* اللهم لا سهل الاما جعلته سهلاً وأنت تجعل الحزن إذا شئت سهلاً *

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Eng. Math II (0301302)
Second Exam 20/04/2015 2:00-3:00

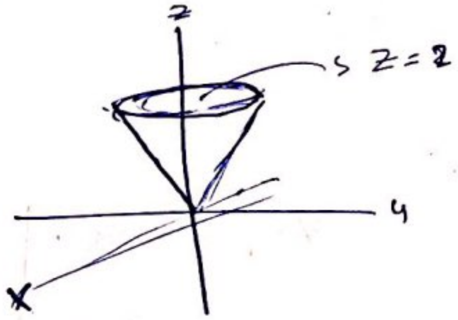
Name: [redacted] Number: [redacted]

Instructor: د. أحمد عبدالله Section and Time: اثنين أربعاء ~~[illegible]~~ 11:00 to 12:30

Q1. (5 points) Using the Divergence Theorem, evaluate $\iint_S \vec{F} \cdot \vec{n}\, dA$, where $\vec{F} = \langle x, x^2, z^2 \rangle$ and S is the surface of the cone $x^2 + y^2 \leq z^2$, $0 \leq z \leq 2$. → Not closed

2.5

$$\iiint_T \operatorname{Div} \cdot F \, dA = \iint_S \vec{F} \cdot \vec{n}\, dA$$



$$\operatorname{Div} \vec{F} = 1 + 0 + 2Z$$

$$= 1 + 2Z$$

$$\int_0^{2\pi}\int_0^{2}\int_0^{2} (1 + 2Z)\, r\, dz\, dr\, d\theta$$

$$\iiint_0^2 r + 2rZ \quad dz\, dr\, d\theta$$

$$\iint 2r + 4r \quad dr\, d\theta$$

$$\iint$$

$$\boxed{\iiint_{S \cup S_2} \qquad - \iint_{S_2}}$$

Q2. (5 points) Using Stokes's Theorem, evaluate $\oint_C \vec{F} \cdot \vec{r}\, ds$, where $\vec{F} = \langle y^2, x^2, z + x\rangle$ and C is the boundary curve around the triangle with vertices $\underset{A}{(0,0,0)}, \underset{B}{(1,0,0)}, \underset{C}{(1,1,0)}$.

$$\oint_c \vec{F}\cdot\vec{r}\, ds = \iint \text{Curl}(F)\cdot n\, dA$$

2.5

$$\text{Curl}\,F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ y^2 & x^2 & z+x \end{vmatrix} \quad 0 - (1-0)\hat{j} + (2x+2y)\hat{k} = 0\hat{i} - 1\hat{j} + (2x+2y)\hat{k}$$

$$N = AB \times AC = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0\hat{i} - 0\hat{j} + 1\hat{k}$$

$$[0, 0, 1]$$

$$\oint \vec{F}\cdot\vec{r} = \iint [0, 1, 2x+2y] \cdot [0,0,1]\, dA$$

$x = r\cos\theta$

$y = r\sin\theta$

$$= \iint 2x + 2y \Rightarrow 2\iint x+y \Rightarrow 2\iint r\cos\theta + r\sin\theta\; r\,dr\,d\theta$$

$$2\int_0^{2\pi}\int_0^{1} r^3(\cos\theta + \sin\theta)\, dr\, d\theta = \frac{r^4}{4} =$$

$$\frac{1}{2}\int_0^{2\pi} \cos\theta + \sin\theta\, d\theta$$

$$\frac{1}{2}\left[\cancel{\sin\theta} - \cos\theta\right]_0^{2\pi} = \boxed{\frac{-1}{2}}$$

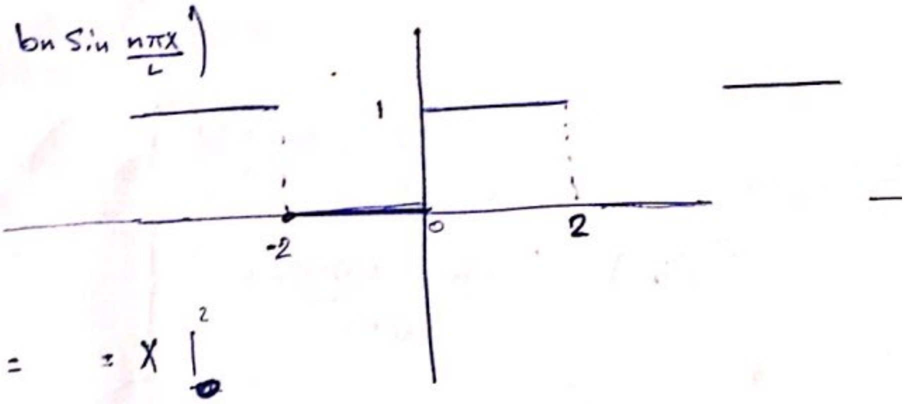
4.5

Q3. (a) (5 points) Show that the Fourier series of $f(x) = \begin{cases} 0, & -2 < x < 0 \\ 1, & 0 < x < 2 \end{cases}$ is $\frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \sin \frac{(2n-1)\pi x}{2}$

Period = 4

$\boxed{L = 2}$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$



$$a_0 = \frac{1}{4}\int_0^2 1 \, dx = x \Big|_0^2 \quad (2+0)$$

$$a_0 = 1/2$$

$$a_n = \frac{1}{2}\int_0^2 \cos\frac{n\pi x}{2} = \frac{1}{2}\left[\frac{2\sin\frac{n\pi x}{2}}{\pi n}\right]_0^2 = 0$$

$$b_n = \frac{1}{2}\int_0^2 \sin\frac{n\pi x}{2}\, dx = \frac{2}{2}\left[\frac{\cos 2\pi n}{\pi n} - 1\right] \quad \text{odd } n \quad (-1)^n$$

$n \to (2n-1)$

$$b_{2n-1} = \frac{2}{(2n-1)\pi}$$

$$\boxed{f(x) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin\frac{(2n-1)x\pi}{2}} \to L$$

(b) (2 points) Find $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1}$.

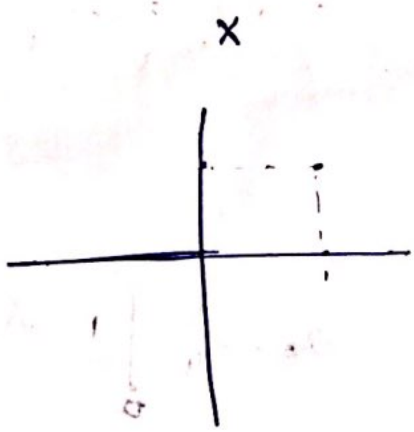
0

$x =$

Q4. (5 points) Show that

$$\int_0^\infty \frac{\sin w - w\cos w}{w^2} \sin xw \, dw = \begin{cases} \pi x/2, & 0 < x < 1 \\ \pi/4, & x = 1 \\ 0, & x > 1 \end{cases}$$

0



$$B(w) = \frac{\sin w - w\cos w}{w^2}$$

Q5. (3 points) Find the Fourier Sine Transform of $f(x) = x \sin x^2$. You can use some of the following results:

$$\mathcal{F}_c(f''(x)) = -w^2 \mathcal{F}_c(f(x)) - \sqrt{\tfrac{2}{\pi}} f'(0),$$
$$\mathcal{F}_c(f'(x)) = w \mathcal{F}_s(f(x)) - \sqrt{\tfrac{2}{\pi}} f(0),$$
$$\mathcal{F}_s(f'(x)) = -w \mathcal{F}_c(f(x))$$
$$\mathcal{F}_c(\sin x^2) = \tfrac{1}{\sqrt{2}} \cos\left(\tfrac{w^2}{4} + \tfrac{\pi}{4}\right).$$

$$F_s\{f(x)\} = \sqrt{\frac{2}{\pi}} \int_0^\infty \underbrace{\underbrace{f(x)}_{odd} \underbrace{\sin wx}_{odd}}_{even} \, dx = ZERO$$

~~Had to solve~~

~~Some difference~~

$$\cancel{f(x) = x \sin x^2}$$

$$\cancel{f'(x) = \sin x^2 + 2x\cos x^2}$$

$$\cancel{f'' = 2x \sin x^2 +}$$

Sine Transform for odd function

we have an even function

So it equals ZERO