19

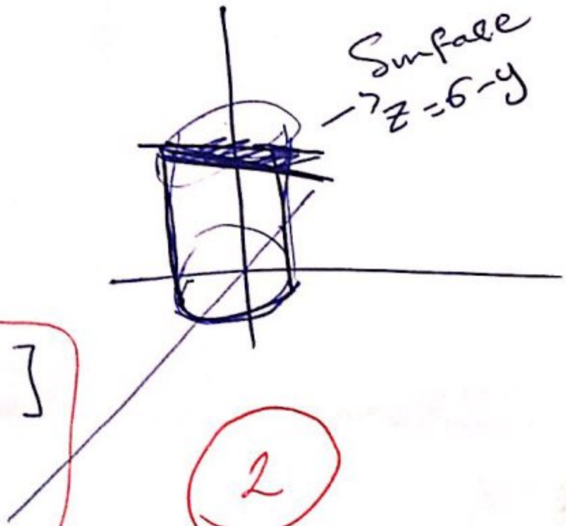
**The University of Jordan**
**Mathematics Department**
**Mathematics for Engineering II . Second Exam**

**Student Name:.** [redacted] **Student number:....** [redacted]

**Lecture time:....** 11 - 12 **........**

**Q1. (6 points) Evaluate** $\iint_S (x+y+z)\,dA$ , $\rightarrow \iint F(r_{u,v}) \cdot \vec{n}$

**where S is the part of the plane** $z = 6 - y$ **that lies in the cylinder** $x^2 + y^2 = 4$ .




$x = 2\cos u$

$y = 2\sin u$

$z = 6 - 2\sin u$

$\vec{N} = \operatorname{grad}(z + y - 6) = [0, 1, 1]$

$|\vec{N}| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2}$

2

$$\int_0^{2\pi}\int_0^{6-2\sin u} (2\cos u + 2\sin u + 6 - 2\sin u) \cdot \sqrt{2}\, du\, dv = \alpha$$

I can't Complete integration

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Q2. (7 points) Use the divergence theorem to evaluate $\iint_S \bar{F} \cdot \vec{n} dA$ ,

where $F(x,y,z) = [z-x, x-y, 2y-z]$ and
S is the boundary of the region between the spheres of radius 2 and 4 centered at the origin.

$$\iint_S \vec{F} \cdot \vec{n}\, dA = \iiint_V \operatorname{div} \vec{F}\, dv$$


$$\operatorname{div} F = \frac{\partial (z-x)}{\partial x}, \ \partial x - y$$

$$\operatorname{div} F = -1 - 1 - 1 = \underline{\underline{-3}}$$

$$dv = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

≈ 7

4×4 = 16×4 = 64

3×64 = 180 192

$$\Rightarrow \underline{\underline{-3}} \int_0^{2\pi}\int_0^{\pi}\int_{\rho=2}^{4} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$\Rightarrow -3 \int_0^{2\pi} d\theta \int_0^{\pi} \sin\phi \, d\phi \int_2^4 \rho^2 \, d\rho \Rightarrow -3(2\pi) * (-\cos\phi)\Big|_0^{\pi} * \left(\frac{\rho^3}{3}\right)\Big|_2^4$$

$$(-1 - 1) * \left(\frac{8}{3} - \frac{64}{3}\right)$$

$$\Rightarrow -3(2\pi) * (-2) * \left(\frac{24 - 192}{9}\right) = \boxed{12\pi * \left(\frac{24-192}{9}\right)}$$

$$12\pi \left( \frac{\rho^3}{3}\Big|_2^4 \right)$$

**Q3. (6 points)** **Show that the integral represents the indicated function**

$$\int_0^\infty \frac{w-\sin w}{w^2}\sin wx\,dw = \begin{cases}\frac{\pi(1-x)}{2}, & 0<x<1,\\ 0, & x\ge 1.\end{cases}$$

$B(w) = \frac{2}{\pi}\int_0^\infty f(x)\sin wx\,dx$

Fourier Sine integral

5.5

$$B(w) = \frac{2}{\pi}\int_0^1 \frac{\pi(1-x)}{2}\sin wx\,dx$$

$$B(w) = \frac{\cancel{2}\cancel{\pi}}{\cancel{\pi}\cancel{2}}\int_0^1 (1-x)\sin wx\,dx$$

by Parts

| | |
|---|---|
| $(1-x)$ + | $\sin wx$ |
| $-1$ − | $-\frac{1}{w}\cos wx$ |
| $0$ + | $-\frac{1}{w^2}\sin wx$ |

$$B(w) = -\frac{(1-x)\cos wx}{w}\Big|_0^1 + \frac{\sin wx}{w^2}\Big|_0^1$$

$$B(w) = \frac{\cos wx - x\cos wx}{w}\Big|_0^1 + \frac{\sin wx}{w^2}\Big|_0^1$$

Zero

$$\frac{1}{w}\left(-1 + (\cancel{\cos x - \cos x})\right) + \left(-\frac{\sin w}{w^2}\right)$$

$$B(w) = -\frac{1}{w} - \frac{\sin w}{w^2} = \frac{w^2 - w\sin w}{w^3} = \frac{w-\sin w}{w^2}$$

So Fourier Sine Transform equals

$$\int_0^\infty \frac{w-\sin w}{w}\sin wx\,dw$$

**Q4.(3 points)** Find $F\{f(x)\}$ for $f(x)=\begin{cases} xe^{-x}, & x>0, \\ 0. & x<0. \end{cases}$.

Use $F\{g(x)\}=\dfrac{1}{\sqrt{2\pi}(1+iw)}$ for $g(x)=\begin{cases} e^{-x}, & x>0, \\ 0. & x<0. \end{cases}$ and $F\{f'\}=iwF\{f\}$.

So, $g'(x) = xe^{-x}$

$g(x) = -e^{-x} = f(x)$

0.5

$$F = iw\{-e^{-x}\} = iw \frac{-1}{\sqrt{2\pi}(1+iw)}$$

$$= \frac{-jw}{\sqrt{2\pi}(1+jw)}$$

$a_0$ $a_n$ $b_n$

**Q5.(3 points)** Let $\frac{\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4}{n^2}(-1)^n\cos(nx)-\frac{2}{n}(-1)^n\sin(nx)$ be the Fourier series for $f(x)=x^2+x$, $-\pi<x<\pi$. $f(x+2\pi)=f(x)$ $\boxed{L=\pi}$

**Evaluate** $\sum_{n=1}^{\infty}\frac{1}{n^2}$

zero

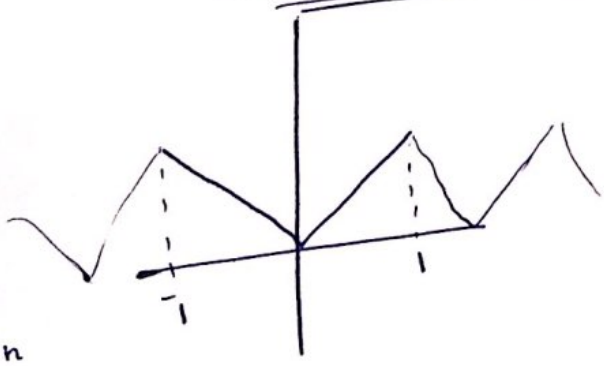
Q6.(5 points) Find the Fourier series for $f(x)=|x|,\ -1<x<1$

$f(x+2)=f(x)$.

& So On !!

$2L = 2$

$\boxed{L = 1}$




it's an even function

so $b_n = ZERO$

now, evalute $a_0$ & $a_n$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx \quad , \quad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$= \frac{2}{2}\int_0^1 f(x)\cos\frac{n\pi x}{L}\,dx$$

$$a_0 = \frac{1}{2}\left[\int_{-1}^{0} -x\,dx + \int_0^1 x\,dx\right] = \frac{1}{2}\left[-\frac{x^2}{2}\Big|_{-1}^{0} + \frac{x^2}{2}\Big|_0^1\right]$$

$$= \frac{1}{2}\left[\frac{+1}{2} + \frac{1}{2}\right] = \frac{+1}{2}$$

4

$$a_n = \cancel{2}(1)\int_{-1}^{0} -x\cos n\pi x\,dx + \int_0^1 x\cos n\pi x\,dx$$

$$a_n = -\frac{x\sin n\pi x}{n\pi}\Big|_{-1}^{0} + \frac{\cos n\pi x}{n^2\pi}\Big|_{-1}^{0} + \frac{x\sin n\pi x}{n\pi}\Big|_0^1 + \frac{\cos n\pi x}{n^2\pi^2}$$

$\boxed{-\cos n\pi = (-1)^{n+1}}$

$$= \cancel{\frac{0\ \sin n\pi}{n\pi}} + \frac{-(\cos n\pi) - 1}{n^2\pi} + \cancel{\frac{0+\sin n\pi}{n\pi}}\ (-1)^n$$

$$+ \frac{1-\cos n\pi}{n^2\pi^2}$$

$$-\frac{(-1)^{n+1}+1}{n^2\pi} + \frac{1-(-1)^n}{n^2\pi} = \frac{2+2(-1)^{n+1}}{n^2\pi^2}$$

(2) by parts

| | |
|---|---|
| $x$ (+) | $\cos n\pi x$ |
| $1$ (−) | $\frac{\sin n\pi x}{n\pi}$ |
| $0$ | $-\frac{\cos n\pi x}{n^2\pi^2}$ |

(1) by parts

| | |
|---|---|
| $-x$ (+) | $\cos n\pi x$ |
| $-1$ (−) | $\frac{\sin n\pi x}{n\pi}$ |
| $0$ | $-\frac{\cos n\pi x}{n^2\pi^2}$ |