* اللهم لا سهل إلا ما جعلته سهلا" وأنت تجعل الحزن إذا شئت سهلا" *

The University of Jordan
Mathematics Department

Eng. Mathematics (II)

13

First exam
5/11/2015

Student name: [redacted]

Student number: [redacted]

Lecture time: 11:00 – 12:00

Q1.(3 points) Given $\vec{F}(x,y,z)=\left[x^2y, xy^2, 2xyz\right]$ and $f(x,y,z)=\dfrac{xy}{z^2}$

Complete:

a) $\nabla\cdot(\nabla\times\vec{F}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & xy^2 & 2xyz \end{vmatrix}$ $\nabla\cdot\left((2xz-0)\hat{i} - (2yz-0)\hat{j} + (y^2-x^2)\hat{k}\right)$ $\boxed{0 = ZERO}$

$= 2z - 2z$

Good

b) $\nabla(\nabla\cdot\vec{F}) = \nabla\left[\cancel{2xy} + \cancel{2xy} + \cancel{2xy}\right]$ $2xy + 2xy + 2xy$ $\nabla(6xy)$ $6y + 6x$ $\boxed{\cancel{6y + 6x}}$

2.5

c) $\nabla\cdot(\nabla f) = \nabla\left[\dfrac{y}{z^2} + \dfrac{x}{z^2} + \dfrac{-2xy}{z^3}\right] = 0 + 0$ $\boxed{\cancel{\dfrac{+2xy}{z^3}}}$ $\boxed{\dfrac{+6xy}{z^4}}$

Q2. (4 points) Evaluate $\oint_C \vec{F}\cdot d\vec{r}$, where C is the ellipse $\dfrac{(x-1)^2}{16}+\dfrac{(z-2)^2}{9}=1$ with $y=2$ and $\vec{F}(x,y,z)=[y, zx, x]$

$zx = 2 + 3\sin t + 8\cos t + 12\cos t\sin t$

$C = r(t)\ [1+4\cos t,\ 2,\ 2+3\sin t]$

$r'(t) = [-4\sin t,\ 0,\ 3\cos t]$

4

$\oint \vec{F}\cdot d\vec{r} = \int_C F(r(t))\cdot r'(t)\,dt.$

$\int_0^{2\pi} [2,\ 2+3\sin t + 8\cos t + 12\cos t\sin t,\ 1+4\cos t]\cdot[-4\sin t, 0, 3\cos t]\,dt$

$= \int_0^{2\pi} [-8\sin t + 0 + 3\cos t + 12\cos^2 t]\,dt = \cancel{+8\cos t}\Big|_0^{2\pi \to \text{zero}} + \cancel{3\sin t}\Big|_0^{2\pi \to \text{zero}} + \dfrac{t}{2}\Big|^{2\pi} + \dfrac{\cos 2t}{2}$

$12\int \dfrac{1}{2} + \dfrac{\sin 2t}{2}$ $\qquad \pi$

$= \pi + \cancel{\ldots}$ $\qquad = 12\pi$

Q3.(5 points) Show that $\vec{F}$ is conservative and evaluate $\int_C \vec{F}\cdot d\vec{r}$, where

$\vec{F}(x,y,z) = [e^y, xe^y + e^z, ye^z + 5]$

C is the line segment from (0,2,0) to (4,0,3)

$$\text{Curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ e^y & xe^y + e^z & ye^z + 5 \end{vmatrix} = (\cancel{e^z - e^z})\hat{i} - (\cancel{0 - 0})\hat{j} + (\cancel{e^y - e^y})\hat{k} = 0$$

Zero Zero Zero

Curl $\vec{F} = \vec{0}$ So it is a Conservative field

~~line Segment → $r(t) = r_0 + (r_1 - r_0)t$~~

~~$[0,2,0] + [4,-2,3]t$~~

~~$t \in [0,1]$~~

~~$r(t) = [4t, 2-2t, 3t]$~~

~~$r'(t) = [4, -2, 3]$~~

Since it's Conservative $F = \nabla f$

≈ 5

$$\vec{F} = \underbrace{\frac{df}{dx}}_{e^y}\hat{i} + \underbrace{\frac{df}{dy}}_{xe^y + e^z}\hat{j} + \underbrace{\frac{df}{dz}}_{ye^z + 5}\hat{k}.$$

So → $f(x,y,z) = \int e^y\,dx = xe^y + g(y,z)$

$\frac{df}{dy} = \int \cancel{xe^y} + g'(y,z) = \int \cancel{xe^y} + e^z = ye^z + l(z)$

$\frac{df}{dz} = \int \cancel{ye^z} + l'(z) = \int \cancel{ye^z} + 5 = 5z + k$

$f(x,y,z) = xe^y + ye^z + 5z + k$

$$\int_{(0,2,0)}^{(4,0,3)} F\cdot dr = f(4,0,3) - f(0,2,0) = \boxed{21}$$

Calculations

$4e^0 + 0 + 15$

19

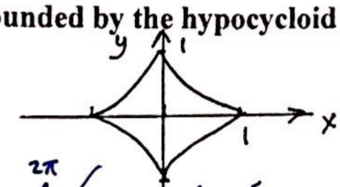
$-2 +$

21

Q4. a) (3 points) Find the area of the region bounded by the hypocycloid

$x = \cos^3 t$ $\quad 0 \le t \le 2\pi$

$y = \sin^3 t$



Area of region: $\frac{1}{2} \oint -y\,dx + x\,dy$ $= \frac{1}{2} \oint_0^{2\pi} \sin^3 t\, dt \;;\; \cos^3 t\, dt$

$z = \sin^3 t$ | $u = \cos t$

$dz = \cos t$ | $du = -\sin t\, dt$

$= \frac{1}{2} \int_0^{2\pi} \sin^3 t\, dt - \cos^3 t\, dt$

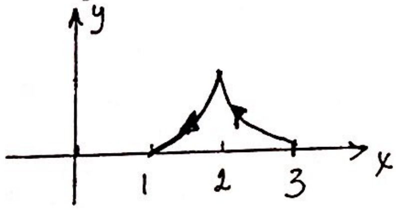
0.5

b)(5 points) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x,y) = \langle \underbrace{\sin^2 x - 3y}_{F_1}, \underbrace{4x + e^{y^2}}_{F_2} \rangle$

C is the upper half of the hypocycloid

$x = 2 + \cos^3 t$ $\quad 0 \le t \le \pi$

$y = \sin^3 t$



* Using Green's thm.

$\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = \underbrace{4-(-3)}_{} = \underline{\underline{7}}$

$\int_0^{\pi} 7\, dt = \boxed{7\pi}$

1