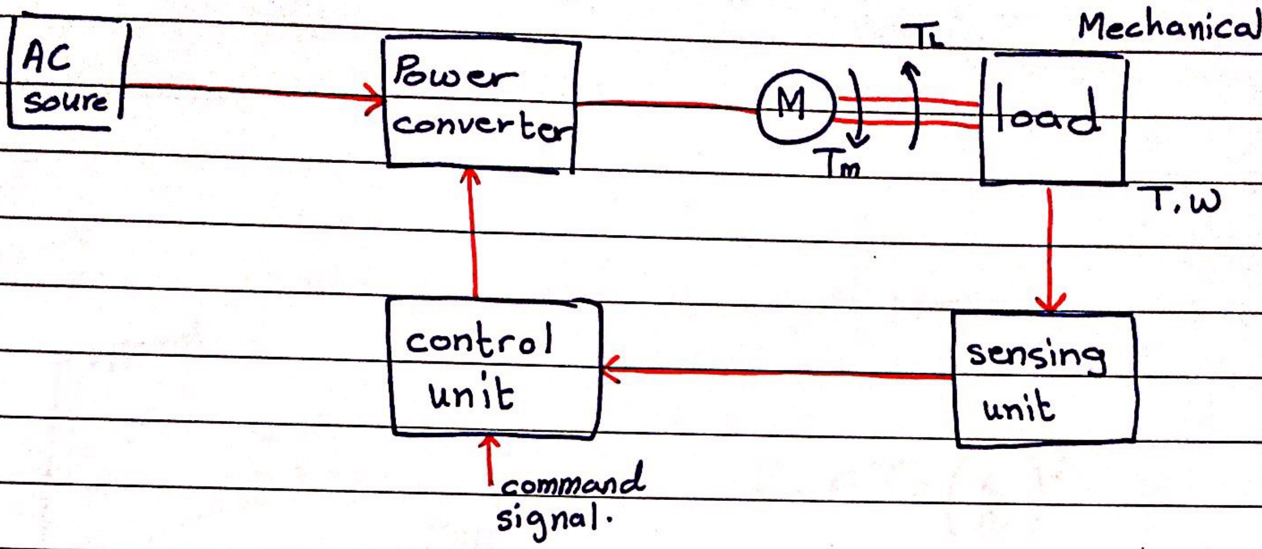
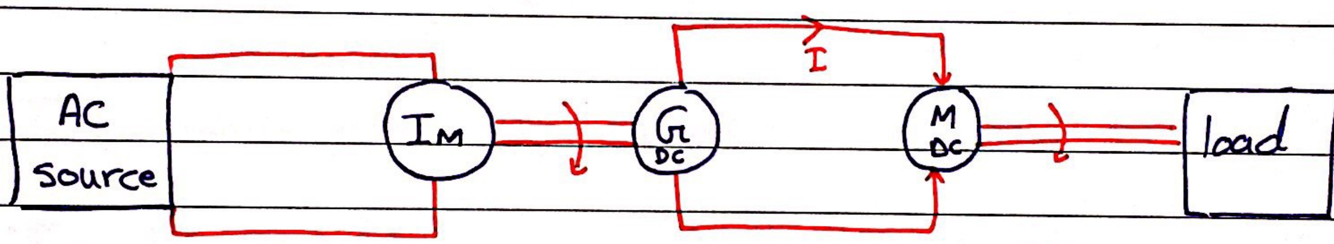
25/9/2016

Electrical drive: control Motors considering mechanical load performance.



old Electrical drive:



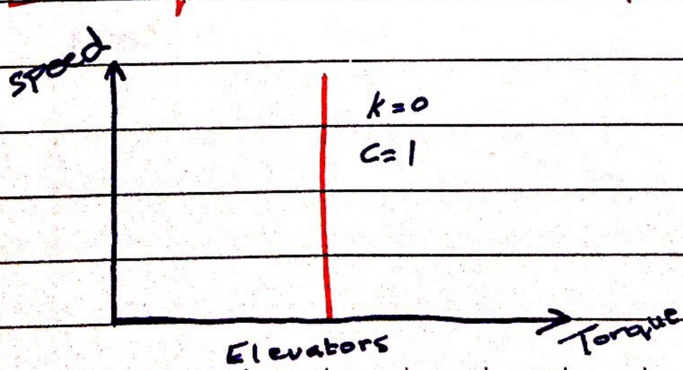
Torque - speed c/s Mechanical loads:

$$T = f(\text{speed})$$

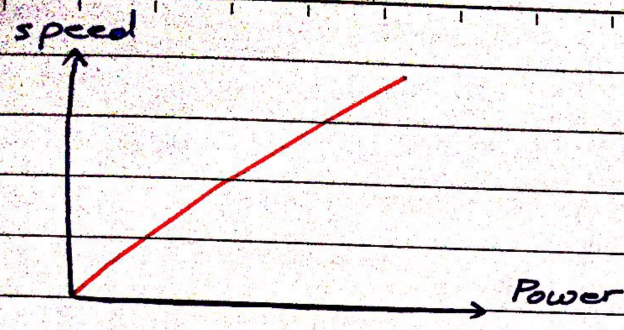
$$T = C\, T_r \left(\frac{n}{n_r}\right)^k \quad k \longrightarrow -1, 0, 1, 2$$

T → torque, C → constant, T_r → rated torque, n → speed, n_r → rated speed

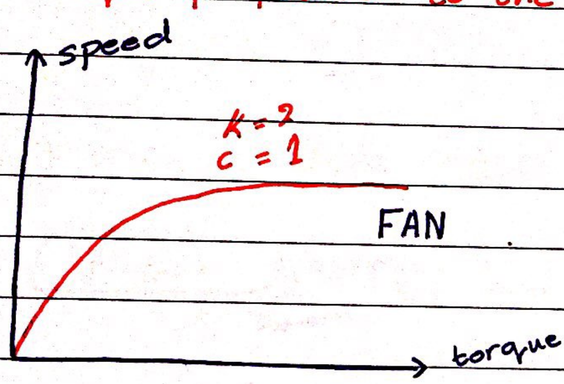
$\boxed{1}$ **Torque - independent of speed:-**



$$P = T\omega$$



[2] Torque propotional to the square of speed :



$$T = c T_r \left(\frac{n}{n_r}\right)^2$$

$$P \propto n^3$$

[3] Torque inversly propotional to speed :

$$T = c T_r \left(\frac{n}{n_r}\right)^{-1}$$

$$P = T\omega$$
$$= \frac{c'}{\cancel{n}} \times \frac{2\pi \cancel{n}}{60}$$
$$= C$$

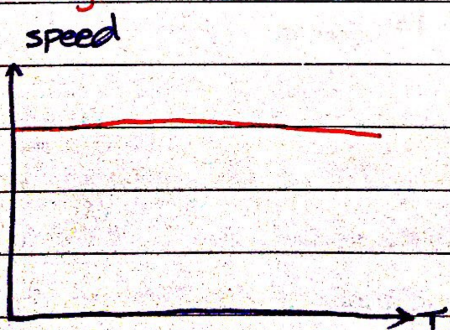
low speed ⇒ large torque

power is constant

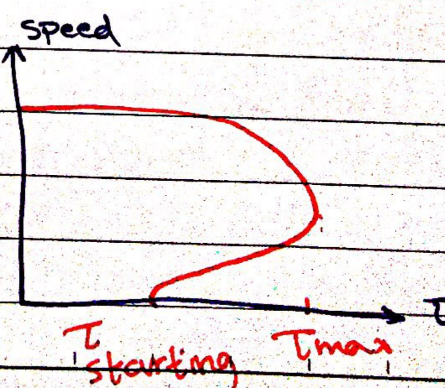
→ Motors speed - Torque c/s

AC Motors :

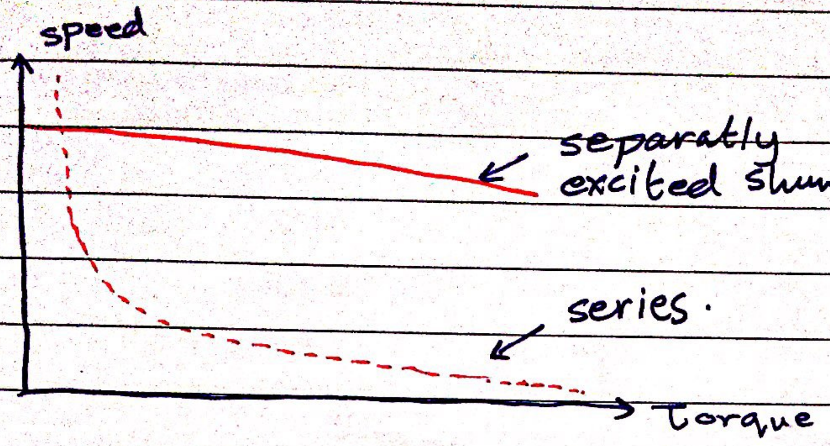
synchronous



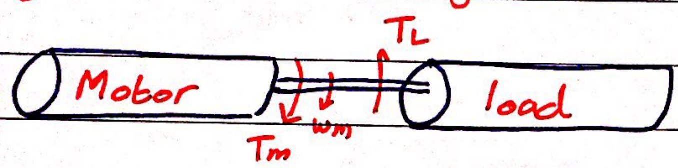
induction



DC motor :



→ Dynamics of drive system



equation of motion

$$\underbrace{J}_{kg\,m^2}\frac{d w_m}{dt} = \underbrace{T_{motor} - T_{load}}_{N\cdot m}$$

moment of inertia

→ T_m & T_L opposite direction

→ T_m & w_m same direction (Motor)

→ T_m & w_m opposite direction (generator)

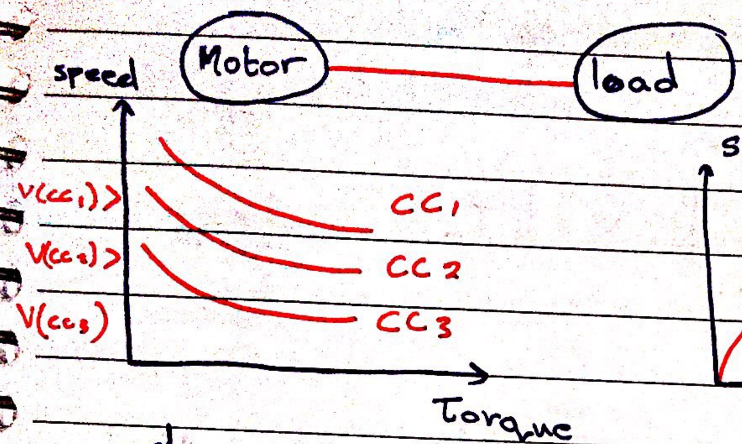
$T_m > T_L \Rightarrow dw/dt = > 0$ acce.

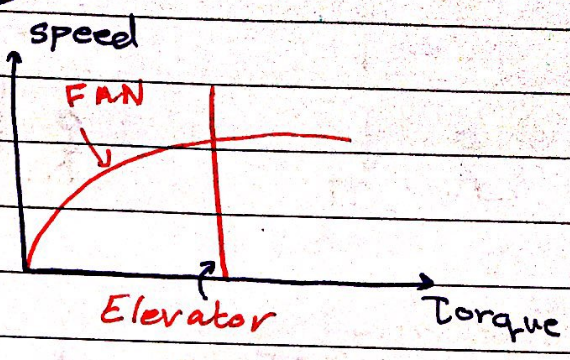
$T_m < T_L \Rightarrow dw/dt = < 0$ deacce.

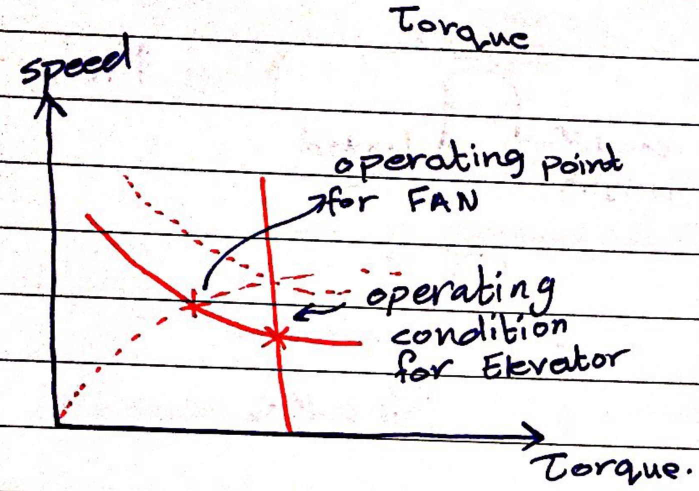
$T_m = T_L \Rightarrow dw/dt = 0$ steady state. → constant speed.
→ Rest.

$J\frac{d w_m}{dt} \cong$ inertia torque.
dynamic " .

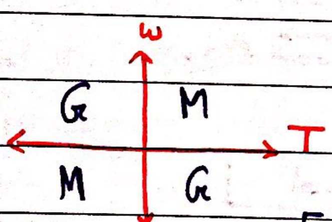
Joint speed c/s of Motor & load.

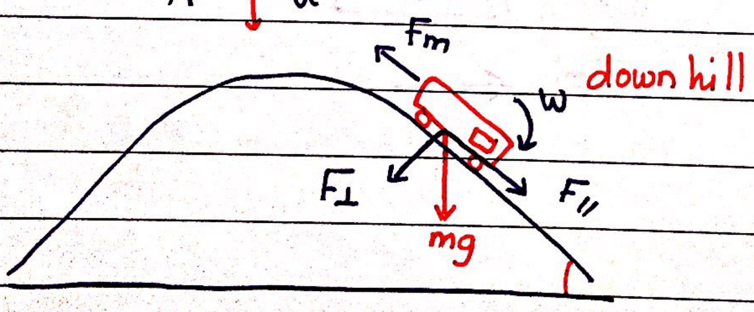


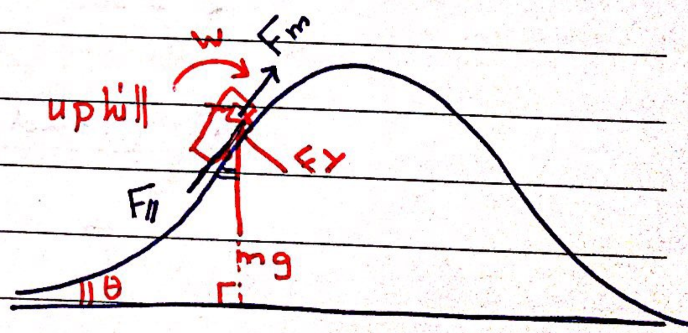




Electrical drive → Bi-directional → load torque, speed
Electrical drive → four quadrant operation.



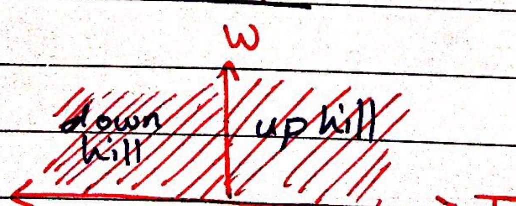




$F_{\parallel} = mg \sin\theta$

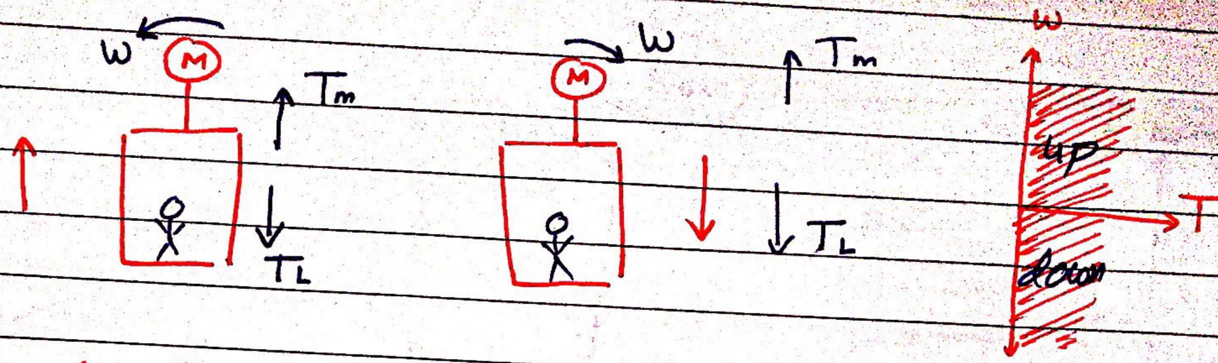
$F_{\perp} = mg \cos\theta$

$T = F \cdot$ wheel radius

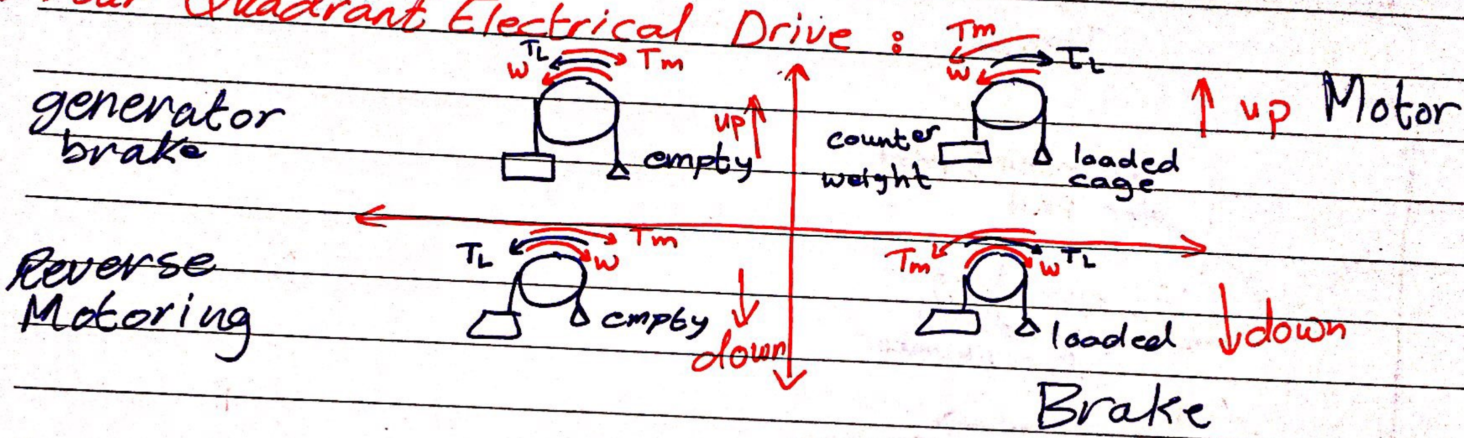


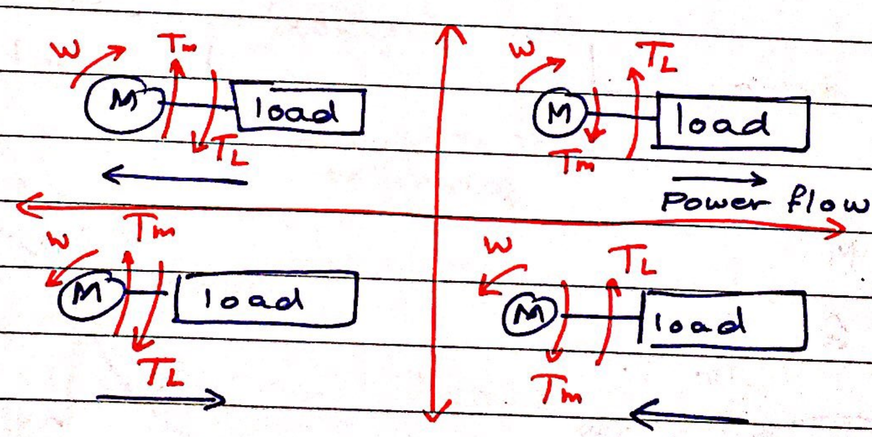
Bi-directional

example : Elevator



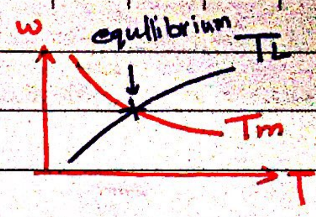
→ Four Quadrant Electrical Drive :





→ Operating point stability :-

* Steady state stability. (small change)
* Transient stability. (heavy change)

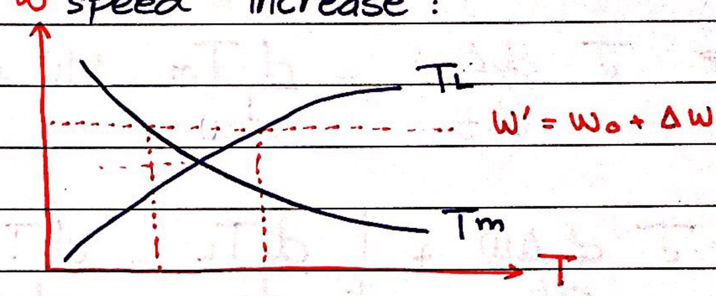


→ Steady state stability.

$$J\frac{dw}{dt} = T_m - T_L$$

if sudden unexpected change happens : → ??

w speed increase :



$$J\frac{dw}{dt} = \underbrace{T_m - T_L}_{-}$$

De-accelerating (Stable)

$$J\frac{dw}{dt} = T_m - T_L$$

$$w = w + \Delta w$$
$$T_m = T_m + \Delta T_m$$
$$T_L = T_L + \Delta T_L$$

w, T_m, T_L equilibrium point

Δw → ΔT_m, ΔT_L

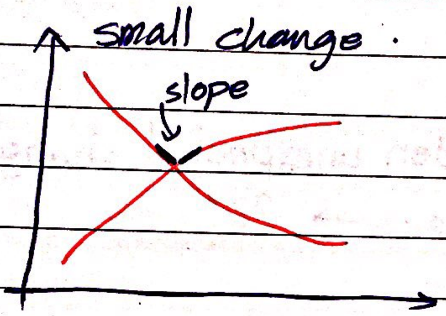
$$J\frac{dw}{dt} + J\frac{d\Delta w}{dt} + T_L + \Delta T_L - T_m - \Delta T_m = 0$$

But, $J\frac{dw}{dt} = T_m - T_L \rightarrow J\frac{dw}{dt} - T_m + T_L = 0$

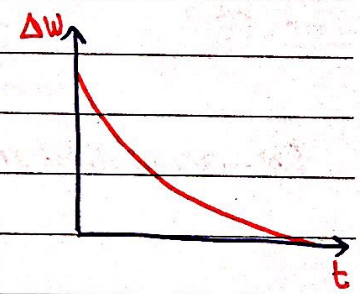
$$J\frac{d\Delta w}{dt} - \Delta T_m + \Delta T_L = 0$$

assume : $T_m = \frac{dT_m}{dw} \cdot \Delta W$

$T_L = \frac{dT_L}{dw} \cdot \Delta W$



$\rightarrow \quad J \cdot \frac{d\Delta w}{dt} - \frac{dT_m}{dw} \cdot \Delta W + \frac{dT_L}{dw} \cdot \Delta W = 0$



$$J \frac{d\Delta W}{dt} + \left[\frac{dT_L}{dw} - \frac{dT_m}{dw} \right] \Delta W = 0$$

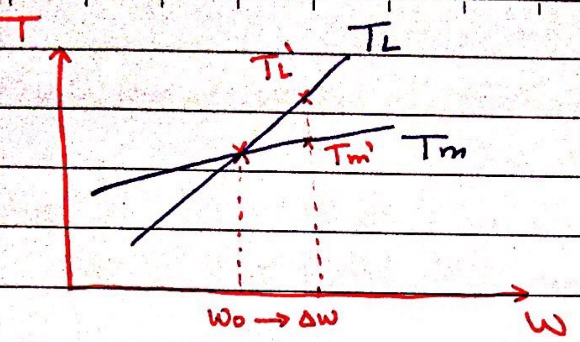
$$\Delta W = \Delta W_o \, e^{-\frac{1}{J}\left[\frac{dT_L}{dw} - \frac{dT_m}{dw}\right]t}$$

$$\frac{dT_L}{dw} - \frac{dT_m}{dw} > 0$$

$\frac{dT_L}{dw} > \frac{dT_m}{dw} \longleftrightarrow$ stable

2/10/2017

Ex :



Δw increase

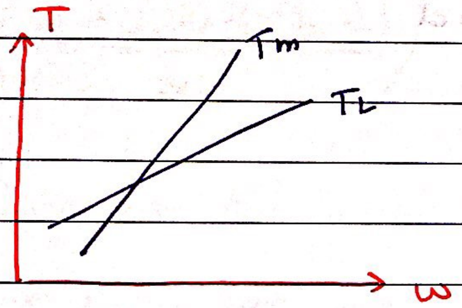
$T_L' > T_m'$

$J \frac{dw}{dt} = T_m - T_L$

deaccelerate

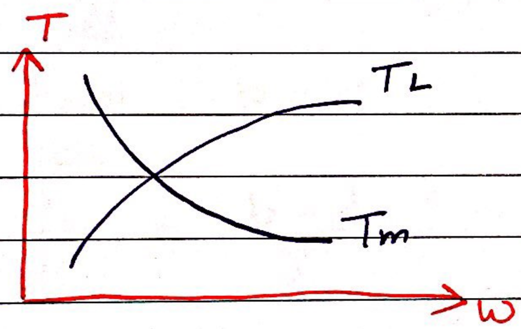
$\frac{dT_L}{dw} > \frac{dT_m}{dw}$ stable

Ex



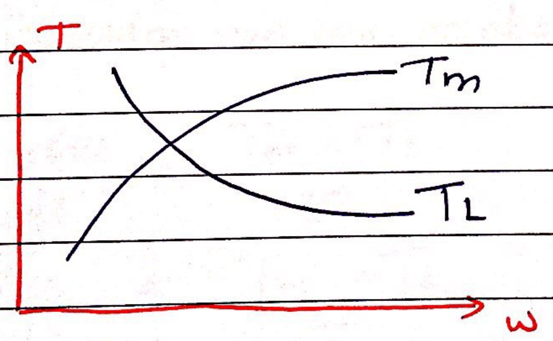
Not stable

Ex



Stable

Ex



Not stable

Ex: $T_m = a\omega + b$, $a,b > 0$
$T_L = c\omega^2 + d$, $c,d > 0$

① Determine the relations between a, b, c, d in order the motor starts together with load & have an equilibrium positive speed ?

Start $\Rightarrow$ $\omega = 0$ $\Rightarrow$ $T_m = b$, $T_L = d$

$$J\frac{d\omega}{dt} = T_m - T_L$$

start (acceleration)

$T_m > T_L$

$b > d$

equilibrium speed

$$T_m = T_L$$

$$a\omega + b = c\omega^2 + d$$

$$c\omega^2 - a\omega - (b - d) = 0$$

$$\omega = \frac{a \pm \sqrt{a^2 + 4c(b-d)}}{2c}$$

$$a^2 + 4c(b-d) > 0$$

$$a > \sqrt{a^2 + 4c(b-d)}$$

$$a^2 > a^2 + 4c(b-d)$$

$0 > c(b-d)$ Not valid

* equilibrium $\omega = \dfrac{a + \sqrt{a^2 + 4c(b-d)}}{2c}$

$$a^2 + 4c(b-d) > 0$$

③ Will the drive be stable @ this speed ?

$$\frac{dT_L}{d\omega} > \frac{dT_m}{d\omega} \qquad dT_L/d\omega = 2c\omega \qquad dT_m/d\omega = a$$

$$2c\omega \overset{??}{>} a$$

$$\cancel{2c}\left(\frac{a+\sqrt{a^2+4c(b-d)}}{\cancel{2c}}\right) \overset{\checkmark}{>} a \qquad \underline{\text{stable}}$$

④ Determine the initial acceleration of the drive ?

$$J\frac{d\omega}{dt} = T_m - T_L$$

$$J\frac{d\omega}{dt} = b - d$$

$$\frac{d\omega}{dt} = \frac{b-d}{J}$$

⑤ Determine the max. accelation of the drive ?

$$\frac{d\omega}{dt} = \frac{T_m - T_L}{J}$$

$$Acc. = \frac{T_m - T_L}{J} \qquad \omega = \frac{a}{2c}$$

$$\frac{d\,Acc}{d\omega} = 0 \Rightarrow \quad Acc\Big|_{max} = \frac{a^2 + 4c(b-d)}{4cJ}$$

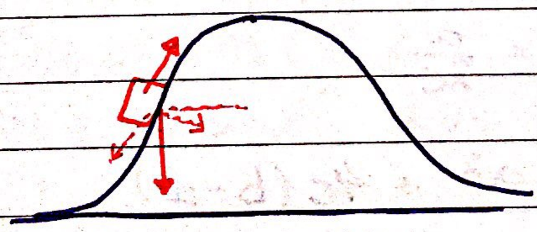
4 / 10 / 2017

→ Fundementals of mechanics

Linear	rotational
Displacement $\vec{d}$, m	- Angular displacement, θ rad
- Rate of change V m/s	- angular velocity ω, rad/s
- acceleration $\vec{a}$ m/s^2	- angular acceleration α rad/s^2
- mass kg	- " second moment of mass" inertia J kgm^2
- F "N"	- Torque T Nm
- Newton second law $F = ma$	- $T = J\alpha$ $T_m - T_L = J\frac{d\omega}{dt}$
- Kinetic energy $\frac{1}{2}mv^2$	- $\frac{1}{2}J\omega^2$
- Power $P = FV$	- $P = T\omega$

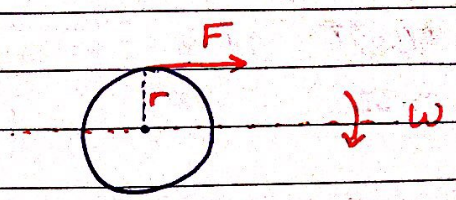
→ Force

- Force of gravity = 9.8m



→ Torque

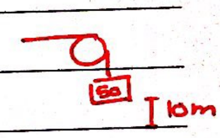
Torque $= \vec{F} \times \vec{r}$ N·m



→ Mechanical Work : (linear)

$$\underbrace{W}_{J} = \underbrace{F}_{N} \cdot \underbrace{d}_{m}$$

Ex: Work done to lift a mass of 50 kg up to 10 m ?

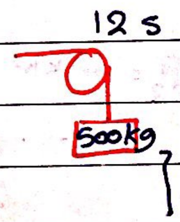


$W = F \cdot d$

$= (50 \times 9.8) \times 10 = 4900\,J$

Power ? $= \dfrac{W}{t}$ Power of motor (hp) = 746 W

Ex: Power delivered by the motor in kW, hp ?



$W = F \cdot d$

$= (500 \times 9.8) \times 30 = 147000\,J$

$P = \dfrac{147000}{12} = 12.25\,kW = 16.4\,hp$

→ Power of motor

$P = T\omega$

W Nm rad/s

$\omega = \dfrac{2\pi n}{60}$ rpm $\omega = \dfrac{n}{9.55}$

Five Apple

$$P = \frac{\overset{rpm}{N} \cdot \overset{Nm}{T}}{9.55} \quad W$$

→ Efficiency of a machine :

$$\eta = \frac{P_o}{P_i} \times 100\%$$

$$\text{thermal Energy} \rightarrow \boxed{\text{Steam turbine}} \rightarrow \text{mechanical energy}$$

$$\eta = 25\% \rightarrow 40\%$$

Electrical Motor :

$$\text{Electrical} \rightarrow \boxed{\text{Motor}} \rightarrow \text{Mechanical}$$

$$\eta = (75\% \rightarrow 98\%)$$

$\frac{150}{0.92} = 163\,kW$

Ex: $\text{Pin ??} \xrightarrow{0.92} (M) \xrightarrow{150kW} P$

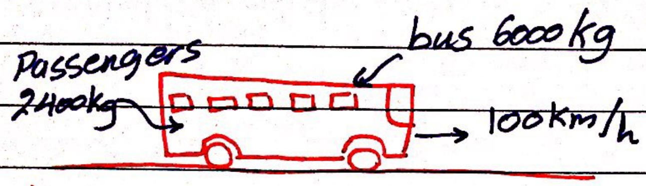
150 kW electrical motor with eff. 92% when it operates at full load calculate the losses ~~of~~ in the machine.

163 kW − 150 kW = 13 kW lossess

Five Apple

→ Kinetic energy of linear motiom $E_k = \frac{1}{2} m V^2$

Ex:



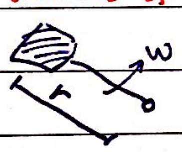
Kinetic energy to stop the bus ??

$$= \frac{1}{2}(2400 + 6000) \times \left(100 * \frac{1000}{3600}\right)^2 = 3.25\ MJ$$

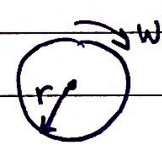
→ Kinetic energy of rotational

$$\Sigma = \frac{1}{2} J w^2$$
$$= \frac{1}{2} J\, n^2 (5.48 \times 10^{-3})$$

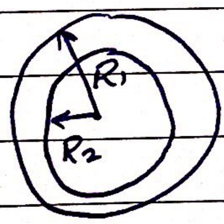
→ Moment of inertia J



$J = mr^2$



$J = \frac{1}{2} mr^2$

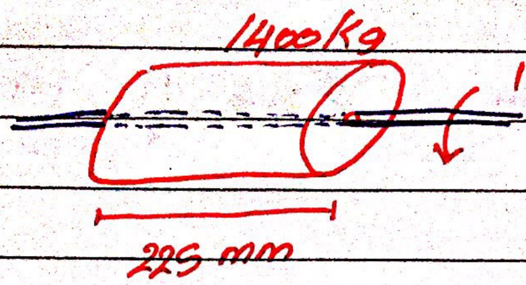


$J = \frac{1}{2} m (R_1 + R_2)^2$

→ Fly wheels: → increase inertia $T = J \frac{dw}{dt}$, $\frac{dw}{dt} = \frac{(T)}{J}$

rotating mech. element → Energy storage $\frac{1}{2} J w^2$

Ex :



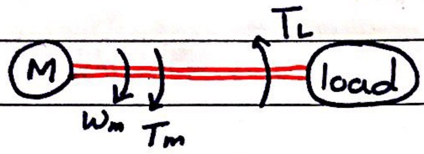
1800 rpm

diameter = 1m

1) $J = \frac{1}{2} m r^2$

2) $KE = \frac{1}{2} J \omega^2$

Dynamics of motor-load combination



$$T_m = T_L + J_{eq} \frac{dW_m}{dt}$$

system Kinetic energy:

$$\frac{1}{2} J_{eq} W_m^2 = \frac{1}{2} J_m W_m^2 + \frac{1}{2} J_L W_m^2$$

$$J_{eq} = J_m + J_L$$

Ex: $J_{eq} = 0.058 \ kg\,m^2$, T_L neglible

Calculate T_m if the speed is required to increase linearly from rest to 1800 rpm in 5 seconds.

$$\frac{dw_m}{dt} = \frac{(1800 - 0)\frac{2\pi}{60}}{5} = 37.7 \ rad/s$$

$$T_m = T_L + J_{eq} \frac{dw_m}{dt} = 2.19 \ Nm$$

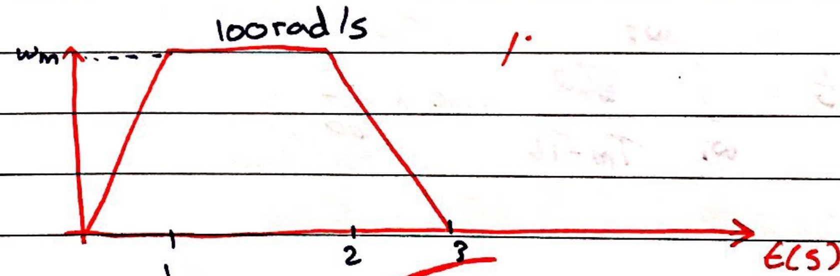
→ Angular displacement:

$$\theta(t) = \theta(0) + \int w_m(t) \, dt$$

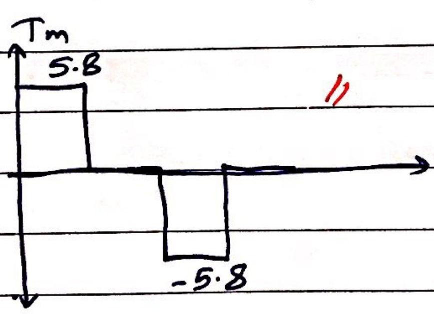
Ex: $J_{eq} = 0.058 \ kg\,m^2$

$T_L = 0$

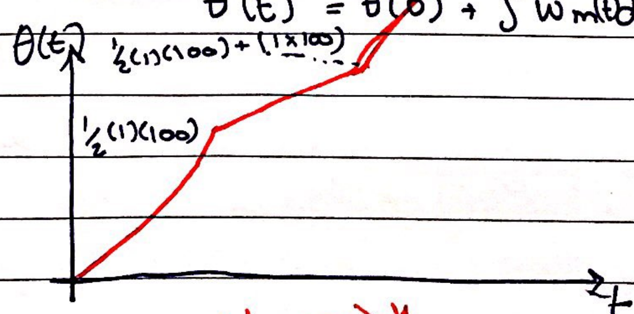
Plot T_m, $\theta(t)$

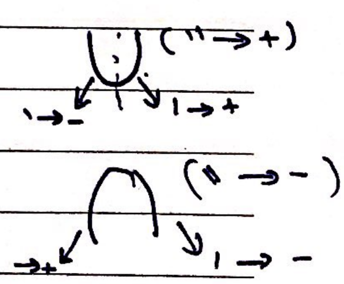


$$T_m = T_L + J_{eq} \frac{dw_m}{dt}$$



$$\theta(t) = \theta(0) + \int w_m(t) dt$$





الإقتران

Ex: A drive has the following parameters :-

$J_{eq} = 10 \text{kg m}^2$, $T_m = 100 - 0.1N \rightarrow \text{rpm}$

$T_L = 0.05Nm$

What is the steady state speed :

$\rightarrow T_m = T_L$

$100 - 0.1N = 0.05Nm$

$N = 666.7 \text{ rpm}.$

Calculate the time required to reverse the motor at the same steady state speed :

$\omega_i = 666.7 \text{rpm}$

$\omega_f = -666.7 \text{rpm}$

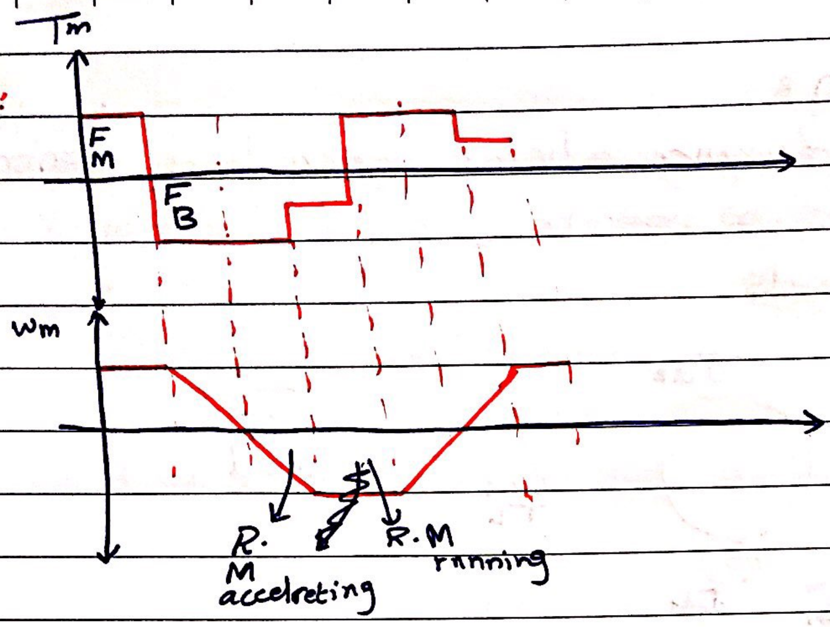
$$J \frac{d\omega_m}{dt} = T_m - T_L$$

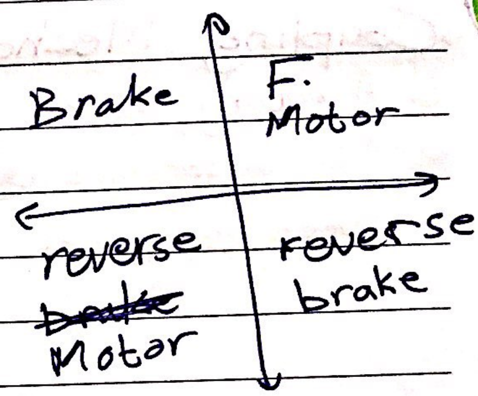
$$J \frac{d\omega_m}{T_m - T_L} = dt$$

$$t = \int_{\omega_i}^{\omega_f} \frac{J d\omega_m}{T_m - T_L}$$ "Travelling time"

$$t = \int_{\omega_i}^{\omega_f} \frac{10}{T_m - T_L} \underbrace{\left(dn \times \frac{2\pi}{60} \right)}_{d\omega_m}$$

Ex:





→ Travelling time

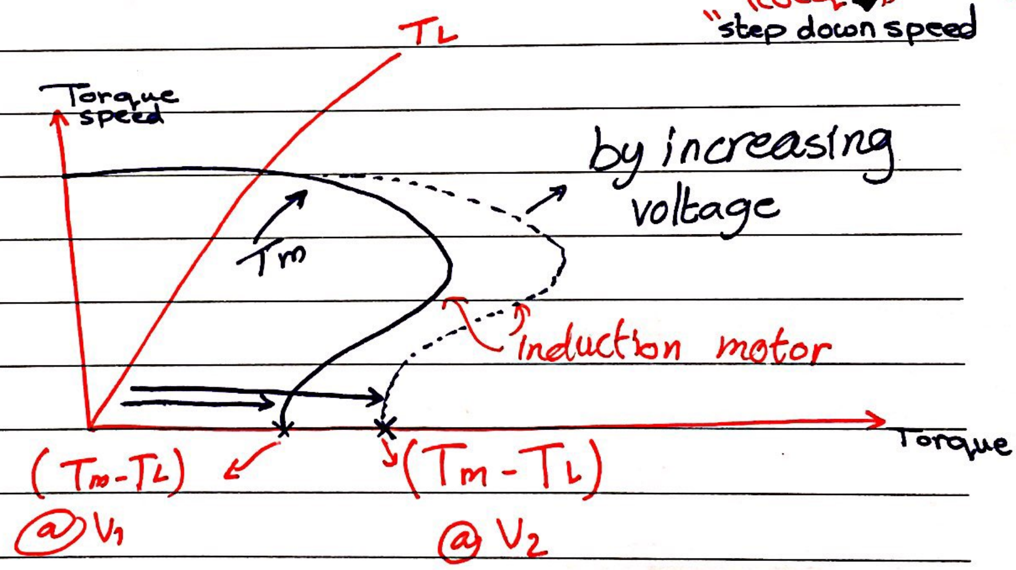
$$t = \int_{w_i}^{w_f} \frac{J_{eq}}{T_m - T_L}\, dw$$

travelling time

1) $T_m - T_L$ ⇈

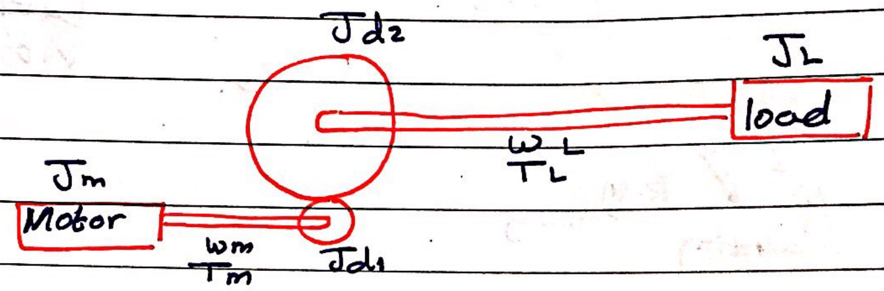
2) reduction gears
((J_{eq} ↓))
"step down speed"

$$J \underbrace{\frac{dw}{dt}}_{\text{dynamic torque speed}} = T_m - T_L$$

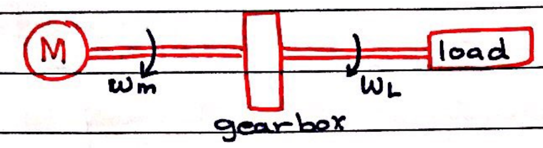


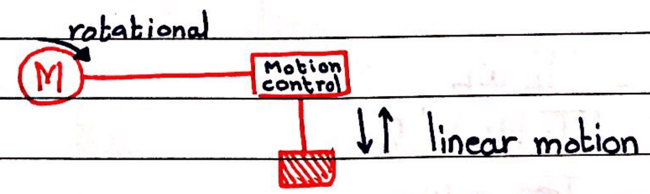
Coupling Mechanism :

* A rotary motor is driving a linear motion load "Motion controll"
* The motors are designed to operate @ higher rotational speed compared to loads.

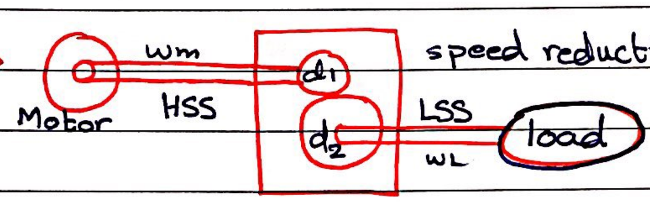


$$\frac{1}{2} J_{eq} w_m^2 = \frac{1}{2} J_m w_m^2 + \frac{1}{2} J_{d1} w_m^2 + \frac{1}{2} J_{d2} w_L^2 + \frac{1}{2} J_L$$

(1) 

(2) 

- Induction gen on doupt fed gen → Wind turbine



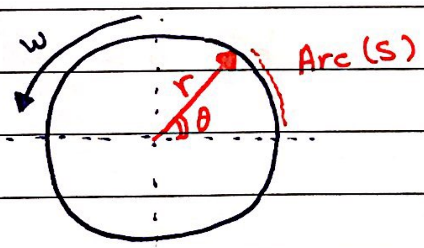
$$T_m - T_L = J_{eq} \frac{dw_m}{dt}$$

?? ??

$$\frac{1}{2} J_{eq} W_m^2 = \frac{1}{2} J_m W_m^2 + \frac{1}{2} J_{d1} W_m^2 + \frac{1}{2} J_d W_L^2 + \frac{1}{2} J_L W_L^2$$

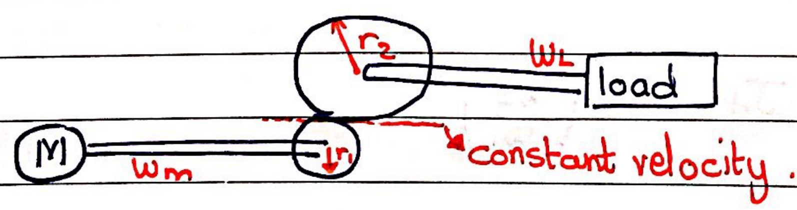
$$\therefore J_{eq} = J_m + J_{d1} + J_{d2}\left(\frac{W_L}{W_m}\right)^2 + J_L\left(\frac{W_L}{W_m}\right)^2$$

GB causes high acceleration



$$\text{linear velocity} = \frac{S}{\text{time}}$$

$$V = \frac{r\theta}{\text{time}} = rw$$



$$r_1 W_m = r_2 W_L$$

$$\therefore W_L = W_m\left(\frac{r_1}{r_2}\right)$$

$$\frac{r_1}{r_2} = \text{gear ratio} = g = a$$

$\therefore J_{eq} = J_m + J_{d1} + J_{d2}(a)^2 + J_L(a)^2$

Impact on the Torque

O/P power of the gear $= T_L \omega_L$

i/P power of the gear $= (T_L \omega_L)/\eta$

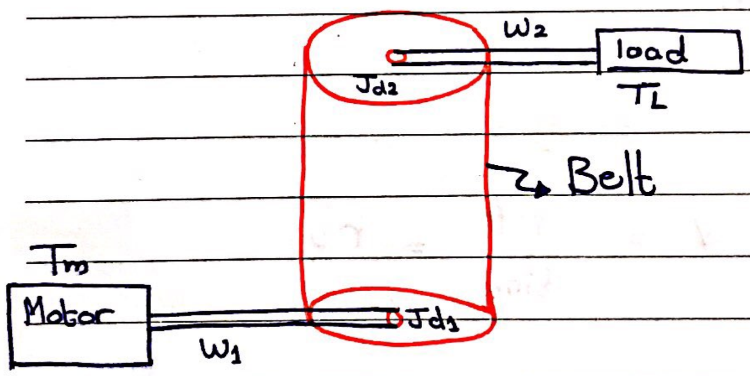
" " " " " $= T_{eq} * \omega_m$

$$T_{eq} * \omega_m = \frac{T_L \omega_L}{\eta}$$

$$\therefore T_{eq} = T_L \left(\frac{\omega_L}{\omega_m}\right) \frac{1}{\eta}$$

- Torque seen by the motor is reduced by a
- Inertia seen by the motor is reduced by a^2

Ex: A simple belt drive system :-

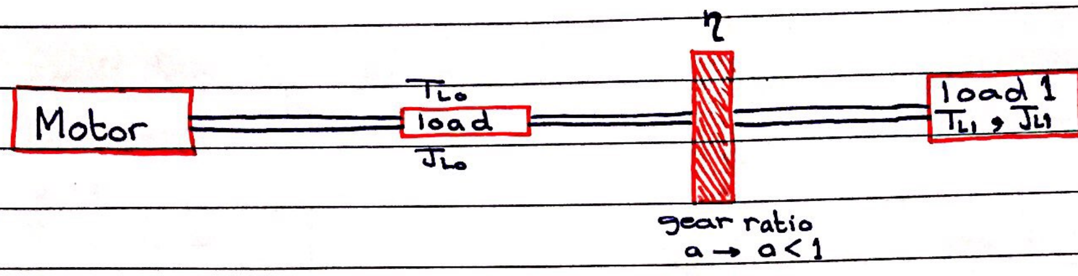


T_L^* seen by motor ?

$$J_{eq} = J_m + J_{d1} + (J_{d2} + J_L)\left(\frac{\omega_2}{\omega_1}\right)^2$$

$$T_L^* = T_L \left(\frac{\omega_2}{\omega_1}\right) \frac{1}{\eta}$$

Ex:

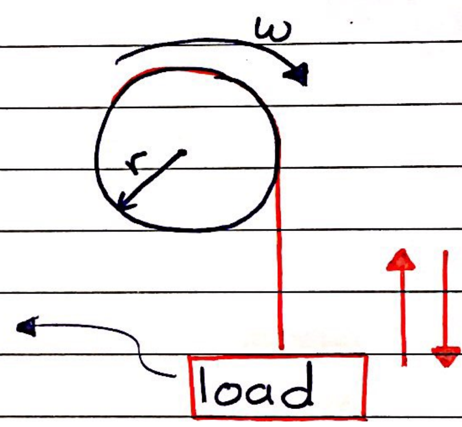


- Torque required by the motor @ steady state

$$T_m = T_{Lo} + \frac{T_{L1} * a}{\eta}$$

$$J_{eq} = J_m + J_{Lo} + J_{L1} * a^2$$

⇒ loads with translation motion



has m
v
F

kinetic energy $= \frac{1}{2} J_{eq} w^2 = \frac{1}{2} m^2 v^2$

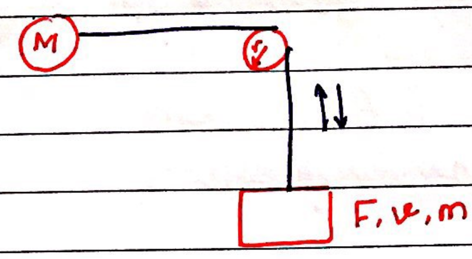
$$\therefore J_{eq} = m\left(\frac{v}{w}\right)^2 = m\left(\frac{rw}{w}\right)^2 = mr^2$$

input power $= T_{eq}\, w$ "rotational"
output power $= F v$ "translation"

$$T_{eq}\, w = \frac{F v}{\eta} \longrightarrow T_{eq} = \frac{F}{\eta}\left(\frac{v}{w}\right) = \frac{F(r)}{\eta}$$

16/10/2017

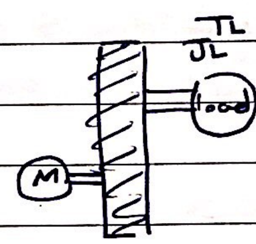
loads with translational motion :



$$J_{eq} = m\left(\frac{v}{\omega}\right)^2 = mr^2$$

$$T_{eq} = \frac{Fv}{\eta\omega} = \frac{F(r\omega)}{\eta r} = \frac{Fr}{\eta}$$

loads with rotational motion (with gearbox)



$$T_{eq} = T_L * a * \frac{1}{\eta}$$

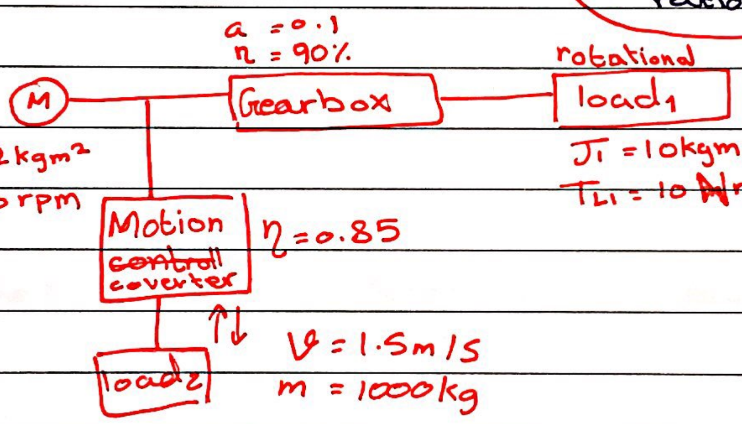
$$J_{eq} = J_L * a^2 * \frac{1}{\eta}$$

$a = \frac{\omega_L}{\omega_m}$

$\omega_L < \omega_m$

gear reduction ratio

Ex:



1) Determine the equ. inertia of the system?

2) Power delivered by motor?

$$J_{eq} = J_m + J_{L1} * a^2 + m\left(\frac{V}{\omega}\right)^2$$

$$= 0.31\ kgm^2$$

@ steady state $T_m = T_L$

$P_m = T_m * \omega_m$

- T_{L1} seen by motor

$$= \frac{10 \times 0.1}{0.9} =$$

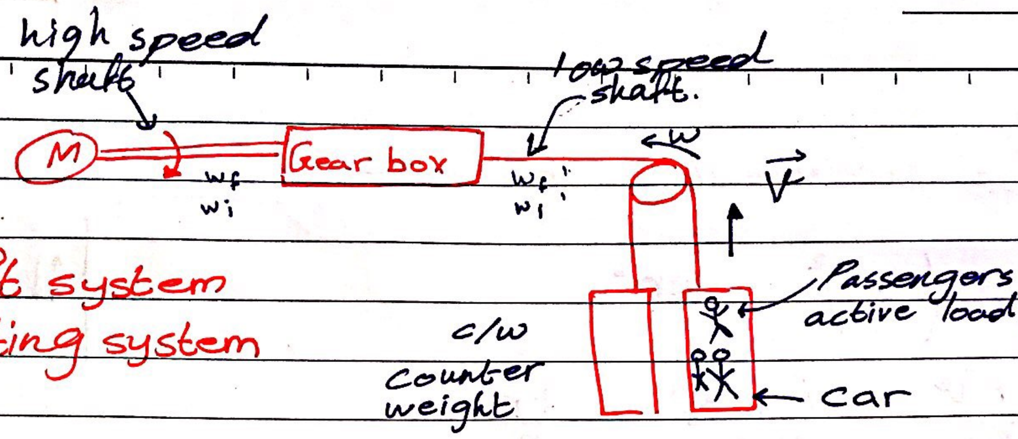
- T_{L2} seen by motor

$$= \frac{F}{\eta} * \frac{v}{\omega}$$

$$T_L = T_{L1} + T_{L2} = 117.55\ Nm$$

$$P_m = T_L * \omega_m = 17.48\ kW.$$

Ex :



lift system
hoisting system

*Calculate the initial acceleration when :
1) Car is full of passengers & moving upwards?
2) " " empty " " " " " ?

load { $Q = 300\,kg$ "Passengers"
$C = 300\,kg$ "car"
$C/w = 50\% = [C + 50\% * Q] = 450\,kg$
$v = 1.2\,m/s$

Pulley { $d = 630\,mm$

Gear box { ~~Gear~~ Gear ratio $74:2 \Rightarrow \frac{W_L}{W_m} = \frac{2}{74} = 0.03$

Motor { rated torque = 25.5 Nm
$J = 0.1\,kgm^2$
speed = 1350 rpm
starting Torque = $2 * T_r$

$\boxed{\eta = 55\%}$

Five Apple

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LSS "low speed shaft"

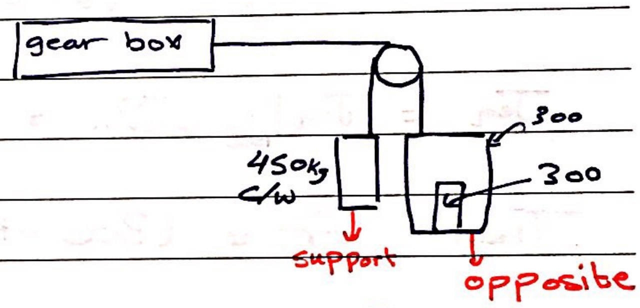
$$J_{eq_{LSS}} = m\left(\frac{v}{\omega}\right)^2 = m\left(\frac{r\omega}{\omega}\right)^2 = mr^2$$

$$= (\underset{car}{300} + \underset{pass.}{300} + \underset{c/w}{450}) * \left(\frac{0.63}{2}\right)^2$$

HSS

$$J_{eq_{HSS}} = J_{eq_{LSS}} * a^2 * \frac{1}{\eta} = 0.138 \ kgm^2$$

$$J_{sys} = J_{eq_{HSS}} + J_m = \underline{\underline{0.238 \ kgm^2}}$$



load

$$T = F\left(\frac{v}{\omega}\right) = Fr$$

F = ??

$$F = (300 + 300 - 450) * 9.8$$

$$_{LSS}T = F \times \left(\frac{0.63}{2}\right)$$

$$T_{Load \ HSS} = T_{L \ LSS} * 0.03 * \frac{1}{0.55} = 22.8 \ Nm$$ seen by motor

Starting Torque = 2*25.5 = 51 Nm

$$J\frac{d\omega_m}{dt} = T_m - T_L$$

$$0.238 \frac{d\omega_m}{dt} = 51 - 22.8 \rightarrow \frac{d\omega_1}{dt} = 118.5 \ rad/s = \alpha$$

* Benifit of counter weight → S. Torque يقل

acc. @LSS α' $= \dfrac{\omega_f' - \omega_i'}{\Delta t} = \dfrac{a(\omega_f - \omega_i)}{\Delta t} = a(118.5)$

$= 0.03 \times 118.5 \text{ rad/s}^2$

linear acc $= \dfrac{V_f - V_i}{\Delta t} = \dfrac{(d/2)\,\omega_f' - (d/2)\,\omega_i'}{\Delta t} = (d/2) * \alpha'$

$= 118.5 * 0.03 * \left(\dfrac{0.63}{2}\right) = 1 \text{ m/s}^2$

2) $J_{eq}\big|_{load}$ (seen by the motor) $= (450 + 300 + 0)\left(\dfrac{V}{\omega}\right)^2 * a^2 * \dfrac{1}{?}$

$= (450 + 300)\left(\dfrac{0.63}{2}\right)^2 * 0.03 * \dfrac{1}{0.55}$

$= 0.0988 \text{ kg m}^2$

$J_{eq} = J_{eq}\big|_{load} + J_m = 0.1988 \text{ kg m}^2$

$T_{load} = Fr = (300 - 450)(9.8)\left(\dfrac{0.63}{2}\right)$

$T_{load \text{ seen by motor}} = (300 - 450)(9.8)\left(\dfrac{0.63}{2}\right)\left(\dfrac{0.03}{0.55}\right) = -22.8 \text{ Nm}$

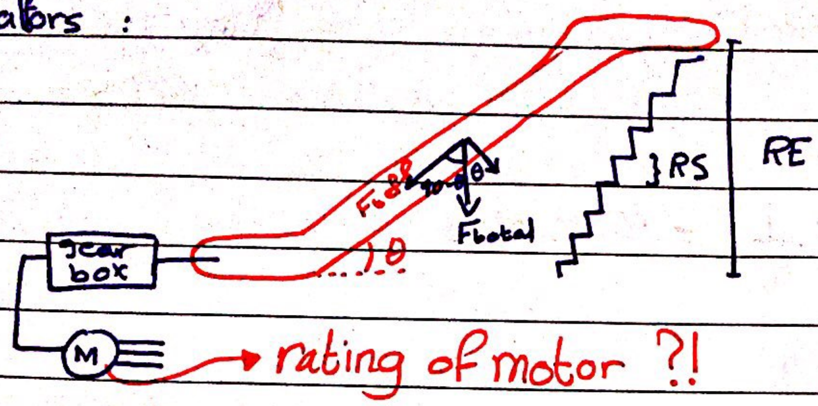
$\alpha = \dfrac{T_m - T_L}{J} = 371.2 \text{ rad/sec}^2$

lin. acc. $= 3.16 \text{ m/s}^2$

23 / 10 / 2017

Escalators :

- Ensure that the motor can move all masses @ rated speed



rating of motor ?!

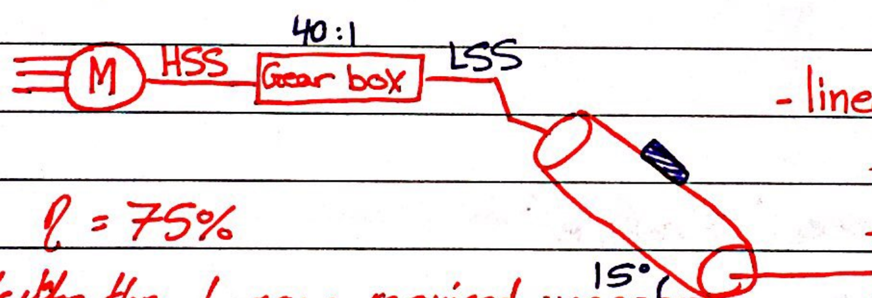
$F_{Total_{eff}} = m * g * n$ ~ # of passenger/step * # of steps * $\sin\theta$

$= m * g * n * \left(\frac{RE}{RS}\right) * \sin\theta$

Power = $F_{eff} * v$

Power = $\frac{F_{eff} * v}{\eta}$

EX:



$\eta = 75\%$

→ Calculate the torque required by the motor @ starting :-

- linear upward
* acceleration 2 m/s²
* Mass 100 kg } load
* Mass belt 20 kg } load
* each pully 10 kg m² } inertia
* Diameter of each pully 2m
* Jm neglected

$J_{eq} \frac{d\omega_m}{dt} = T_m - T_L$

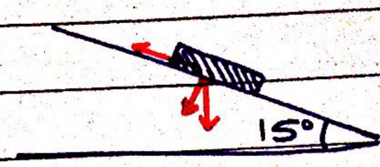
$J_{Load} = m\left(\frac{v}{\omega}\right)^2 = mr^2 = 100(1)^2 = 100 \text{ kg m}^2$

$J_{belt} = 20 \text{ kg m}^2$

$J_{eq}|_{LSS} = 100 + 20 + 10 + 10 = 140 \text{ kg m}^2$

$J_{eq}|_{HSS} = 140 * \left(\frac{1}{40}\right)^2 * \frac{1}{0.75} = 0.116 \text{ kg m}^2$

$J_{eq}|_{sys} = 0.116 \text{ kg m}^2 + \cancel{J_m}$



$T_{load} = (100 * 9.8 * \sin\theta) * r$

$T_{load}|_{LSS} = 253.9\ Nm^2$

$T_{HSS} = 253.9 * \left(\frac{1}{40}\right) * \frac{1}{\eta} = 8.46\ N.m$

linear acceleration = 2 m/s

$V = r\omega$ → see

$Ang.\ acc.|_{LSS} = \frac{2\ m/s^2}{1} = 2\ rad/sec^2$

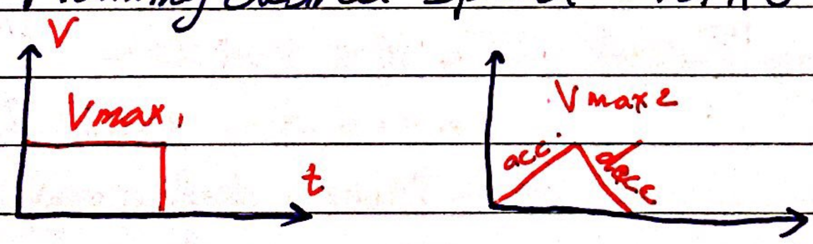
$Ang.\ acc.|_{HSS} = 2 * 40 = 80\ rad/sec.$

$$J_{eq} \frac{d\omega_m}{dt} = T_m - T_L$$

$$\boxed{T_m = 17.79\ Nm}$$

Motion profile

"Planning desired speed Profile".



Optimal

- Min. Journey time
- " Energy consumption
- Max. Safety
- Passenger comfort.

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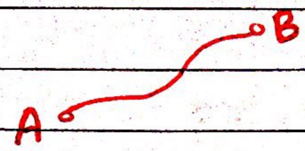
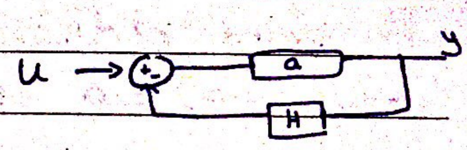
25 / 10 / 2017

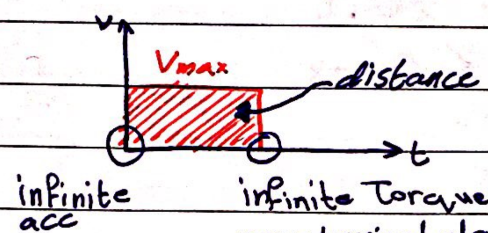
Motion Profile :





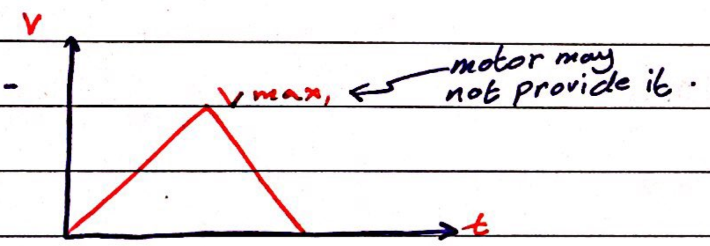
Motion profile → triangular
→ trapezoidal
→ S-curve
→

- Why not



*mechanical damage.
*human constraints.

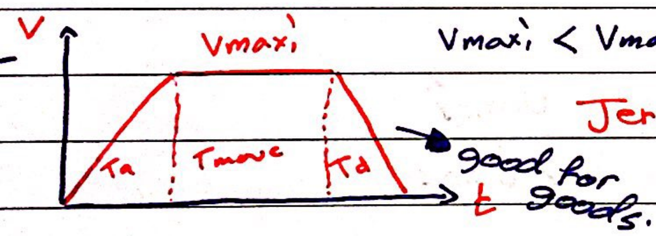
-



Triangular

$$a = \frac{V_{max} - 0}{T/2} \leq a_{max}$$

-



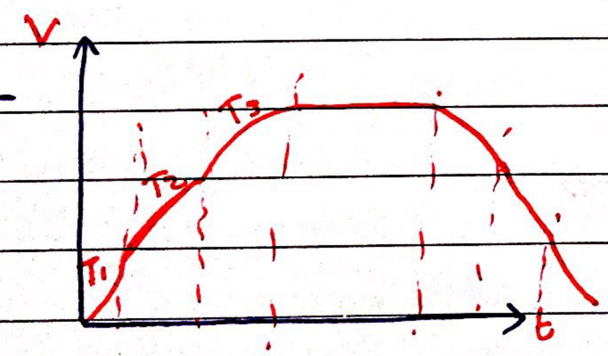
$V_{maxi} < V_{max1}$

Trapezoidal

$$Jerk = \frac{d\,acc}{dt}$$

→ goods are relaxed
→ human constraints.

-



S-curve

$$V(t) = c_1 t^2 + C_2 t + C_3$$

if $j(t) \leq j_{max}$

$$j(t) = 2C_1 < j_{max}$$

$$C_1 \leq \frac{j_{max}}{2}$$

Five Apple

Boundary conditions :

* $V(t=0) = 0 \Rightarrow c_3 = 0$

* $a(t) \leq a_{max}$

$a(t) = 2c_1 t + c_2 \Rightarrow a(t=0) \Rightarrow \boxed{c_2 = 0}$

bcz $c_2 = 0 \Rightarrow a(t) = 2c_1 t$

$2c_1 t_1 \leq a_{max}$

$$\therefore t_1 \leq \frac{a_{max}}{2c_1}$$

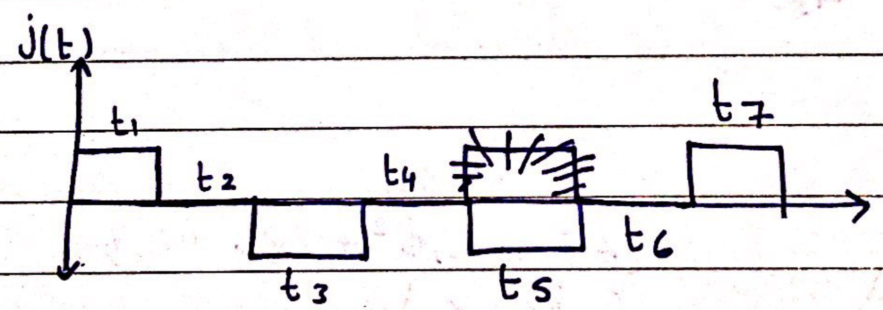
$$t_1 = \frac{a_{max}}{j_{max}}$$

* Constraints $\Rightarrow a_{max}$ & j_{max}

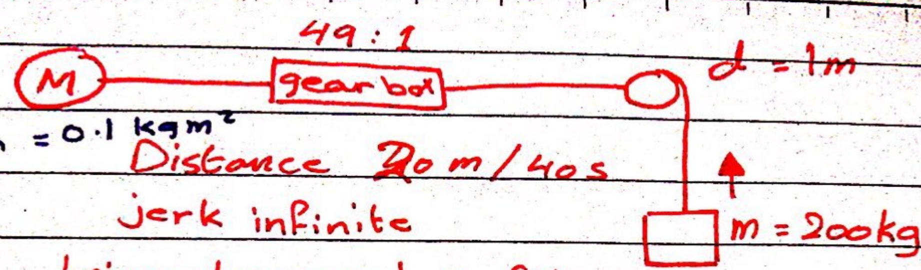
$$0 \leq t \leq t_1$$

$$V(t) = \frac{j_{max}}{2} t^2$$

$$V(t_1) = \frac{j_{max}}{2}\left(\frac{a_{max}}{j_{max}}\right)^2 = \frac{a_{max}^2}{2 j_{max}}$$



Ex:



$j_m = 0.1 \ kgm^2$

Distance 20 m / 40 s

jerk infinite

triangular speed profile

Find required starting motor torque , $\eta = 0.85$