Department of Electrical Engineering
Faculty of Engineering & Technology
University of Jordan

Course Title: Probability & Random Variables	Exam: 2^{nd} Exam	Date: Apr 13/2016
Course No.: 0903321	Semester: 2^{nd} Term 2015-2016	Time Period: 1:30 Hr.

Instructor:
Dr. Ahmad Atieh

Q. 1	Q. 2	Q. 3	Q. 4	Total /30
2.5	0.5	8.5	2	13.5 / 30

Student Name: [redacted]

Student Number:

Section: 2 11:00 – 12:30

$$G(u) = \int_{\alpha(u)}^{\beta(u)} H(x,u)\, dx$$

$$\frac{dG(u)}{du} = H[\beta(u),u]\frac{d\beta(u)}{du} - H[\alpha(u),u]\frac{d\alpha(u)}{du} + \int_{\alpha(u)}^{\beta(u)} \frac{\partial H(x,u)}{\partial u} dx$$

$$\frac{d\, ln\,[\,u(x)]}{dx} = \frac{1}{u(x)}\,\frac{du(x)}{dx}$$

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Q1) (5 marks)

The envelope of a noise filtered by a bandpass filter in a radar communication system has a Rayleigh probability density function given by

$$f_X(x) = \begin{cases} \frac{2}{b}(x-a)e^{-(x-a)^2/b} & x \geq a \\ 0 & x < a \end{cases}$$

Assume $a = 0$, $b = 2\sigma_X^2$, and σ_X^2 represents the power of filtered noise. If the filtered noise signal is detected by a diode with square-law characteristics such that a new variable $Y = cX^2$ is created, where c is constant.

Find the probability density function of Y.

$\boxed{\text{Non-monotoinc}}$

② $y = c x^2 \longrightarrow x = \pm\sqrt{\frac{y}{c}}$

$$f(x) = \frac{2}{2\sigma_x^2}(x)e^{-(x)^2/2\sigma_x^2} = \frac{2x}{2\sigma_x^2}e^{-(x)^2/2\sigma_x^2}$$ $\boxed{x \geq 0}$

①

$$f_Y(y) = \sum \frac{f_X(x)}{\left|\frac{T(x)}{dy}\right|_{x=x_i}}$$

$$f_Y(y) = \frac{x\, e^{-(x)^2/2\sigma_x^2}}{2\sigma_x^2\sqrt{cy}} + \cancel{\frac{x\, e^{-(x)^2/2\sigma_x^2}}{2\sigma_x^2\sqrt{cy}}}$$

substitute

$$f_Y(y) = \frac{2x\, e^{-(x)^2/2\sigma x^2}}{2\sigma_x^2\sqrt{cy}}$$

$$= \frac{x\, e^{-(x)^2/2\sigma x^2}}{\sigma_x^2\sqrt{cy}}$$

$$\frac{x/\sigma_x^2\, e^{-(x)^2/2\sigma_x^2}}{|2cx_1|} + \frac{x/\sigma_x^2\, e^{-(x)^2/2\sigma_x^2}}{|2cx_2|}$$

$2c\sqrt{\frac{y}{c}}$

2/5

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Q2) (5 marks)

The characteristic function of a Poisson random variable X is given by

$$\phi_X(\omega) = e^{\lambda(e^{j\omega}-1)}$$

Find the mean and variance of the random variable?

$$\boxed{M_n = (-j)^n \frac{d^n \phi_x(w)}{dw^n}\Big|_{w=0}}$$

$$m_1 = (-j)^1 \frac{d(e^{\lambda(e^{jw}-1)})}{dw}$$

$$m_1 = (-j) \frac{d\, e^{\lambda(e^{jw}-1)}}{dw}\Big|_{w=0} = (-j) \frac{d e^{(\lambda e jw - \lambda)}}{dw}\Big|_{w=0}$$

$$\frac{d \ln(u(x))}{dx} = \frac{1}{u(x)} \cdot \frac{du(x)}{dx}$$

take ln

$$\Rightarrow (-j) \frac{d\, \lambda e^{jw} - \lambda}{dx}$$

$$= \cancel{(-j)} \frac{1}{\lambda e^{jw} - \lambda} \cdot \left(\frac{\lambda}{j}\right)\Big|_{w=0} = \frac{1}{e^{j}} = e^{-j}$$

$$M_2 = (-j)^n \frac{d^2 e^{\lambda(e^{jw}-1)}}{dw^2} = (-j)^2 \frac{d\, e^{-j}}{dx} = \text{ZERO}$$

0.5

0

$$\frac{d e^{f(x)}}{dx} = f'(x)\, e^{f(x)}$$

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Q3) (10 marks)

The following sequence of letters A, B, C, and D is generated by a musical instrument.

AAAAAAAAAAAAAAABAAAAABDAAAAAAAACCAADDA AAAAAAAABBBB = 50 letter

Typically Data compression for the given sequence is achieved using Huffman code mapping:

A ⟶ 0
B ⟶ 10
C ⟶ 110
D ⟶ 111

$A = \frac{39}{50} = 0.78$
$B = \frac{6}{50} = 0.12$
$C = \frac{2}{50} = 0.04$
$D = \frac{3}{50} = 0.06$

Find the following:

PDF

a) The probability density function for the generated sequence?
b) The mean code length for the produced sequence (bits/letter)?
c) Calculate the entropy of the musical instrument?

$\hookrightarrow H_{entropy} = -\sum_{i=0}^{L} P(x_i) \log_2(P(x_i))$

3

$f_x(x) = P(x_i)\,\delta(x - x_i)$

ⓐ $f_x(x) = 0.78\,\delta(x-0) + 0.12\,\delta(x-10) + 0.04\,\delta(x-110) + 0.06\,\delta(x-111)$

$\hookrightarrow$ from mappings

ⓑ $A = \frac{39}{50}$, $C = \frac{6}{50}$

$B = \frac{12}{50}$, $D = \frac{9}{50}$

$\boxed{= 1.32}$

2

ⓒ $H_{entropy} = -\sum P(x_i) \log_2(P(x_i))$

$= -\left((0.78)\log_2(0.78) + (0.12)\log_2(0.12) + (0.04)\log_2(0.04) + (0.06)\log_2(0.06)\right)$

$= -((-0.28) + (-0.367) + (-0.186) + (-0.244))$

$= 1.077$

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Q4) (10 marks)

Let $X_1, X_2, \dots, X_n$ are independent random variables, each having a uniform probability density function over (0, 1). Let M = maximum ($X_1, X_2, \dots, X_n$).

1) Show that the probability distribution function $\mathbf{F_M(\dots)}$ is given by:

$$\mathbf{F_M(\dots) = x^n}, \qquad 0 \le x \le 1$$

2) What is the probability density function for M?

$$f_x(x) = \begin{cases} \frac{1}{b-a} & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

① $F_m(\dots) = (X_1)(X_2)\cdots(X_N) = X^n$

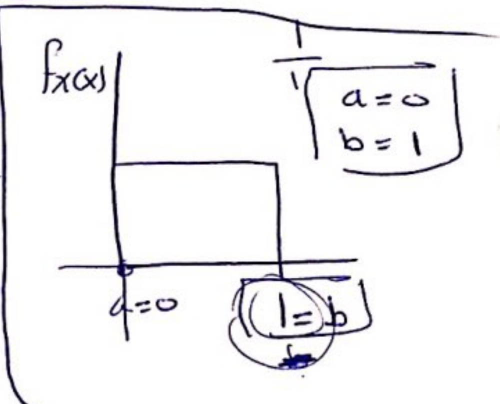
→ independent R.V

$\boxed{F_m(x) = X}$

$F_M(x) = \int_0^1 \frac{1}{b-a}\,dx = \boxed{x} = \boxed{X}$

2/3

→ $F_m(\dots) = (X)(X)\cdots(X) = \boxed{X^n}$



② $f_m(\dots) = \frac{d F_m(\dots)}{dx}$

$$\boxed{f_m(\dots) = \begin{cases} n X^{(n-1)} & \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}}$$

$XX = x^2 X = X^3 \dots \rightarrow \boxed{X^n}$

$$F_x(x) = \begin{cases} X, & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

1.3

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