Department of Electrical Engineering
Faculty of Engineering & Technology
University of Jordan

Course Title: Probability & Random Variables	Exam: 1^{st} Exam	Date: Mar 07/2016
Course No.: 0903321	Semester: 2^{nd} Term 2015-2016	Time Period: 1:00 Hr.
Instructor: Dr. Ahmad Atieh		

Q. 1	Q. 2	Q. 3	Q. 4 (Bonus)	Total /20
1.5	1	2.5	0.5	5.5/20

Student Name: [redacted]

Student Number: 0[redacted]

Section: 2 Mon, Wed
11:00 - 12:30

$$Q(x) \cong \frac{1}{(1-a)\,x + a\sqrt{x^2+b}} \frac{e^{x^2/2}}{\sqrt{2\pi}}$$

Where a = 0.339 and b = 5.510

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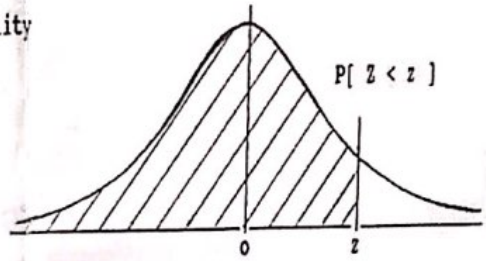
STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z i.e.

$$P[\, Z < z \,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

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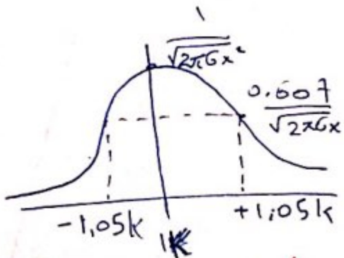
Q1) (7 marks)

A factory produces resistors with 1 kΩ nominal values. It was clear that the probability of actual measured resistors follow Gaussian distribution. If about 60.7% of produced resistors have values within ±1.05 kΩ. Assume that 1,000,000 resistors are used in the analysis and only measured resistors within ±3σ are shipped to customers.

1500

a) Calculate the tolerance (Prob.) of shipped resistors?
b) The total number of shipped resistors?
c) The probability of producing resistors with no more than 1.1 kΩ?

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma_x} e^{\frac{-(x-a_x)^2}{2\sigma_x^2}}$$



(a)

$\sigma_x = 50$ ✓

$a_x = 1000$ ✓

~~$\pm 3(50) = \pm 150$ are shipped~~

P{shipped resistors} = Q(x)

$$F\left(\frac{1150-1000}{50}\right) = F(3)$$

$Q(3) = 10.967$

6/7

1×10^{-6} *

$$Q(x) = \frac{1}{(1-a)x + a\sqrt{x^2+b}} \cdot \frac{e^{x^2/2}}{\sqrt{2\pi}}$$

~~$a = 0.339$~~
~~$b = 5.510$~~

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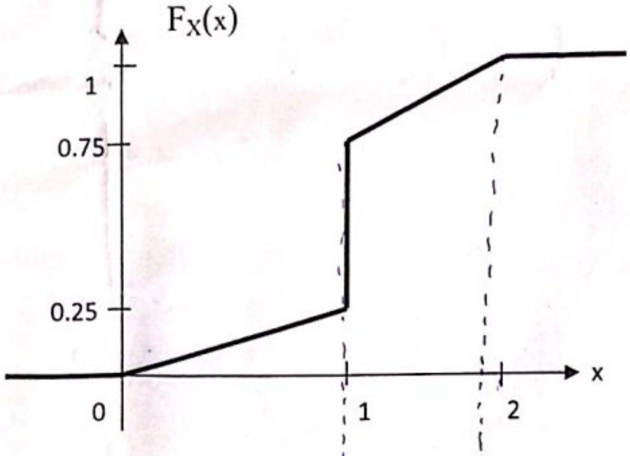
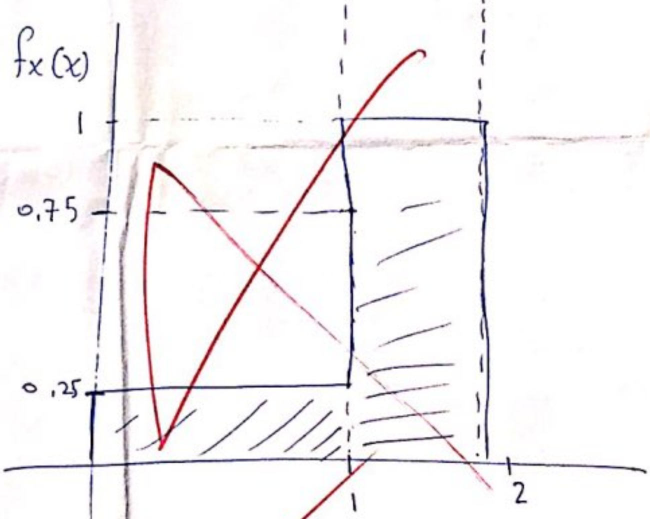
Q2) (5 marks)

a) Sketch the probability density function (pdf) for the cumulative distribution function (cdf) shown in the figure below?

b) Validate the pdf?

$f_x(x) = \frac{d F_x(x)}{dx}$





PDF

① $f_x(x) \geq 0$ ✓ from the graph

② $\int_{-\infty}^{\infty} f_x(x)\, dx =$

1.0/5

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Q3) (8 marks)

~~PDF~~ CDF

a) Verify the validity of the following probability distribution function?

b) If it is not valid, go to part (c)? If it is valid go to part (d).

c) Modify the distribution function to make it valid?

d) Sketch its probability **density** function? PDF

e) Find the P [$-\frac{1}{2} \le X < \frac{1}{2}$]

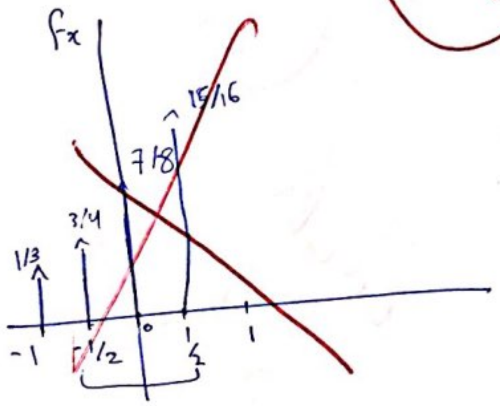
$$F_x[x_i] = \begin{cases} 0 & x < -1 \\ \frac{1}{3} & -1 \le x < -\frac{1}{2} \\ \frac{3}{4} & -\frac{1}{2} \le x < 0 \\ \frac{7}{8} & 0 \le x < \frac{1}{2} \\ \frac{15}{16} & \frac{1}{2} \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

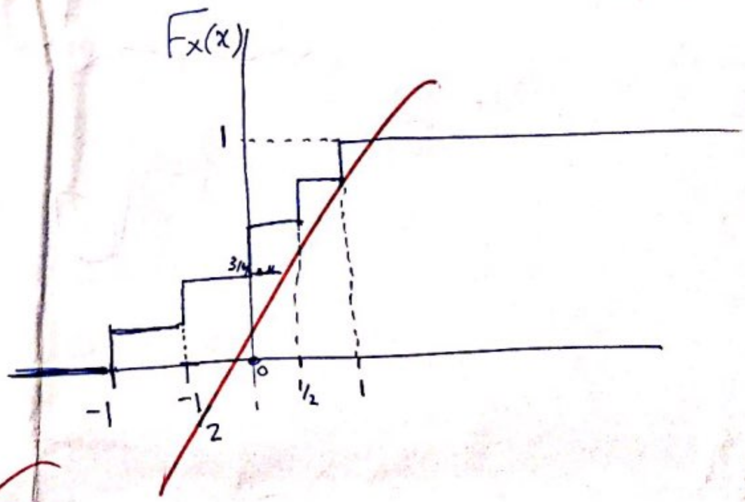
(a)
① $F_x(-\infty) = 0$
② $F_x(\infty) = 1$
③ it's increasing function
④ $F(x^+) \neq F(x^-)$

none decreasing

(b) it's Valid

(d)





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(e) $P[-\frac{1}{2} \le X \le \frac{1}{2}]$

=

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Q4) Bonus (3 marks)

At a certain military installation, six similar radars are placed in operation. It is known that a radar's probability of failing to operate before 500 hours of "ON" have accumulated is 0.06. What is the probability that; before 500 hours have elapsed, only one radar will fail?

~~P{500}~~

P{~~Fail < 500~~}

N # trials 6
k # success 1
p = 0.06

$P(\text{Fail}) = 0.06$

$P(\text{Success}) = 0.94 \rightarrow$ after 500

$P\{\text{one radar will fail before 500}\} = \binom{N}{k} p^k (P-1)^{N-k}$

~~$\binom{6}{5}(0.94)^5(0.06)$~~

$\rightarrow P\{\text{one radar will fail before 500}\} = \binom{6}{5}(0.94)^5(0.06)$

$\simeq 0.26421$

$\binom{6}{1} 0.06^1 0.94^5$

wrong

0.5/3

+0.5

6

4

64

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