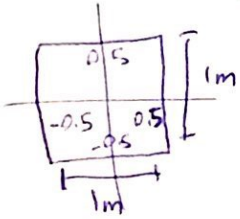


Q.1 (5 Points)

Find the emf voltage generated at the terminals of a 50 turns square coil of area 1m^2 centered at the origin in x-y plane if the magnetic flux density is given by:



$$B = 4|x||y|\cos(2\pi \times 10^3 t) \mathbf{a}_z$$

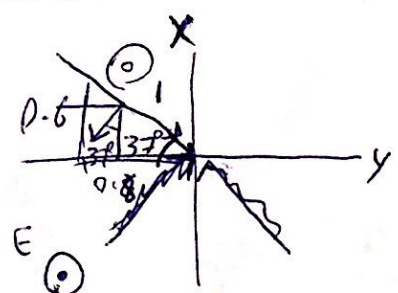
$$\begin{aligned} v_{emf} &= -N \frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{s} \\ &= -50 \times \frac{d}{dt} \int_{-0.5}^{0.5} \int_{-0.5}^{0.5} 4|x||y| \cos(2\pi \times 10^3 t) dx dy \\ &= 1.256 \text{ M} \int_{-0.5}^{0.5} x dx \int_{-0.5}^{0.5} y dy \sin(2\pi \times 10^3 t) \\ &= 0.25 \times 1.256 \text{ M} \times 0.25 \times 0.25 \times 1.256 \text{ M} \times \sin(2\pi \times 10^3 t) \\ &= 78539 \sin(2\pi \times 10^3 t) \end{aligned}$$

$$\begin{aligned} \frac{dB}{dt} &= \frac{d}{dt} [4|x||y|\cos(2\pi \times 10^3 t)] \\ &= 4|x||y|(-2\pi \times 10^3) \sin(2\pi \times 10^3 t) \\ &= -8\pi \times 10^3 \sin(2\pi \times 10^3 t) |x||y| \\ &= -8\pi \times 10^3 \sin(2\pi \times 10^3 t) \int_{-0.5}^{0.5} x^2 dx \int_{-0.5}^{0.5} y^2 dy \\ &= -8\pi \times 10^3 \sin(2\pi \times 10^3 t) \left[\frac{x^3}{3} \right]_{-0.5}^{0.5} \left[\frac{y^3}{3} \right]_{-0.5}^{0.5} \\ &= -8\pi \times 10^3 \sin(2\pi \times 10^3 t) \times \frac{0.125}{3} \times \frac{0.125}{3} \\ &= -8\pi \times 10^3 \sin(2\pi \times 10^3 t) \times 0.001736 \\ &= -13.8 \sin(2\pi \times 10^3 t) \end{aligned}$$

Q.2 (5 Points)

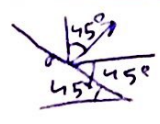
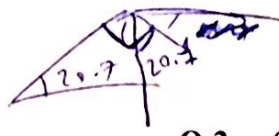
In free space at 6GHz the direction of propagation is in $(0.6\mathbf{a}_x - 0.8\mathbf{a}_y)$ if the power density is $100\text{mW}/\text{m}^2$ and the electric field is in the $+z$ direction, Find E and H.

$$\begin{aligned} f &= 6 \text{ GHz} \quad \omega = 120\pi \times 10^9 \\ p &= 100 \text{ mW}/\text{m}^2 = 0.1 \quad \beta = \frac{\omega}{c} = \frac{2\pi \times 6 \times 10^9}{3 \times 10^8} = 40\pi \\ p &= \frac{|E_0|^2}{2\eta} \\ 0.1 &= \frac{|E_0|^2}{2 \times 120\pi} \rightarrow E_0 = 8.68 \text{ V/m} \end{aligned}$$



$$\begin{aligned} \mathbf{E} &= E_0 \cos(\omega t - \beta r) \mathbf{a}_z \\ &= 8.68 \cos(2\pi \times 6 \times 10^9 t - 40\pi (0.6x - 0.8y)) \mathbf{a}_z \\ \mathbf{H} &= \frac{E_0}{\eta} \cos(\omega t - \beta r) (-0.6\mathbf{a}_y - 0.8\mathbf{a}_x) \\ &= \frac{8.68}{120\pi} \cos(2\pi \times 6 \times 10^9 t - 40\pi (0.6x - 0.8y)) (-0.6\mathbf{a}_y - 0.8\mathbf{a}_x) \end{aligned}$$

$$\begin{aligned} \mathbf{E} &= 8.68 \cos(2\pi \times 6 \times 10^9 t - 40\pi (0.6x - 0.8y)) \mathbf{a}_z \\ \mathbf{H} &= -0.6\mathbf{a}_y - 0.8\mathbf{a}_x \end{aligned}$$



$$\beta_1 = \frac{\omega}{c} = \frac{2\pi \times 6 \times 10^9}{3 \times 10^8}$$

$$\beta_2 = \frac{\omega}{v} = \frac{2\pi \times 6 \times 10^9 \times 2}{2.3 \times 10^8}$$

Q.3 (6 Points)

A plane wave at 6GHz is incident at angle of 45 from normal from air into a dielectric with relative permittivity of 4. If the incident electric field amplitude is 10^{-6} V/m in +y direction. Find the reflected wave and transmitted wave (both E and H).

$f = 6 \text{ GHz}$ $\theta_i = 45^\circ$
 $|E_i| = 10^{-6} \text{ V/m } \vec{a}_y$
 $n_1 = 120\pi$ $n_2 = \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{120\pi}{2} = 60\pi$

$n_1 \sin(\theta_i) = n_2 \sin(\theta_t)$

$\frac{n_1}{n_2} \sin(\theta_i) = \sin(\theta_t)$

$\frac{\sqrt{\epsilon_r} n_1}{n_2} \sin(45) = \sin(\theta_t)$

$\theta_t = 20.7^\circ$

$E_t = 0.55 \times 10^{-6} \cos(2\pi \times 6 \times 10^9 - 80\pi (\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}z)) \vec{a}_y$
 $H_t = \frac{0.55 \times 10^{-6}}{60\pi} \cos(2\pi \times 6 \times 10^9 - 80\pi (\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}z)) (-0.93 \vec{a}_x + 0.35 \vec{a}_z)$

$\Gamma_{\perp} = \frac{n_2 \cos(\theta_i) - n_1 \cos(\theta_t)}{n_2 \cos(\theta_i) + n_1 \cos(\theta_t)}$
 $= \frac{60\pi \cos(45) - 120\pi \cos(20.7)}{60\pi \cos(45) + 120\pi \cos(20.7)}$
 $\Gamma_{\perp} = -0.45$
 $T_{\perp} = 0.55$
 $E_r = \Gamma_{\perp} E_0 \cos(\omega t - \beta_1 (\sin(45^\circ)x - \cos(45^\circ)z)) \vec{a}_y$
 $= -0.45 \times 10^{-6} \cos(2\pi \times 6 \times 10^9 - 40\pi (\frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}z)) \vec{a}_y$
 $H_r = \frac{-0.45 \times 10^{-6}}{120\pi} \cos(2\pi \times 6 \times 10^9 - 40\pi (\frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}z)) (\frac{1}{\sqrt{2}} \vec{a}_x + \frac{1}{\sqrt{2}} \vec{a}_z)$

$\cos(20.7) = 0.93$
 $\sin(20.7) = 0.35$

$(1.6)^2 = 3^2 + 2^2 - 2 \times 2 \times 3 \cos(\theta_x)$
 $\theta_x = 132.8^\circ$
 $\theta_r = 132.8 - 180^\circ = -47.15^\circ$
 $\Gamma = \frac{n_2 - 120\pi}{n_2 + 120\pi} = \Gamma = \frac{2}{3} \angle -47.15^\circ$

Q.4 (6 Points)

Given the following standing wave, find x and the characteristic impedance for the second media if the first media is free space.

$\frac{\lambda}{2} = 20 \text{ cm}$

$\lambda = 40 \text{ cm}$
 $E_{max} = 5$
 $E_{min} = 1$

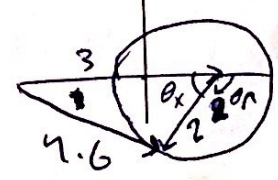
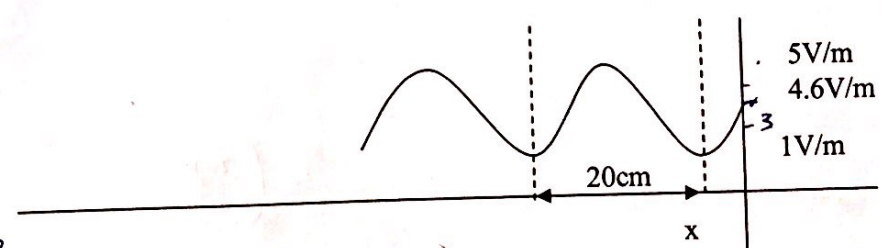
$\sqrt{r} = \frac{5-1}{1+5} = \frac{4}{6} = \frac{2}{3}$

$\eta_2 = 788.35 \angle -60.4^\circ$

$x = 7.4 \text{ cm} \angle 7.38 \text{ cm}$

$0.5 - 0.315 = 0.185 \lambda = 7.4 \text{ cm}$

$\eta_2 = 120\pi + \eta_2 (\frac{2}{3} \angle -47.15^\circ) + 120\pi (\frac{2}{3} \angle -47.15^\circ)$
 $\eta_2 = 788.35 \angle -60.4^\circ$



$(1.6)^2 = 3^2 + 2^2 - 2 \times 2 \times 3 \cos(\theta_x)$
 $\theta_x = 132.8^\circ$
 $\theta_r = 132.8 - 180^\circ = -47.15^\circ$
 $\Gamma = \frac{n_2 - 120\pi}{n_2 + 120\pi} = \Gamma = \frac{2}{3} \angle -47.15^\circ$

Q.5 (8 Points)

Use series stub matching to match a load of $200-j70$ if the T.L impedance is 100Ω (solve using S.C stub and repeat for O.C stub)

$$Z_L = 200 - j70$$

$$Z_{T.L} = 100\Omega$$

$$z_L = \frac{200 - j70}{100} = 2 - 0.7j$$

$$0.342\lambda$$

$$1 - 0.9j \rightarrow d = 0.342\lambda - 0.28\lambda$$

$$1 + 0.9j \rightarrow d = 0.062\lambda$$

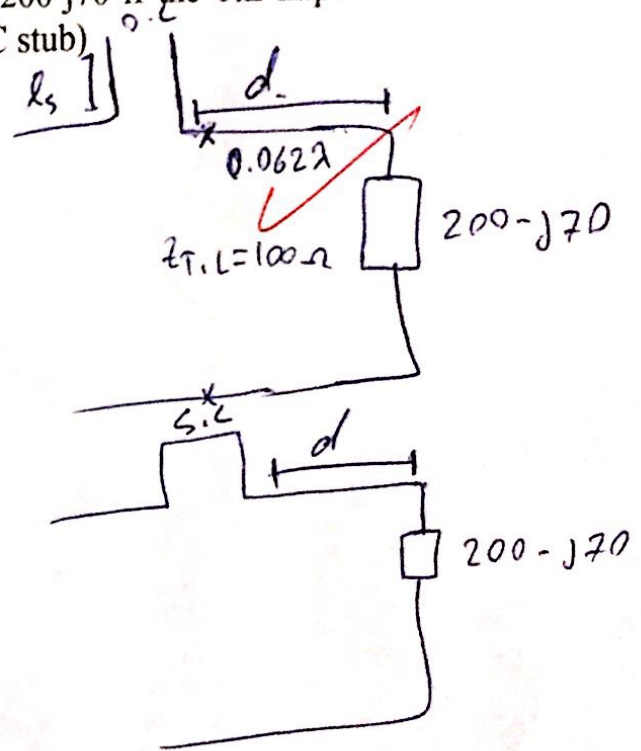
$$+0.9j \rightarrow 0.117\lambda$$

for short circuit stub

$$l_s = 0.117 \quad \checkmark$$

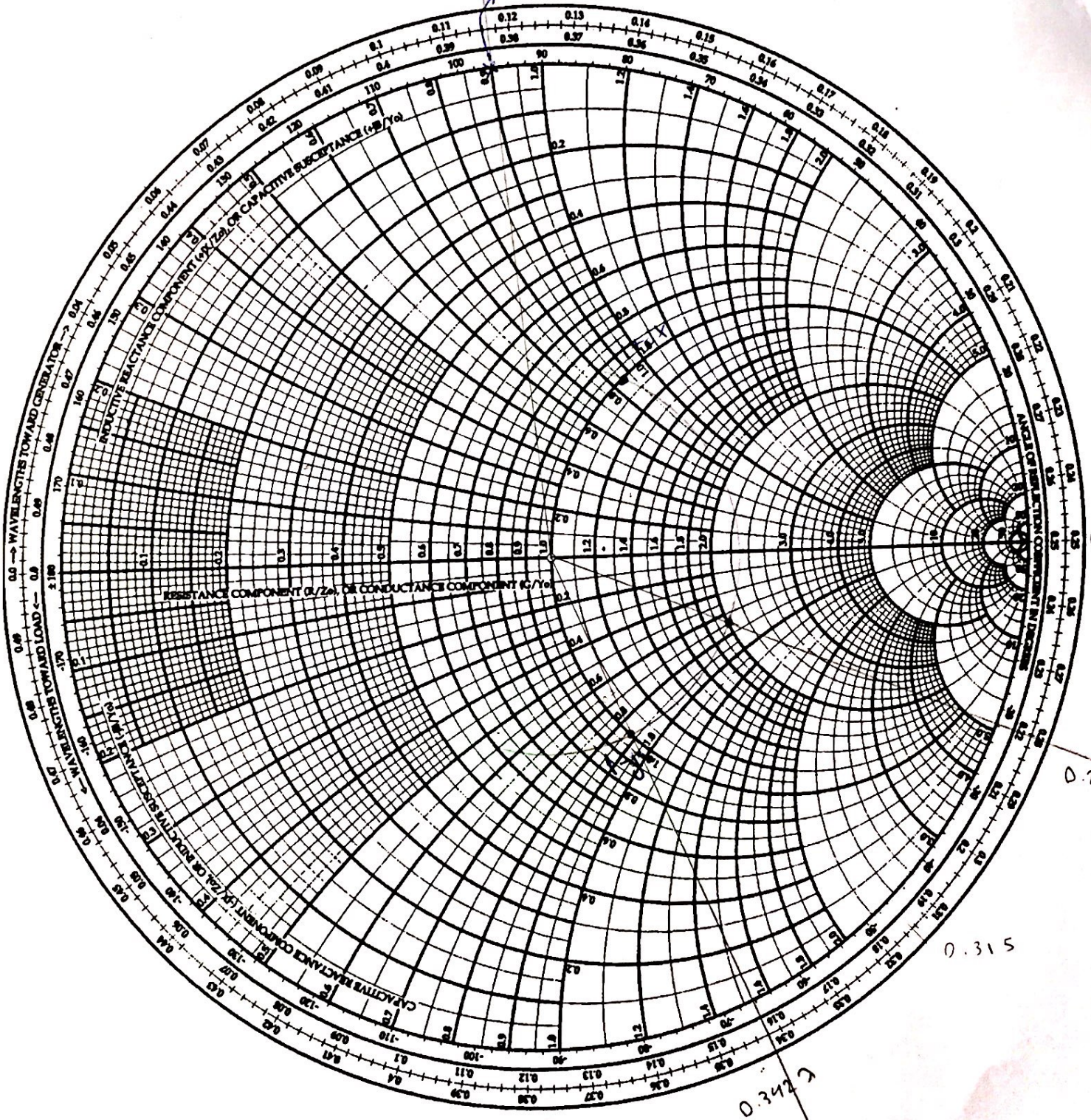
for open circuit stub

$$l_s = 0.25\lambda + 0.117\lambda = 0.367\lambda \quad \checkmark$$



8

E
=
H.



0.1177
0.9

0.29

0.315

0.29