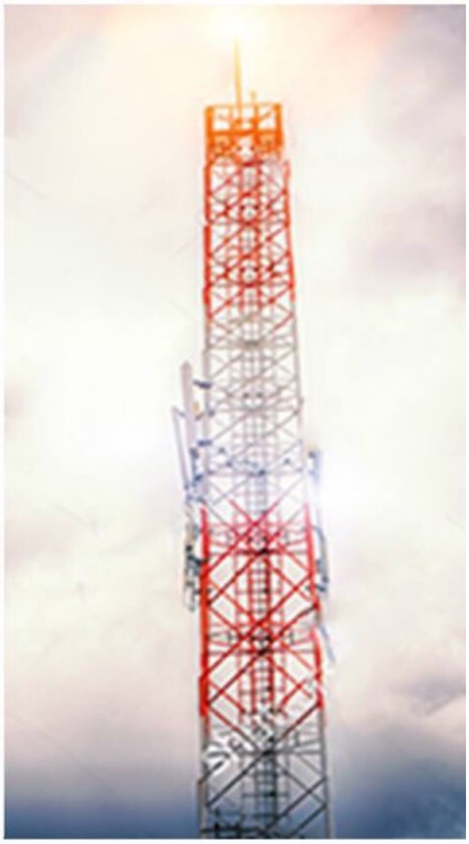
SIGNALS

Dr.Ghazi AlSukkar
By: Lama Al-louzi



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- Signals :

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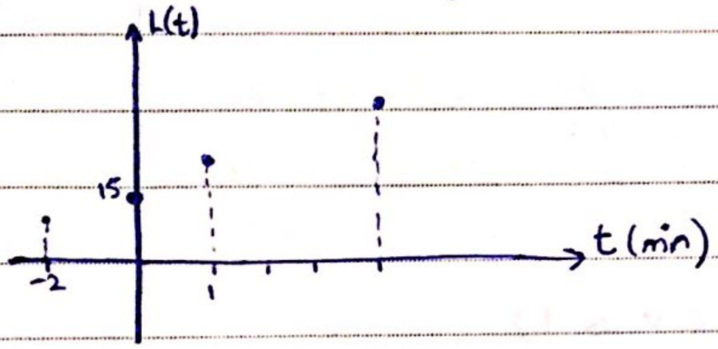
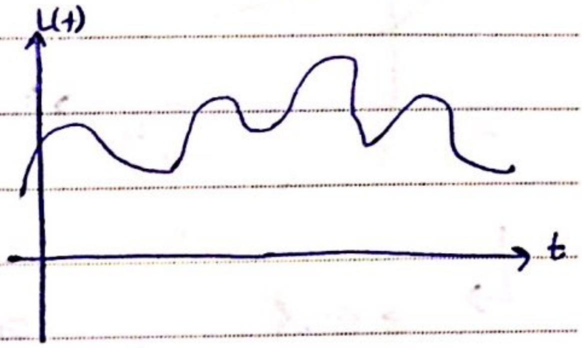
signal : Mathematical function $f(x)$

modeling : convert any function of time to physics signal



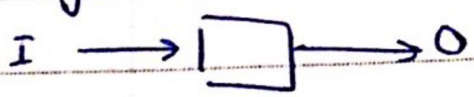


Discrete Time (DT) Continous Time (CT)

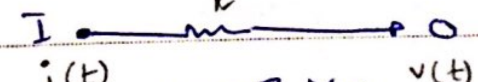
* signals → Traffic signal
  → speed
  → stockmarket

* $L(x, y, z, t)$ 4 independent variables
* $z = L(t)$
  → Dependent variable

* Systems :

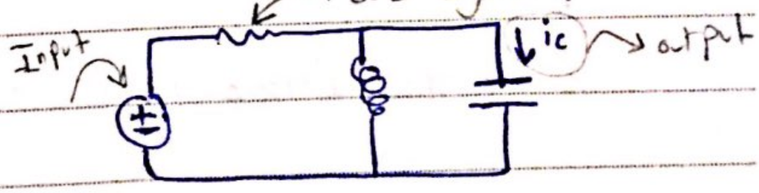


system (Resistor)

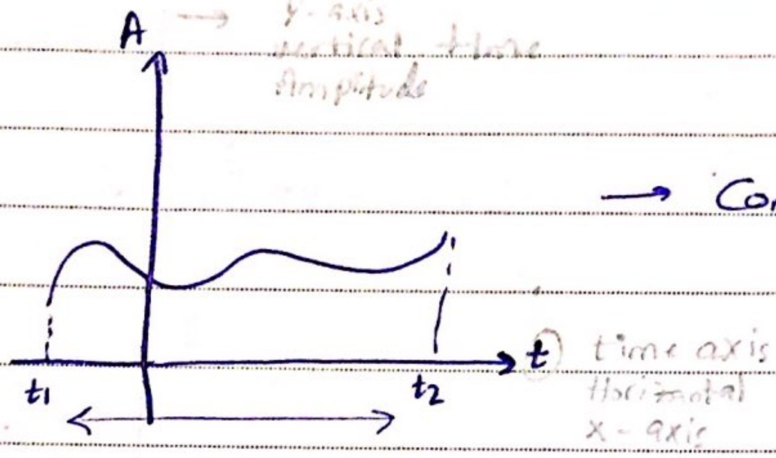


$v(t) = R\, i(t)$

sub-system.



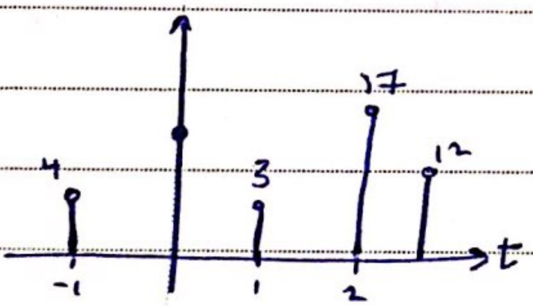
physical system
↓ modeling
mathematical equation



→ Continous Time signal

→ Discrete Time
→ continuous Time

↳ $t \in \bar{R}$, $t_1 \le t \le t_2$

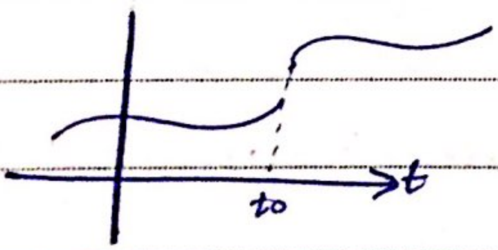


→ Discrete Time signal

$x(t) = \ln(t)$ → continuous Real value

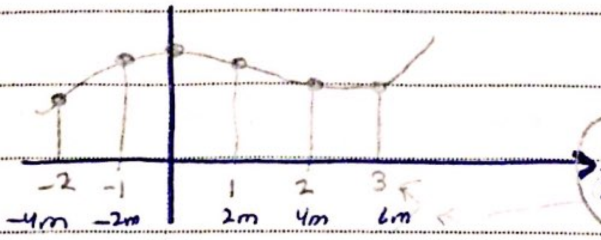
$x(t) = \underbrace{\ln(t)}_{\text{Real value}} + j\, e^{-t}$ → Imaginary

↳ complex operator $j = \sqrt{-1}$



→ continuous Time signal
→ discontinuous function

* continuous time signal could be discontinuous function

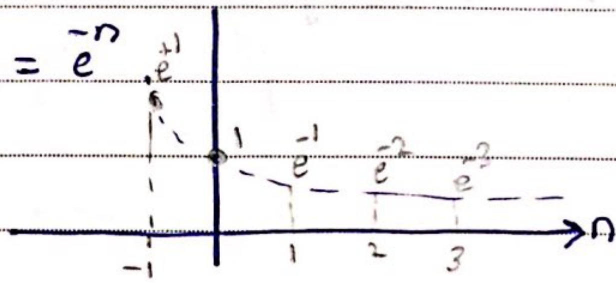


$\frac{t}{2m} \rightarrow n \rightarrow n \in \mathbb{Z}$ (Z: integers)

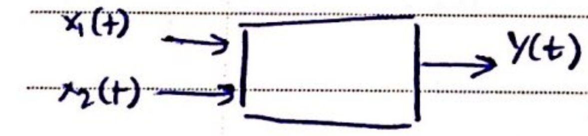
$x(t) \rightarrow$ continuous Time signal

$x[t] \rightarrow$ Discrete Time signal

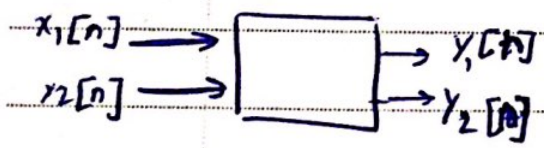
$\boxed{ex}$ $x[n] = e^{-n}$



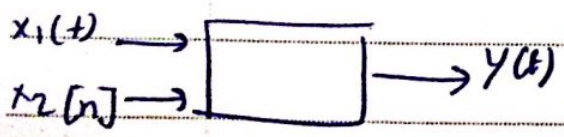
$x[n] = e^{-n} + j\cos(n^2)$, $n \in \mathbb{Z}$ $\rightarrow$ Discrete Time



$\rightarrow$ continuous Time system



$\rightarrow$ Discrete Time system



$\rightarrow$ Hybrid system

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Analog vs Digital :

- D and DT → finite set (has a start and end and countable)
- A and CT
- A and DT → infinite set (doesn't has start or end or has both but not countable)
- D and CT

$A \in Z$ $\{A : -2 < A \leq 3\}$

$A = \{-1, 0, 1, 2, 3\}$ → Digital

Countable → can be mapped with a subset of integers

$\{----, 2, 4, 6, 8, ------\}$

$---3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad --$

---

slide 19 :

- continuous time system $\xrightarrow{\text{output}}$ Differantial equation
- Discrete time system $\xrightarrow{\text{output}}$ Difference equation

$$y[n] = 2x[n-1] + 5y^2[n-3]$$

$x[t] \rightarrow \boxed{\text{Sampler}} \rightarrow x[n]$ (Hybrid system)

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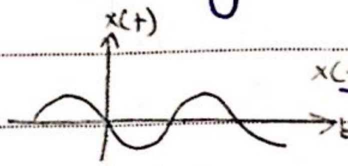
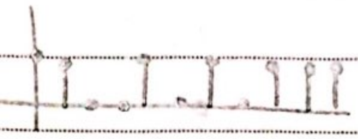
* Analog to Digital Converter (ADC)

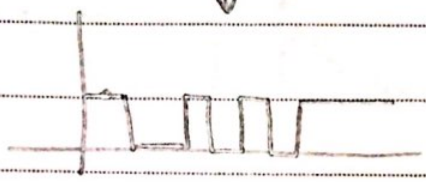


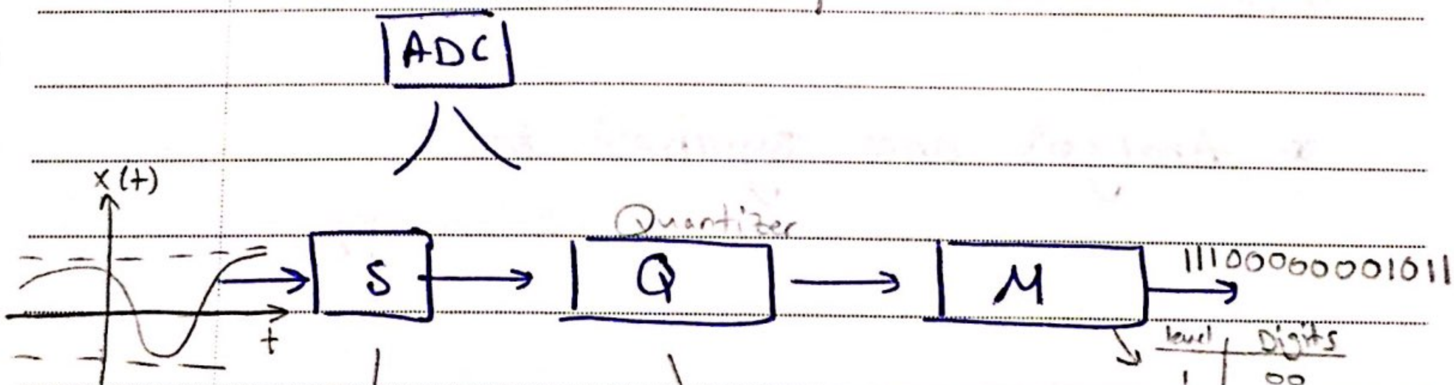
CT and A

x(t) → ADC → 1100101011 (practically)



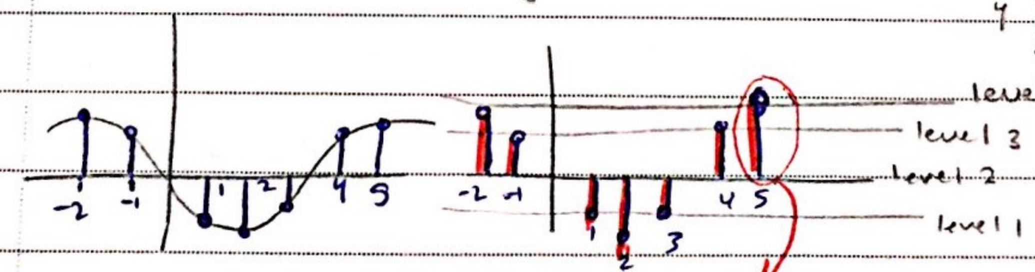
DT and D X





level	Digits
1	00
2	01
3	10
4	11

CT and A



DT and A

4 levels → 2-bit for each sample

8 levels → 3-bit for each sample

16 levels → 4-bit for each sample

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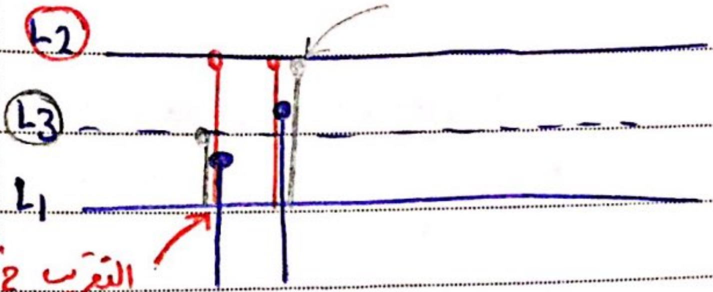
* Quantaization :

لو ضفت level جديدة اسمها L3



ممكن نقرب ال sample ب ٣ طرق

ceiling / floor / average

التقريب ع طول كأنه
عندي نقط 2levels

* Analysis and synthesis :-
→ (Design)

* Transformation :-

Time domain $\longleftrightarrow$ frequency domain

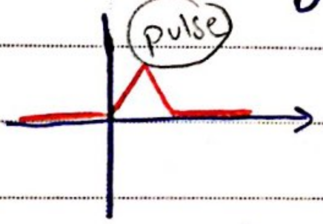
$x(t) \longleftrightarrow x(f),\ x(\omega),\ x(S),\ x(Z)$

** Lecture 2 :-

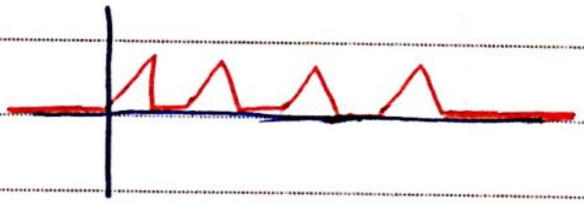
signal classification

- periodicity
  - Periodic
  - Non-periodic $\equiv$ aperiodic
- Deterministic / probabillistic (Random)
- energy or power or neither
- even or odd or neither

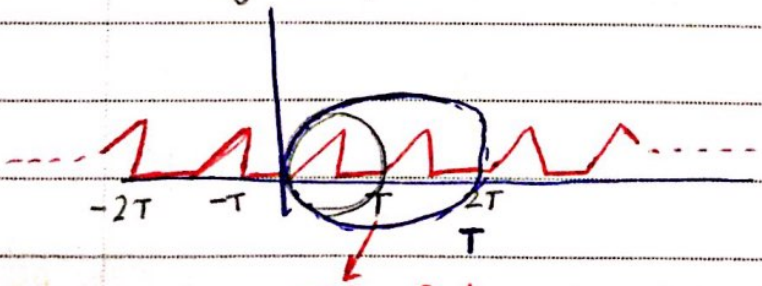
① Periodicity :



non-Periodic
(Time limited signal)



non - periodic



Periodic (Time unlimited Signal)

$T_0 \rightarrow$ fundemental period (أصغر T بتكرر فيها)

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$x(t) = x(t+T_0)$ → if true, the signal is periodic

$t + T_0$ $\qquad$ $t + T_0$

$x(t+T_0) = x(t+2T_0)$

$x(t) = x(t+2T_0)$

$x(t+T_0) = x(t+3T_0)$

$\vdots$

$x(t) = x(t+nT_0) \quad , \quad n \in \mathbb{Z}$

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$x(t) \stackrel{?}{=} x(t + nT_0)$, $n \in \mathbb{Z}$

T_0 : fundamental Period

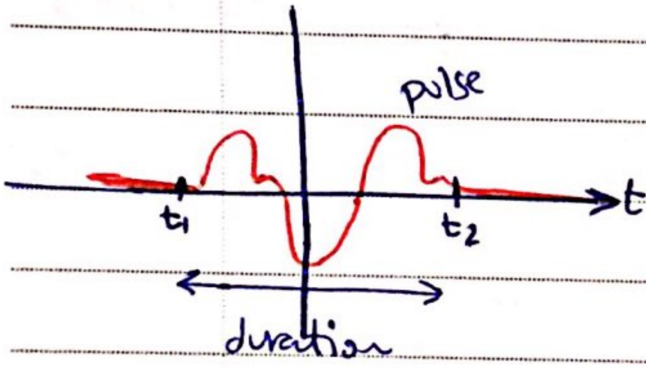
$\frac{1}{T_0}$: fundamental frequency (f_0)

$f_0 = 3\ Hz$

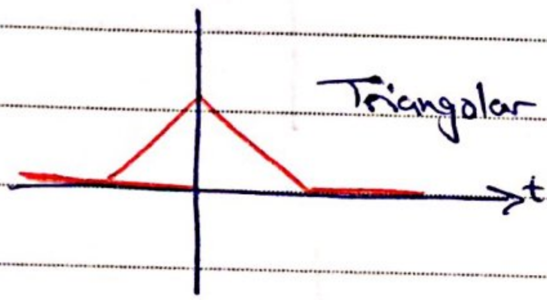
عدد التكرارات في الثانية

→ Sawtooth signal

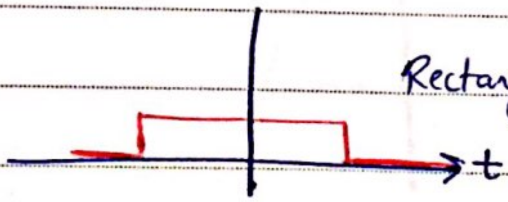
$w_0 = \frac{2\pi}{T_0} = 2f_0$ (fundamental angular frequency) radian



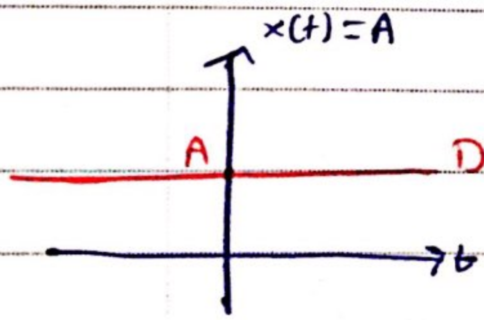
Time Limited



Triangolar Pulse



Rectangular Pulse



$\left.\begin{array}{l} f_0 = 0\ Hz \\ T_0 = \infty \end{array}\right]$ DC signal (Non-Periodic)

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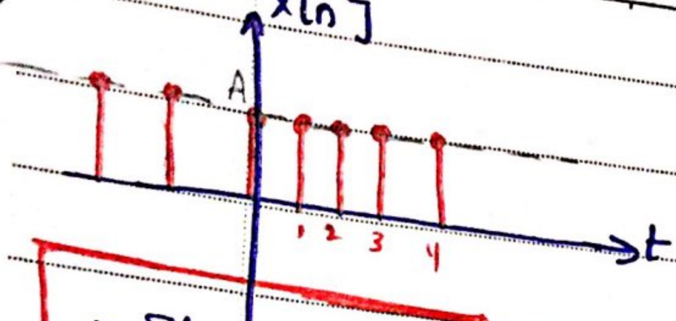


* Sinusoidal :-

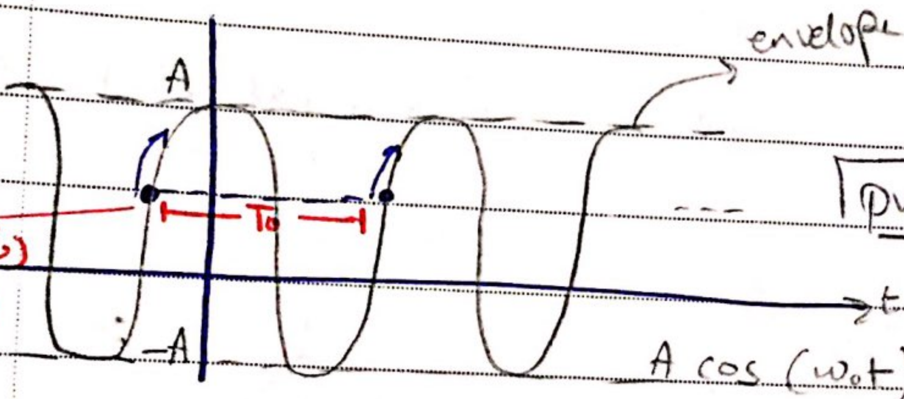
$$x(t) = \underline{A} \; \begin{matrix}\cos \\ \sin\end{matrix} \; (\overbrace{\omega_0 t + \theta}^{\text{argument}})$$

- magnitude
- Peak value
- Amplitude

θ → phase-shift

$\omega_0 = \dfrac{2\pi}{T_0}$

envelope

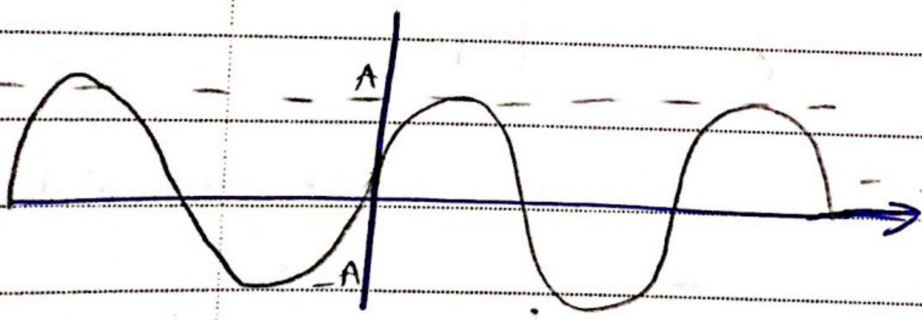


نقطتين متماثلتا
(متساويات ع ...
والميل (...) و ...
بنفس ...

$\boxed{\text{pure cos}}$

phase shift = 0

$A \cos(\omega_0 t)$



$\boxed{\text{pure sin}}$

$A \sin(\omega_0 t)$

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* Sinusoidal signals in Discrete time :-

$$x[n] = A \begin{matrix}\cos\\ \sin\end{matrix} (\Omega n + \phi) \quad , n \in \mathbb{Z}$$

Periodic non-Periodic

⊛ Non-periodic ± Non-periodic = Non periodic

$\ln(t) \pm e^{-t}$

⊛ Non-periodic ± DC = Non-periodic

⊛ periodic ± Dc = periodic

⊛ Periodic ± Periodic = { periodic / Non-periodic

T_1 ± T_1 = periodic T_1

T_1 T_2 , $T \neq T_2$

$\boxed{ex}$ $x(t) = \underset{T_1}{x_1(t)} \pm \underset{T_2}{x_2(t)} \pm \underset{T_3}{x_3(t)} \pm \underset{T_4}{x_4(t)}$

$\frac{T_1}{T_2}$ $\frac{T_1}{T_3}$ $\frac{T_1}{T_4}$

$\frac{int}{int} = \frac{T_2}{T_1} \leftarrow \frac{int}{int} = \frac{T_1}{T_2}$ إذا كان

$\frac{T_2}{\cancel{T_1}} \times \frac{\cancel{T_1}}{T_3} \rightarrow \frac{T_2}{T_3} = \frac{int}{int}$

$\frac{T_2}{T_1}$ $\frac{T_2}{T_3}$ $\cancel{\frac{T_2}{T_4}}$

$\cancel{\frac{T_3}{T_1}}$ $\frac{T_3}{T_2}$ $\cancel{\frac{T_4}{T_3}}$

# of ratio = # of signals − 1

$\cancel{\frac{T_4}{T_1}}$ $\cancel{\frac{T_4}{T_2}}$ $\cancel{\frac{T_4}{T_3}}$

FIVE APPLE

π, e^{1}, e^{-1} --- irrational numbers

→ $\frac{10}{5}$, $\frac{2}{3}$ متقاربة لـ ratio عدد صحيح

$$T_0 = T_1 * LCM(1, 3)$$

Example slide 98

is $x(t) = \cos\left(\frac{\pi t}{2}\right) + \cos\left(\frac{\pi t}{3}\right)$ Periodic?

find T_0 ?!

$$\cos\left(\frac{\pi t}{2}\right) + \cos\left(\frac{\pi t}{3}\right)$$

$\frac{\pi}{2} \rightarrow \omega_1$ $\qquad$ $\frac{\pi}{3} \rightarrow \omega_2$

$$\omega_1 = \frac{2\pi}{T_0} \qquad \frac{\pi}{3} = \frac{2\pi}{T_2}$$

$$\frac{\pi}{2} = \frac{2\pi}{T_1} \qquad \boxed{T_2 = 6}$$

$$\boxed{T_1 = 4}$$

$*$ $\frac{T_1}{T_2} = \frac{4}{6} = \frac{2}{3}$ ✓

$$T_0 = 4 \, LCM(3)$$
$$= 4 * 3 = 12$$

إذا كان رقم واحد → LCM
فهو نفسه LCM

or $T_0 = 6 \, LCM(2)$

$$6 * 2 = 12$$

No. 11. Feb. 2018

Is $x(t) = \cos(3.5t + \sin(2t) + \cos\left(\frac{7t}{6}\right)$ periodic?
find T_0 ?

عشان تكون ال period irrational

عشان أقدر أطلع ال ratios

$\omega_1 = 3.5 = \frac{2\pi}{T_1}$

$T_1 = \frac{2\pi}{3.5} = \frac{4\pi}{7}$

$\omega_2 = 2 = \frac{2\pi}{T_2} \quad T_2 = \pi$

$\omega_3 = \frac{7}{6} = \frac{2\pi}{T_3} \Rightarrow T_3 = \frac{12\pi}{7}$

① $\frac{T_1}{T_2} = \frac{4\pi}{7} \cdot \frac{1}{\pi} = \boxed{\frac{4}{7}}$

② $\frac{T_1}{T_3} = \frac{4\pi}{7} \cdot \frac{7}{12\pi} = \boxed{\frac{1}{3}}$

2 ratios (# of terms − 1)

$T_0 = \frac{4\pi}{7}\ \text{LCM}(7, 3)$ prime numbers so LCM = ◯ × ◯

لازم اللي كنت أحطه بالبسط لما أطلع ال ratio = T_1

$= \frac{4\pi}{\cancel{7}} \times \cancel{7} \times 3$

$\boxed{T_0 = 12\pi}$

* least common multiper :-

Find LCM for (95, 70, 222, 21) :

95	5 × 19
350	2 × 5 × 5 × 7
21	3 × 7
222	2 × 3 × 37

← بأكبر عدد تكرار لل 5 تكررت مرتين

$\text{LCM} = 5^2 \times 19^1 \times 2^1 \times 7^1 \times 3^1 \times 37$

No. 13. feb. 2018

- Non periodic $\times/$ Non periodic $=$ Non
- Periodic $\times/$ DC $=$ periodic
- periodic $\times/$ periodic $=$ Periodic

$$x(t) = e^{\sin(t)}$$

$$x(t) \stackrel{?}{=} x(t + nT_0) \quad , n \in \mathbb{Z}$$

$$e^{\sin(t)} \stackrel{?}{=} e^{\sin(t+nT_0)}$$

$$e^{x} = e^{y}$$
$$x = y \quad \#$$

$$\hookrightarrow \sin(t) = \sin(t + nT_0)$$

yes (sin is periodic signal)

$$\omega_0 = 1 = \frac{2\pi}{T_0} \rightarrow \boxed{T_0 = 2\pi}$$

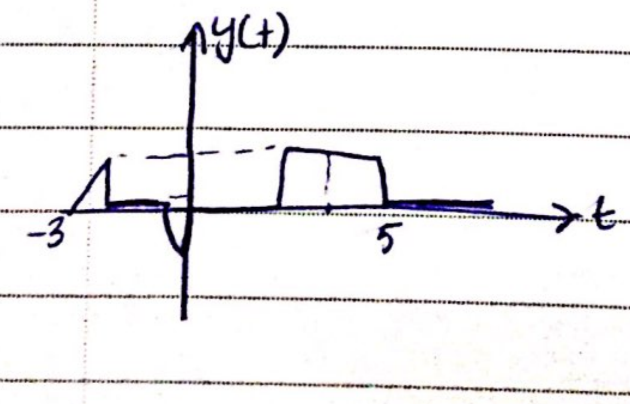
* Deterministic vs Random

يمكن أحدد قيمتها في لحظة من الزمن

ما بقدر أعرف قيمتها بالزبط في المستقبل
بس يمكن أرسم الجزء الي صار (مثل الزلازل)

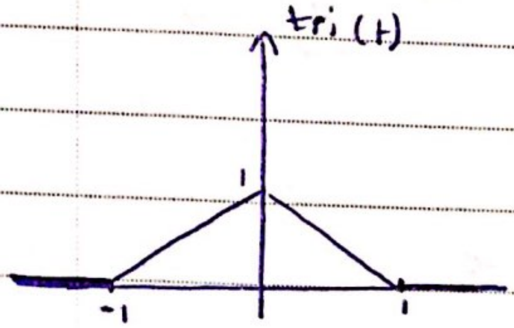
$$x(t) = 2\cos(5t + 30^\circ) \longrightarrow \text{Deterministic}$$

$$x(3) \quad , \quad x(-30)$$

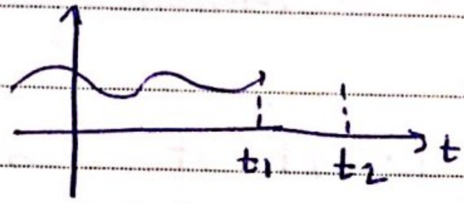


No. 13. feb. 2018

$$\Delta(t) = tri(t) = \begin{cases} 1 - |t| , & |t| \leq 1 \\ 0 , & \text{otherwise} \end{cases}$$



+ Random signals :-

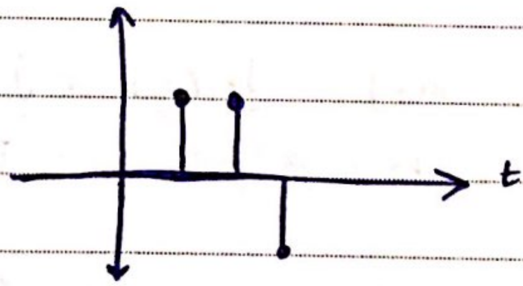


مستحيل أعرف قيمة الـ signal عند (t_2) بس ممكن أحط احتمالات معينة
أعرف الـ avg أو mean الـ! بس مش ممكن أحددها.

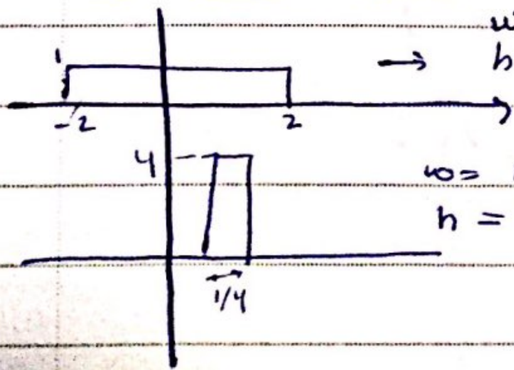
* Noise is Random signal

Slide 10

Head → 1
tail → -1
} Random variables



* Energy vs Power

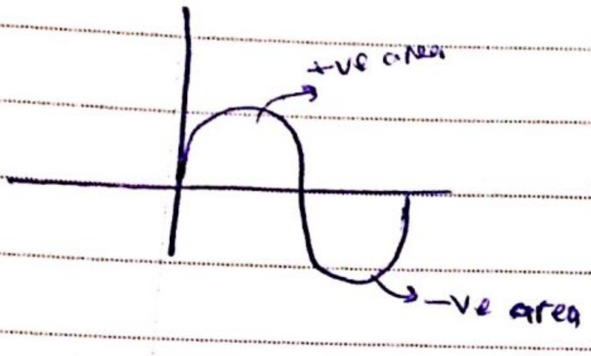


→ w = 4 → area = 4 × 1 = 4
h = 1

area = w . h

w = 1/4 → area = $\frac{1}{4} \times 4 = 1$
h = 4

No. 13 - feb. 2018



$$\rightarrow E_{Total} = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \text{ Joule}$$

CT $\longrightarrow$ DT

integration $\longrightarrow$ summation Σ

differential $\longrightarrow$ difference $-$

$$E_T = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \longrightarrow E_T = \sum_{n=-\infty}^{\infty} |x[n]|^2 \text{ Joule}$$

Real $E_T = \int_{-\infty} x(t)^2 \, dt$

$E_T =$

$$\underbrace{x(t) = \ln(t)}_{\text{Real Part}} + \underbrace{j e^{t^2}}_{\text{imaginary Part}}$$

Real value

$E_T = \int_{-\infty}^{\infty} x^2(t) \, dt$

$E_T = \Sigma \, x^2[n]$

$$x^*(t) = \ln(t) - j e^{t^2}$$

$$|x(t)|^2 = \sqrt{(\ln(t))^2 + (e^{(t^2)})^2}^{\,2}$$

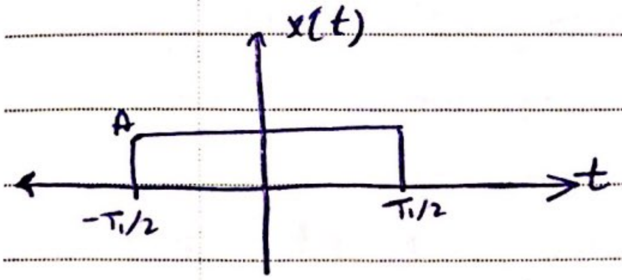
$$\downarrow$$

$$|x(t)|^2 = \ln(t)^2 + (e^{(t^2)})^2$$

No. 13. feb. 2018

Example slide 12 :

$$x(t) = \begin{cases} A , & |t| < T_1/2 \\ 0 , & \text{otherwise} \end{cases}$$

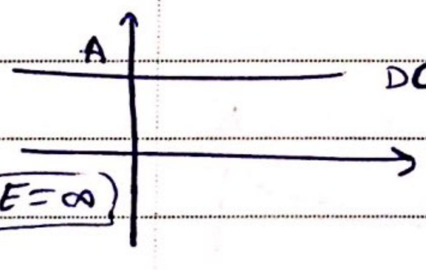


$$E_T = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-\infty}^{-T_1/2} \cancel{0}\, dt + \int_{-T_1/2}^{T_1/2} A^2\, dt + \int_{T_1/2}^{\infty} \cancel{0}\, dt$$

so

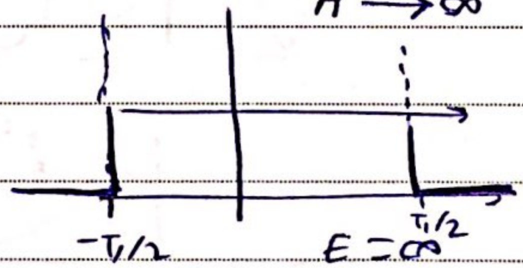
$$\# = \int_{-T_1/2}^{T_1/2} A^2\, dt = A^2 t \Big]_{-T_1/2}^{T_1/2} = A^2 T_1$$

$T_1 \longrightarrow \infty$



$\boxed{E = \infty}$

$A \longrightarrow \infty$



$E = \infty$

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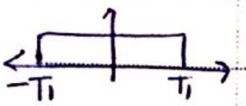
$\boxed{\text{* Bounded}}$

Yes → Energy or power

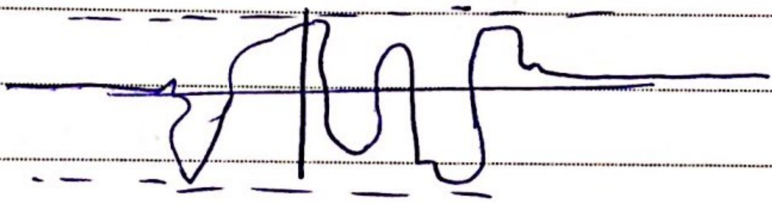
No → Neither energy nor power

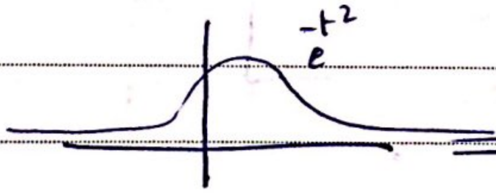
Energy or power:

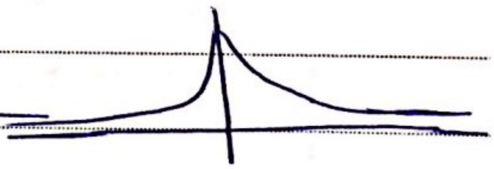
- Time limited → Energy
- Time unlimited:
  - Energy
  - power غالبا

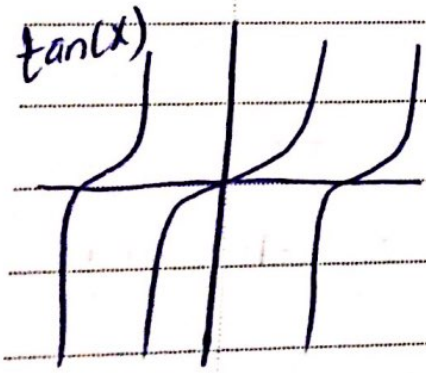


if $\lim_{|t| \to \infty} x(t) = 0 \longrightarrow$ Energy signal









→ Time unlimited

unbanded (Neither energy nor power)

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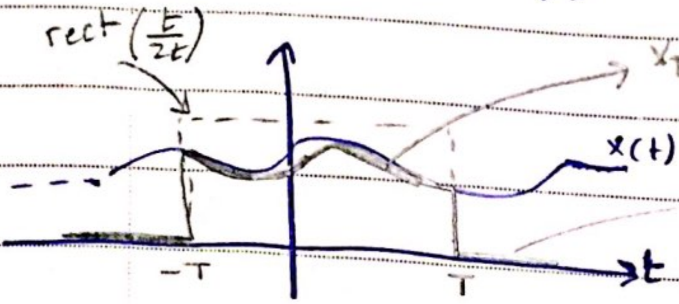
$$P = \frac{\Delta E}{\Delta t} \qquad P = \frac{de}{dt}$$



→ Truncated version of $x(t)$

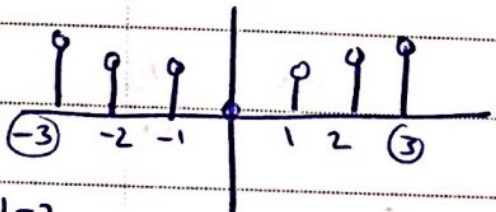
$$x_T(t) = \begin{cases} x(t), & -T<t<T \\ 0, & \text{otherwise} \end{cases}$$

$$E = \int_{-\infty}^{\infty} |x_T(t)|^2 \, dt = \int_{-T}^{T} |x(t)|^2 \, dt$$

$$P_T = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \rightarrow \text{watt} \quad (\text{average power})$$

total

$$P_T = \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{n=-N}^{N} |x[n]|^2$$



$N=3$

$$P_T = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N-1} |x[n]|^2$$

$\boxed{\text{rms}}$ → root, mean, square

Average (mean)

DC

$x(t)$

$$\underline{Av} = \frac{1}{t_2 - t_1} \int_{T_1}^{T_2} x(t) \, dt \rightarrow \text{energy}$$

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) \, dt \longrightarrow Av = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t) \, dt$$

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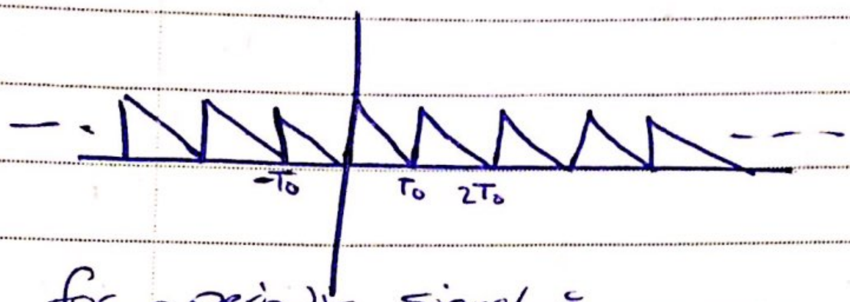
$$\sqrt{\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)^2 \, dt} \longrightarrow rms = \sqrt{P_T}$$



for a periodic signal :

$$P_T = \frac{1}{T_0}\int_{T_0} |x(t)|^2 \, dt$$

مش مهم شو حدود التكامل ← T_0
المهم يكون الفرق بينهم T_0

* Sinusoidal signal :-

$$x(t) = A\cos(\omega_0 t + \phi) \qquad \omega_0 = \frac{2\pi}{T_0}$$

$$P_T = \frac{1}{T_0}\int_{T_0} (A\cos(\omega t + \phi))^2 \, dt$$

ما في داعي | |
لأنه Real signal

$$P_T = \frac{1}{T_0}\int_{T_0} A^2 \cos^2(\omega t + \phi) \, dt$$

$$= \frac{A^2}{T_0}\int \left[\frac{1}{2} + \frac{1}{2}\cos(2\omega_0 t + 2\phi)\right] dt$$

$$\frac{A^2}{2T_0}\int 1 + \cos(2\omega_0 t + 2\phi) \, dt$$

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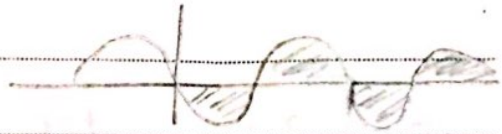
$$= \frac{A^2}{2T_0}\left[\int_{T_0} 1\, dt + \underbrace{\int_{T_0} \cos(2\omega_0 t + 2\phi)\, dt}_{\text{zero}}\right]$$

period = T = $\frac{T_0}{2}$

$$\frac{A^2}{2\cancel{T_0}}\left[\cancel{T_0} + 0\right]$$

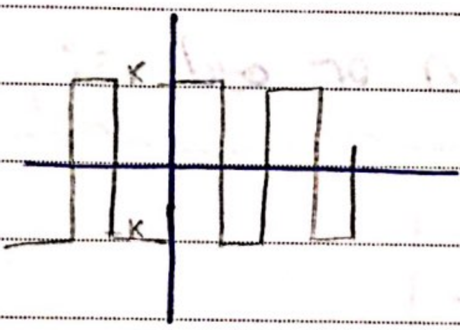
$$= \frac{A^2}{2}$$



$$rms = \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}}$$

integration $= 0$ [over one period or integer × period]

* TTL signal :-



$T_0 = 2$

$$p = \frac{1}{2}\int_2 x^2(t)\, dt =$$

$$\frac{1}{2}\int_{-1}^{1} x^2(t)\, dt$$

$$= \frac{1}{2}\left[\int_{-1}^{0} (-k)^2\, dt + \int_0^1 (k)^2\, dt\right]$$

$$= \frac{1}{2}\left[k^2 t\Big]_{-1}^{0} + k^2 t\Big]_0^1\right] = k^2$$

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$$\frac{E}{P} \propto M^2$$

Energy or avg power directly proportional to the (magnitude)^2 of the signal

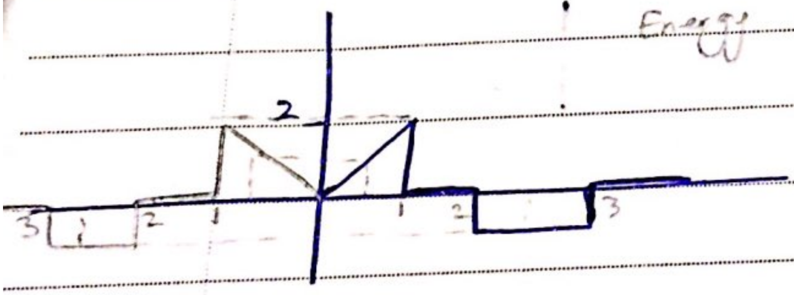
$E = c M^2$

$P = A M^2$

constants

* [power for the energy signal $= 0$
  Energy for the power signal $= \infty$

* even or odd signals :

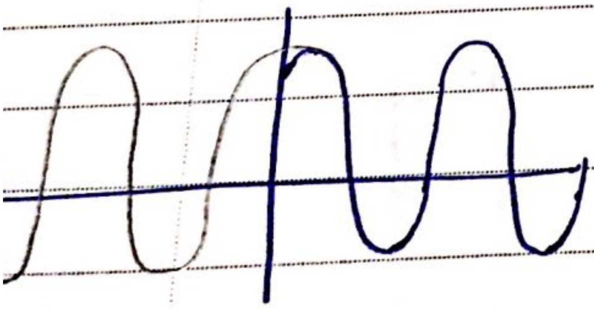


Energy signal / $\boxed{\text{even signal}}$

$x(1) = x(-1)$

$x(\frac{1}{2}) = x(-\frac{1}{2})$

$x(t) = x(-t)$, $\forall t$

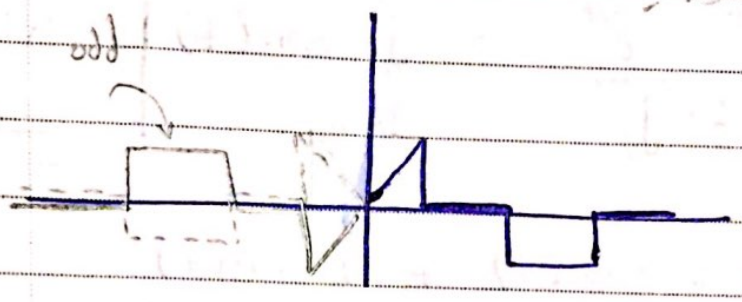


→ pure cos → even signal

odd signal $\rightarrow$ $x(t) = -x(-t)\ ,\ \forall t$

(symmetric around 45°)

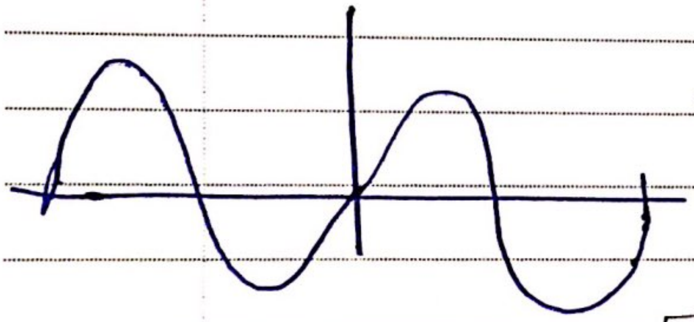
x بتعامل even ويعمل Reflection



Average value of any odd signal $= 0$

$x(t) = -x(-t)\ ,\ \forall t$

$-x(t) = x(-t)$



$\rightarrow \sin(\theta) = -\sin(-\theta)$

$\sin(\omega_0 t) = -\sin(-\omega_0 t)$

even component

odd component

$x(t) = x_e(t) + x_o(t)$ --- ①

$x(-t) = x_e(-t) + x_o(-t)$

$x_e(t) = x_e(-t)$

$-x_o(t) = x_o(-t)$

①+② $+$

$x(-t) = x_e(t) - x_o(t)$ --- ②

$x(t) + x(-t) = 2(x_e(t)) \rightarrow x_e(t) = \dfrac{x(t) + x(-t)}{2}$

$x(t) - x(-t) = 2x_o(t) \longrightarrow \dfrac{x(t) + -x(-t)}{2}$

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$$x(t) = e^{jt}$$

$$x(-t) = e^{-jt}$$

$$x_e(t) = \frac{e^{jt} + e^{-jt}}{2} = \cos(t)$$

$$x_o(t) = j \; \frac{e^{jt} - e^{-jt}}{2 \cdot j} = j \sin(t)$$

$$\cosh(t) = \frac{e^{t} + e^{-t}}{2}$$

$$\sinh(t) = \frac{e^{t} - e^{-t}}{2}$$

So, $x(t) = \cos(t) + j \sin(t)$

even + odd → Neither odd Nor even.

* Average value for even signal $\neq 0$