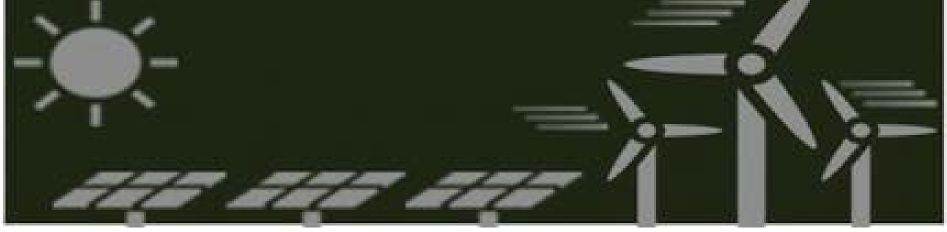
Topics in Power

Dr.Mhmd Haj-Ahmad

By: Hamza Abedrabbo

powerunit-ju.com



PU
2018

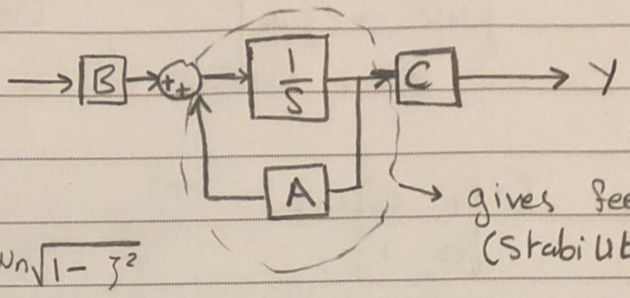
$S^2 + 2\zeta \omega_n S + \omega_n^2 \rightarrow$ C.E (characterestic equation)

Previous state state matrix state

$\dot{x} = Ax + Bu$

$Y = Cx$

output



gives feedback (stability)

$S_1, S_2 = -\zeta \omega_n \mp j\omega_n \sqrt{1-\zeta^2}$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $x_1 = \Delta\delta$

$x_2 = \Delta\omega = \Delta\dot{\delta}$

$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$ $\dot{x}_1 = x_2$

$\dot{x}_2 = \ddot{x}_1 = -\omega_n^2 x_1 - 2\zeta\omega_n x_2$

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\omega_n \frac{d\Delta\delta}{dt} + \omega_n^2 \Delta\delta = 0$$

$$\Delta\ddot{\delta} = \frac{d^2\Delta\delta}{dt^2} = -2\zeta\omega_n \Delta\dot{\delta} - \omega_n^2 \Delta\delta$$

$$\dot{x}_2 = -2\zeta\omega_n \dot{x}_1 - \omega_n^2 x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$\dot{x} = Ax$

$\dot{x}(t) = A\,x(t)$

$S\,X(S) - x(0) = A\,X(S)$

$S\,X(S) - A\,X(S) = X(0)$

$(SI - A)\,X(S) = x(0)$

$X(S) = (SI - A)^{-1} x(0) \rightsquigarrow$ بتمثل السيارة ← المطب (Disturbance)

* laplace transfer → Steady state domain

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}$$

*The Idea is to see what will the Disturbance do to the System.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$SI = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} \rightarrow SI - A = \begin{bmatrix} S & -1 \\ \omega_n^2 & S + 2\zeta\omega_n \end{bmatrix}$$

$$(SI - A)^{-1} = \frac{1}{S^2 + 2\zeta\omega_n S + \omega_n^2} \cdot \begin{bmatrix} S + 2\zeta\omega_n & 1 \\ -\omega_n^2 & S \end{bmatrix}$$

$$X(t) = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \Delta\delta(t) \\ \Delta\omega(t) \end{bmatrix}$$

$$X(S) = (SI - A)^{-1} X(0) \rightarrow \begin{bmatrix} \Delta\delta_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta(S) \\ \Delta\omega(S) \end{bmatrix} = \frac{1}{S^2 + 2\zeta\omega_n S + \omega_n^2} \cdot \begin{bmatrix} S + 2\zeta\omega_n & 1 \\ -\omega_n^2 & S \end{bmatrix} \cdot \begin{bmatrix} \Delta\delta_0 \\ 0 \end{bmatrix}$$

$$\Delta\delta(S) = \frac{(S + 2\zeta\omega_n)\,\Delta\delta_0}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

$$\Delta\omega(S) = \frac{-\omega_n^2\,\Delta\delta_0}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

Taking the inverse laplace transformation.

$$\Delta \delta(t) = \frac{\Delta \delta_o}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t + \theta)$$

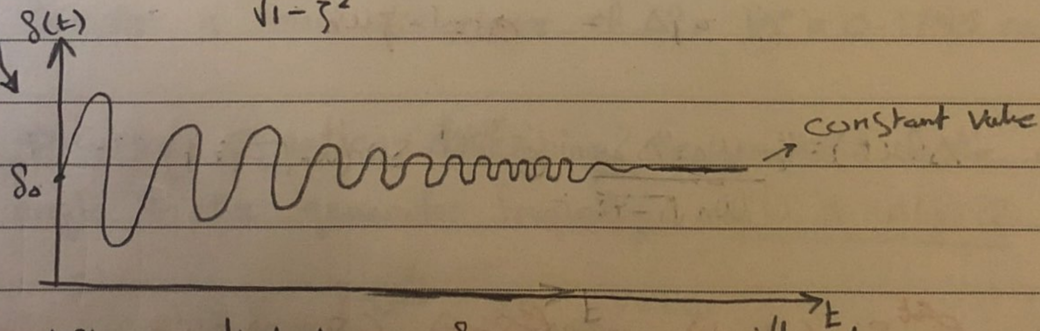
$$\Delta \omega(t) = \frac{-\omega_n \Delta \delta_o}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t)$$

where $\theta = \cos^{-1} \zeta$

The total response

$$\delta = \delta_o + \frac{\Delta \delta_o}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t + \theta)$$

$$\omega = \omega_o - \frac{\omega_n \Delta \delta_o}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t)$$



* After a disturbance δ changes rapidly to a constant Value; this is due to the Insertion of a damping winding.

From $\Delta\omega(S)$ to ω

$$\Delta\omega(S) = \frac{-\omega_n^2\,\Delta\delta_o}{S^2+2\zeta\omega_n S+\omega_n^2+(-\zeta^2\omega_n^2+\zeta^2\omega_n^2)}$$

$$= \frac{-\omega_n^2\,\Delta\delta_o}{\left[(S+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)\right]\times\omega_n\sqrt{1-\zeta^2}} \times \omega_n\sqrt{1-\zeta^2}$$

$$= \frac{-\omega_n\,\Delta\delta_o}{\omega_n\sqrt{1-\zeta^2}} * \frac{\omega_n\sqrt{1-\zeta^2}}{(S+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)}$$

$$e^{at}\sin bt \longrightarrow \frac{b}{(S-a)^2+b^2}$$

$$b=\omega_n\sqrt{1-\zeta^2}$$

$$a=-\zeta\omega_n$$

$$\Delta\omega(t) = \frac{-\omega_n\,\Delta\delta_o}{\omega_n\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t}\,\sin(\omega_n\sqrt{1-\zeta^2}\,t)$$

$$e^{at}\sin(at+b) \longrightarrow \cancel{(S-a)}\ \frac{(S-C)\sin b + a\cos b}{(S-C)^2+a^2}$$

Ch3, Ch8, Ch11
Ch16

4/3

Suggested Problems

Ch8: 6, ~~8~~, 9, 10, 11, 12

Ch11: 1, 2, 3

Stevenson

Ch16: 1, 2, 3

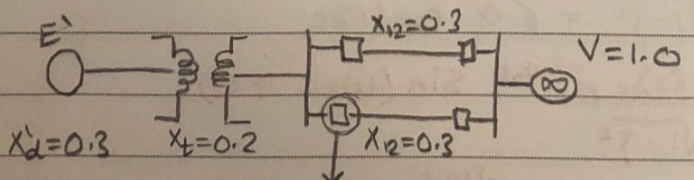
$$\frac{H}{\pi f_0}\cdot\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + P_s\,\Delta\delta = 0$$

(last required material for First exam)

Ch 11 H. Saadat

Ex. 60 Hz Synch. generator with Inertia constant $H = 9.94$ MJ/MVA & $X'_d = 0.3$ pu is connected to Infinite bus through a purely reactive Circuit as Shown (all in P.U). the generator delivers 0.6 pu real power at 0.8 pf lagging, to the Infinite bus. assume $D = 0.138$ pu damping power coefficient. Consider a Small disturbance of $\Delta\delta = 10° = 0.1745$ rad

⇒ obtain equation describing the motion of the rotor angle & the generator frequency $\delta(t)$ & $\omega(t)$??



open & close of this CB ; for example (Before the PM takes action)

$X_{total} = j0.2 + j0.3 + j0.15 = j0.65$ pu

$$\delta = \overset{?}{\delta_0} + \frac{\overset{10}{\Delta\delta_0}}{\sqrt{1-\zeta^2}}\,e^{-\zeta\omega_n t}\sin(\omega_d t + \theta)$$

$$\omega = \underset{60}{\omega_0} - \frac{\omega_n\,\Delta\delta_0}{\sqrt{1-\zeta^2}}\,e^{-\zeta\omega_n t}\sin(\omega_d t)$$

$$S = \frac{0.6}{0.8} = 0.75\angle 36.87^\circ \text{ pu}$$

$$I = \frac{S^*}{V^*} = 0.75\angle -36.87^\circ \text{ pu}$$

$V^* \rightarrow 1\angle 0$

$$E' = V + jX_{total} I$$

$$= 1\angle 0 + j0.65 \times 0.75\angle -36.87^\circ$$

$$= 1.35\angle 16.69^\circ \text{ pu}$$

$16.69^\circ \rightarrow \delta_0$

$$P_s = P_{max} \cos \delta_0$$

$$= \frac{1.35 \times 1}{0.65} \cos(16.69)$$

E', V_∞, X_{total}

$$= 1.9884 \text{ pu} \Rightarrow P_s = 1.9884 \text{ pu}$$

Synch. Power

$$\omega_n = \sqrt{\frac{\pi f_0}{H} P_s} = 6.1405 \text{ rad/s}$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_0}{H P_s}} = 0.212 \rightarrow \theta = \cos^{-1}(\zeta) = 77.69^\circ$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 6.0 \text{ rad/sec}$$

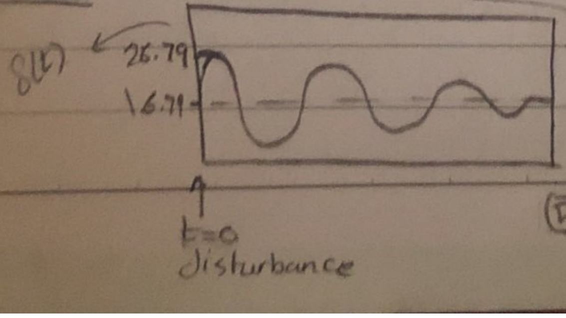
$$\delta(t) = \delta_0 + \frac{\Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \theta)$$

$$\omega(t) = \omega_0 - \frac{\omega_n \Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t)$$

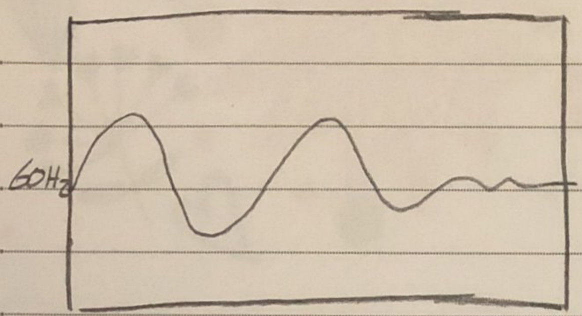
$$\delta(t) = 16.69 + 10.234 e^{-1.3t} \sin(6t + 77.69^\circ)$$

Hz Frequency $\leftarrow f(t) = 60 - 0.1746 e^{-1.3t} \sin(6t)$

$$0.1746 \rightarrow \left(\frac{6.1405 \times}{2\pi} \frac{10^\circ}{\sqrt{1-.212^2}} \times \frac{180}{\pi}\right)$$



51



*~~Home work #2~~

* Rotor angle response to Small power Input:-

$$\frac{H}{\pi f_0} \cdot \frac{d^2 \Delta\delta}{dt^2} + D \frac{d\Delta\delta}{dt} + P_s \Delta\delta = \Delta P$$

$\Delta P \rightarrow$ Small Increase/decrease In PM torque

$$\frac{d^2\Delta\delta}{dt^2} + \frac{\pi f_0 D}{H} \cdot \frac{d\Delta\delta}{dt} + \frac{\pi f_0 P_s}{H} \Delta\delta = \frac{\pi f_0}{H} \Delta P$$

Input

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\omega_n \frac{d\Delta\delta}{dt} + \omega_n^2 \Delta\delta = \Delta u$$

In State Space representation:

$x_1 = \Delta\delta$, $x_2 = \Delta W = \Delta\dot{\delta}$

$\dot{x}_1 = x_2$

$\dot{x}_2 = -\omega_n^2 - 2\zeta\omega_n x_2 + \Delta u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta u$$

$\dot{x} = Ax + Bu$

→ going to S-domain:

$$S X(S) - \cancel{x(0)}^{\,0} = Ax(S) + B\Delta U(S)$$

$$S X(S) = A X(S) + B \Delta U(S)$$

$$X(S) = (SI - A)^{-1} B \Delta U(S)$$

$$\left(\begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \right)^{-1} = \begin{bmatrix} S & -1 \\ \omega_n^2 & S + 2\zeta\omega_n \end{bmatrix}^{-1} = \frac{1}{AD - BC} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta(S) \\ \Delta\omega(S) \end{bmatrix} = X(S) = \frac{\begin{bmatrix} S + 2\zeta\omega_n & 1 \\ -\omega_n^2 & S \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{\Delta u}{S}}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

$\rightarrow \begin{bmatrix} 1 \\ S \end{bmatrix}$

Scanned by CamScanner

$$\Delta\delta(s) = \frac{\Delta u}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$$

$$\Delta\omega(s) = \frac{\Delta u}{s^2+2\zeta\omega_n s+\omega_n^2}$$

Time domain:

$$\Delta\delta(t) = \frac{\Delta u}{\omega_n^2}\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t+\theta)\right]$$

$$\Delta\omega(t) = \frac{\Delta u}{\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\omega_d t$$

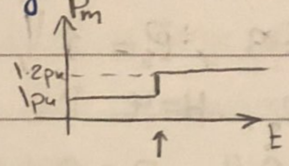
Full Response:

$$\delta(t) = \delta_0 + \frac{\pi f_0 \Delta P}{H\omega_n^2}\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t+\theta)\right]$$

$$\omega(t) = \omega_0 + \frac{\pi f_0 \Delta P}{H\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\omega_d t$$

Ex. The same generator studied with $\delta_0 = 16.79°$, the Input power Increased by $\Delta P = 0.2$ pu, obtain the (Step-response) for the rotor angle & frequency.

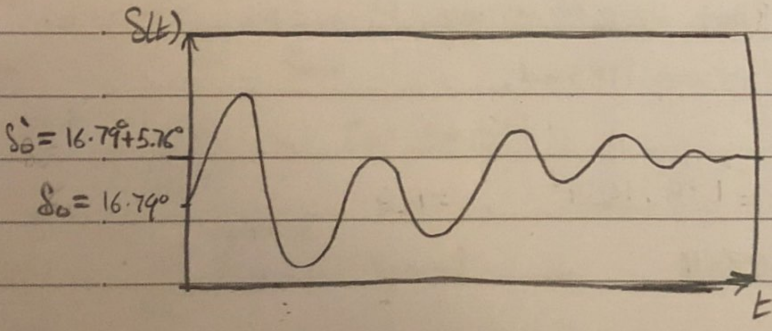
$$\left[\begin{array}{l} H = 9.94 \text{ MJ/MVA} \\ \zeta = 0.2131 \\ \omega_n = 6.1405 \text{ rad/s} \end{array}\right]$$



$$\delta(t) = \delta_0 + \frac{\pi f_0 \Delta P}{H \omega_n^2}\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t + \theta)\right]$$

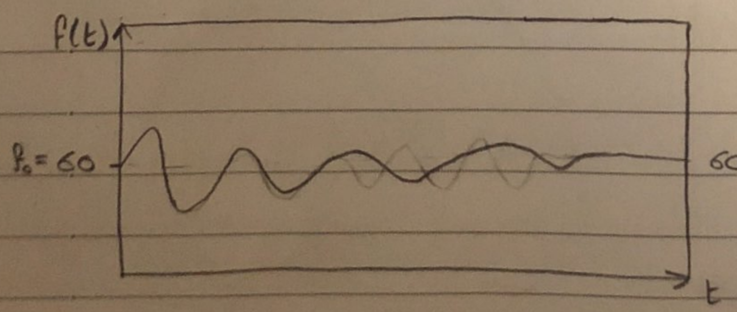
$$= 16.79 + \frac{180° \times (60) \times (0.2)}{9.94 \times (6.1405)^2}\left[1 - \frac{1}{\sqrt{1-(0.2131)^2}} e^{-0.2131(6.14)t} \sin(6t + 77.7°)\right]$$

$$= 16.79° + 5.763\left[1 - 1.0235 e^{-1.3t} \sin(6t + 77.7°)\right]$$



$$f = 60 + \frac{60(0.2)}{2(9.94)(6.14)\sqrt{1-(0.2131)^2}} e^{-1.3t} \sin 6t$$

$$f = 60 + 0.1 e^{-1.3t} \sin 6t$$



Note

$V = 4.44 \phi f k$

$\frac{V}{f} = 4.44 \phi k$ → constant

$f\downarrow, \phi\uparrow \rightarrow$ Saturation

Your load might not handle it

* Homework #2

Part A → Draw the rotor angle & frequency (Hz) responses using matlab in example 11.2 :- [Small disturbance]

a) as in the example

b) $P_s = 3$, $P_s = 1$, $P_s = -2$, $P_s = 0$

c) $H = 2$, $H = 4$

d) $D = 0.4$, $D = 0$.

e) $\Delta P = 0.2$ pu

Part B → Solve ex 11.3 using Simulink with $\Delta P = 0.15$ pu

Due date: 18/3

a) Ex. $f_0 = 60Hz$, $X_d' = 0.3pu$, $H = 9.94$ MJ/MVA, $P_m = 0.6$ Pu ; 0.8 PF lag.

$X_{eq} = 0.65pu$, $\Delta\delta = 10° = 0.1745rad$,

$I_a = \left(\frac{S}{V}\right)^*$

$E\angle\delta = V_t + jX_{eq} I_a = 1.35 \angle 16.79°$, $V = 1\angle 0$

$P_s = P_{max} \cos\delta_0 = 1.9884$

$\delta(t) = \delta_0 +$

Transient Stability:-
Equal Area Criterion

⇒ The generator is subjected to a large disturbance.

⇒ There is a possibility to lose synchronism.

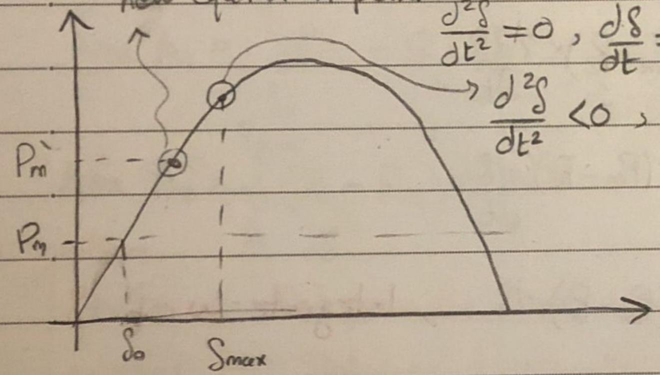
⇒ $\delta = \delta_0 + \Delta\delta$ is not accurate anymore

⇒ Kinetic energy exchange will help in solving the problem

new equilibrium point

$\frac{d^2\delta}{dt^2} = 0$, $\frac{d\delta}{dt} \neq$ Synch. Speed

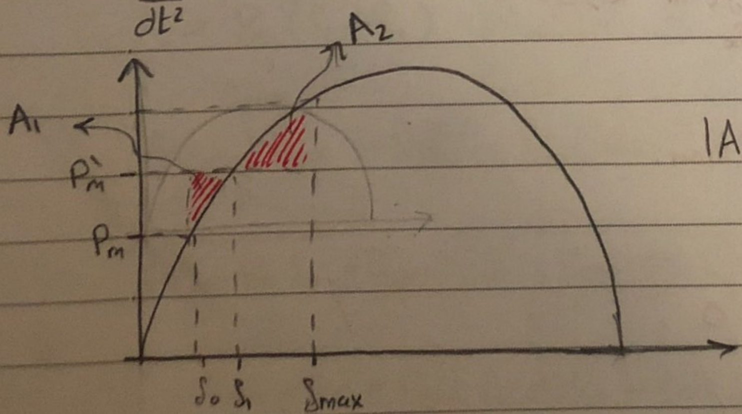
$\frac{d^2\delta}{dt^2} < 0$, $\frac{d\delta}{dt}$ = Synch. Speed (not equilibrium point)



$\frac{d^2\delta}{dt^2} > 0$ [at $t = 0^+$]

$\frac{d\delta}{dt} = \uparrow$ relative synch. Speed

$\frac{d^2\delta}{dt^2} = 0$ [at $t = 0^-$]



|Area A_1| = |Area A_2|

⇒ Graphical representation of energy exchange.

→ Considering a Single machine Connected to an Infinite bus.

→ Swing equation neglecting damping!

$$\frac{H}{\pi f_0} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e = P_a$$

$$\left[\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H}(P_m - P_e)\right] \times 2\frac{d\delta}{dt}$$

$$2\frac{d\delta}{dt} \cdot \frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H}(P_m - P_e) \times 2\frac{d\delta}{dt}$$

$$\frac{d}{\cancel{dt}}\left(\left(\frac{d\delta}{dt}\right)^2\right) = \frac{2\pi f_0}{H} \cdot (P_m - P_e)\frac{d\delta}{\cancel{dt}}$$

$$d\left(\left(\frac{d\delta}{dt}\right)^2\right) = \frac{2\pi f_0}{H}(P_m - P_e)\,d\delta$$; Integrate w.r.t δ

$$\left[\frac{d\delta}{dt}\right]^2 = \frac{2\pi f_0}{H}\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta$$

rotor Speed:

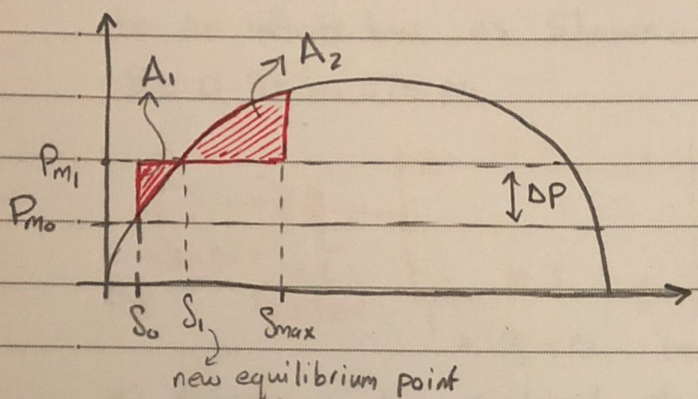
$$\frac{d\delta}{dt} = \sqrt{\frac{2\pi f_0}{H}\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta}$$

For Stable Condition:

relative synch. speed ← $\frac{d\delta}{dt} = 0$

$$\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta = 0$$

* Suppose a Sudden Increase in Input Power:



* why not Stop at δ_1 (why get to δ_{max}) because of the increase in energy. (remember the car example) ⇒ You want to go from 80 → 100 ; You'll get to 105 or So, then You'll get to 100. (Swinging)

$$\text{Area } A_1 = \int_{\delta_0}^{\delta_1} (P_{m_1} - P_e)\, d\delta$$

$P_e \to P_{max} \sin\delta$

$$\text{Area } A_2 = \int_{\delta_1}^{\delta_{max}} (P_e - P_{m_1})\, d\delta$$

objective

→ Goal : Determine the max Input power P_m, which can be applied for Stability to be maintained.

what if $A_2 > A_1$??

The generator will lose Synchronism.

→ Goal : to find largest Area A_2 Such that Stability will be maintained

$$A_1 = P_{m_1}(\delta_1 - \delta_0) - \int_{\delta_0}^{\delta_1} P_{max} \sin\delta \, d\delta$$

$$A_2 = \int_{\delta_1}^{\delta_{max}} P_{max} \sin\delta \, d\delta - P_{m_1}(\delta_{max} - \delta_1)$$

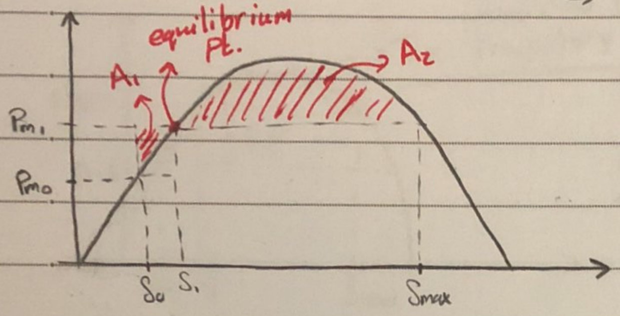
$$\underbrace{\cancel{P_{m_1}\delta_1} - P_{m_1}\delta_0 + \cancel{P_{max}\cos\delta_1} - P_{max}\cos\delta_0}_{A_1} = \underbrace{-P_{max}\cos\delta_{max} + \cancel{P_{max}\cos\delta_1} - P_{m_1}\delta_{max} + \cancel{P_{m_1}\delta_1}}_{A_2} \quad \div P_{m_1}$$

$$\delta_{max} - \delta_0 = P_{max}(\cos\delta_0 - \cos\delta_{max})$$

$$P_{m_1} = P_{max} \sin\delta_1$$

*Application of Sudden Increase in input power!

⇒ what is maximum amount of Input power (mechanical), before the generator losses Synchronism. → Maximum of Area 2



$$(\delta_{max} - \delta_0) P_{m_1} = P_{max} (\cos\delta_0 - \cos\delta_{max})$$

$$P_{m_1} = P_{max} \sin\delta_1$$

$$= P_{max} \sin\delta_{max} \quad ; \quad [\delta_1 = \pi - \delta_{max}]$$

$$\Downarrow$$

$$(\delta_{max} - \delta_0) \sin\delta_{max} + \cos\delta_{max} = \cos(\delta_0)$$

$$f(\delta_{max}) = C$$

Solve for δ_{max}, then

maximum mech. Power to be applied to the gen. before loss of Synchronism. ← $\boxed{P_{m_1} = P_{max} \sin\delta_1}$

This equation can be solved iteratively using Newton - Raphson method

① Start with an Initial guess

$$\frac{\pi}{2} < \delta_{max}^{(k)} < \pi$$

$$\Delta \delta_{max}^{(k)} = \frac{c - f(\delta_{max}^{(k)})}{\left.\frac{df}{d\delta_{max}}\right|_{\delta_{max}^{(k)}}}$$

& $$\left.\frac{df}{d\delta_{max}}\right|_{\delta_{max}^{(k)}} = (\delta_{max}^{(k)} - \delta_0) \cos \delta_{max}^{(k)}$$

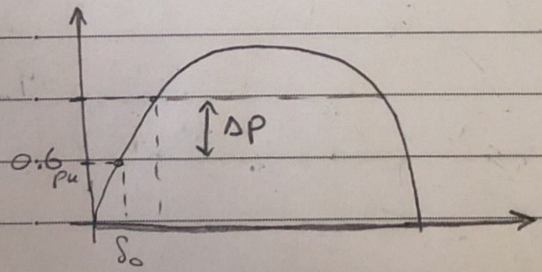
$$\delta_{max}^{k+1} = \delta_{max}^{(k)} + \Delta \delta_{max}^{(k)}$$

Stop when

$$|\delta_{max}^{(k+1)} - \delta_{max}^{(k)}| \leq \epsilon$$

Ex. The machine of previous example is delivering a real power of 0.6 pu at 0.8 pf lagging to the Infinite bus with 1pu Voltage!

⇒ Find maximum Input power that can be applied before loss of Synchronism.



$$P_{max} = \frac{|E||V|}{X_s} = \frac{1.35 \times 1}{X_s} = 2.077 pu$$

$$P_m = P_{max} \sin \delta_1$$

$$\delta_1 = \pi - \delta_{max}$$

$$\delta_0 = 16.79°$$

$$\left(\delta_{max} - 16.79 \times \frac{\pi}{180}\right) \sin \delta_{max} + \cos \delta_{max} = \cos 16.79°$$

$$\delta_{max} = 125.84°$$

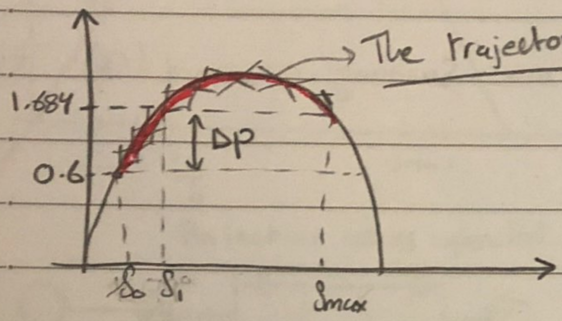
$$\delta_1 = 180 - 125.84° = 54.12°$$

$P_m = 2.077 \sin(54.12°)$

$= 1.684$ p.u

maximum Sudden power

$\Delta P = 1.684 - 0.6 = 1.082$ pu



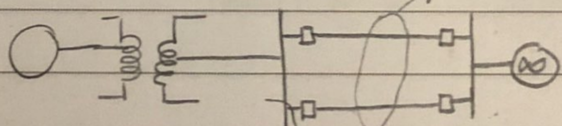
* Equal area criterion applied to a 3ϕ fault:

→ P_m is constant

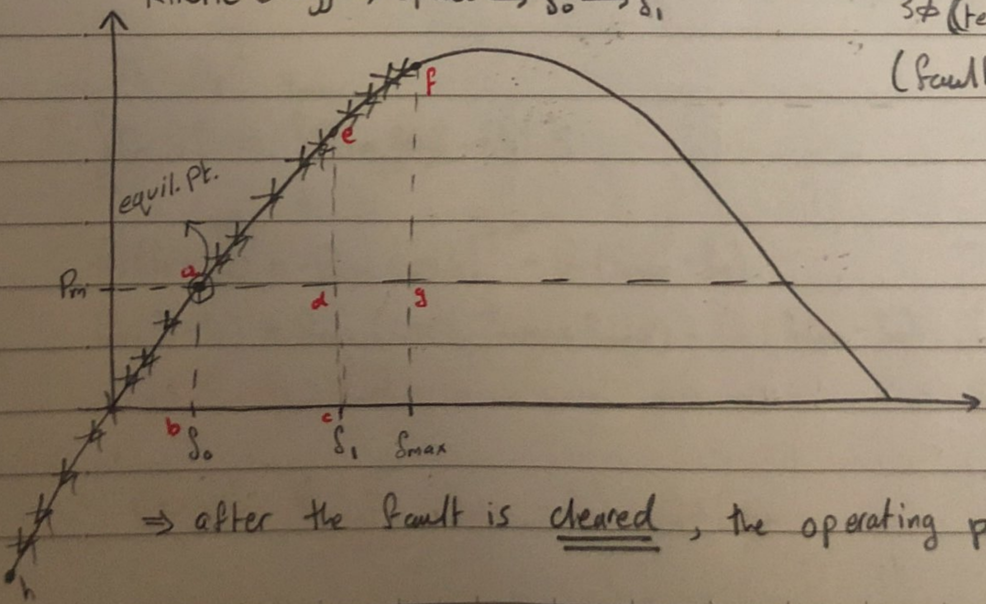
→ when 3ϕ fault happens $[P_e = 0]$, $[V = 0]$ at fault location.

→ P_m is all accelerating power.

$$\frac{H}{\pi f_0} \cdot \frac{d^2\delta}{dt^2} = P_m - \cancel{P_e}^{\,0}$$



Kinetic energy → speed → $\delta_0 \rightarrow \delta_1$



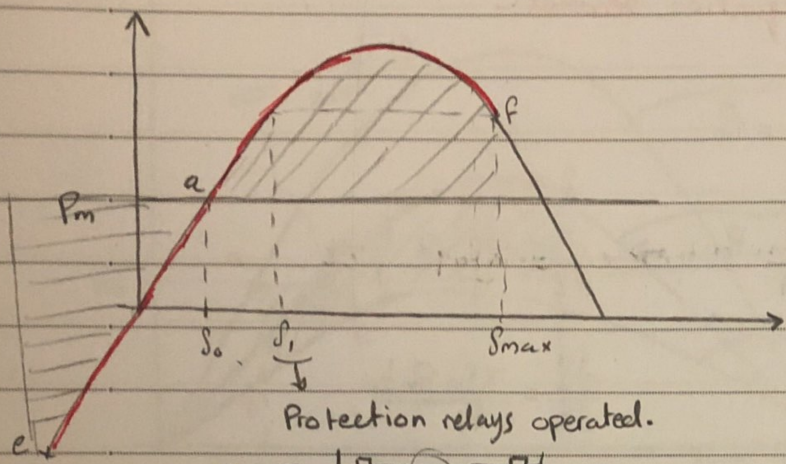
⇒ after the fault is cleared, the operating point is $\underline{\underline{e}}$

⇒ $P_e > P_m$, δ is Still Increasing, but the rotor is decelarating.

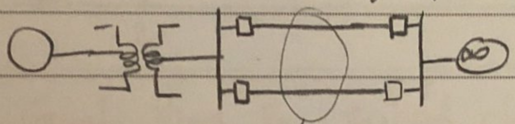
⇒ at f the Speed is Synch. but not an equilibrium point, δ goes back

⇒ The System will continue Swinging around (a) till it damped (f8h)

*Application of equal area criterion to 3ϕ Systems:

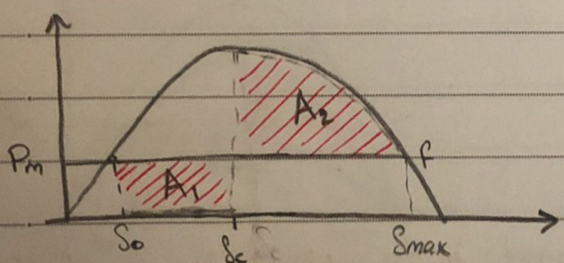


Protection relays operated.



both lines are intact

what if the protection action is delayed??



δ_c critical clearing angle (after this angle, Stability will be lost)

$\delta_{max} = \pi - \delta_o$

$A_1 = A_2$

$$\int_{\delta_o}^{\delta_c} P_m \, d\delta = \int_{\delta_c}^{\delta_{max}} (P_{max}\sin\delta - P_m)\, d\delta$$

$$P_m(\delta_c - \delta_o) = -P_{max}\cos\delta \Big|_{\delta_c}^{\delta_{max}} - P_m(\delta_{max} - \delta_c)$$

$$P_m(\delta_c - \delta_o) = P_{max}(\cos\delta_c - \cos\delta_{max}) - P_m(\delta_{max} - \delta_c)$$

⇒ Solving for δ_c

$$\cos\delta_c = \frac{P_m}{P_{max}}(\delta_{max} - \delta_o) + \cos\delta_{max}$$

However, for the Critical case of severe 3ϕ faults $[P_e=0]$

⇒ The Swing equation becomes:-

$$\frac{H}{\pi f_o} \cdot \frac{d^2\delta}{dt^2} = P_m$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_o \cdot P_m}{H} \Rightarrow \frac{d\delta}{dt} = \frac{\pi f_o P_m}{H} \int_0^t dt$$

$$\frac{d\delta}{dt} = \frac{\pi f_o P_m t}{H}$$

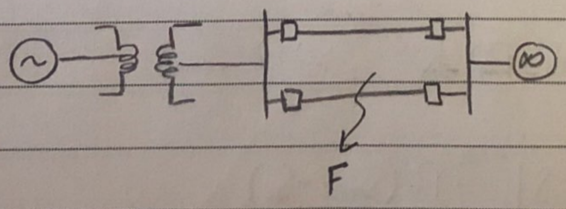
$$\boxed{\delta = \frac{\pi f_o P_m}{2H} t^2 + \delta_o}$$

If we take δ_o as δ_c

$$t_c = \sqrt{\frac{2H(\delta_c - \delta_o)}{\pi f_o P_m}}$$

↳ Critical clearing time

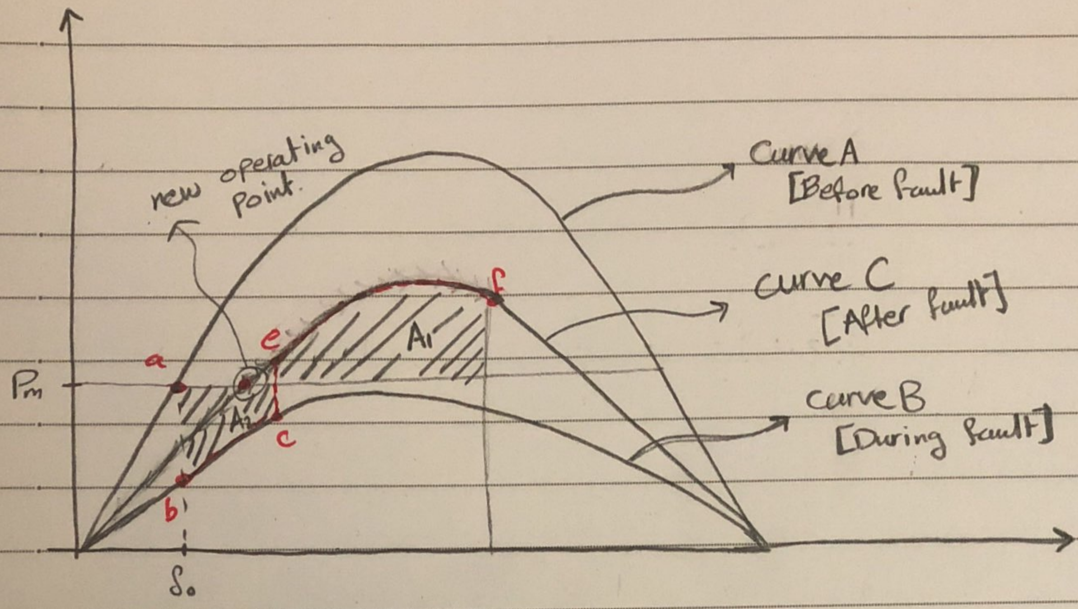
what if the fault is in the middle of one of the lines & the line has to be disconnected.



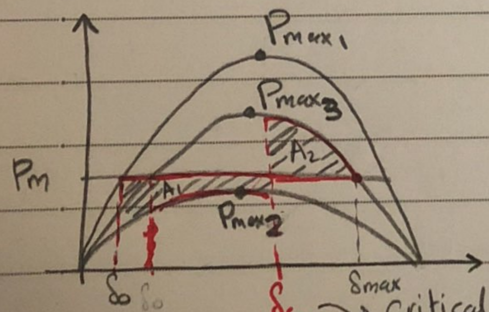
⇒ P_m is constant (Prime mover)

⇒ During the fault, the electric curve is B

⇒ After the fault is cleared, & the fault line is disconnected the curve of power transfer is C.



Goal: find the critical clearing angle before which the machine will stay in stability.



$\underline{\underline{\delta_c}} \rightarrow$ critical clearing angle

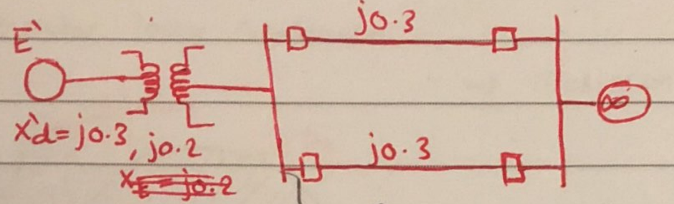
Applying equal area criterion

$$P_m[\delta_c - \delta_0] - \int_{\delta_0}^{\delta_c} P_{max2} \sin\delta \, d\delta = \int_{\delta_c}^{\delta_{max}} P_{max3} \sin\delta \, d\delta - P_m(\delta_{max} - \delta_c)$$

Solving for δ_c:

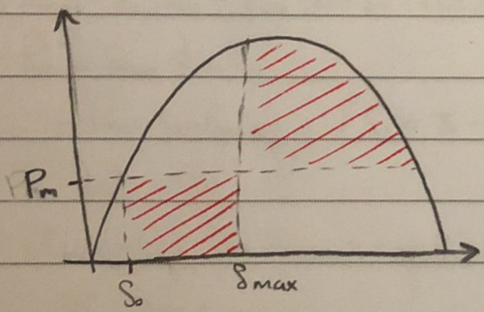
$$\cos\delta_c = \frac{P_m[\delta_{max} - \delta_0] + P_{max2}\cos\delta_{max} - P_{max2}\cos\delta_0}{P_{max3} - P_{max2}}$$

Ex. 60Hz generator with $H = 5 MJ/MVA$, $X'_d = 0.3$ is connected to an infinite bus as shown, the generator delivers $S = 0.8 + j0.074$ pu



→ $P_e = 0$ (equi. point / operating pt. is the same)

a) determine the critical clearing angle & the critical fault time when a temporary 3ϕ fault at the sending end Occurred & both lines are back to service!



⇒ find δ_0 / operating conditions

$$I = \left(\frac{S}{V}\right)^* = \frac{0.8 - j0.074}{1\angle 0} = 0.8 - j0.074 \text{ p.u}$$

$$X_{tot} = j0.3 + j0.2 + \frac{j0.3}{2} = j0.65 \text{ pu}$$

$$E' = V + jX_{tot} I_a$$
$$= 1\angle 0 + j0.65 * [0.8 - j0.074]$$
$$= 1.17 \angle 26.387°$$

$$P_{max} = \frac{|E||V|}{X_s} = \frac{1.17 \times 1}{0.65} = 1.8 \text{ pu} \; ; \; \delta_{max} = 180 - 26.387 = 153.612°$$

$$\cos\delta_c = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos(\delta_{max}) = \frac{0.8}{1.8}[153.612 - 26.387] \times \frac{\pi}{180}$$
$$+ \cos(153.61)$$
$$= 0.09106$$

$$\delta_c = \cos^{-1}(0.09106)$$
$$= 84.776^\circ = 1.48 \text{ rad}$$

$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f_0 P_m}}$$
$$= \sqrt{\frac{2*5\,(1.48 - 0.46055)}{\pi(60)(0.8)}}$$
$$= 0.26 \text{ Sec.}$$

unrelated

Relay Companies

SIEMENS

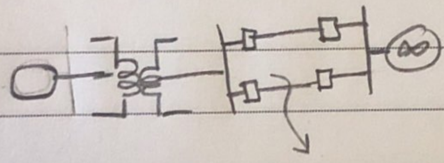
ABB

SEL

AREVA

→ Relay operating time: 0.25 cycle.

b) Determine the critical clearing angle when a 3ϕ fault ~~occurs~~ occurred in the middle of one of the lines, & the line is isolated.



① Before fault

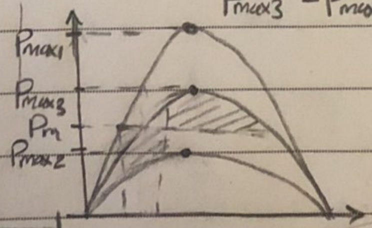
$P_{max1} = 1.8$

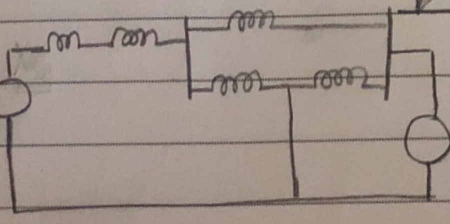
$\delta_0 = 26.4^\circ = 0.4605 \text{ rad.}$

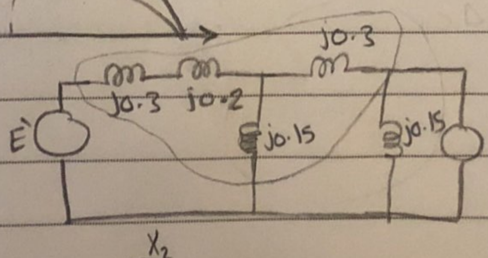
② During fault

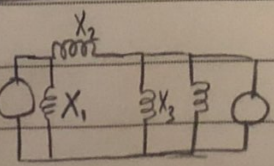
$$P_{max2} = \frac{EV}{X_{s2}}$$

$$\cos(\delta_c) = \frac{P_m(\delta_{max} - \delta_0) + P_{max3}\cos\delta_{max} - P_{max2}\cos\delta_0}{P_{max3} - P_{max2}}$$









$$P_{max2} = \frac{E' V}{X_{12}}$$

$$X_2 = \frac{(0.5)(0.15) + (0.5)(0.3) + (0.15)(0.3)}{0.15}$$

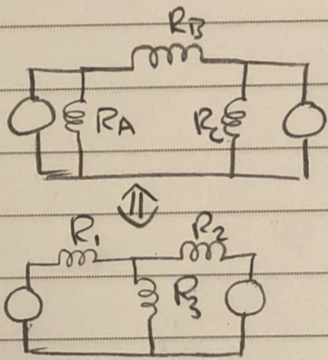
$X_2 = 1.8\ pu$

$$X_1 = \frac{(0.5)(0.15) + (0.5)(0.3) + (0.15)(0.3)}{0.3}$$

$X_1 = 0.9\ pu$

$$X_3 = \frac{(0.5)(0.15) + (0.5)(0.3) + (0.15)(0.3)}{0.5}$$

$X_3 = 0.54\ pu$



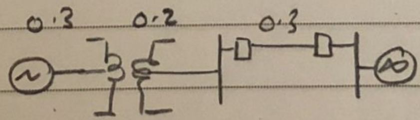
$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$P_{2max} = \frac{(1.17)(1)}{1.8} = 0.65\ pu$$

X_2

③ after fault



; $X_{s_3} = 0.3 + 0.2 + 0.3 = 0.8$

$$P_{3max} = \frac{1.17(1)}{0.8} = 1.4625\ pu$$

$$\cos\delta_c = \frac{\overset{0.8}{P_m}(\overset{2.5628}{\delta_{max}} - \overset{26\times\pi/180}{\delta_0}) + \overset{1.4625}{P_{3max}}\cos\delta_{max} - \overset{0.65}{P_{2max}}\cos\delta_0}{P_{3max} - P_{2max}}$$

$\delta_{max} = 180 - \delta_0 \rightarrow \sin^{-1}\left(\frac{P_m}{P_{3max}}\right)$

$= 146.8^\circ$

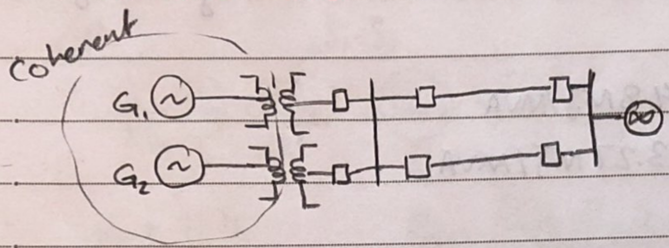
$= 2.5628$

$\cos(\delta_c) = -0.15356 \longrightarrow \boxed{\delta_c = 98.843^\circ}$

Suggested Problems.

Ch11: 1, 2, 3, 5, 6, 7, 8, 10, 12

* 2 Synch. machines Swing together [coherent Set]



→ Same prime mover (not neccesarily)

→ Same location

for generator 1

$$\frac{H_1}{\pi f_0} \cdot \frac{d^2 \delta_1}{d^2 t} = P_{m_1} - P_{e_1}$$

for generator 2

$$\frac{H_2}{\pi f_0} \frac{d^2 \delta_2}{dt^2} = P_{m_2} - P_{e_2}$$

→ If the two generators are considered as coherent set, then:

$$\delta_1 = \delta_2 = \delta$$

$$\frac{H}{\pi f_0} \frac{d^2 \delta}{dt^2} = P_m - P_e$$

where
$H = H_1 + H_2$
$P_m = P_{m_1} + P_{m_2}$
$P_e = P_{e_1} + P_{e_2}$

Ex. two 60 Hz generators Operating in parallel with the same power plant & have the following ratings:

G_1: 500MVA, H_1 = 4.8 MJ/MVA

G_2: 1333MVA, H_2 = 3.27 MJ/MVA

Calculate the equivalent H constant for the two units on a 1000 MVA base:-

⇒ The total kinetic energy of rotation of the two machines

$$K.E = 500 * 4.8 + 1333 * 3.27$$

$$= 6759 \text{ MJ}$$

H for a 1000 MVA base

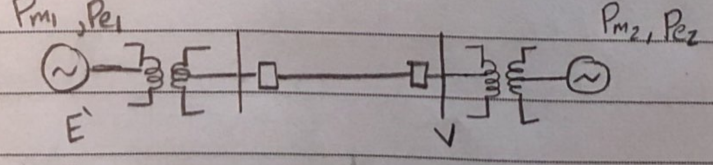
$$H = 6.759 \text{ MJ/MVA}$$

For non-coherent sets:-

→ Different location

→ Different prime mover

[2 machines only in the system, not connected to an Infinite bus.]



$$\frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2} = \pi f_0 \left[\frac{P_{m1} - P_{e1}}{H_1} - \frac{P_{m2} - P_{e2}}{H_2} \right]$$

multiply by $\frac{H_1 H_2}{H_1 + H_2}$

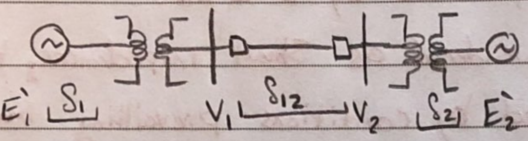
$$\frac{1}{\pi f_0}\left[\frac{H_1 H_2}{H_1+H_2}\right]\frac{d^2(\delta_1-\delta_2)}{dt^2} = \frac{P_{m1}H_2 - P_{m2}H_1}{H_1+H_2} - \frac{P_{e1}H_2 - P_{e2}H_1}{H_1+H_2}$$

OR

$\delta_1-\delta_2$

$$\frac{1}{\pi f_0} H_{12} \frac{d^2\delta_{12}}{dt^2} = P_{m12} - P_{e12}$$

$$H_{12} = \frac{H_1 H_2}{H_1+H_2} \qquad P_{m12} = \frac{P_{m1}H_2 - P_{m2}H_1}{H_1+H_2} \qquad P_{e12} = \frac{P_{e1}H_2 - P_{e2}H_1}{H_1+H_2}$$



If $\delta_1=\delta_2 \Rightarrow$ Power flow = zero ; Power flow $\delta_1 \longrightarrow \delta_2$; $\delta_1>\delta_2$

Special Case of Interest [Generator, Motor set]

⇛ lossless System.

$$P_{m1} = -P_{m2} = P_m$$
$$P_{e1} = -P_{e2} = P_e$$

$$P_{m12} = P_m \quad , \quad P_{e12} = P_e$$

$$\frac{H_{12}}{\pi f_0}\cdot\frac{d^2\delta_{12}}{dt^2} = P_m - P_e$$

Multi - Machine Stability : more than 1 machine

Swings versus a infinite bus

⇒ Conditions:

① P_m is Constant

② Damping is negligible

③ Each machine is represented by E' & X_d'.

④ All loads are considered as Shunt Impedances, to ground with Values determined by conditions prevailling Immediately Prior to the transient condition.

→ Calculate the transient internal Voltage of each generator

$$E' = V_t + jIX_d'$$

→ Each load is converted into a constant admittance to ground, at its bus using

$$Y_L = \frac{P_L - jQ_L}{|V_L|^2}$$

→ The new dimensions of the modified Y bus matrix then corresponds to the number of generators.

→ Find P_{ei}

~~where~~

~~$P_{ei} = |E_i|^2 Y_{ii} +$~~

where

$$P_{ei} = |E_i'|^2 Y_{11} + |E_1'||E_2'| Y_{12} \cos(\delta_{12} - \theta_{12}) + |E_1'||E_3'| Y_{13} \cos(\delta_{13} - \theta_{13}) + \dots$$

$\delta_{12} = \delta_1 - \delta_2$

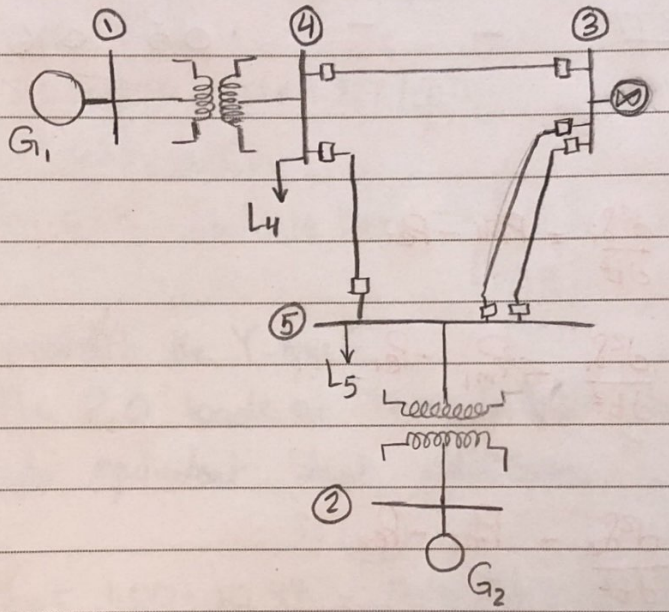
$\delta_{13} = \delta_1 - \delta_3$

$$\boxed{\frac{H_i}{\pi f_0}\frac{d^2\delta_i}{dt} = P_{mi} - P_{ei}}$$

Scanned by CamScanner

2013

Ex. 60 Hz, 230 kV, transmission system Shown in figure has two generators of ~~infinite~~ finite inertia & Infinite bus, all data are given In the tables, a 3ϕ occurs In line ④-⑤ near bus 4 find the Swing equation for each machine during the fault period, every reactance & H constant is expressed on a 100 MVA base.



Generator1: 400MVA, 20 kV, $X'_d = 0.067$ pu, H = 11.2 MJ/MVA

Generator 2: 250MVA, 18 kV, $X'_d = 0.1$ pu, H = 8 MJ/MVA

	Z: R	Z: X	Y: B
TR ①-④	-	0.022	-
TR ②-⑤	-	0.04	-
Line ③-④	0.007	0.04	0.082
Line ③-⑤ 182	0.008	0.047	0.098
Line ④-⑤	0.018	0.11	0.226

	Voltage	Generation P	Generation Q	Load P	Load Q
1	$1.03\angle 8.88^\circ$	3.5	0.712	–	–
2	$1.02\angle 6.38^\circ$	1.85	0.298	–	–
3	$1.0\angle 0$	–	–	–	–
4	$1.018\angle 4.68^\circ$	–	–	1.00	9.44
5	$1.011\angle 2.77^\circ$	–	–	0.5	0.16

$$\frac{H_i}{\pi f_0} \cdot \frac{d^2\delta_i}{dt^2} = P_{mi} - P_{ei}$$

$$\frac{H_1}{\pi f_0} \frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} \quad ??$$

$$\frac{H_2}{\pi f_0} \frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} \quad ??$$

$$P_{ei} = |E_1'|^2 G_{11} + |E_1'||E_2'|\,|Y_{12}| \cos(\delta_{12} - \theta) + |E_1'||E_3'|\,|Y_{13}| \cos(\delta_{13} - \theta) + \dots$$

→ find E_1' & E_2'

$$I_1 = \frac{(P_1 + jQ_1)^*}{V_1^*} = \frac{3.5 - j0.712}{1.03\angle -8.88^\circ} = 3.468\angle -2.619^\circ \text{ pu}$$

$$I_1 = \frac{V_1 - V_4}{TR①-④} = \frac{1.03\angle 8.88 - 1.018\angle 4.68}{j0.022} = 3.45\angle -2.3^\circ \text{ pu}$$

!!

$$I_2 = \frac{(P_2 + jQ_2)^*}{V_2^*} = \frac{1.85 - j0.298}{1.02\angle{-6.38}} = 1.837\angle{-2.711} \text{ pu}$$

$$E_1' = V_1 + jX_d' I_1$$
$$= 1.03\angle{8.88} + j[3.468\angle{-26.19^\circ}] * 0.067$$
$$= 1.1\angle{20.82^\circ} \text{ pu}$$

$$E_2' = V_2 + jX_d' I_2$$
$$= 1.02\angle{6.38} + j0.1 * 1.837\angle{-2.711}$$
$$= 1.065\angle{16.19^\circ} \text{ pu}$$

$E_3' = 1.0\angle{0^\circ}$ (Infinite bus) $\rightarrow \delta_{13} = \delta_1$ $\quad(\delta_3 = 0)$

$\delta_{23} = \delta_2$

→ Construct the Y-bus :-

⇒ The P, Q loads at buses ④ & ⑤ Should be converted Into equivalent Shunt admittances.

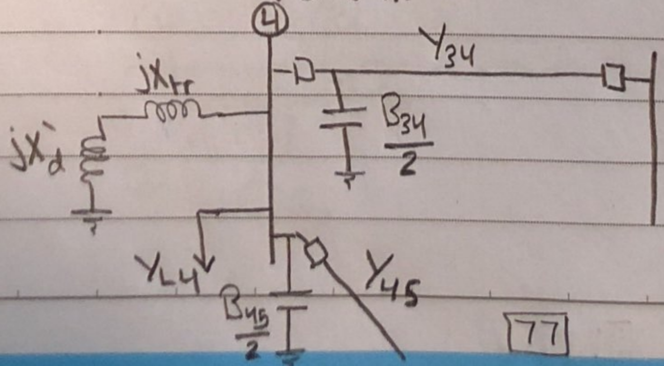
$$Y_{L4} = \frac{1.00 - j0.44}{(1.018)^2} = 0.9649 - j0.4246 \text{ pu}$$

$$\boxed{Y_L = \frac{P_L - jQ_L}{|V_L|^2}}$$

$$Y_{L5} = \frac{0.5 - j0.16}{(1.011)^2} = 0.4892 - j0.1565 \text{ pu}$$

→ How to construct the Y-bus elements

$$Y_{44} = \frac{1}{jX_d' + jX_{tr.}} + Y_{34} + Y_{45} + \frac{B_{34}}{2} + \frac{B_{45}}{2} + Y_{L4}$$



$$Y_{44} = \frac{1}{j0.067 + j0.022} + \frac{1}{0.007 + j0.04} + \frac{1}{0.018 + j0.11}$$

$$+ \frac{j0.226}{2} + \frac{j0.082}{2} + 0.9649 - j0.4246$$

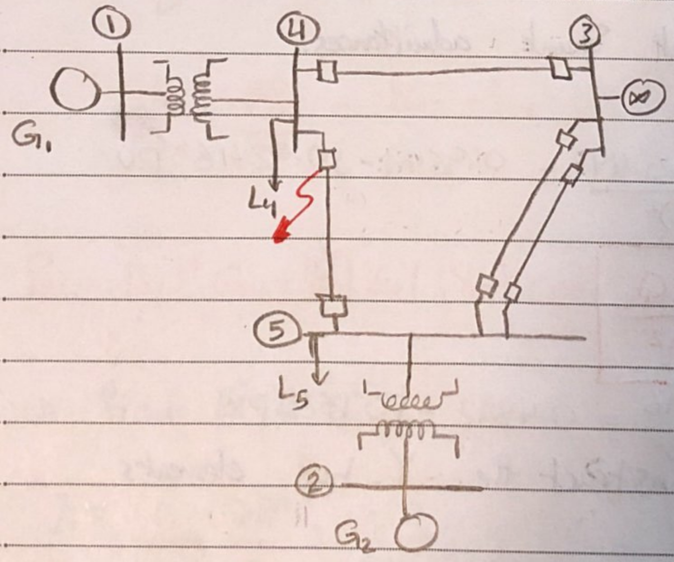
$$= 6.6587 - j44.6175 \text{ P.U}$$

Continuing the example — 22/7

$$Y_{bus} = \begin{bmatrix} -j11.236 & 0 & 0 & j11.236 & 0 \\ 0 & -j7.1429 & 0 & 0 & j7.1429 \\ 0 & 0 & 11.284 - j65.47 & -4.24 + j24.25 & -7.039 + j41.35 \\ j11.236 & 0 & -4.24 + j24.25 & 6.65 - j44.61 & -1.44 + j8.86 \\ 0 & j7.1429 & -7.039 + j41. & -1.44 + j8.85 & 8.977 - j57.29 \end{bmatrix}$$

Fault @ bus 4

Pivot cell



$$\frac{H_1}{\pi f_0} \cdot \frac{d^2 \delta_1}{dt^2} = P_{m_1} - P_{e_1}$$

$$\frac{H_2}{\pi f_0} \cdot \frac{d^2 \delta_2}{dt^2} = P_{m_2} - P_{e_2}$$

$$P_{e1} = |E_1|^2 Y_{11} + |E_1||E_2| Y_{12} \cos(\delta_{12} - \theta_{12}) + |E_1||E_3| Y_{13} \cos(\delta_{13} - \theta_{13})$$

* We need to connect Buses 1,2,3 directly.

$\Rightarrow$ Use Kron reduction:-

$$Y_{jk(new)} = Y_{jk(old)} - \frac{Y_{jp} Y_{pk}}{Y_{pp}} \longrightarrow \text{Pivot cell}$$

* $Y_{PP} = Y_{55}$ here

* for example $Y_{33(new)} = Y_{33(old)} - \dfrac{Y_{35} Y_{53}}{Y_{55}}$

$$= 11.284 - j65.47 - \frac{(-7.039 + j41.35)(-7.039 + j41.35)}{8.977 - j57.29}$$

$$= 5.798 - j35.63$$

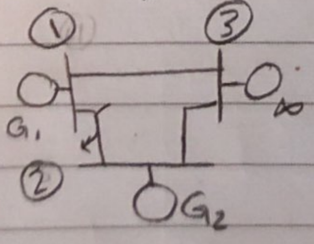
$$Y_{12(new)} = Y_{12(old)} - \frac{Y_{15} Y_{52}}{Y_{55}}$$

$$= 0 - \frac{0 * (j7.1429)}{8.977 - j57.29}$$

$$= 0$$

The new Y-matrix

$$Y_{[new]} = \begin{bmatrix} -j11.236 & 0 & 0 \\ 0 & 0.1362 - j6.27 & -0.0681 + j5.166 \\ 0 & -0.681 + j5.166 & 5.79 + j35.63 \end{bmatrix}$$



$\rightsquigarrow$ Our equivalent diagram.

Notes: G1 delivers 0 power to the ∞ bus.

G2 delivers 0 power to bus G1 Since Voltage is zero on faulted bus.

G2 delivers its power to the Infinite bus

$\Rightarrow P_{e1} = 0$

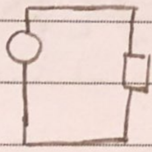
$$\frac{d^2\delta_1}{dt^2} = \frac{\pi f_0}{H_1}(P_{m1} - P_{e1})$$

accelaration of δ during fault

$$\frac{d^2\delta_1}{dt^2} = \frac{\pi(60)}{11.2}(3.5 - 0)$$

$$P_{e2} = |E_2'|^2 G_{22} + |E_2'||E_1'| \underbrace{Y_{21}}_{\text{zero}} \cos(\delta_{21} - \theta_{21}) + |E_2'||E_3'| Y_{23}\cos(\delta_{23} - \theta_{23})$$

$$= (1.065)^2 \times 0.1362 + 0 + (1.065)(1.0)[-0.681 + j5.166)\cos(\delta_2 - \theta_{23})$$



1.065

$Y = 0.1362 - j6.27$ S

$P = 1.065^2 * 0.1362$

$\theta_{23} = 90.755°$

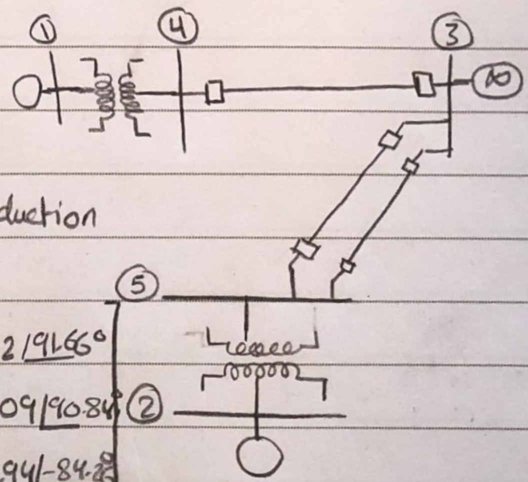
$$P_{e2} = 0.1545 + 5.5023\sin(\delta_2 - 0.755°)$$

$$\frac{d^2\delta_2}{dt^2} = \frac{180 f}{8} * \left[\overbrace{1.85 - 0.1545}^{1.6955} - 5.5023\sin(\delta_2 - 0.755)\right]$$

swing equation for generator 2 elec-degree / s^2

<u>Ex.</u> Determine the Swing equation for the post fault condition [If line ④ - ⑤ is disconnected.]

→ Rewrite the Y_{bus} matrix



$$Y_{45} = Y_{54} = 0$$

$[5\times5] \rightarrow [4\times4] \rightarrow [3\times3]$: Kron reduction

$$Y_{bus} = \begin{bmatrix} 0.5005 - j7.789 & 0 & 7.632\angle 91.66^\circ \\ 0 & 6.1189\angle -88.51^\circ & 6.09\angle 90.84^\circ \\ 7.632\angle 91.66 & 6.098\angle 90.84^\circ & 13.94\angle -84.86^\circ \end{bmatrix}$$

$$P_{e1} = |E_1'|^2 G_{11} + |E_1'||E_3'||Y_{13}| \cos(\delta_{13} - \theta_{13})$$

$$= (1.1)^2 * (0.5005) + (1.1)(1)(7.632) \cos(\delta_1 - 91.66^\circ)$$

$$= 0.6056 + 8.3955 \sin(\delta_1 - 1.66^\circ)$$

$$\frac{d^2\delta_1}{dt^2} = \frac{180f}{H}\left[3.5 - P_{e1}\right] \quad \text{elec-degree}/s^2$$

$$P_{e2} = |E_2'|^2 G_{22} + |E_2'||E_3'||Y_{23}| \cos(\delta_{23} - \theta_{23})$$

$$= (1.065)^2 * \underbrace{6.1189 \cos(-88.51)}_{\text{real part}} + (1.065)(1)(6.09) \cos(\delta_2 - 90.84^\circ)$$

$$\frac{d^2\delta_2}{dt^2} = \frac{\pi f_0}{H}\left(1.85 - P_{e2}\right) \quad \text{elec-degree}/s^2$$