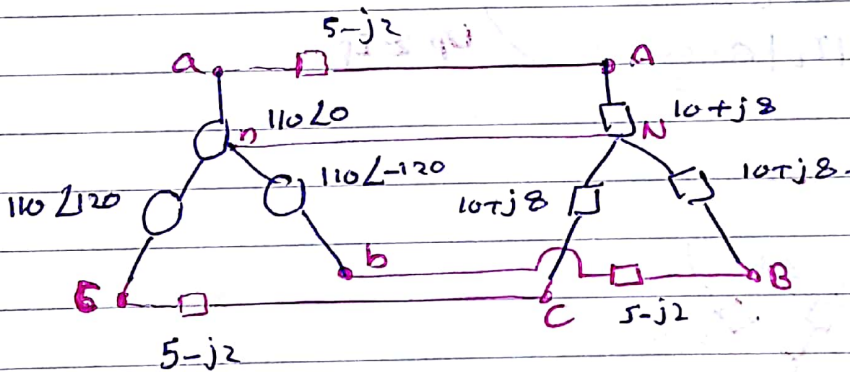
Using line:-

$$P_{3\phi} = \sqrt{3} |U_L| |I_L| \cos \theta$$

$$= \sqrt{3} * 100 * \sqrt{3} * 3 * \cos (30 + 60) = \underline{\underline{Zero}}$$

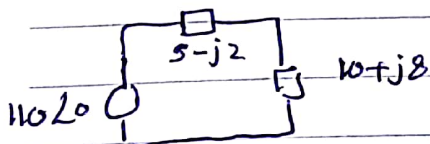
Ex.

given the ckt shown find the source complex power & the load complex power & the T.L losses.



* Single phase equivalent ckt

→ Take phase A.



$$I_a = \frac{110 \angle 0^\circ}{15 + j6} = 6.8 \angle -21.8^\circ$$

$$S_{(source)} = 3 * U_p * I_p^*$$

$$= 3 * 110 \angle 0^\circ * 6.8 \angle 21.8^\circ$$

$$= 2247 \angle 21.8^\circ$$

$$= 2087 + j834.6$$

$P_{3\phi}$ \rightarrow $Q_{3\phi}$

$S(\text{load})$

$$V_{AN} = (\text{voltage division}) = \frac{110 \angle 0^\circ * (10 + j8)}{15 + 6j}$$

$$\begin{aligned} S_{\text{load}} &= 3 V_p I_p^* \\ &= 3 Z_p * I_p * I_p^* \\ &= 3 Z_p |I_p|^2 \end{aligned}$$

$$\begin{aligned} &= 3 * (10 + j8) * (6.8)^2 \\ &= 1392 + j1113 = 1782 \angle 38.66^\circ \end{aligned}$$

* power measurement.

→ power is measured in the lab using a device called wattmeter.

How does wattmeter work:

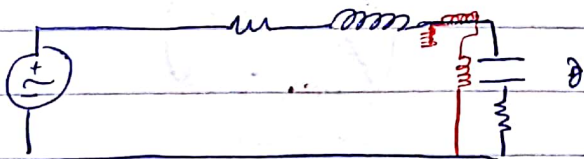
(displacement of pointer)
Deflection.

$$P_{\text{measured}} \propto V, I, \cos \theta.$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta - \phi)$$

Ex.

find the wattmeter reading (find the real power absorbed by the load)

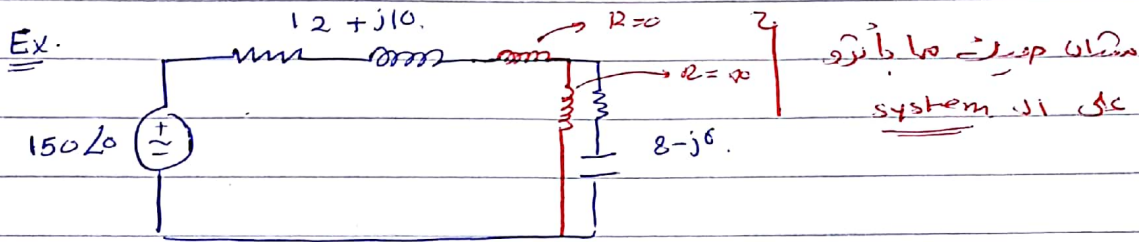
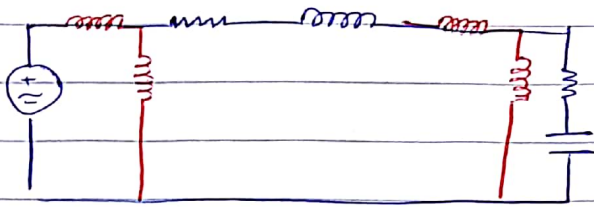


2 coils → parallel (voltage) coil
↳ Series (current) coil

Current coil → load current.

Voltage coil → load voltage.

load w power ji wattmeter
 source w power ji
 coils



$$P = V_{rms} I_{rms} \cos(\theta - \phi)$$

$$V_{load} \rightarrow \text{voltage division} = \frac{150 \angle 0 * (8 - j6)}{(20 + j4)} = 73.5 \angle -84^\circ$$

$$I_{load} \rightarrow \frac{150 \angle 0}{20 + j4} = 7.35 \angle -11.3^\circ$$

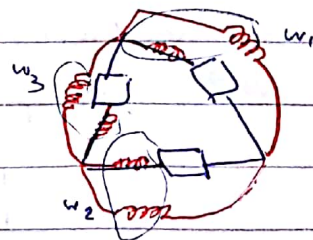
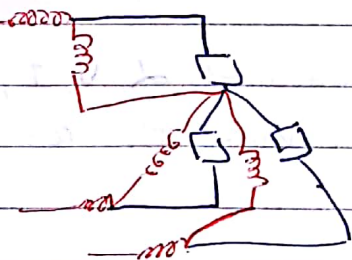
$$\begin{aligned} W_{reading} &= P(\text{load}) = V_{rms} I_{rms} \cos(\theta - \phi) \\ &= 73.5 * 7.32 * \cos(-84 + 11.3) \\ &= 432.1 \text{ W} \end{aligned}$$

* Power measurement in 3 phase system.

$$P_{3\phi} = W_1 + W_2 + W_3$$

if the load is Balanced

$$P_{3\phi} = 3W_1$$



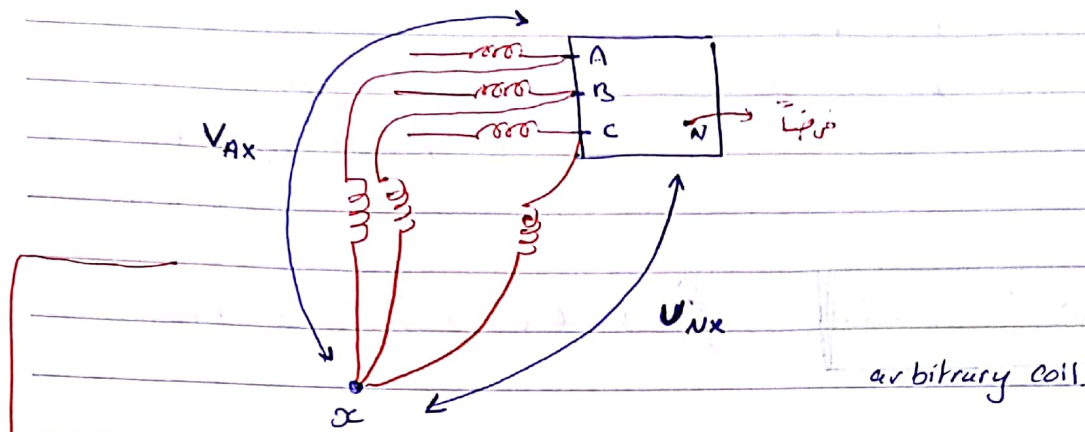
* for a Δ connected load.

$$P_{3\phi} = W_1 + W_2 + W_3$$

if the load is Balanced

$$P_{3\phi} = 3W$$

(the neutral point is not accessible)

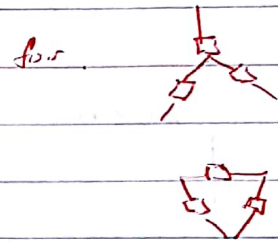


$$P_{3\phi} \stackrel{??}{=} \omega_1 + \omega_2 + \omega_3$$

$$\omega_1 = \frac{1}{T} \int_0^T U_{AN}(t) \cdot i_a(t) \cdot dt$$

$$\omega_2 = \frac{1}{T} \int_0^T U_{BN}(t) \cdot i_b(t) \cdot dt$$

$$\omega_3 = \frac{1}{T} \int_0^T U_{CN}(t) \cdot i_c(t) \cdot dt$$



$$\omega_1 = \frac{1}{T} \int_0^T U_{Ax}(t) \cdot i_a(t) \cdot dt$$

$$\omega_2 = \frac{1}{T} \int_0^T U_{Bx}(t) \cdot i_b(t) \cdot dt$$

$$\omega_3 = \frac{1}{T} \int_0^T U_{Cx}(t) \cdot i_c(t) \cdot dt$$

$$\begin{aligned} U_{Ax} &= U_{AN} + U_{Nx} \\ U_{Bx} &= U_{BN} + U_{Nx} \\ U_{Cx} &= U_{CN} + U_{Nx} \end{aligned}$$

$$\begin{aligned} P_{3\phi} &= \frac{1}{T} \int_0^T (U_{AN} + U_{Nx}) \cdot i_a(t) \cdot dt + \frac{1}{T} \int_0^T (U_{BN} + U_{Nx}) \cdot i_b(t) \cdot dt \\ &\quad + \frac{1}{T} \int_0^T (U_{CN} + U_{Nx}) \cdot i_c(t) \cdot dt \end{aligned}$$

$$= \frac{1}{T} \left[U_{AN}(t) \cdot i_a(t) + U_{BN}(t) \cdot i_b(t) + U_{CN}(t) \cdot i_c(t) + U_{Nx} (i_a + i_b + i_c) \right]$$

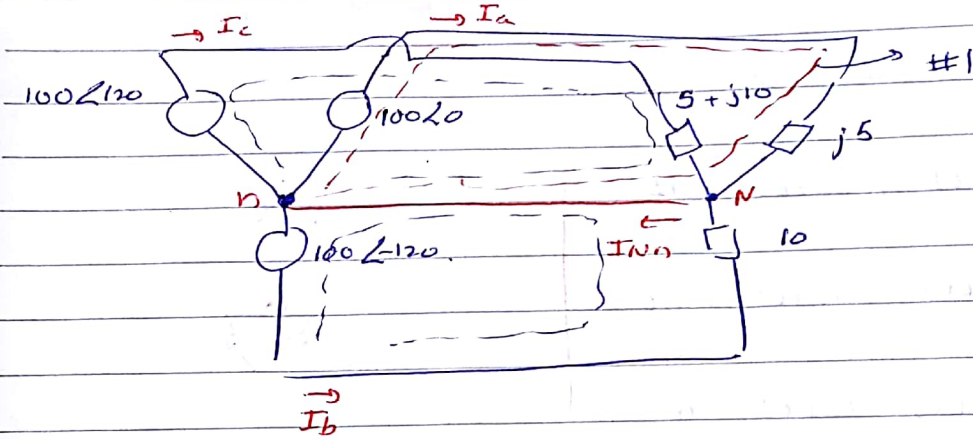
$$= \frac{1}{T} \int_0^T U_{AN}(t) \cdot i_a(t) \cdot dt + \frac{1}{T} \int_0^T U_{BN}(t) \cdot i_b(t) \cdot dt + \frac{1}{T} \int_0^T U_{CN}(t) \cdot i_c(t) \cdot dt$$

= zero (in Δ 2 y without neutral)

so.

$P_{3\phi} = w_1 + w_2 + w_3$
The ~~choice~~ of point x will not affect
the total power (3 ϕ power).

Unbalanced load. find I_a, I_b, I_c & I_{Nn} .



KVL at #1.

$$-100 \angle 0 + j5 I_a = 0$$

$$I_a = \frac{100 \angle 0}{j5} = 20 \angle -90^\circ \text{ A}$$

KVL

$$I_b = \frac{100 \angle -120}{10} = 10 \angle -120 \text{ A}$$

KVL

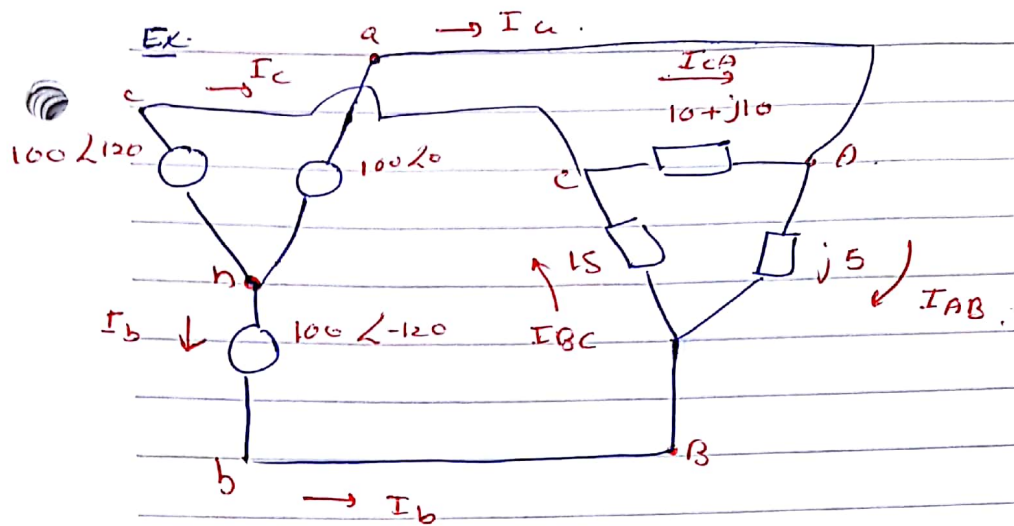
$$I_c = \frac{100 \angle +120}{5 + j10} \neq$$

$$I_{Nn} = I_a + I_b + I_c = 20 \angle -90^\circ + 10 \angle -120^\circ + \frac{100 \angle 120}{5 + j10} \neq \underline{\underline{\text{zero}}}$$

$$V_N = V_n = 0$$

always = zero

تا م نینک غیر ذلت



find phase
& line
currents.

$$I_{AB} = \frac{U_{AB}}{j5} = \frac{U_{ab}}{j5} = \frac{100\sqrt{3}}{j5} \angle 30 = 20\sqrt{3} \angle -60 \text{ A.}$$

$$I_{BC} = \frac{U_{BC}}{15} = \frac{U_{bc}}{15} = \frac{100\sqrt{3}}{15} \angle -90 = 6.66\sqrt{3} \angle -90 \text{ A.}$$

$$I_{CA} = \frac{U_{CA}}{j10} = \frac{U_{ca}}{10 + j10} = \frac{\sqrt{3} * 100 \angle 150}{10 + j10} \#$$

120°

→ for Balanced load

$$I_{line} = \sqrt{3} I_{phase} \angle -30$$

→ for here (non balanced load) we applied **KCL**

$$I_a + I_{CA} = I_{AB}$$

$$I_a = I_{AB} - I_{CA}$$

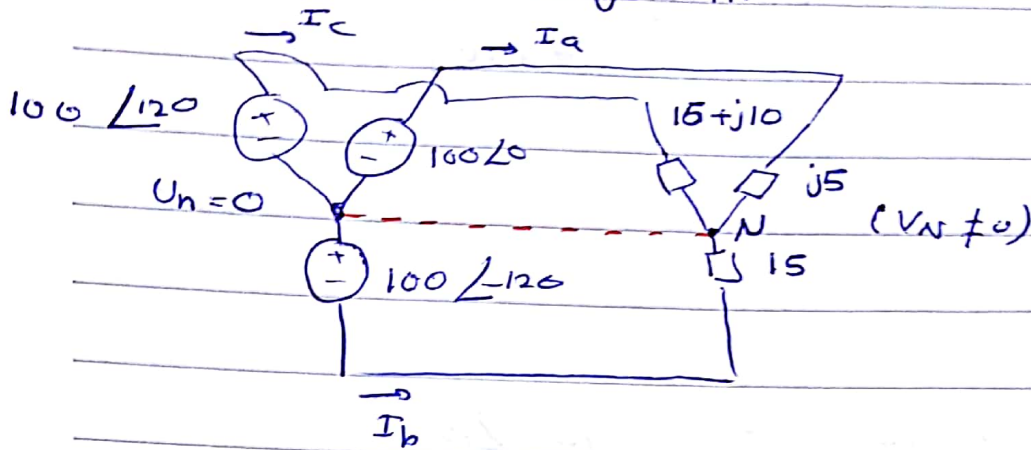
$$I_b + I_{AB} = I_{BC}$$

$$I_b = I_{BC} - I_{AB}$$

$$I_c + I_{BC} = I_{CA}$$

$$I_c = I_{CA} - I_{BC}$$

* unbalanced system:-



KVL. (first way).

$$1 \quad -100 \angle 0^\circ + j5 - 15 I_b + 100 \angle 120^\circ = 0$$

$$2 \quad -100 \angle 120^\circ + I_c (10 + j10) + 15 I_b + 100 \angle -120^\circ = 0$$

$$3 \quad \dots$$

* ~~first way~~ second way:

KCL.

$$I_a + I_b + I_c = 0, \quad U_N \neq 0$$

nN ~~is not zero~~ * ~~is not zero~~

$$I_a + I_b + I_c = I_N$$

$$U_N = \underline{\underline{Zero}}$$

~~is not zero~~ *

$$I_a + I_b + I_c = 0$$

$$U_N \neq 0$$

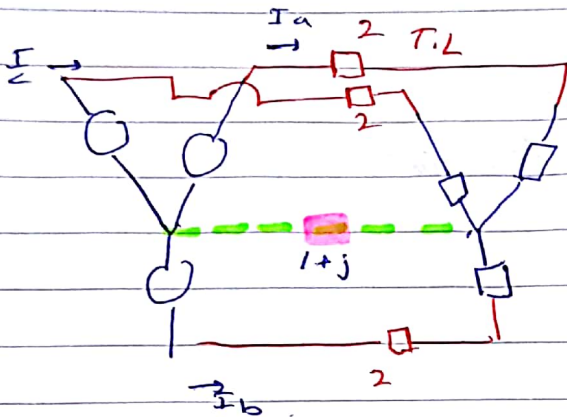
$$* I_a = \frac{100 \angle 0 - U_N}{j5}$$

$$* I_b = \frac{100 \angle -120 - U_N}{15}$$

$$* I_c = \frac{100 \angle 120 - U_N}{10 - j10}$$

$$\frac{100 \angle 0 - U_N}{j5} + \frac{100 \angle -120 - U_N}{15} + \frac{100 \angle 120 - U_N}{10 + j10} = 0$$

* find U_N then I_a, I_b, I_c .



$$I_a = \frac{100 \angle 0 - U_N}{2 + j5}$$

$$I_b = \frac{100 \angle -120 - U_N}{12}$$

$$I_c = \frac{100 \angle 120 - U_N}{17 + j10}$$

(لو صحت الجواب)

$$I_a = \frac{100 \angle 0}{2 + j5}, \quad I_b = \frac{100 \angle -120}{12}, \quad I_c = \frac{100 \angle 120}{17 + j10}$$

(لو صحت الجواب على صحت)

$$U_N \neq 0$$

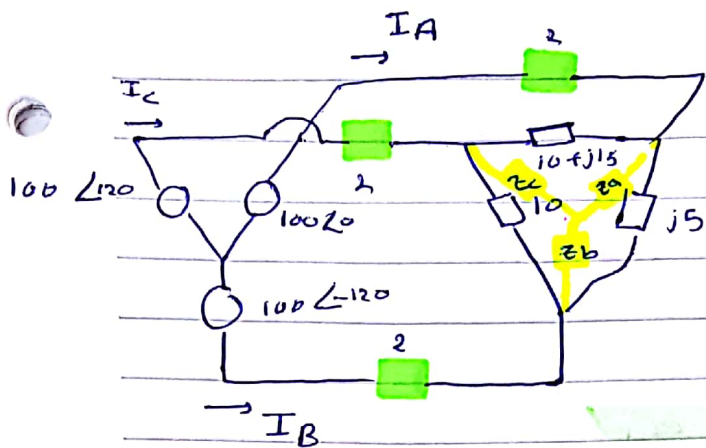
$$I_{Nn} = \frac{U_N}{1 + j} = I_a + I_b + I_c$$

$$I_a = \frac{100 \angle 0 - U_N}{2 + j5}$$

$$I_b = \frac{100 \angle -120 - U_N}{12}$$

$$I_c = \frac{100 \angle 120 - U_N}{17 - j10}$$

$$\frac{100 - U_N}{2 + j5} + \frac{100 \angle -120 - U_N}{12} + \frac{100 \angle 120 - U_N}{17 + j10} = \frac{U_N}{1 + j}$$



* Without T.L impedance:-

$$I_{AB} = \frac{U_{AB}}{j5}, \quad I_{BC} = \frac{U_{BC}}{10}$$

$$I_{AB} = \frac{\sqrt{3} \cdot 100 \angle 30^\circ}{j5}, \quad I_{BC} = \frac{\sqrt{3} \cdot 100 \angle -90^\circ}{10}$$

$$I_{AB} = \frac{U_{AB}}{j5} = \frac{\sqrt{3} \cdot 100 \angle 30^\circ}{j5}$$

$$I_{BC} = \frac{U_{BC}}{10} = \frac{\sqrt{3} \cdot 100 \angle -90^\circ}{10}$$

$$I_{CA} = \frac{U_{CA}}{10 + j15} = \frac{\sqrt{3} \cdot 100 \angle 150^\circ}{j15 + 10}$$

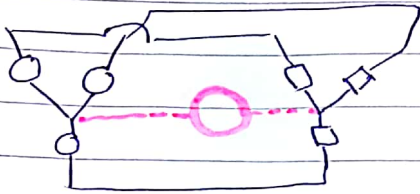
* with T.L

$$U_{ab} \neq U_{AB}$$

Δ-Y transformation.

$$Z_a = \frac{j5(10 + j5)}{j5 + 10 + j5 + 10}$$

$$I_a = \frac{100 \angle 0^\circ - U_n}{2 + Z_a}$$



(A)

short circuit

(there is Nn line)

(V)

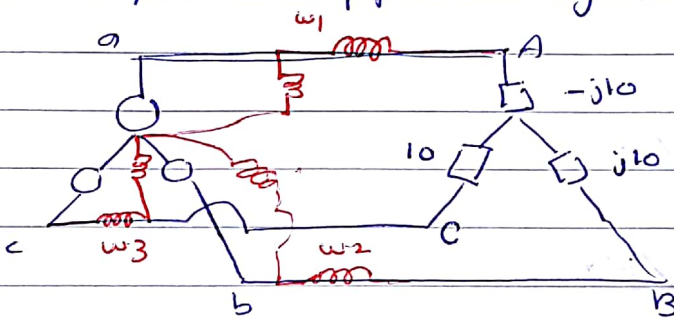
open ckt

(there is no Nn line)

Given that $U_{AB} = 230 \angle 0$

find the reading at each

Find the reading of the 3 wattmeters & find the total power supplied by the source.



$$U_{ab} = 100 \angle 0$$

$$W_1 \Rightarrow V_{an} \text{ \& \ } I_a, \quad V_{an} = (100/\sqrt{3}) \angle -30$$

$$W_2 \Rightarrow V_{bn} \text{ \& \ } I_b, \quad V_{bn} = (100/\sqrt{3}) \angle -150$$

$$W_3 \Rightarrow V_{cn} \text{ \& \ } I_c, \quad V_{cn} = (100/\sqrt{3}) \angle 90$$

$$\frac{100 \angle -30}{\sqrt{3}} \cdot (-j10) + \frac{100 \angle -150}{\sqrt{3}} \cdot (j10) + \frac{100 \angle 90}{\sqrt{3}} \cdot 10 = 0$$

find U_N then find I_a, I_b, I_c

$$* I_a = 19.32 \angle 15$$

$$* I_b = 19.32 \angle 16.2$$

$$* I_c = 10 \angle -90$$

$$w_1: p_1 = U_{an} * I_a * \cos(\theta_{U_{an}} - \phi_{I_a})$$

$$= \frac{100}{\sqrt{3}} * 19.32 * \cos(-30 - 15) = 788.7 \text{ W}$$

$$w_2: p_2 = U_{bn} * I_b * \cos(\theta - \phi)$$

$$= \frac{100}{\sqrt{3}} * 19.32 * \cos(-150 - 165)$$

$$= 788.7 \text{ W}$$

$$w_3: p_3 = U_{cn} * I_c * \cos(\theta - \phi)$$

$$= \frac{100}{\sqrt{3}} * 10 * \cos(90 + 90)$$

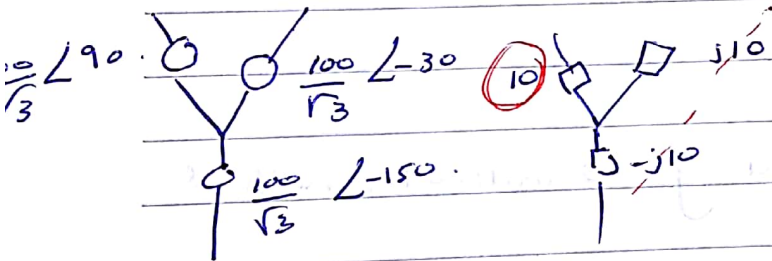
$$= -577.4 \text{ W}$$

total power supplied by the source is:

$$P_{3\phi} = p_1 + p_2 + p_3$$

$$= w_1 + w_2 + w_3$$

$$= 788.7 + 788.7 - 577.4 = 1000 \text{ W}$$



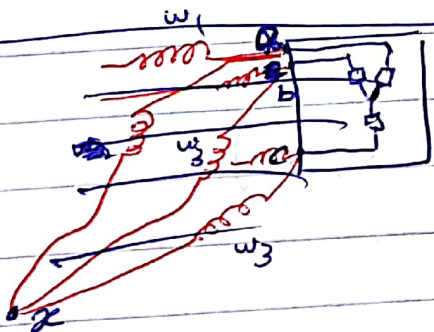
Real power only on 10Ω

$$P_{3\phi} = (I_c)^2 * R$$

$$= (10)^2 * 10$$

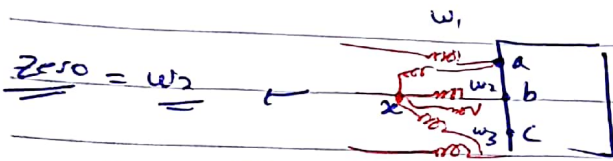
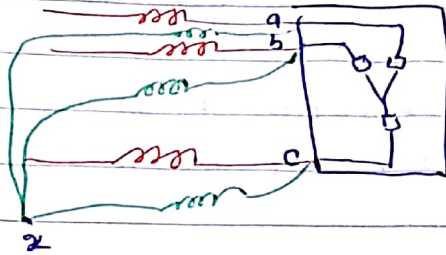
$$= 1000 \text{ W} \dots \#$$

The two wattmeter method :-

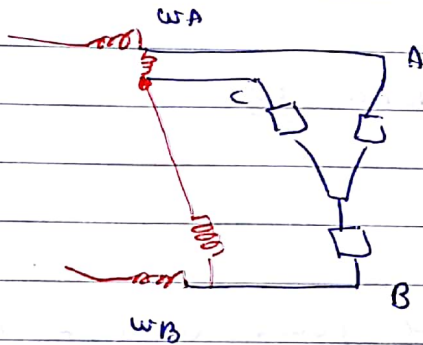


The Two wattmeter method:

α : what if α is chosen such that α is put on one of phases.

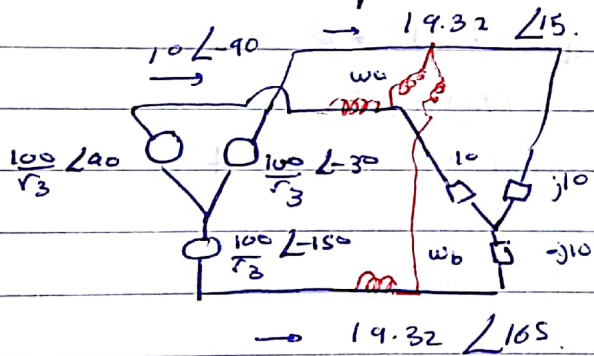


Two wattmeters.



$W_A \Rightarrow U_{AC}, I_A$
 $W_B \Rightarrow U_{BC}, I_B$

* Solve the previous exp. using 2 wattmeter method.



$w_a : U_{ca}, I_c$
 $w_b \Rightarrow U_{ba}, I_b$

$U_{AB} = \sqrt{3} V_{an} \angle 30$

$V_{ab} = 100 \angle 0 \quad (V_{ba} = -V_{ab} = 100 \angle 180)$

$V_{bc} = 100 \angle -120$

$V_{ca} = 100 \angle 120$

$w_a : P_a : |V_{ca}| |I_c| \cos(\theta_{V_{ca}} - \phi_{I_c}) = 100 * 10 * \cos(120 + 90) = -886$
 $w_b : P_b : |V_{ba}| |I_b| \cos(\theta_{V_{ba}} - \phi_{I_b}) = 100 * 19.32 \cos(180 - 15) = 1886 \text{ W}$

$1886 - 886 = 1000 \text{ W} \neq$

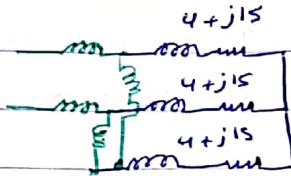
* Given that $V_{AB} = 230 \angle 0^\circ$

→ find the reading of each wattmeter.

& the total power supplied to the load.

$W_A : U_{AC} : I_A$

$W_B : U_{BC} : I_B$



$V_{AB} = 230 \angle 0^\circ$

$V_{BC} = 230 \angle -120^\circ$

$V_{CA} = 230 \angle 120^\circ, U_{AC} = 230 \angle 120^\circ - 180^\circ$

$$I_a = \frac{V_{AN}}{4 + j15} = \frac{(V_{AB} / \sqrt{3}) \angle -30^\circ}{4 + j15} = \frac{230 \angle -30^\circ}{\sqrt{3} (4 + j15)} = 8.554 \angle -105.1^\circ$$

$I_b = 8.554 \angle -106 - 120$

$I_c = 8.554 \angle -105.1 + 120$

$W_1 : P_1 = 230 * 8.554 \cos(-60 + 105.1) = 1389 \text{ W}$

$W_2 : P_2 = 230 * 8.554 \cos(-120 + 125.1) = -512 \text{ W}$

$P_T = P_1 + P_2 = 876.5 \text{ W}$

$P_T = 3 |I|^2 R = 3 + (8.554)^2 * 4 \text{ W}$

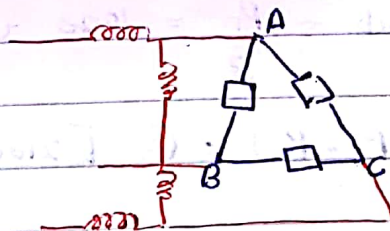
How to calculate the pF from the reading of wattmeter.

$$W_A = |U_{AB}| |I_A| \cos(\theta_{U_{AB}} - \phi_{I_A})$$

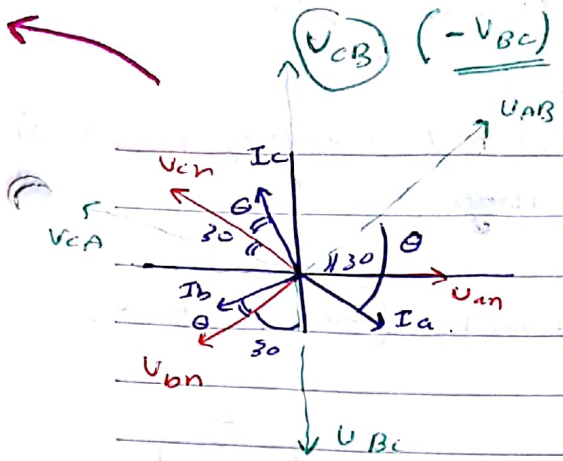
$$= |U_{LL}| |I_L| \cos(\theta_{U_{AB}} - \phi_{I_A})$$

$$W_B = |U_{CB}| |I_C| \cos(\theta_{U_{CB}} - \phi_{I_C})$$

$$= |U_{LL}| |I_L| \cos(\theta_{U_{CB}} - \phi_{I_C})$$



⇒ consider inductive load (voltage leads current)



$$P_A = V_{LL} I_L \cos(30 + \theta)$$

$$P_B = V_{LL} I_L \cos(30 - \theta)$$

$$\frac{P_A}{P_B} = \frac{|V_{LL}| |I_L| \cos(30 + \theta)}{|V_{LL}| |I_L| \cos(30 - \theta)}$$

$$\frac{P_A}{P_B} = \frac{\cos 30 \cos \theta - \sin 30 \sin \theta}{\cos 30 \cos \theta + \sin 30 \sin \theta}$$

$$= \left(\frac{\frac{\sqrt{3} \cos \theta - \frac{1}{2} \sin \theta}{2}}{\frac{\sqrt{3} \cos \theta + \frac{1}{2} \sin \theta}{2}} \right) + \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} + \frac{\cos \theta}{\cos \theta}$$

$$\frac{P_A}{P_B} = \frac{\sqrt{3} - \tan \theta}{\sqrt{3} + \tan \theta}$$

$$P_A (\sqrt{3} + \tan \theta) = P_B (\sqrt{3} - \tan \theta)$$

$$P_A \sqrt{3} + P_A \tan \theta = P_B \sqrt{3} - P_B \tan \theta$$

$$\tan \theta [P_A + P_B] = \sqrt{3} [P_B - P_A]$$

$$\tan \theta = \frac{\sqrt{3} [P_B - P_A]}{[P_A + P_B]}$$

$$\rightarrow \tan \theta = \frac{\sqrt{3} [P_2 - P_1]}{[P_1 + P_2]}$$

$P_2 = P_1$
(Resistive load)

$P_2 > P_1$
(Inductive load)

$P_2 < P_1$
(Capacitive load)

* واحد من القوتين يكون عاليه والآخر منخفض

P_1 ← الموصول

P_2 ← الموصول

$$\begin{aligned}
 P_T &= P_1 + P_2 = V_{LL} I_L \cos(30 + \theta) + V_{LL} I_L \cos(30 - \theta) \\
 &= V_{LL} I_L \cos 30 \cos \theta - V_{LL} I_L \sin 30 \sin \theta + V_{LL} I_L \cos 30 \cos \theta \\
 &\quad + V_{LL} I_L \sin 30 \sin \theta \\
 &= 2 V_{LL} I_L \cos 30 \cos \theta = \boxed{\sqrt{3} V_{LL} I_L \cos \theta} = P_T
 \end{aligned}$$

$$\begin{aligned}
 \rightarrow \sqrt{3} [P_2 - P_1] &= \sqrt{3} V_{LL} I_L \cos(30 - \theta) - \sqrt{3} V_{LL} I_L \cos(30 + \theta) \\
 &= \sqrt{3} V_{LL} I_L \cos 30 \cos \theta + \sqrt{3} V_{LL} I_L \sin 30 \sin \theta \\
 &\quad - \sqrt{3} V_{LL} I_L \cos 30 \cos \theta + \sqrt{3} V_{LL} I_L \sin 30 \sin \theta \\
 &= \sqrt{3} V_{LL} I_L \sin \theta = Q_T
 \end{aligned}$$

$ \begin{aligned} P_T &= P_2 + P_1 \\ Q_T &= \sqrt{3} [P_2 - P_1] \\ S &= \sqrt{P_2^2 + P_1^2} \\ \tan \theta &= \frac{Q_T}{P_T} = \frac{\sqrt{3} [P_2 - P_1]}{[P_2 + P_1]} \end{aligned} $
--

Ex. Two wattmeters produce readings $P_1 = 1560$, $P_2 = 2100$ w when connected to 3 ϕ load, find:

[1] $P_T = P_1 + P_2 = 1560 + 2100 = 3660$ w

if the load is balanced,

$$P_{3\phi} = \frac{P_T}{3} = 1220 \text{ w}$$

[2] $Q_T = \sqrt{3} [P_2 - P_1] = \sqrt{3} [2100 - 1560] = 935 \text{ VAR}$

if the load is balanced,

$$Q_{3\phi} = \frac{Q_T}{3} = 311.67 \text{ VAR}$$

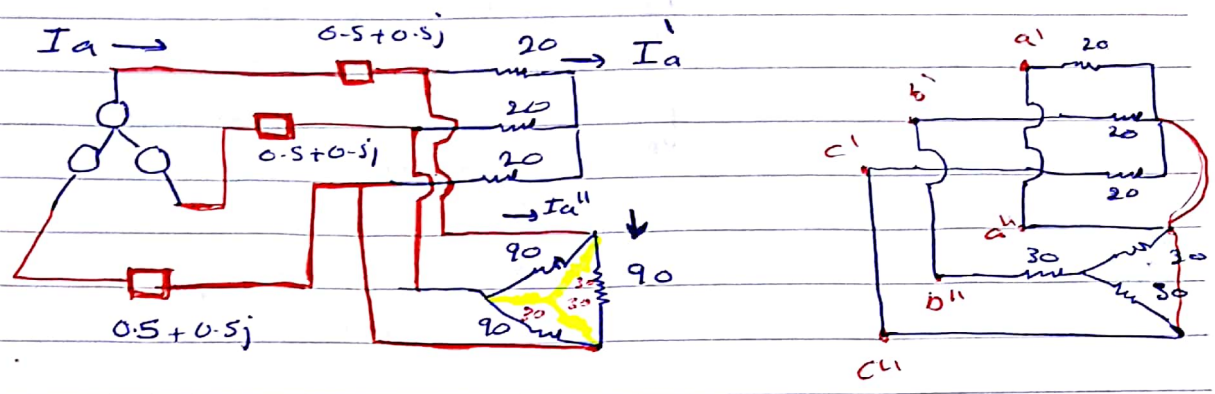
[3] $PF = \cos \left[\tan^{-1} \left[\frac{\sqrt{3} [2100 - 1560]}{2100 + 1560} \right] \right]$
 $= 0.9689$ lagging ($P_2 > P_1$)

$\angle 0 \rightarrow$ reference
 \rightarrow RMS.
 \rightarrow line to line voltage (use $\sqrt{3}$)

Example:-

Voltage at generator = 208 Volt

Find the line currents & phase currents (away from)



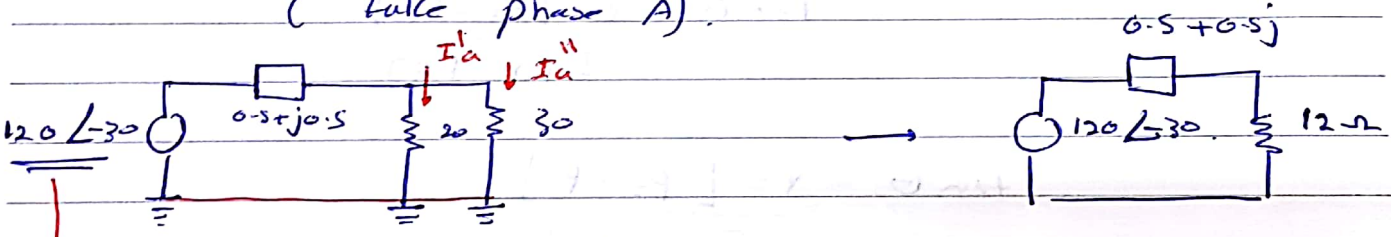
* Convert Δ to Y .

$$Z_Y = \frac{1}{3} Z_D \text{ (balanced load)}$$

$$= 30 \Omega$$

* then single phase equivalent circuit.

(take phase A)



Take V_{AB} as our reference
 $V_{AB} = 208 \angle 0$

$$I_a = \frac{120 \angle -30}{12 + 0.5 + 0.5j}$$

$$V_{AN} = \frac{V_{AB}}{\sqrt{3}} \angle -30 = 120 \angle -30$$

$$I_a' = \frac{30}{50} I_a$$

$$I_a'' = \frac{20}{50} I_a$$

Current division