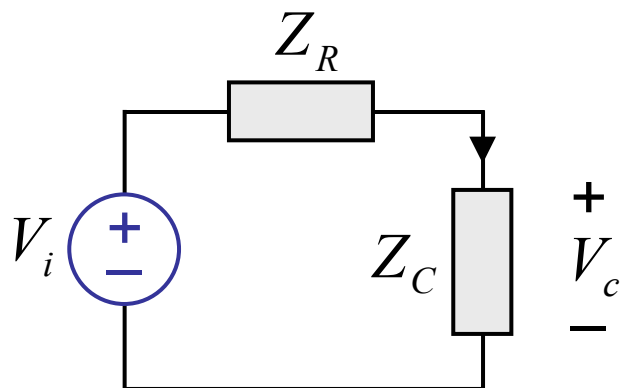
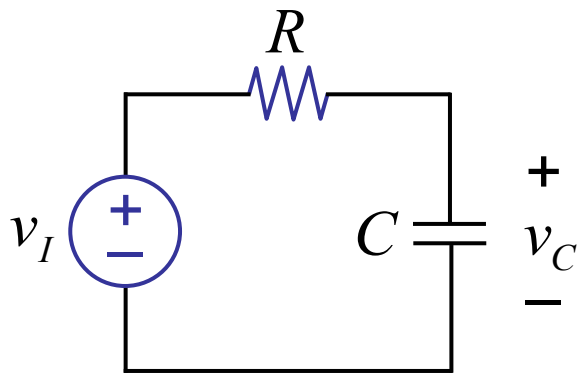


6.002

**CIRCUITS AND
ELECTRONICS**

Filters

Review

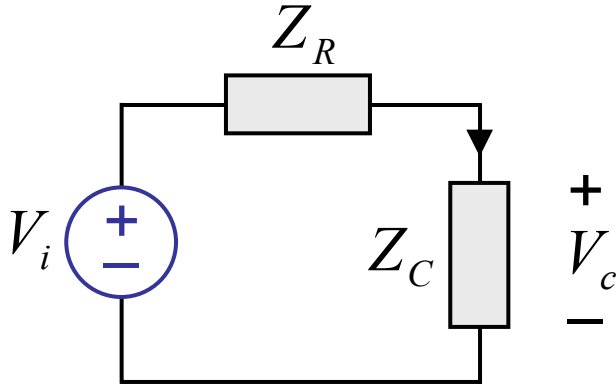


$$V_c = \frac{Z_C}{Z_C + Z_R} \cdot V_i$$

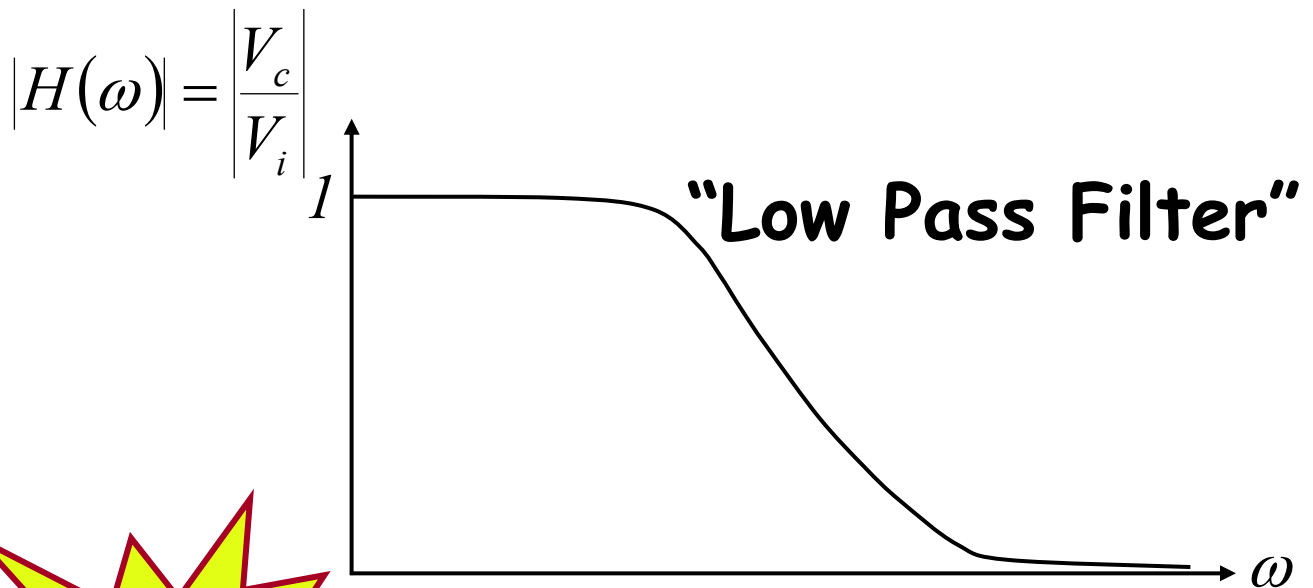
$$\frac{V_c}{V_i} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega RC}$$

Reading: Section 14.5, 14.6, 15.3 from A & L.

A Filter

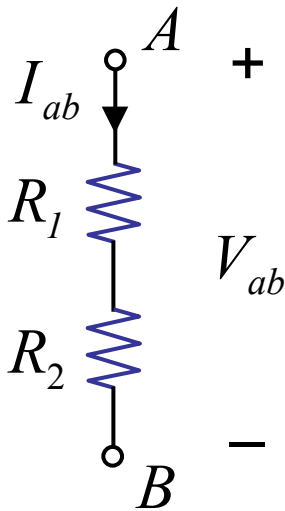


$$V_c = \frac{Z_C}{Z_C + Z_R} \cdot V_i = \frac{1}{1 + j\omega RC}$$

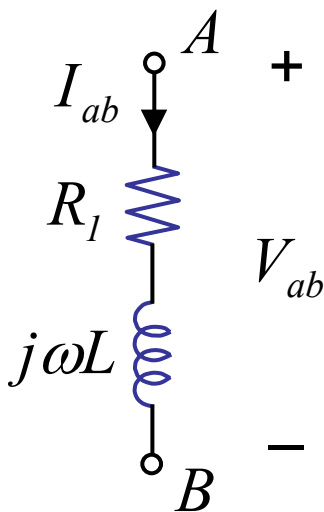


Quick Review of Impedances-

Just as



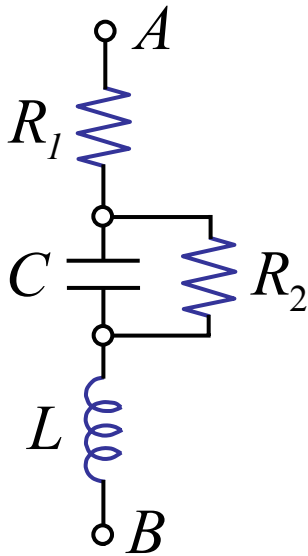
$$R_{AB} = \frac{V_{ab}}{I_{ab}} = R_1 + R_2$$



$$Z_{AB} = \frac{V_{ab}}{I_{ab}} = R_1 + j\omega L$$

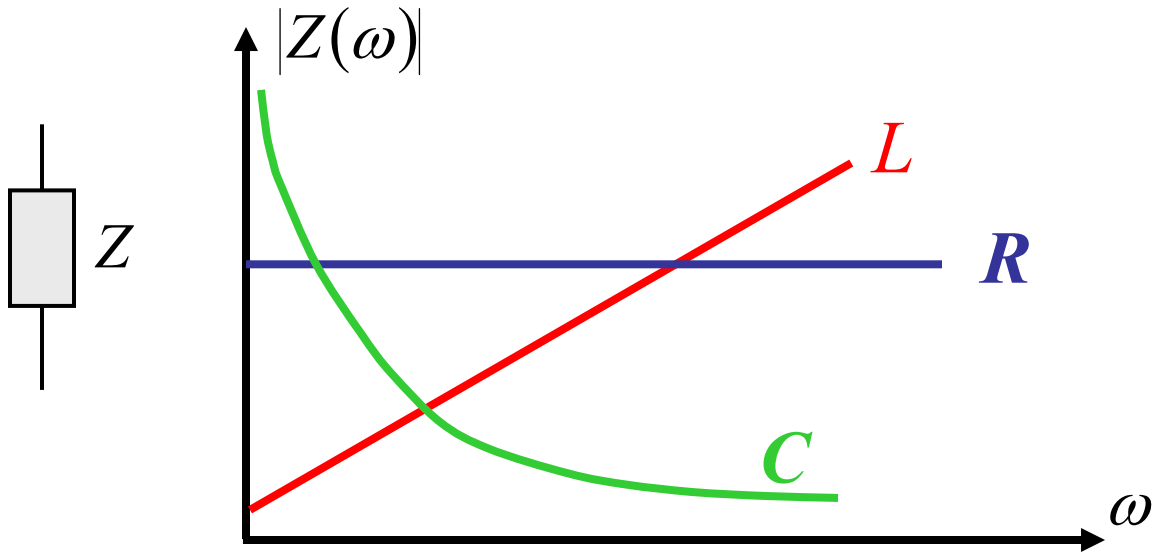
Quick Review of Impedances

Similarly

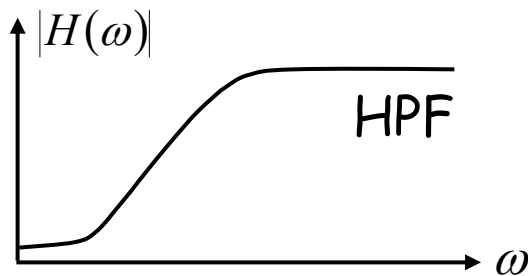
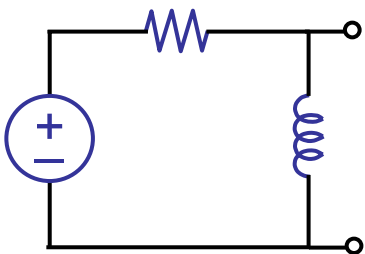
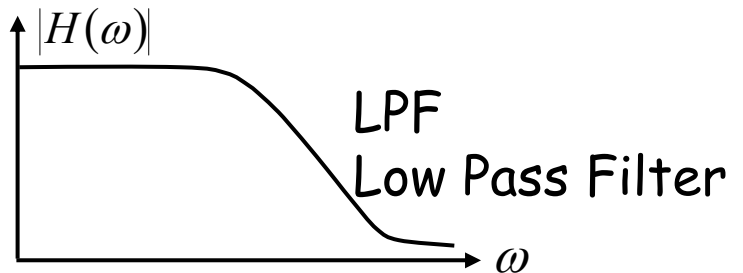
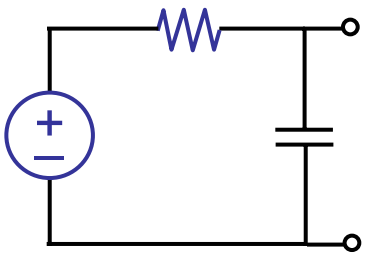
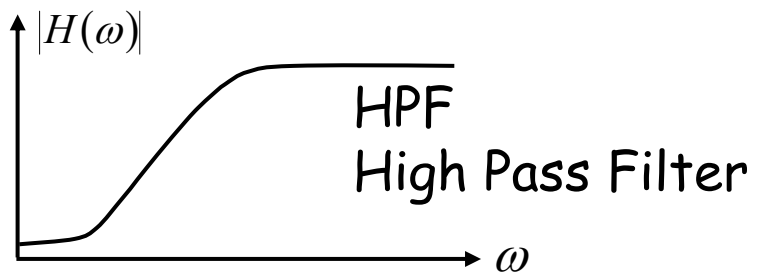
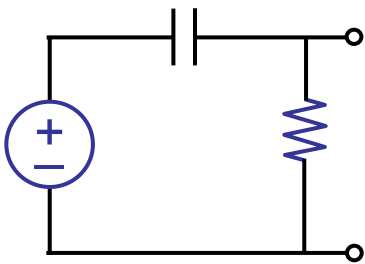
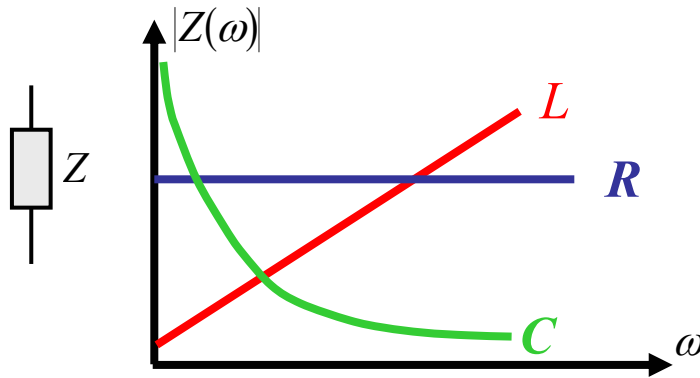


$$\begin{aligned} Z_{AB} &= R_1 + Z_C \parallel R_2 + Z_L \\ &= R_1 + \frac{Z_C R_2}{Z_C + R_2} + Z_L \\ &= R_1 + \frac{R_2}{1 + j\omega C R_2} + j\omega L \end{aligned}$$

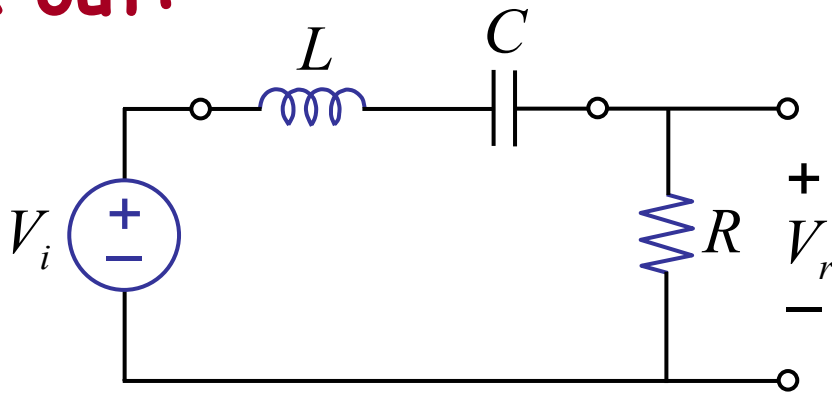
We can build other filters by combining impedances



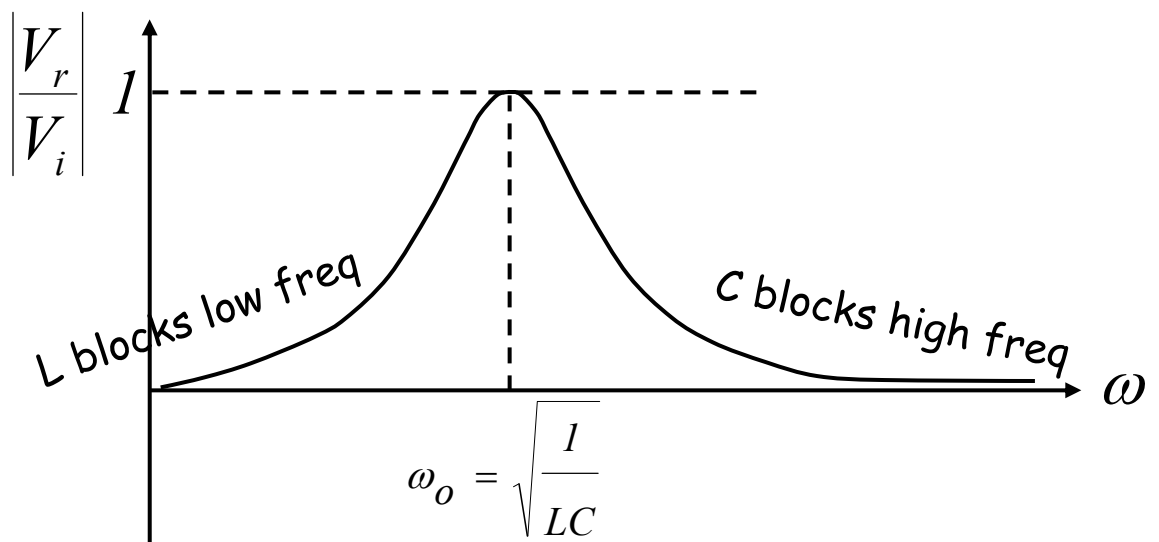
We can build other filters by combining impedances



Check out:



Intuitively:

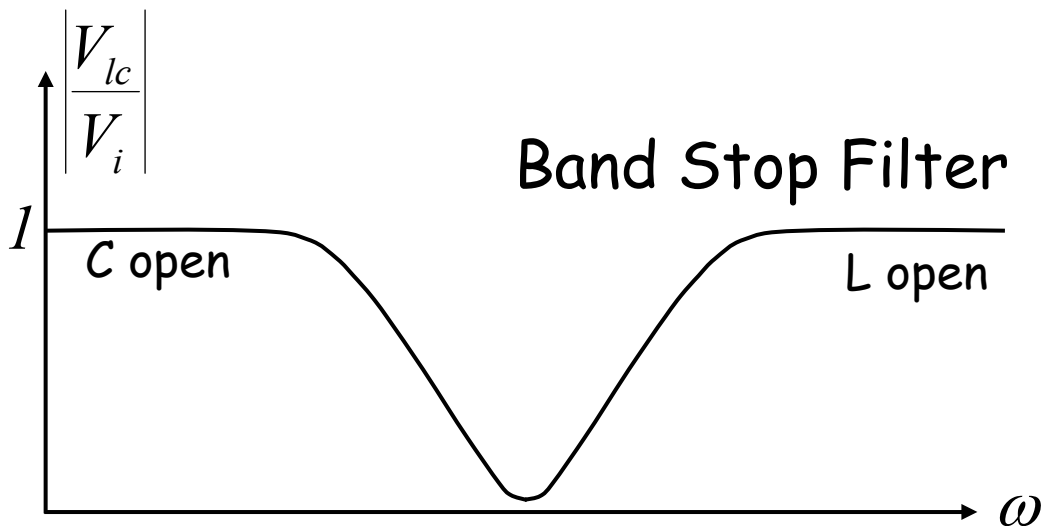
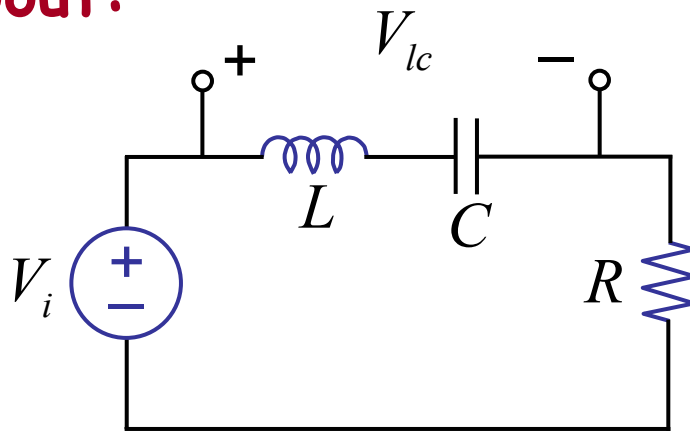


$$\begin{aligned}\frac{V_r}{V_i} &= \frac{R}{j\omega L + \frac{1}{j\omega C} + R} \\ &= \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC}\end{aligned}$$

$$\left| \frac{V_r}{V_i} \right| = \frac{\omega RC}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}}$$

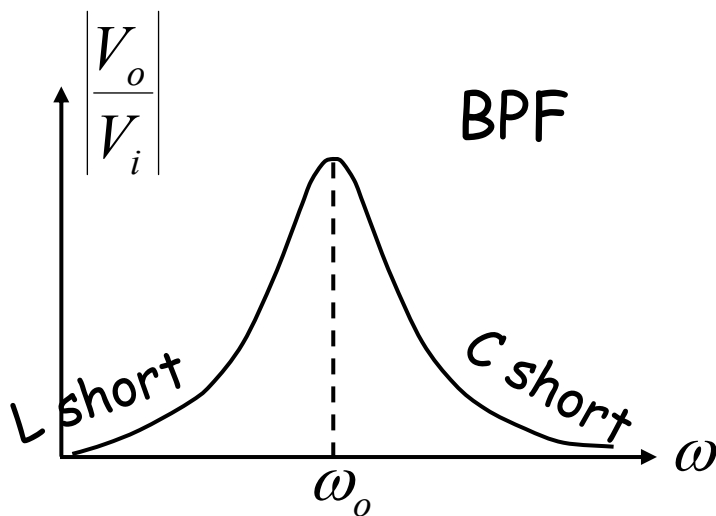
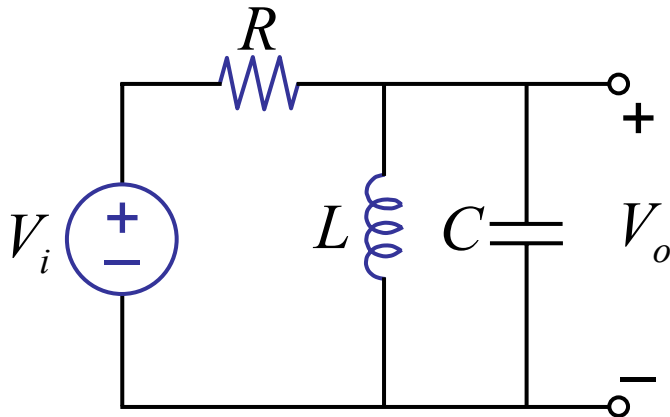
★ At resonance,
 $\omega = \omega_o$
and
 $Z_L + Z_C = 0$,
so V_i sees
only R !
More later...

What about:



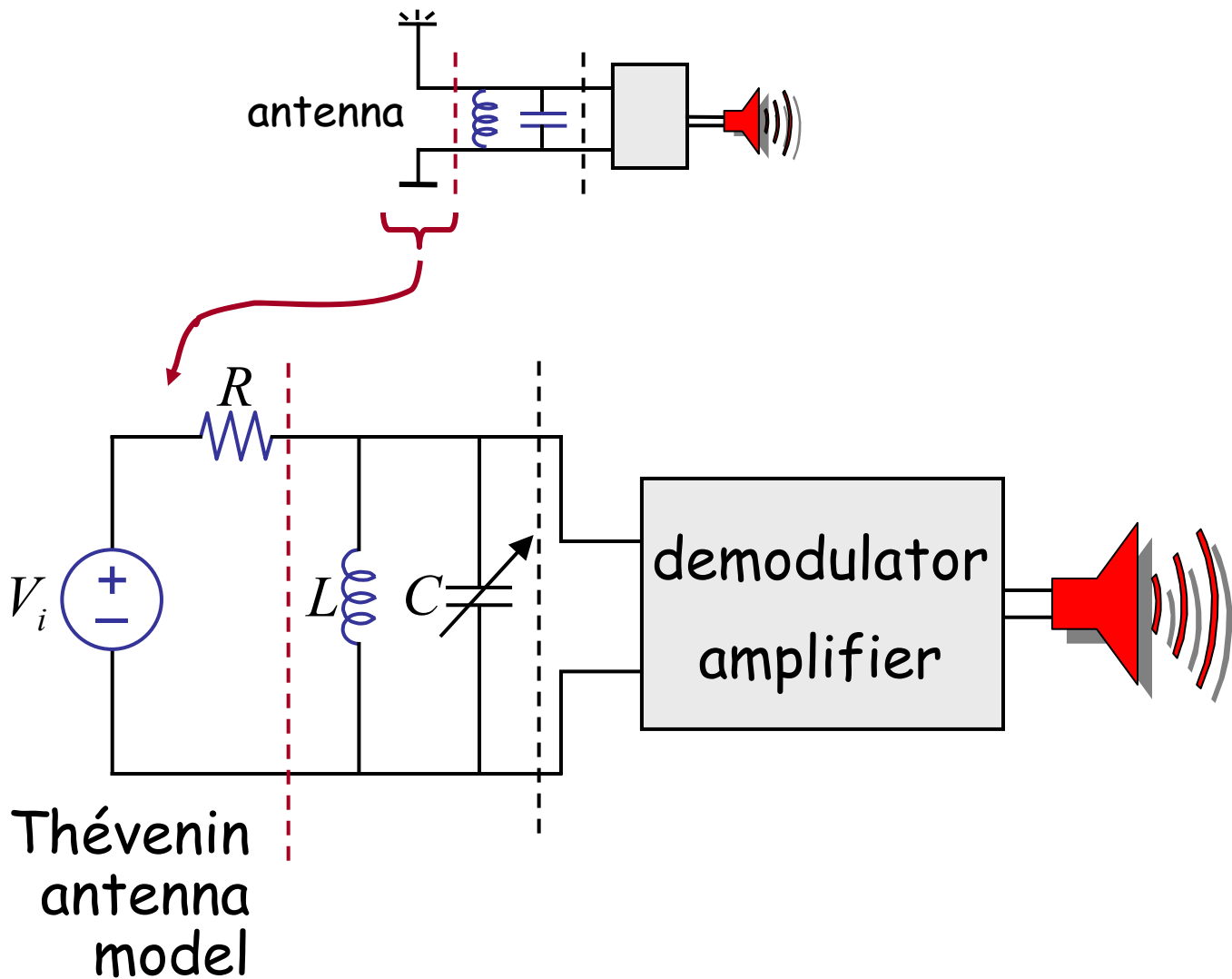
Check out V_l and V_c in the lab.

Another example:



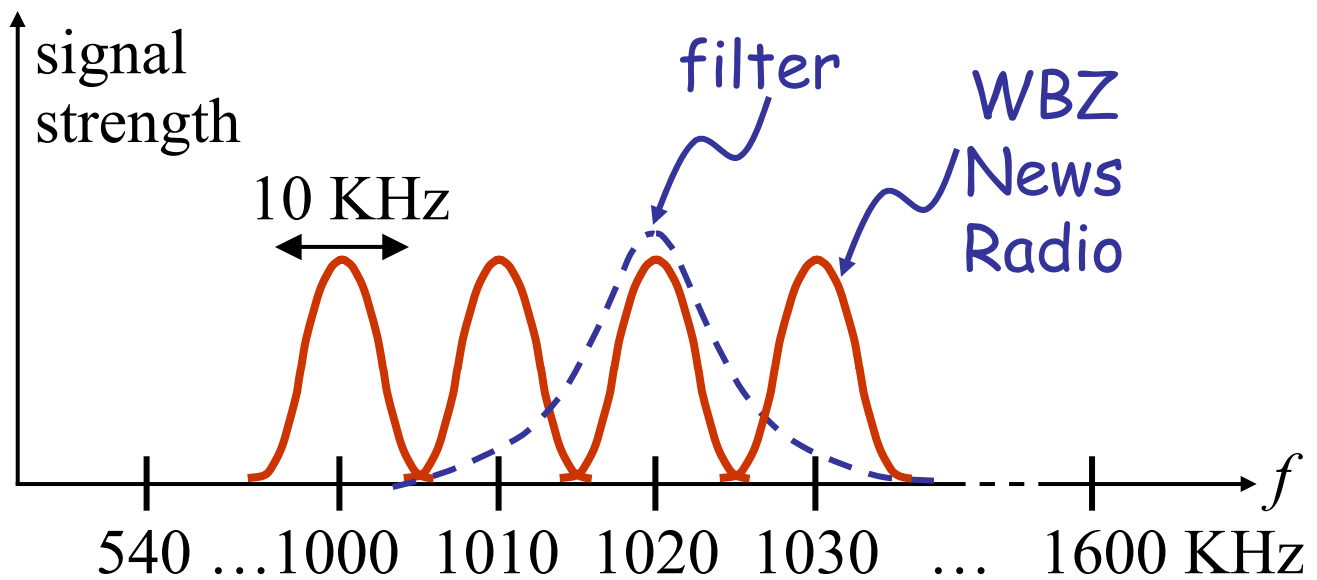
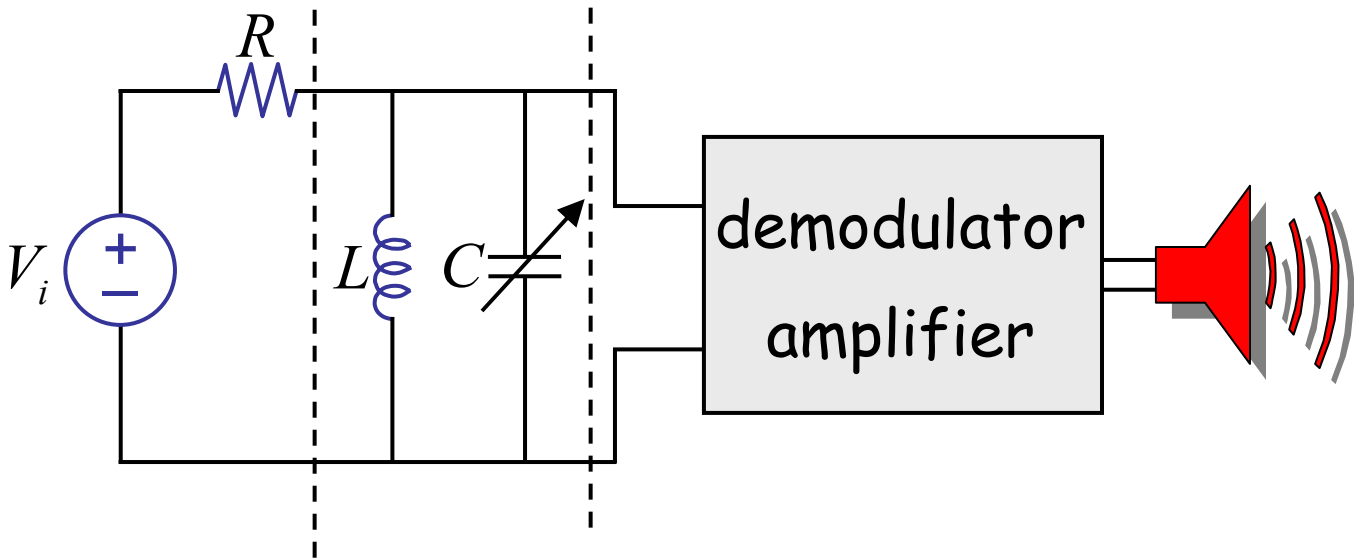
Application: see AM radio coming up shortly

AM Radio Receiver



crystal radio demo

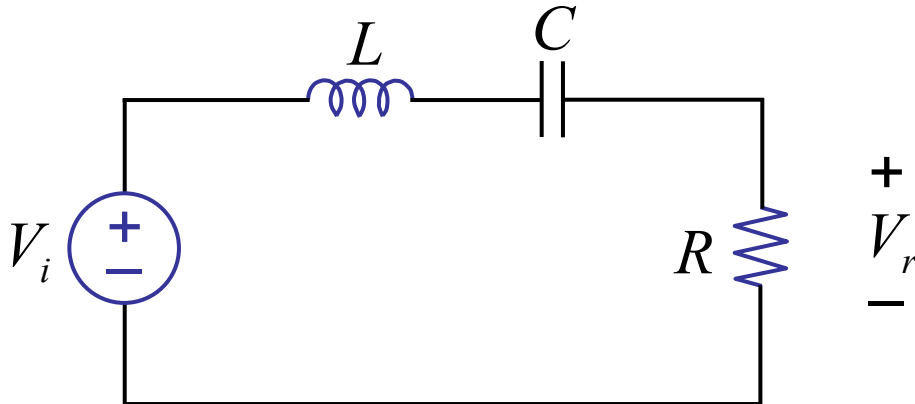
AM Receiver



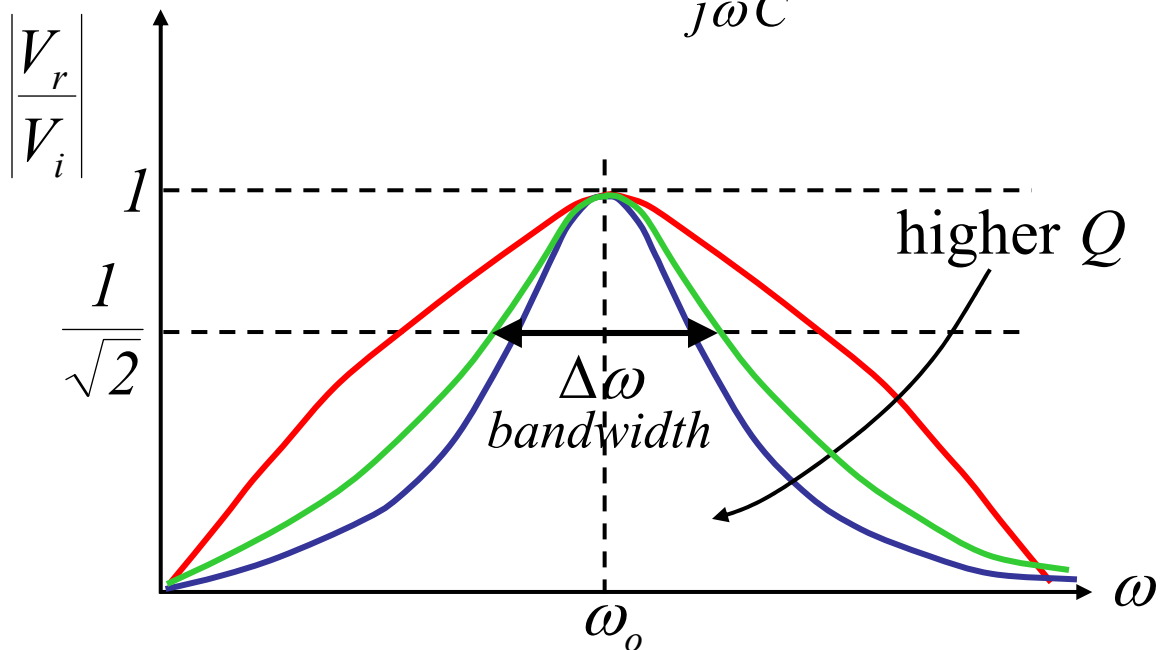
"Selectivity" important —
relates to a parameter " Q " for the filter. Next...

Selectivity:

Look at series RLC in more detail



Recall,
$$\frac{V_r}{V_i} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}}$$



Define $Q = \frac{\omega_0}{\Delta\omega}$ *quality factor*

high $Q \Rightarrow$ more selective

Quality Factor Q

$$Q = \frac{\omega_o}{\Delta\omega}$$

ω_o :

$$\frac{V_r}{V_i} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{1 + j\left(\omega \frac{L}{R} - \frac{1}{\omega CR}\right)}$$

at $\omega_o = 0$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$\Delta\omega$?

Quality Factor Q

$$Q = \frac{\omega_o}{\Delta\omega}$$

$\Delta\omega$:

Note that abs magnitude is $\frac{1}{\sqrt{2}}$

when
$$\frac{V_r}{V_i} = \frac{1}{1 + j\left(\omega \frac{L}{R} - \frac{1}{\omega CR}\right)} = \frac{1}{1 \pm j1}$$

i.e. when
$$\frac{\omega L}{R} - \frac{1}{\omega CR} = \pm 1$$

$$\omega^2 \mp \frac{\omega R}{L} - \frac{1}{LC} = 0$$

Looking at the roots of both equations,

$$\omega_1 = \frac{R}{2L} + \frac{1}{2} \sqrt{\frac{R^2}{L^2} + \frac{4}{LC}} \quad \omega_2 = -\frac{R}{2L} + \frac{1}{2} \sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}$$

$$\Delta\omega = \omega_1 - \omega_2 = \frac{R}{L}$$

Quality Factor Q

$$Q = \frac{\omega_o}{\Delta\omega}$$

$$Q = \frac{\omega_o}{R} = \frac{\omega_o L}{R}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

The lower the R (for series R),
the sharper the peak

Quality Factor Q

Another way of looking at Q :

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy lost per cycle}}$$

$$= 2\pi \frac{\frac{1}{2} L |I_r|^2}{\frac{1}{2} |I_r|^2 R \frac{2\pi}{\omega_0}}$$

$$Q = \frac{\omega_0 L}{R}$$