Topics in Power

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powerunit-ju.com

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Topics In power 30/1/2018

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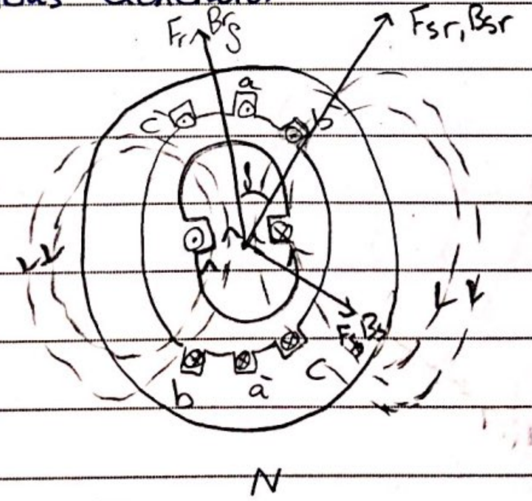
Required books
- Power System Analysis by H. Saadat
Chapters 3, 8, 10, 11, 12
- Power System Stability by P. kundur
- Power System Analysis by Stevenson
chapters 3, 16

*Stability
- Synchronous Generator



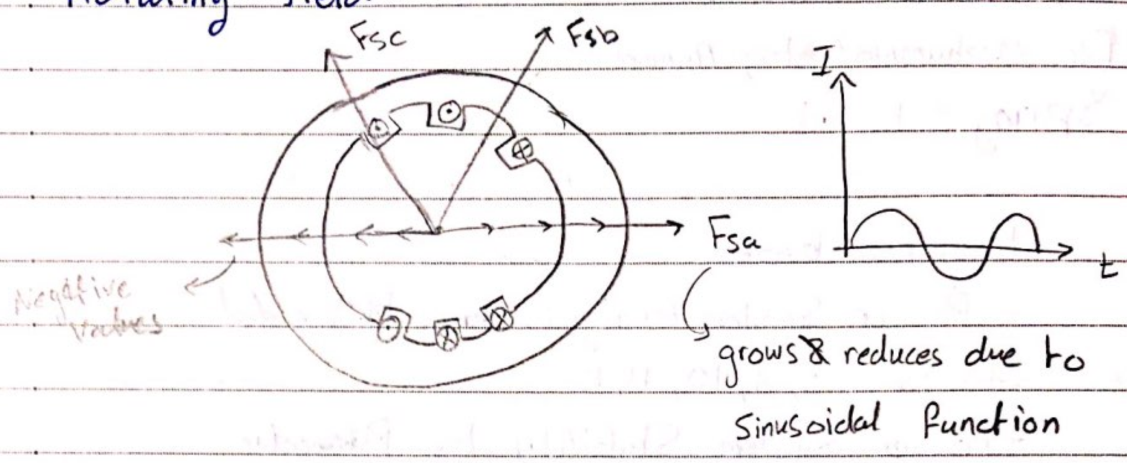
F_r, B_r : Rotor flux density
" magnetic force

B_s : Stator flux density

F_{ss}, B_{sr} : equivalent flux density
" magnetic force

δ : the degree between rotor & equivalent

Rotating field



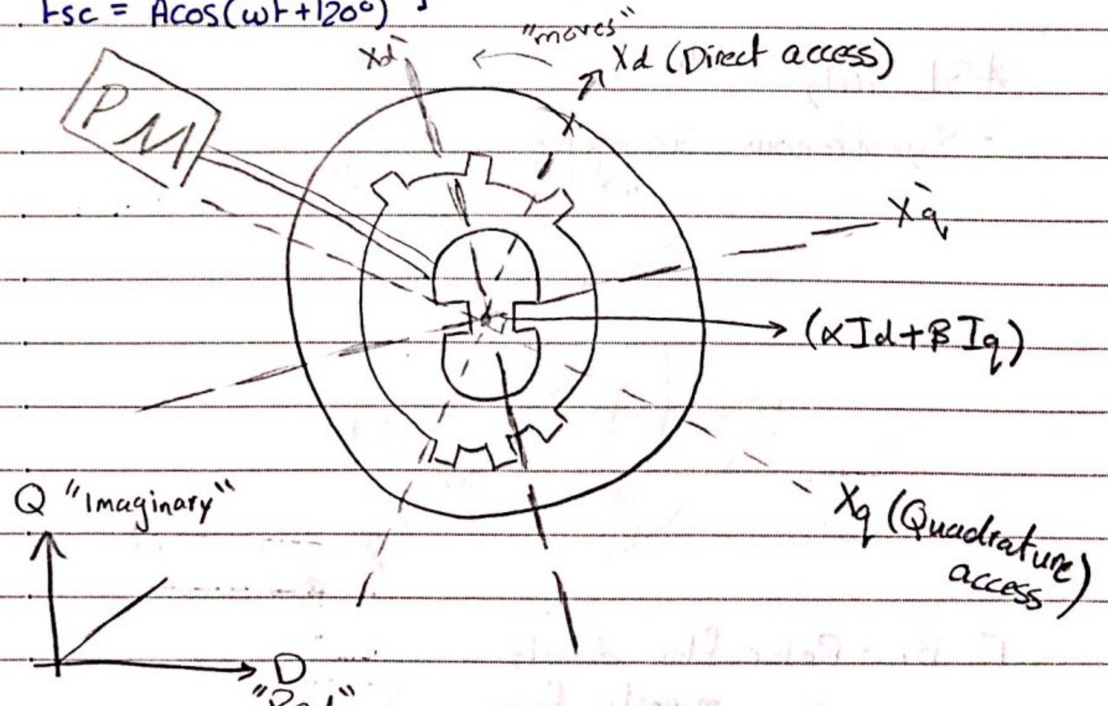
$F_s = F_{sa} + F_{sb} + F_{sc}$

$F_{sa} = A\cos(\omega t)$

$F_{sb} = A\cos(\omega t - 120°)$

$F_{sc} = A\cos(\omega t + 120°)$

$\longrightarrow F_s = \frac{3}{2} A\cos(\omega t)$



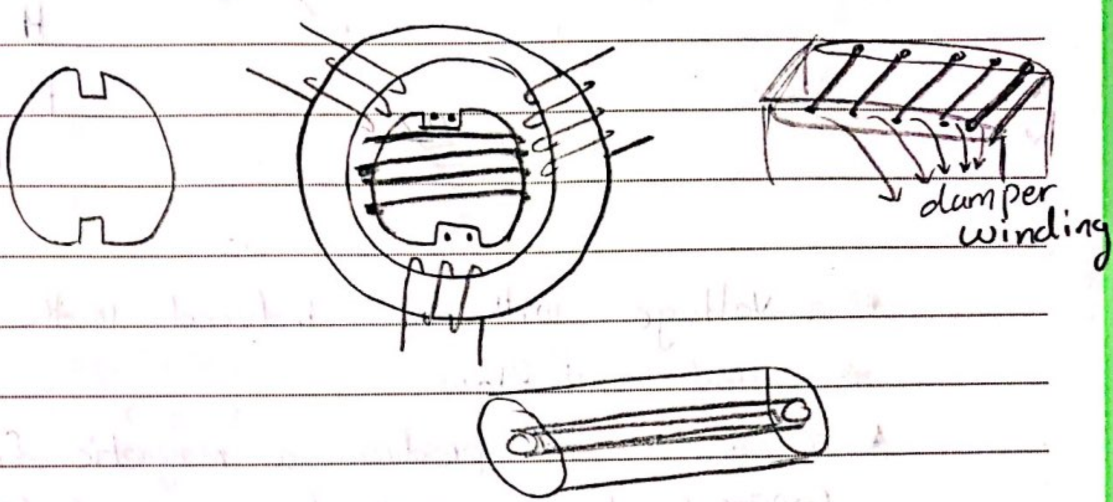
* Prime mover

Source of mechanical power; wind, Solar, Hydro, gas

* Excitation: field system [magnetic field]

$V = N \frac{d\phi}{dt}$

* Damper Winding: Short ckt winding on the rotor Structure

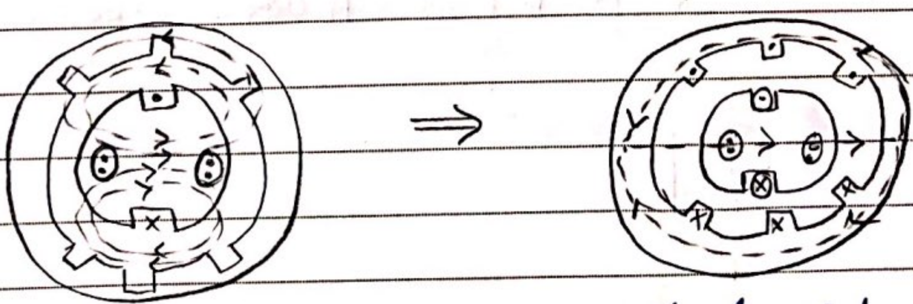


- In motor: they help in Starting
  (Produce Unidirectional magnetic field)

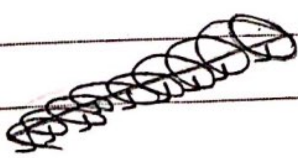
- In generator: It helps Improve machine Stability

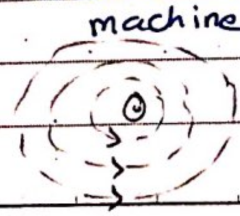
$n = n_{sync}$ $\qquad$ $n \neq n_{sync}$

- No Voltage Induced in the damper winding
- No effect in Steady State operation

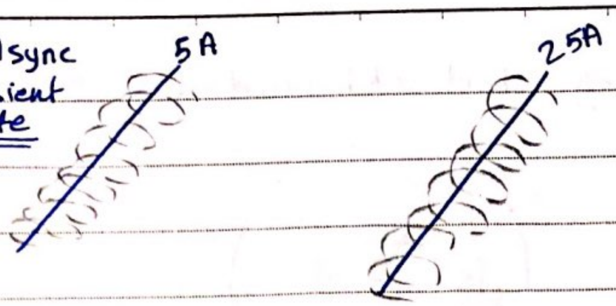


at the time when excitation system starts $\qquad$ Steady State operation [for loaded or unloaded machine]





$n \neq n_{sync}$
Transient
state



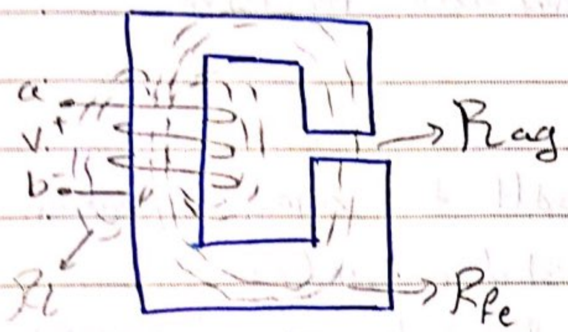
~~A~~ ~~increases~~
H Increases

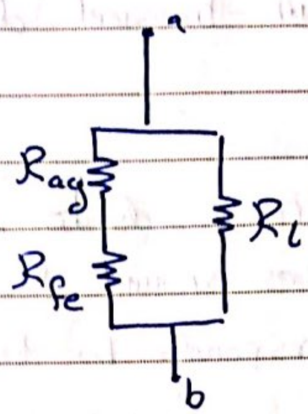
* a Voltage will be Induced in the damper winding
⇒ current will flow
* a current will produce a magnetic field
(opposite to the original magnetic field)
⇒ This magnetic field will interact with the main field and the Interaction will oppose the main field, and it will decrease/Increase the machine speed.

Damper winding works to maintain machine Stability in the transient state → sudden change In load
→
→
→

the 4 cases

In gradual change In load AWG works the machine.

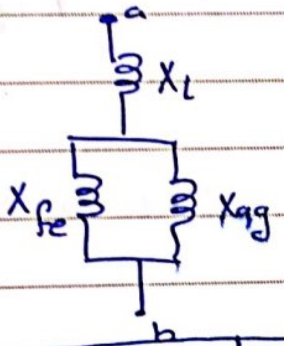




eq. magnetic circuit

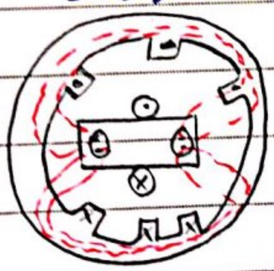
↓ eq. electric circuit

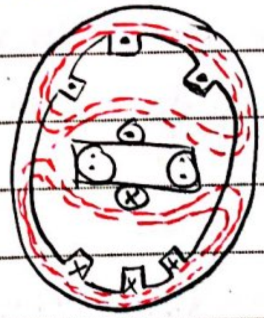
$$\boxed{L = \frac{N^2}{R}}$$

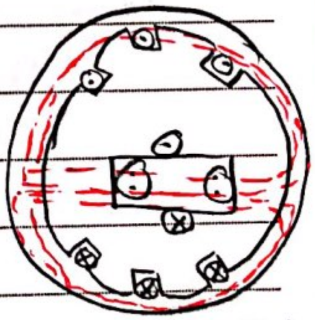


* what will happen ~~to~~ In the machine Immediately after fault ??

Transient ~~Sub-transient~~	Sub-transient	Steady State





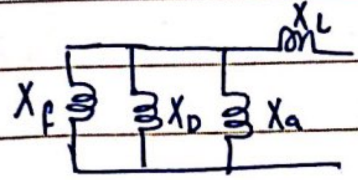


- largest air gap
- largest Reluctance
- lowest Reactance

- lowest Reluctance
- largest Reactance

$X_d = X_l + X_a$

$$X_d'' = X_l + \frac{1}{\frac{1}{X_a} + \frac{1}{X_D} + \frac{1}{X_f}}$$



X_d'': Sub-transient direct-axis reactance

X_d':

X_d:

X_L: Corresponds to path that armature leakage flux takes around the stator winding

X_D: Corresponds to the flux path around Damper winding

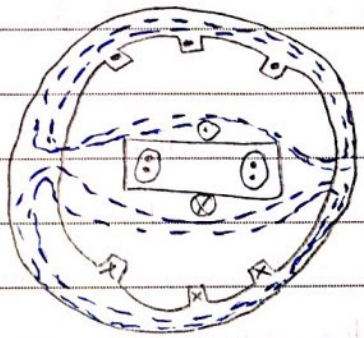
X_f: " " " " " " field "

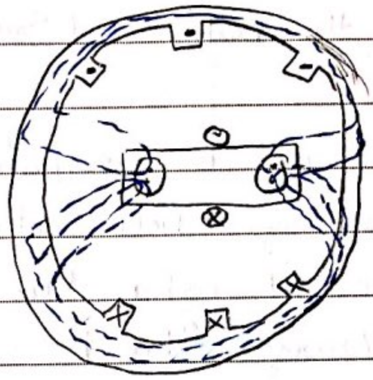
X_a: " " " " " Airgap "

Immediately after the fault

Sub - transient

Transient





largest Reluctance

lowest Reactance

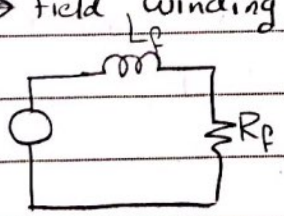
$$X''_d = X_L + \cfrac{1}{\cfrac{1}{X_D} + \cfrac{1}{X_f} + \cfrac{1}{X_a}}$$

↳ Sub transient direct access reactance

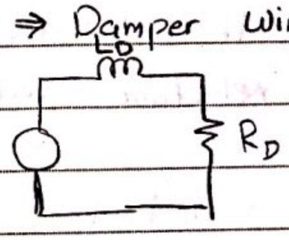
$$X'_d = X_L + \cfrac{1}{\cfrac{1}{X_a} + \cfrac{1}{X_f}}$$

↳ Transient direct access reactance

→ field winding



⇒ Damper winding



$R_D > R_f$

decay faster in winding

This topic is mentioned in Power System Dynamics, Stability & control Machowski after fault

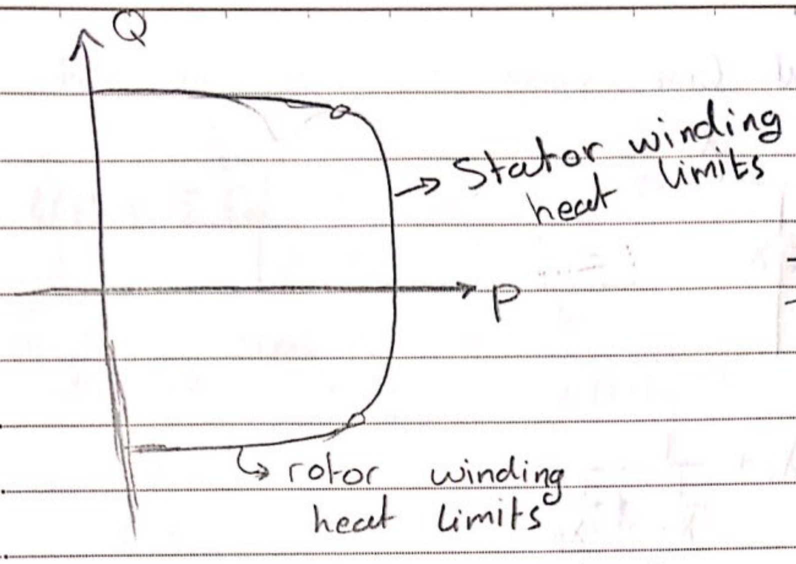
→ Current Induced In both rotor field & damper winding forces the armature reaction flux completly out of the rotor (Sub - transient State)

→ Energy is dissipated In the resistor of rotor windings (field & damper), the currents maintaining constant rotor flux linkages decay with time allowing flux to enter the windings

→ The resistor of damper winding is larger than the resistor of field winding.
The damper current is the first to decay, allowing the armature flux to enter the rotor pole face, But still forced out of the field winding itself.
(Transient State)

→ The field current then decays with time to its steady state value allowing the armature reaction flux eventually to enter the whole rotor & assume the minimum reluctance path. (Steady State)

$X_d'' \rightarrow$ Protection
$X_d' \rightarrow$ Stability

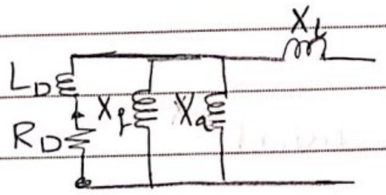


two circles

* Power System Analysis by H. Saadat chapters 3, 8

* 3rd Harmonics affects Power quality not Stability.

Sub transient time:

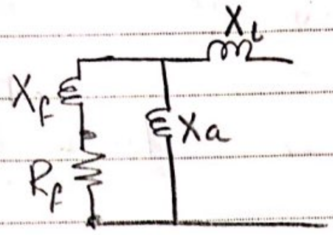


* equivalent electric circuit *

$$\boxed{X_d'' = X_L + \frac{1}{\frac{1}{X_D} + \frac{1}{X_f} + \frac{1}{X_a}}}$$ (0.07 - 0.12) pu

$$\boxed{\tau_d'' = \frac{X_D + \frac{1}{\frac{1}{X_a} + \frac{1}{X_f} + \frac{1}{X_L}}}{\omega R_D}}$$ $\approx 35\,mS$

Transient time



$$X_d' = X_l + \frac{1}{\frac{1}{X_a} + \frac{1}{X_f}}$$

$$\boxed{T_d' = \frac{\left(X_f + \frac{1}{\frac{1}{X_a} + \frac{1}{X_l}}\right)}{\omega R_f}}$$

(1-2) Sec

"Protection System works In 35mS (Sub-transient) In case it did not work, the protection must work in (1-2) sec for after that you lose Stability."

"Xd` & Xd`` after large disturbances"

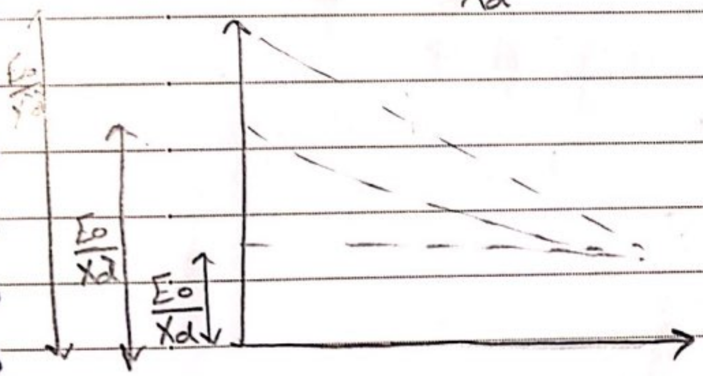
For an unloaded generator :

$$i(t) = \sqrt{2}\, E_0 \Big[\underbrace{\left(\frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-t/T_d''}}_{\text{sub}} + \underbrace{\left(\frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-t/T_d'}}_{\text{Transient}} + \underbrace{\frac{1}{X_d}}_{\text{Steady state}} \Big] \sin(\omega t - \delta)$$

$i(t)$ → AC Short circuit current

$$i(t) = \frac{V(P.u)}{X_d'}$$

(ch8 H. Saadat)



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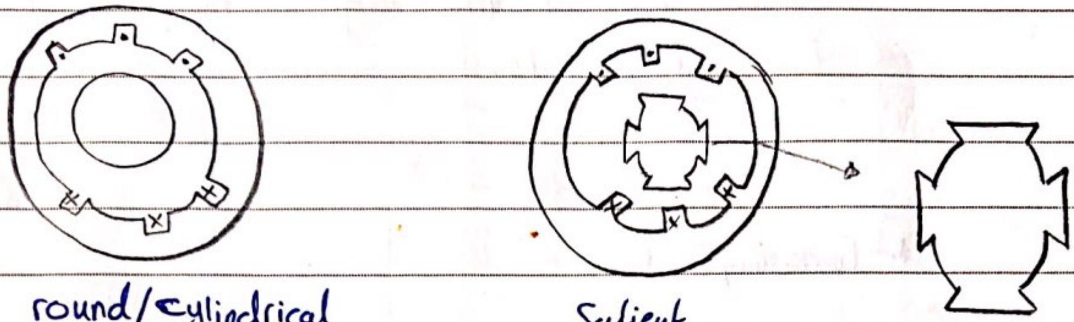
Salient & cylindrical rotors:



round/cylindrical

Salient
(slow Prime movers)
Hydro

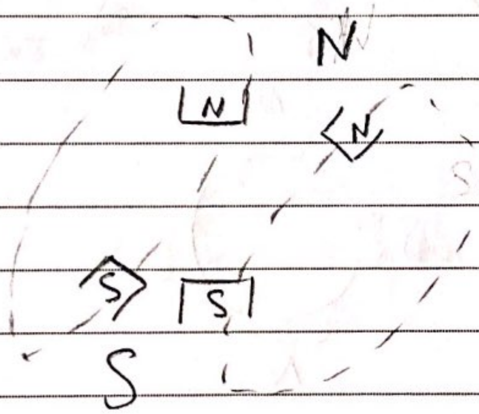
$f = \frac{nP}{60}$

$n = \frac{f \times 60}{P}$ → Pole pairs

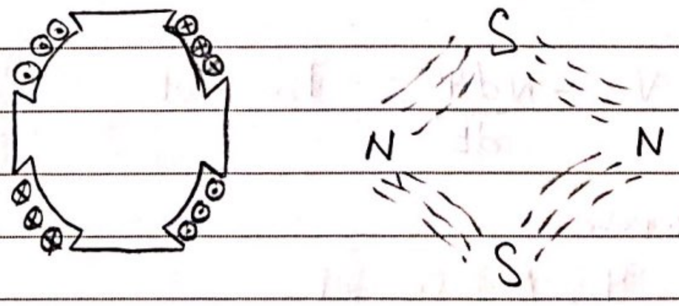
f: electric frequency

n: mechanical ~~frequen~~ speed

$f = \frac{n \times P}{60}$



⇒ Structure for 2 pole pair



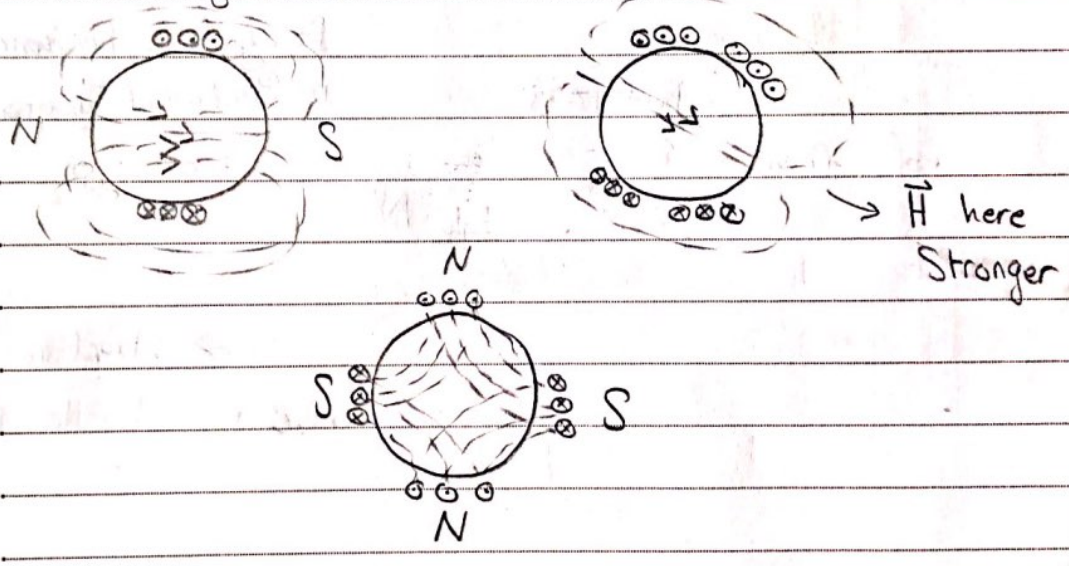
In round/cylindrical → uniform field

In Salient → there are differences

Problem (Salient pole): non-Uniform Airgap
$\Rightarrow$ need different modelling technique, specially for transient State.

~~* Faraday's law :~~

For Cylindrical



* Faraday's law

$$V = -N\frac{d\Psi}{dt} \rightarrow \Phi_m \cos\omega t$$

Suppose

$$\Psi = N\Phi\cos\omega t$$

$$V = \underbrace{\omega N\Phi_m}_{E_{max}} \sin\omega t$$

$$E_{max} = \omega N\Phi_m$$

$$= 2\pi f N\Phi_m$$

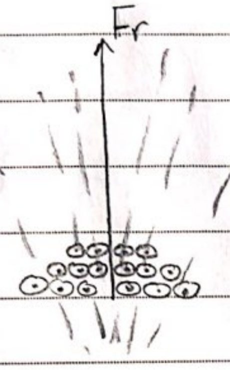
$$E_{rms} = \frac{E_{max}}{\sqrt{2}}$$

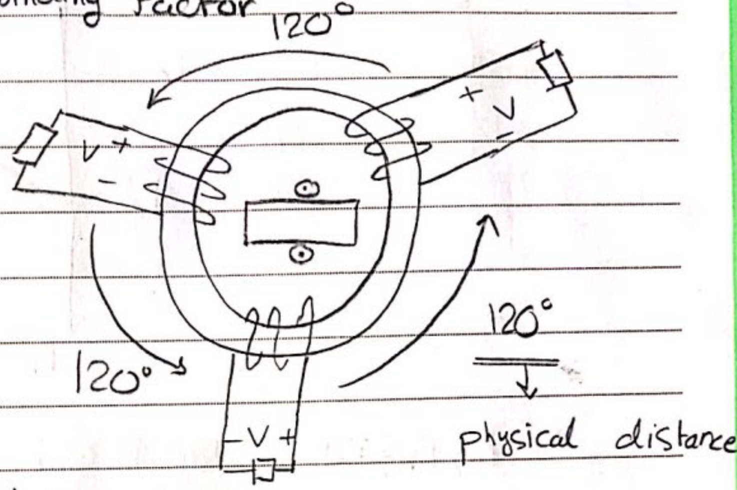
$$E_{rms} = 4.44\, K_w f N \Phi$$

Stator winding

field

winding factor



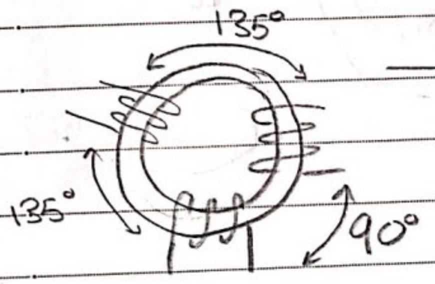


$$i_a = I_{max} \sin(\omega t - \psi)$$

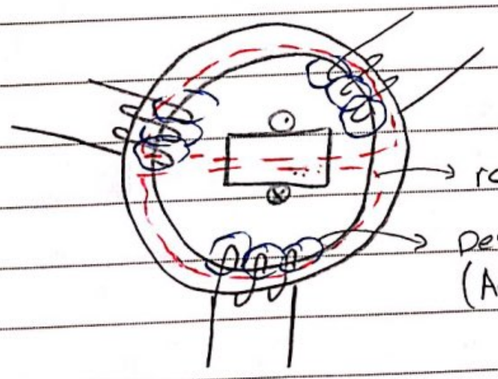
$$i_b = I_{max} \sin\left(\omega t - \psi - \frac{2\pi}{3}\right)$$

$$i_c = I_{max} \sin\left(\omega t - \psi + \frac{2\pi}{3}\right)$$

$$\boxed{P_{3\phi}(t) = \text{Constant}}$$



→ Vibration problems
(undesired)

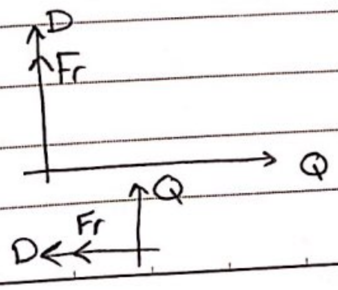


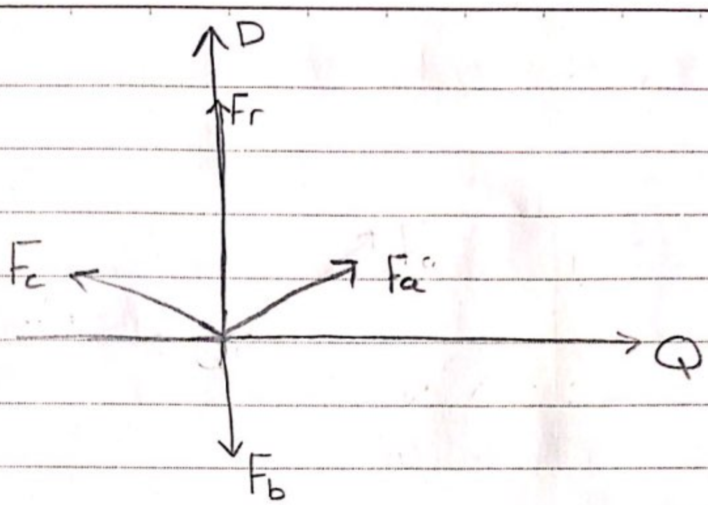
Stator winding factor

$$F_a = K i_a = F_m \sin(\omega t - \psi)$$

$$F_b = K i_b = F_m \sin(\omega t - \psi - 2\pi/3)$$

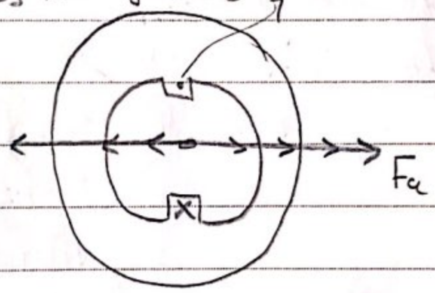
$$F_c = K i_c = F_m \sin(\omega t - \psi + 2\pi/3)$$



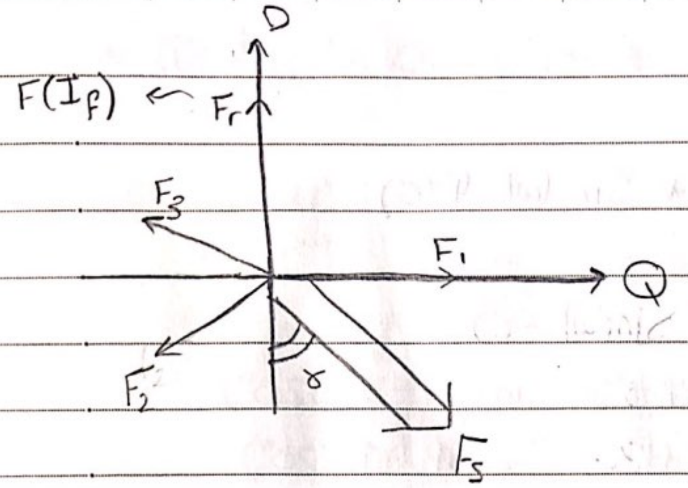


F_a, F_b, F_c : Stays in Same place
changes in magnitude

$I_m \sin(\omega t - \psi)$



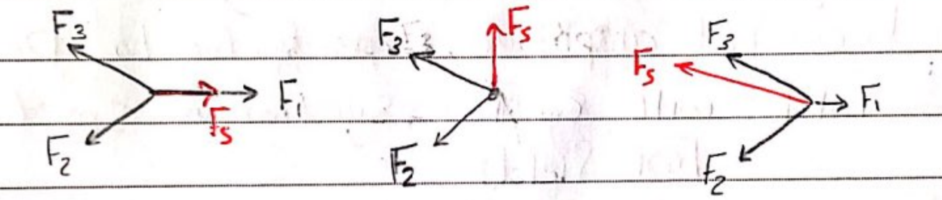
$F = F_a + F_b + F_c$



$F_s = F_1 + F_2 + F_3$

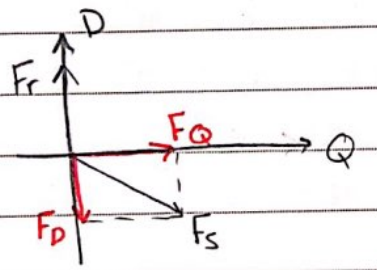
$*$ F_s moves with axis

t=0 t= 0.005 t= 0.01



t= 0.015 t= 0.02

F_s rotates depending on changing of values of F_1, F_2, F_3



$F_a = K I_{max} \sin(\omega t - \psi) = F_m \sin(\omega t$

$F_b = K I_{max} \sin(\omega t - \psi - \frac{2\pi}{3})$

$F_c = K I_{max} \sin(\omega t - \psi + \frac{2\pi}{3})$

phase shift

$F_D = F_a + F_b + F_c \quad * \cos(\omega t - \gamma - \theta)$

$= F_m \sin(\omega t - \psi) \cos(\omega t - \gamma)$

$+ F_m \sin(\omega t - \psi - 120^\circ) \cos(\omega t - \gamma - 120^\circ)$

$+ F_m \sin(\omega t - \psi + 120^\circ) \cos(\omega t - \gamma + 120^\circ)$

F_D = zero

$\cancel{F_Q = F_m \sin \omega t}$

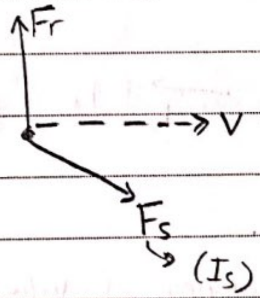
$$F_Q = F_a + F_b + F_c \ * \ \sin(\omega t - \psi \pm \theta)$$

$$= F_m \sin(\omega t - \psi) \sin(\omega t - \gamma)$$
$$+ F_m \sin(\omega t - \psi - 120^\circ) \sin(\omega t - \gamma - 120^\circ)$$
$$+ F_m \sin(\omega t - \psi + 120^\circ) \sin(\omega t - \gamma + 120^\circ)$$

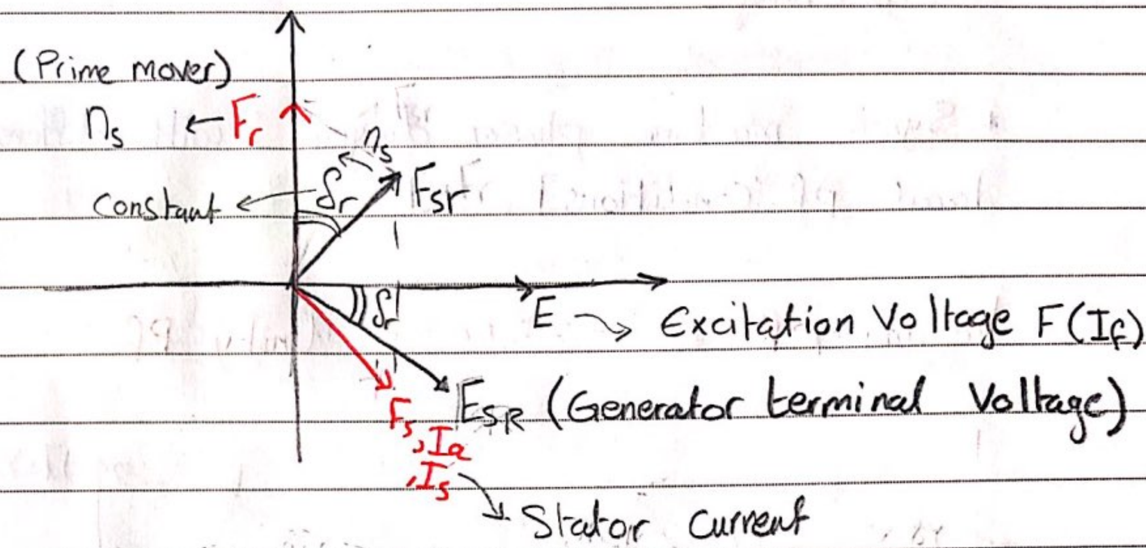
$$F_Q = \frac{3}{2} F_m \cos(\gamma - \omega t)$$

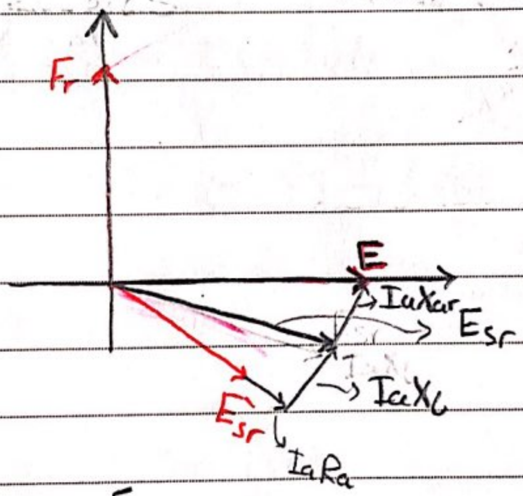
↳ constant amplitude, Perpendicular to D-axis
rotates with Constant Synchronous Speed
[rot field]
or [rotor field]

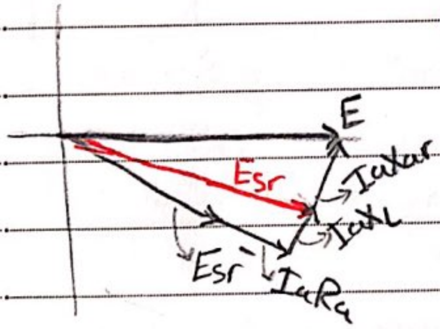
$v = -N \frac{d\phi}{dt}$



* Electrical Circuit model of Synch. gen.







→ leakage flux
→ Stator resistance (R_a)
→ Armature reaction

$\delta_r = \delta$ (In Case we neglect leakage flux component)
↳ Power angle

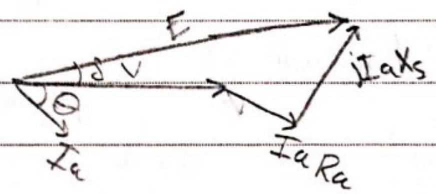
$$E = E_{sr} + j I_a X_{ar}$$
$$= E_{sr}' + [jX_{ar} + jX_l + R_a] I_a$$
$$= V + [R_a + jX_s] I_a$$

Synchronous reactance
→ AR
→ leakage

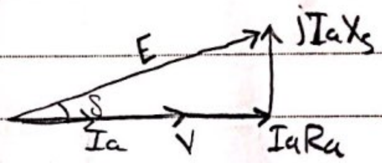
In the change of load $\rightarrow F_s$ changes $\rightarrow F_{sr}$ changes $\rightarrow \delta$ changes

* Synch. machine phasor diagram [with different load pf Conditions]

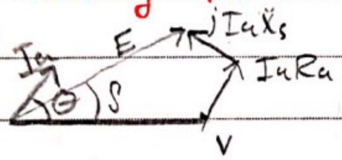
lagging pf



Unity Pf



leading pf



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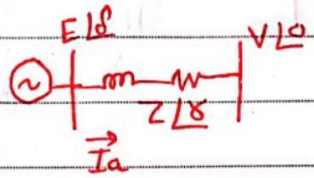
Power factor Control:

- It is not load pf
- P & Q produced by the generator



$$S_{3\phi} = 3 V I_a^*$$

$$I_a = \frac{E\angle\delta - V\angle 0}{Z\angle\gamma}$$

$$S_{3\phi} = \frac{3|E||V|}{Z}\angle\gamma-\delta \;-\; \frac{3|V|^2\angle\gamma}{Z}$$

neglect $R_a \rightarrow Z\angle\gamma = jX_s$ $[\gamma = 90°]$

$$P_{3\phi} = \frac{3|E||V|}{Z}\cos(\gamma-\delta) - \cancel{\frac{3|V|^2 \overset{\text{Zero}}{\cos\gamma}}{Z}} = \frac{3|E||V|}{X_s}\sin\delta$$

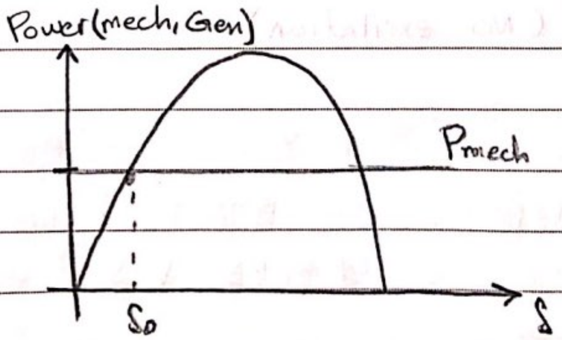
$\cos(90-\delta) \longrightarrow \sin\delta$, $\cos 90 = 0$

$$Q_{3\phi} = \frac{3|E||V|}{Z}\sin(\gamma-\delta) - \frac{3|V|^2}{Z}\sin\gamma$$

when $R_a = 0$

$$= \frac{3|V|}{X_s}\left[|E|\cos\delta - |V|\right]$$

$\sin(90-\delta) \longrightarrow \cos\delta$



⇒ As long as P_{mech} (τ) is constant, P_{Gen} is constant.

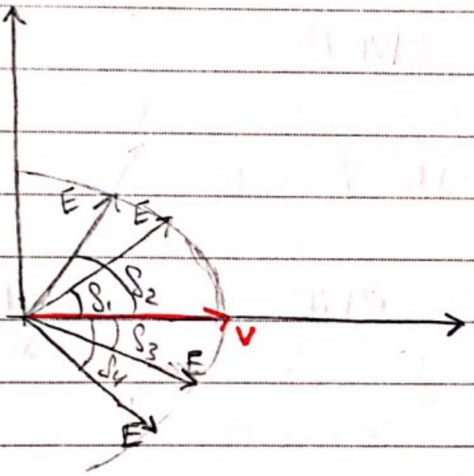
- Generators are usually connected to Infinite ~~busses~~ buses
"V is Constant"

$$P_{3\phi} = \frac{3\overset{\text{field circuit}}{|E|}\,\overset{\text{Infinite bus}}{|V|}}{\underset{\text{Synchronous reactance}}{X_s}} \sin\delta$$

$\lvert E\rvert$	Controled by field circuit
$\lvert V\rvert$	Infinite bus voltage
X_s	Synchronous Reactance

Change the P_{Gen} Set Point

* Change in $P_{3\phi}$ → Change in δ



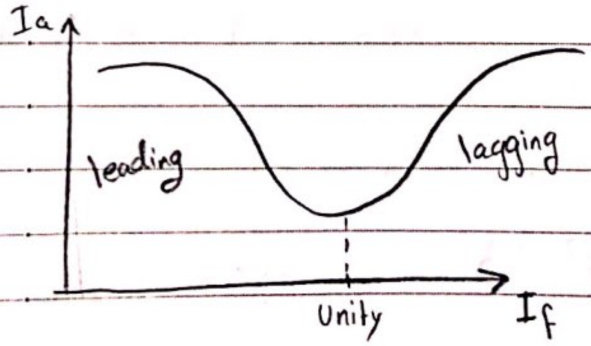
$$Q_{3\phi} = \frac{3|V|}{X_s}\left[|E|\cos\delta - |V|\right]$$

$|E|\cos\delta - |V| > 0$ (overexcited generator)

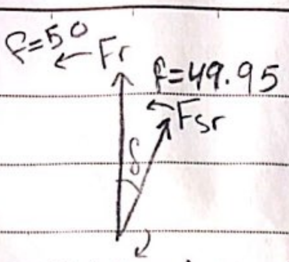
$|E|\cos\delta - |V| < 0$ (Underexcited generator)

Q is delivered to the generator.

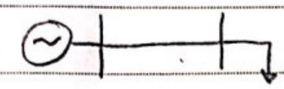
$|E|\cos\delta - |V| = 0$ (No excitation)



* You can change Q by changing the field while keeping V & P constant



f= 49

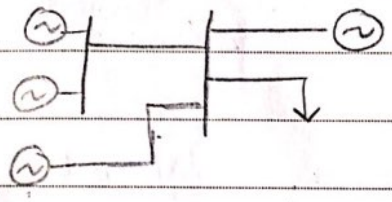


@ f = 25 Hz you will notice the difference

Single Generator ; no problem is change in frequency

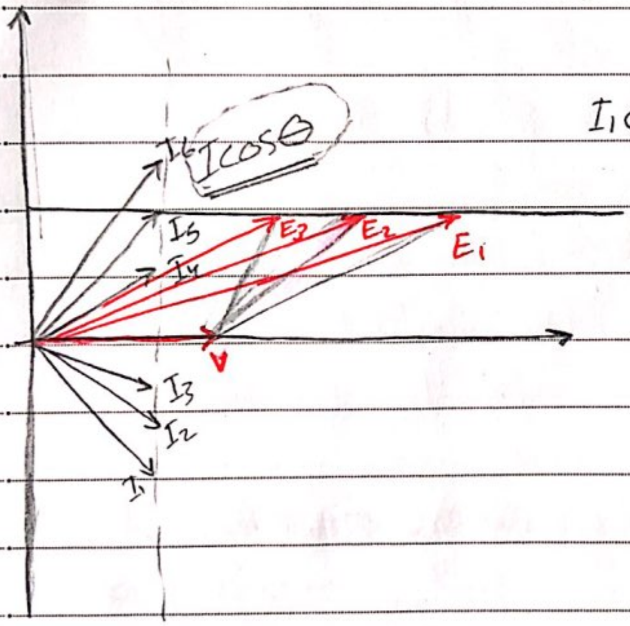
Problem: loss of synchlonism

f=49.98 ⟶ acceptable but should be changed to 50 Hz

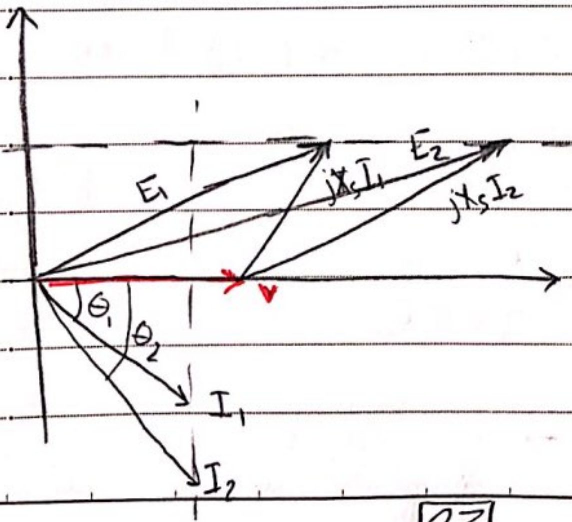


V & P Constant

$$P = 3\frac{|E||V|}{X_s}\sin\delta = 3VI\cos\theta \longrightarrow \text{must be constant}$$



$I_1\cos\theta_1 = I_2\cos\theta_2 = \dots$



Ex 3.1 (H. Saadat), Infinite bus voltage (V_{LL}) [take it as reference]

Rated S

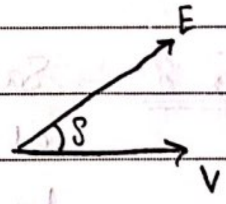
50 MVA, 30 kV, 3ϕ, 60 Hz Synch. Gen. connected to an Infinite bus through $X_s = 9\,\Omega$, $R_a = 0\,\Omega$. It is delivering full load current at 0.8 lagging Pf.

a) Find E & δ

$$S_{3\phi} = 50 \angle \cos^{-1} 0.8$$
$$= 40\text{ MW} + 30\text{ MVAr}$$

$$I_a = \frac{S^*_{3\phi}}{3 V^*_{LN}} = \frac{(50 \angle -36.9) \times 10^6}{3 \times \frac{30 \times 10^3}{\sqrt{3}}} = 962.25 \angle -36.87 \text{ A}$$

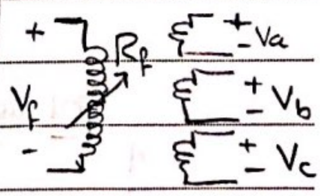
$$E = V + I_a * jX_s$$
$$= \frac{30 \times 10^3}{\sqrt{3}} + 962.25 \angle -36.87 * j9$$
$$= \underbrace{23.56}_{E} \underbrace{\angle 17.1}_{\delta} \text{ kV (L-N)}$$



b) The Generator now is delivering 25 MW & the excitation is kept constant, find Pf & I_a

(could not be done unless the mechanical torque is changed)

excitation is changed by field circuit

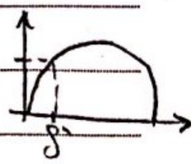


$$P_{3\phi} = \frac{3 |V|_{LN} |E|_{LN}}{X_s} \sin\delta$$

Change in $P_{3\phi}$ is changed by δ

$$25\text{ M} = \frac{3 \times \frac{30}{\sqrt{3}}\text{K} \times 23.56\text{K}}{9} \sin\delta' \longrightarrow \boxed{\delta = 10.59°}$$

$$E = 23.56 \angle 10.59°$$

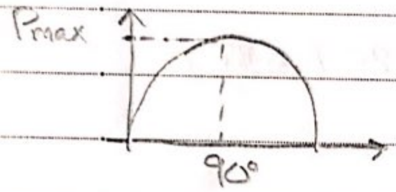


$$I_a = \frac{E - V}{jX_s} = \frac{23.56 \angle 10.59 - 17.32 \angle 0}{j9}$$

$$\boxed{I_a = 807.48 \angle -53.43^\circ}$$

$\boxed{Pf = \cos(53.43) = 0.596 \text{ lagging}}$ → as seen by the terminal of the gen.

c) Maximum Power that may be delivered by the generator



$$P_{max\,(3\phi)} = \frac{3|V||E|}{X_s} = 136\,MW$$

This number is not Practical only theoritical

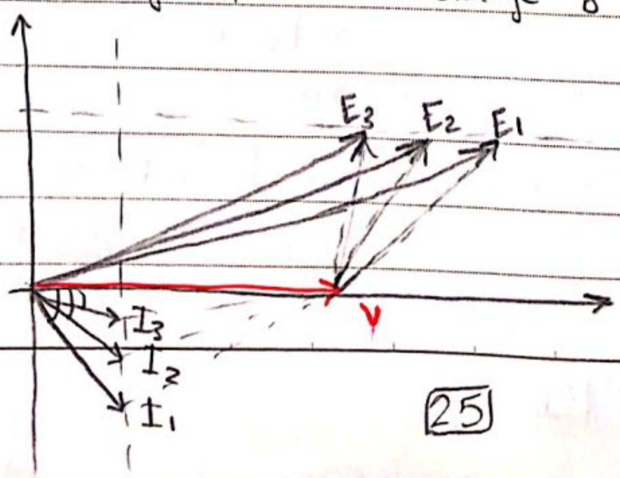
changed by Prime mover

Ex 3.2 Same generator as above is delivering 40MW at 30 kV, The excitation Voltage is reduced to 79.2%, find the new Power angle δ''.

$$|E'| = 0.792 \times 23.56\,kV = 18.66\,kV$$

$$\delta'' = \sin^{-1}\left(\frac{P_{3\phi} * X_s}{|E'| * |V|}\right) = \sin^{-1}\left(\frac{40 \times 9}{18.66 \times \frac{30}{\sqrt{3}}}\right) = 21.8^\circ$$

Change in E will change δ as $P_{3\phi}$ is kept constant



Find I_a & pf

$$I_a = \frac{18.66 \angle 21.8^\circ - 17.32 \angle 0^\circ}{j9} = 769.8^\circ \angle 0 \text{ A}$$

$Pf = \cos(0) = 1$ (No excitation)

* Suggested Problems :-

3.1, 3.2, 3.3, 3.4

HW1 : $X_d'' = 0.15$ pu , $X_d' = 0.4$ pu , $X_d = 1.2$ pu , $\delta = 0$

$\tau_d'' = 0.035$ sec

$\tau_d' = 1$ sec

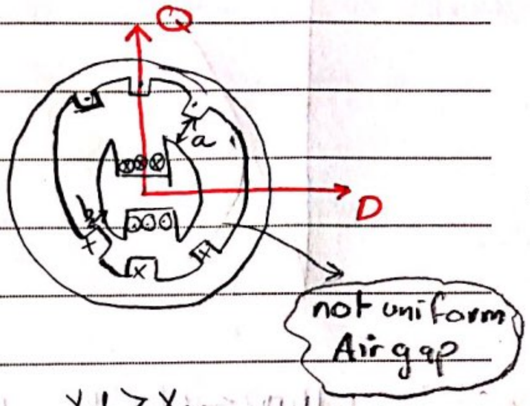
$E = 1$ pu

$\Rightarrow$ Draw the graph for the Short ckt current (use m-file).

*** modelling of Salient pole generator:**
"Used for Hydro units" → "to speed up rpm"

- D-axis centered magnetically

In the centre of North pole



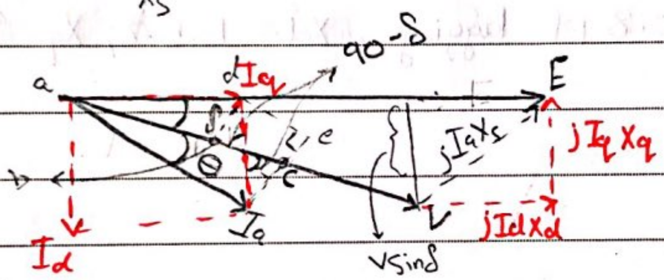
- Q-axis 90° degrees (ahead) of the D-axis

$X_d > X_q$

Reluctance of X_d < Reluctance of X_q (Bigger Airgap)

(ch8)

$$P = \frac{3\,|V|\,|E|}{X_s}\sin\delta$$



$$X_s = \sqrt{X_d^2 + X_q^2}$$

* X_d, X_q will not be required to find their values; advanced topics in power

$$|E| = |V|\cos\delta + X_d I_d$$

$$E = V + \underbrace{jX_d I_d + jI_q X_q}_{jI_a X_s}$$

$$P = 3|V|\,|I_a|\cos\theta$$

$$I_a\cos\theta = ab + bc$$
$$= ab + de$$
$$= I_q\cos\delta +$$

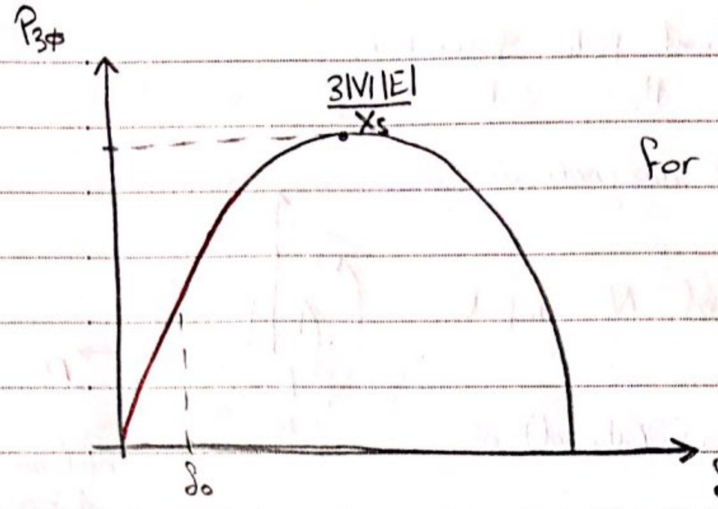
$$P = 3|V|\left(I_q\cos\delta + I_d\sin\delta\right)$$

But $|V|\sin\delta = X_q I_q$

$$I_d = \frac{|E| - V\cos\delta}{X_d}$$

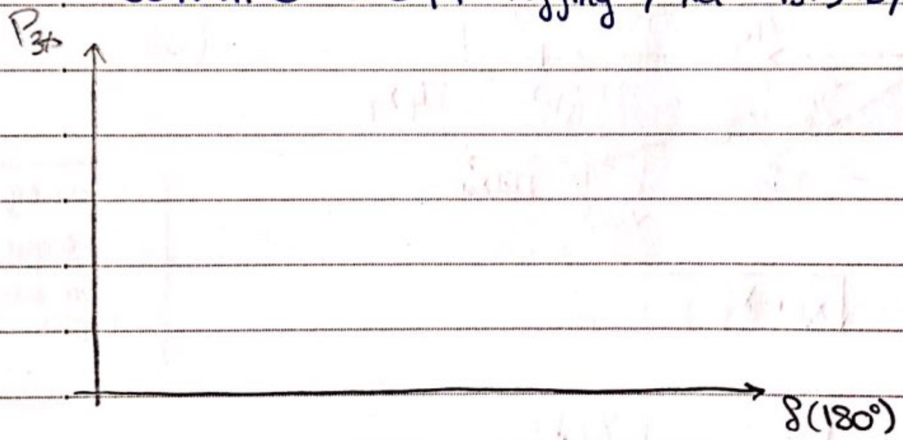
$$\boxed{P_{3\phi} = 3\,\frac{|V|\,|E|}{X_d}\sin\delta + 3|V|^2\left(\frac{X_d - X_q}{2X_dX_q}\right)\sin 2\delta}$$

→ Reluctance Power ($X_d \neq X_q$)

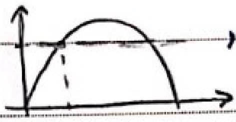


For cylindrical rotor Generator

HW2: Plot the Power Curve for a Salient pole generator
[is Connected to Infinite bus with 34.64 KV, delivering
60 MVA @ 0.8 pf lagging, $X_d = 13.5\Omega$, $X_q = 9.33\Omega$]



Comment on the Stability robustnes for a Salient pole & a Cylindrical rotor generator (Homework)

Stability: The property of a power system that enables it to remain in a state of operating equilibrium under normal operating conditions  , and to regain an acceptable state of equilibrium after being subjected to a disturbance.

※ maintaining Synchronous Operation ※

→ Loss of Voltage is an Instability condition

$P \leftrightarrow f$
$\Updownarrow$
$Q \leftrightarrow V$

Automatic generation Control

Small Disturbances:

- → Increase / Decrease of load
- → Switching of Small lines
- → Switching of Small generation

Large Disturbances:

- → Electric faults
- → Switching of large generation
- → Switching of large lines
- → Abrupt change In load / Generation

18/2

Power System Stability

→ Kundur - Chapter 2

1- Rotor angle Stability (0.75%) → f = 49.85

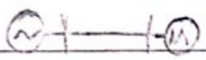
⇒ maintain Synchronism

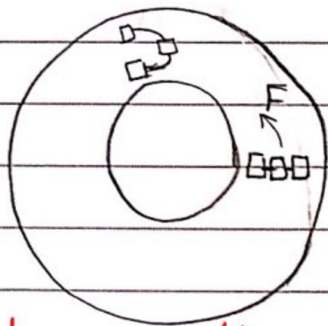
a. Steady State (Small Disturbances)

b. Transient (Large Disturbances) (0.1 - 5 Sec)

* Inrush, Starting current do not affect stability of a system.

a1. Oscillatory Instability	a2. Non-Oscillatory Instability
Insufficient Synchronizing torque	Insufficient damping torque





Formula 1 example

2- Voltage Stability (10%)

→ here, no rotor angle stability

(1 - Several minutes)

→ ability to maintain Steady State acceptable power!

⇒ Reactive power balance

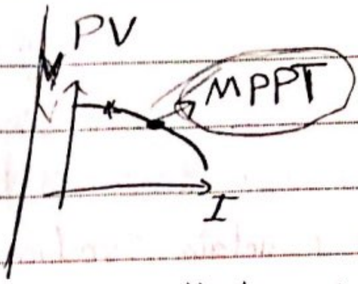
→ large Disturbance Voltage Stability
- large Disturbance
- ULTC, load fluctuation
- Protection & control coordination

→ Small Disturbance Voltage Stability
- PE
- Drives

Unrelated

$C = \frac{V}{f}$

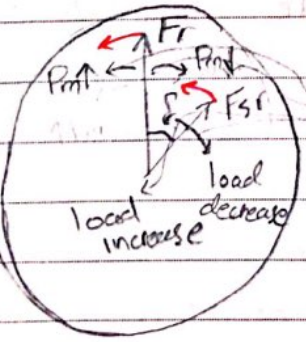
$V = 4.44 k f \Phi$



$\uparrow \frac{V}{f} = 4.44 k \, \Phi \uparrow$; fluxing, $f\downarrow$, $V\uparrow$, excitation Increases which is a problem

---

Mid-term Stability	long-term Stability
→ Protection/control coordination	→ Boiler Dynamics of thermal units.
→ Insufficient P/Q reserve	→ Penstock Dynamics of Hydro units.
	→ AGC (Automatic generation Control)
	→ Protection/Control Coordination



→ not direction, speed up/down

→ Any change will cause delaccelaration (+/−)

Swinging !!

→ Change in P $\overset{\text{تلقائياً}}{\leftrightarrow}$ change in $\underline{\delta^\circ}$

→ change in δ° may exceed the corresponding change in Power

Torque $\Leftrightarrow$ Power

$$\Delta \tau_e = \tau_S \Delta \delta + \tau_D \Delta \omega$$

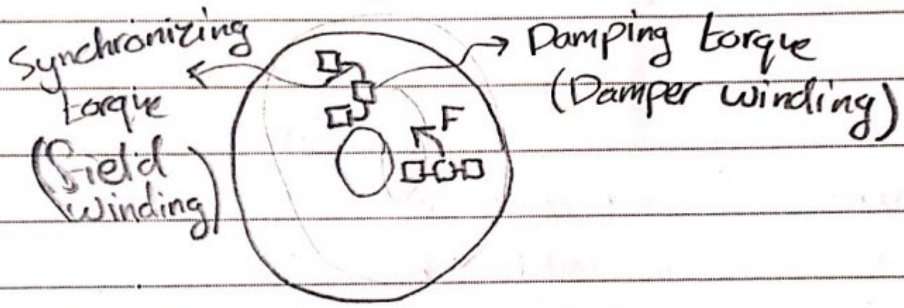
τ_e: electrical torque

$\tau_e = \tau_m$ (Steady)

τ_S: Synchronizing torque

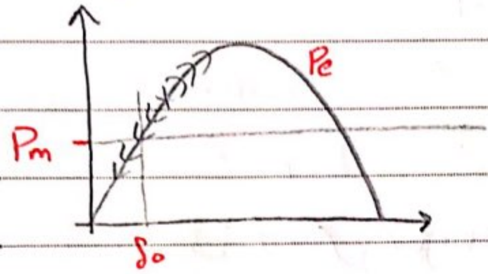
τ_D: Damping torque

$$P_{3\phi} = \frac{3|V||E|}{X_s} \sin \delta$$



⇒ Subject to a disturbance, a change in T_m or T_e will take place

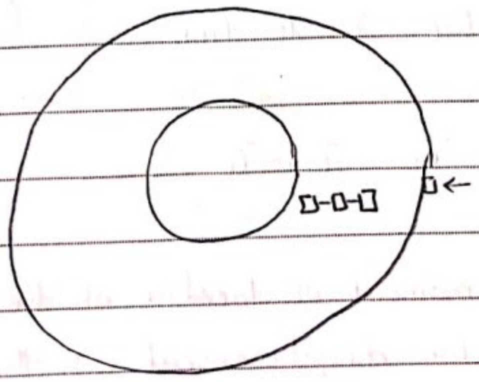
⇒ This change will directly change the value of power angle



⇒ This generator /system will always work to hence $[T_e = T_m]$ ⇒ Steady State

⇒ If we Study the problem from mechanical point of view, we will see that δ is Swinging around the new equilibrium point.

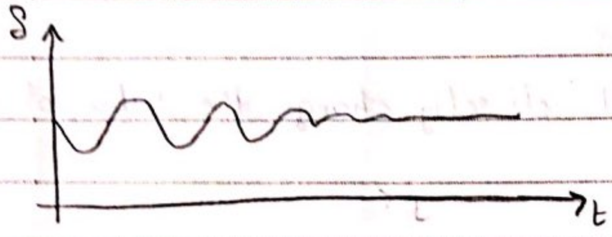
⇒ This Behaviour is Studied by a Swinging equation model.



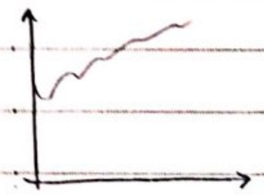
⇒ after a change. one of two Situations will take place.

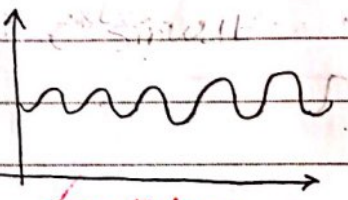
The two Situations

① Back to Stability



② Instable condition





Non-oscillatory Instability — loss of T_s

Oscillatory Instability — loss of T_d

* **Swing Equation:** the equation governing rotor angle motion of synchronous generator. It is based on the elementary principles of dynamics which states that the accelerating torque is the product of the moment of Inertia of the rotor times its angular acceleration

$$J\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e$$

where:

J: The total moment of Inertia of the rotor masses. (Kgm^2)

θ_m: The angular displacement of the rotor with respect to a Stationary ~~axcess~~ axis. (rad)

Prime mover ← T_m: mechanical torque

load ← T_e: Electrical torque

T_a: net torque

→ Steady State

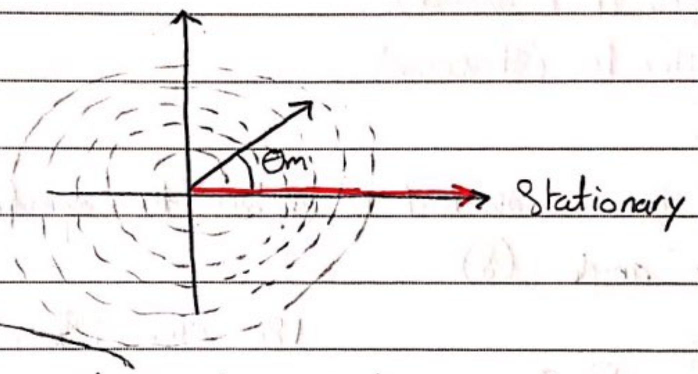
$T_e = T_m \rightsquigarrow T_a = 0$

If a disturbance happens

$T_a = T_m - T_e \neq 0$

$T_m > T_e \Rightarrow$ accelarating

$T_m < T_e \Rightarrow$ decelarating



with respect to

θ_m is measured w.r.t a Stationary reference axis on Stator, So it is continuously Increasing. with time even at constant Synchronous Speed.

→ it is better to measure the relative rotor position w.r.t a reference axis which rotates at Synch. Speed.

$$\theta_m = \omega_{sm} t + \delta_m$$

$$\frac{d^2\theta_m}{dt^2} = \frac{d \cdot \delta_m{}^2}{dt^2}$$

$$J \cdot \frac{d^2\delta_m}{dt^2} = T_a = T_m - T_e \quad \text{(multiply by } \omega_m\text{)}$$

M (Inertia constant)

$$J\omega_m \frac{d^2\delta_m}{dt^2} = T_m\omega_m - T_e\,\omega_m = P_m - P_e$$

$$\boxed{M \frac{d^2\delta_m}{dt^2} = P_m - P_e}$$

But $W_K = \frac{1}{2} J \omega_m^2 = \frac{1}{2} M \omega_m$

Kinetic energy

$M = 2 W_K / \omega_m = 2 W_K / \omega_{sm}$

$\rightarrow$ $\omega_m = \omega_{sm}$ (Steady State Condition)

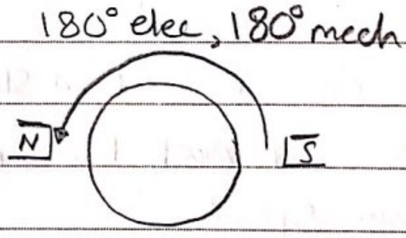
* Chapter 11 (Saadat)
* Chapter 16 (Stevenson)

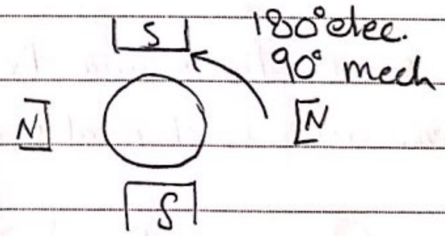
$\rightarrow$ Its more convenient to express the equation with electrical Power angle (δ)

number of Poles

$$\delta = \frac{P}{2} \delta_m$$

$$\omega = \frac{P}{2} \omega_m$$





$$\delta = \frac{P}{2}\,\delta_m$$

$$\omega = \frac{P}{2}\,\omega_m$$

$[\omega_{sm} = \omega_s]$

$\frac{2\omega_s}{\omega_{sm}}$

$$\Rightarrow \frac{2}{P}\, m \frac{d^2\delta}{dt^2} = P_m - P_e$$

⇒ we define H constant (usually used in stability studies)

$$H \triangleq \frac{W_k}{S_B} \equiv \frac{\text{Stored kinetic energy in mega joules @ Synch. Speed}}{\text{machine ratings in MVA}}$$

$$H = \frac{\frac{1}{2} J \omega_{sm}^2}{S_{mach}} = \frac{\frac{1}{2} m\, \omega_{sm}}{S_{mach}} \quad MJ/MVA$$

In per unit

$$\frac{2}{P}\left(\frac{2W_k}{\omega_{sm} \cdot S_{mach}}\right)\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{S_{mach.}} \quad ; \quad \frac{W_k}{S_{mach}} = H$$

$$\frac{2}{P}\left(\frac{2H}{\omega_{sm}}\right)\frac{d^2\delta}{dt^2} = P_{m.pu} - P_{e.pu} \quad ; \quad \omega_{sm} = \frac{2}{P}\,\omega_s$$

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e \quad (pu)$$

electric radians

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

electric degrees

$$\frac{H}{180 f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

* lets study the behaviour of the machine for a small change of $\Delta\delta$.

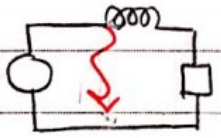
$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$= P_m - P_{max}\sin\delta$$

$$\frac{|E||V|}{X_s} \qquad \frac{|E'||V|}{X_{12}}$$

$$X_{12} = X_d' + X_t$$

* Short Circuit with loaded generators



when there is a pre fault load. The voltage before the fault is

$E'' = V_T + jI_a X_d''$ (Voltage behind Sub-transient Reactance)

for stability studies, since we are interested in transient reactance.

$E' = V_T + jI_a X_d'$ (Voltage behind transient reactance)

In Steady State

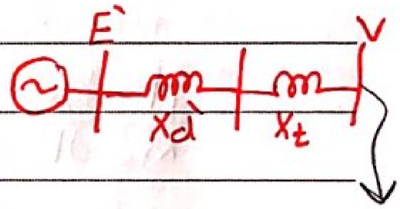
$E = V_T + jI_a X_d$ (Voltage behind Steady State reactance)

Ex 13.8 KV (V_B), 100MVA (S_B), 0.8 pf generator, with:
$X_d = 1$ pu, $X_d' = 0.25$ pu, $X_d'' = 0.12$ pu is connected through a 13.8/220 KV ~~TR~~ transformer with $X_t = j0.2$, find the generator transient fault.

Pu
Do not Divide by $\sqrt{3}$

$$I_L = \frac{S^*}{V^*} = \left(\frac{1\angle -36.87}{1\angle 0}\right)^* = 1\angle -36.87^\circ \text{ pu}$$

$$E' = V + j(X_d' + X_t)\, I_L$$
$$= 1\angle 0 + j(0.25 + 0.2) \times 1\angle -36.87^\circ$$
$$= 1.32\angle 15.83^\circ \text{ pu}$$

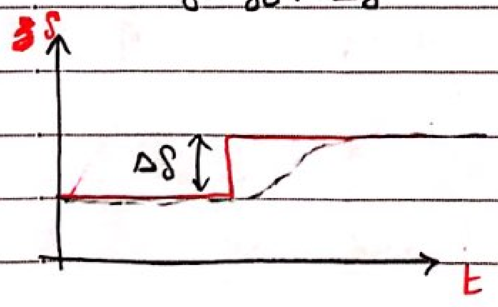


$$I_g' = \frac{E'}{j(X_d' + X_t)} = 2.93\angle -74.17^\circ \text{ pu}$$

* Back to the behaviour of a machine for a small change of $\Delta\delta$

For a small change, it is still correct to linearize the swing equation.

$$\delta = \delta_0 + \Delta\delta$$



$$\frac{H}{\pi f_0} \cdot \frac{d^2(\delta_0 + \Delta\delta)}{dt^2} = P_m - P_{max}\sin(\delta_0 + \Delta\delta)$$

$$\frac{H}{\pi f_0}\left[\frac{d^2\delta}{dt^2}+\frac{d^2\Delta\delta}{dt^2}\right]=P_m-P_{max}\left[\sin\delta_0.\overset{1}{\cancel{\cos\Delta\delta}}+\cos\delta_0.\overset{\Delta\delta}{\cancel{\sin\Delta\delta}}\right]$$

$$\sin(\Delta\delta)=\Delta\delta$$

$$\boxed{\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2}}+\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2}=\boxed{P_m-P_{max}\sin\delta_0}-P_{max}\cos\delta_0*\Delta\delta$$

$$\frac{H}{\pi f_0}.\frac{d^2\Delta\delta}{dt^2}+\underbrace{P_{max}.\cos\delta_0}_{P_s}.\Delta\delta=0$$

P_s → Synchronizing Power coefficient

→ Slope of Power angle curve at δ_0

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$\delta_0 \longrightarrow \delta_0 + \Delta\delta$

$$\frac{H}{\pi f_0} \frac{d^2 \Delta\delta}{dt^2} + \underbrace{P_{max} (\cos \delta_0)}_{P_S} \Delta\delta = 0$$

$P_S \rightsquigarrow$ Synchronizing Power coefficient

$$P_S = \left.\frac{dP}{d\delta}\right|_{\delta_0} = P_{max} \cos(\delta_0)$$

$$\frac{H}{\pi f_0} \cdot \frac{d^2 \Delta\delta}{dt^2} + P_S \Delta\delta = 0$$

- Laplace Domain (S)

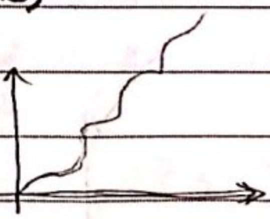
Time	S
$f'(t) \longrightarrow$	$SF(s) - f(0)$
$f''(t) \longrightarrow$	$S^2F(s) - S f(0) - f'(0)$
$1 \longrightarrow$	$1/S$
$e^{at} \longrightarrow$	$1/(S-a)$
$\sin(at) \longrightarrow$	$a/(S^2+a^2)$
$\cos(at) \longrightarrow$	$S/(S^2+a^2)$

$$\frac{H}{\pi f_0} S^2 \Delta\delta(S) + P_S \Delta\delta(S) = 0$$

$$\frac{H}{\pi f_0} S^2 + P_S = 0 \quad ; \quad S^2 = \frac{-P_S * \pi f_0}{H}$$

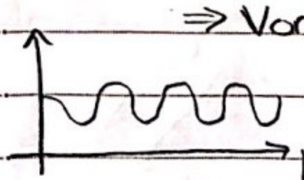
P_S is negative S_1, S_2 (both $-ve$ or both $+ve$)

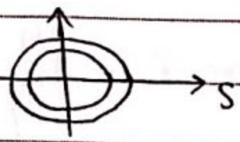
$e^{+S_1 t}, e^{+S_2 t} \Rightarrow$ Non Stable behaviour (non-Oscillatory)



P_S is positive S_1, S_2 (both pure Imaginary)

$\Rightarrow$ Vortex





$\Rightarrow$ Non Stable (Oscillatory)

$$af''(t) + bf'(t) + f(t) = 0 \qquad \nearrow IC$$

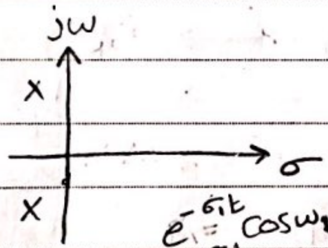
$$aS^2 F(S) + bS F(S) + F(S) = C_1$$

$$F(S)[aS^2 + bS + C] = C_1$$

$$F(S) = \frac{C_1}{aS^2 + bS + C}$$

$$F(S) = \frac{A}{S-\alpha} + \frac{B}{S-\beta}$$

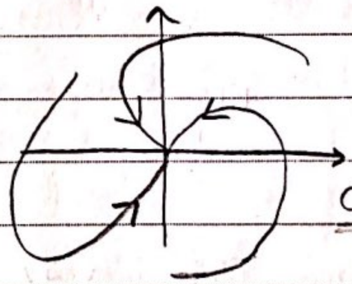
$$f(t) = Ae^{-\alpha t} + Be^{-\beta t} \quad ; \quad S = \sigma \pm jw$$



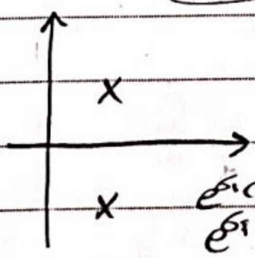
$e^{-\sigma_1 t} \cos w_1 t$

$e^{-\sigma_1 t} \cos w_2 t$

$w_2 = -w_1$

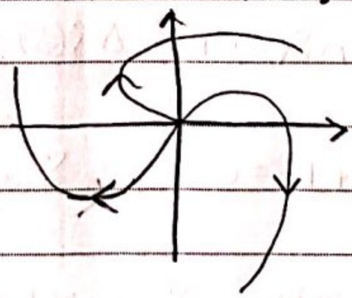


⇒ Stable focus



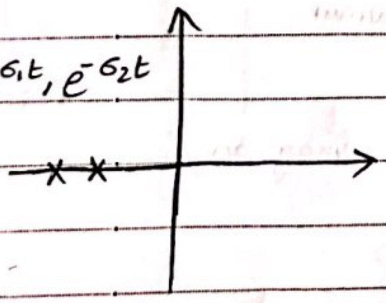
$e^{\sigma_1} \cos w_1 t$

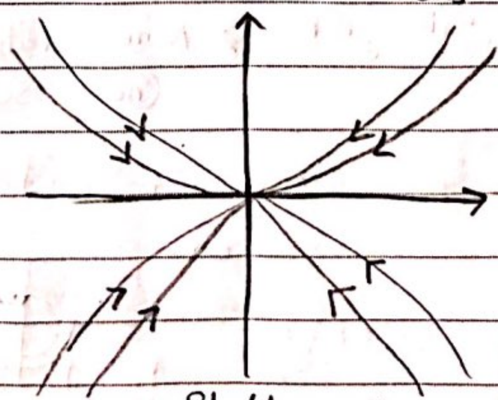
$e^{\sigma_1} \cos w_2 t$



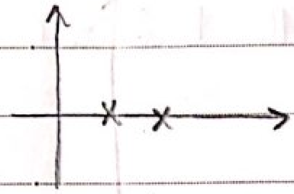
⇒ Non Stable focus

$e^{-\sigma_1 t}, e^{-\sigma_2 t}$

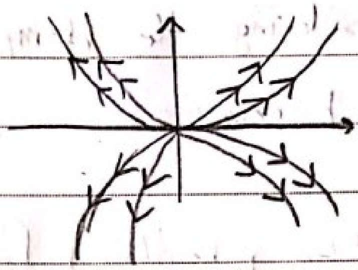




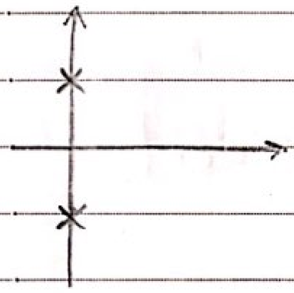
⇒ Stable node

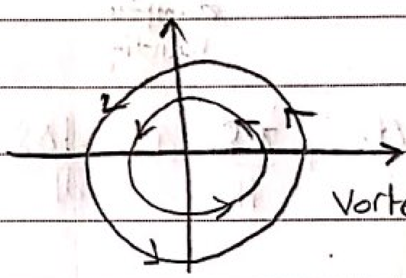


$e^{\sigma_1 t}, e^{\sigma_2 t}$



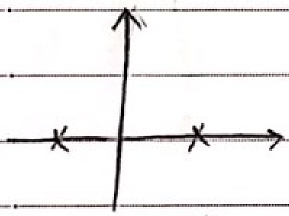
$\Rightarrow$ unstable node



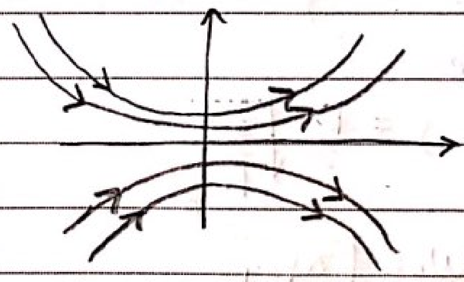


Vortex

$\Rightarrow$ marginally Stable



$e^{\sigma_1 t} + e^{-\sigma_2 t}$



Saddle point

$\Rightarrow$ The machine has tendency to return to synchronism by means of damper winding, that causes what is so called damping torque.

$\Rightarrow$ Additional damping torque is caused by the prime mover but neglected here!

$$P_D = \overbrace{D}^{\text{Defined by manufacturer}} \underbrace{\frac{d\delta}{dt}}_{\text{Speed}}$$

⇒ considering the damping torque, the linearized swing equation becomes:

$$\left[\frac{H}{\pi f_0} \cdot \frac{d^2 \Delta \delta}{dt^2} + \underbrace{D \frac{d\Delta\delta}{dt}}_{\text{damper winding}} + P_s \, \Delta\delta = 0\right] * \frac{\pi f_0}{H}$$

$$\frac{d^2 \Delta\delta}{dt^2} + \frac{\pi f_0}{H} * D \, \frac{d\Delta\delta}{dt} + \frac{\pi f_0}{H} \cdot P_s \cdot \Delta\delta = 0$$

$$\frac{d^2 \Delta\delta}{dt^2} + 2\zeta \omega_n \frac{d\Delta\delta}{dt} + \omega_n^2 \, \Delta\delta$$

where

$$\zeta = \frac{D}{2} \sqrt{\frac{\pi f_0}{H P_s}}$$

$$\omega_n = \sqrt{\frac{\pi f_0}{H} P_s}$$

⇒ characterestic equation:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

by design:

$$\zeta = \frac{D}{2} \sqrt{\frac{\pi f_0}{H P_s}} < 1$$

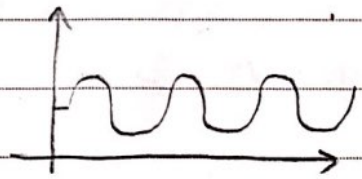
$$s_1, s_2 = -\zeta\omega_n \pm j\omega_n \sqrt{1-\zeta^2}$$
$$= -\zeta\omega_n \pm j\omega_d$$

where

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

$P_s > 0$!!

$\Rightarrow \delta(t)$, $\omega(t)$??



$\Rightarrow$ To find a general Solution In time domain, we Introduce the System in State Space representation.

$$\dot{x} = Ax + Bu$$
$$Y = Cx + Du$$

car : combustion, Previous combustion
fuel, Speed

$\dot{x} \equiv$ Previous combustion

$x \equiv$ combustion

$u \equiv$ fuel

$Y \equiv$ Speed