University of Jordan
Elect. Eng. Dept.
Second Exam

Control Systems 903441 Tuesday 2/8/2016
Student Name: [redacted] Reg. No. 0137524 Mark out of 30 **20**

Answers should be written in ink **Exam Duration: 70 min**

Q1) A second order system is described by the following system of differential equations

$$\frac{dx}{dt} = y \quad ; \quad x(0) = (0) = 0$$

$$\frac{dy}{dt} = -5x - 2y + 10\,u(t)$$

$sX(s) = Y(s) \rightsquigarrow X(s) = \frac{Y(s)}{s}$

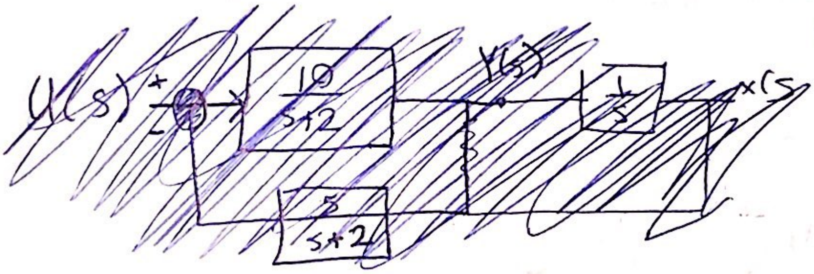
$sY(s) = -5x(s) - 2y(s) + \frac{10}{s}$

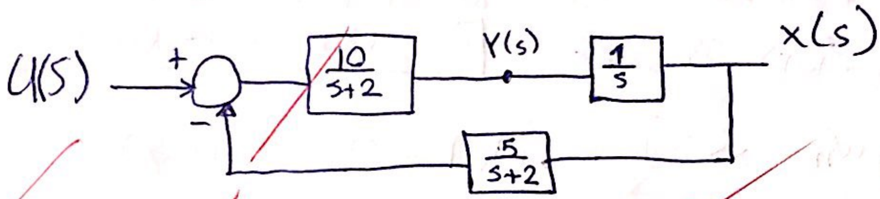
i) Obtained a detailed block diagram with $X(s)$ as output and $U(s)$ as set value, clearly indicating on the diagram where to measure $Y(s)$.

$sY(s) + 5x(s) + 2y(s) = \frac{10}{s}$

$(s+2)\,y(s) = \frac{10}{s} - 5x(s)$

$y(s) = \frac{10}{s+2} - \frac{5x(s)}{s+2}$ ($u(s)$)





3

ii) By reducing the block diagram or otherwise, determine the **closed loop** transfer function.

$$\frac{X(s)}{U(s)} = \frac{G(s)}{1+G(s)H(s)} \Rightarrow \frac{10/s^2+2s}{1+\frac{10}{s^2+2s}\cdot\frac{5}{s+2}} = \frac{10}{s^2+2s+10+5s+20}$$

$$\boxed{= \frac{10\,(s+2)}{s^2+7s+20}}$$

1

iii) Determine ω_n, ζ, and ω_d of the closed loop system. Hence, or otherwise determine the **unit step** response of the closed loop system by any method you know.

$\boxed{\omega_n = \sqrt{20}}$

$7 = 2\zeta\omega_n \rightarrow \boxed{\zeta = 0.7826}$ $\frac{7\sqrt{5}}{20}$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$ $\boxed{= 2.7839}$

$\Rightarrow \frac{\sqrt{31}}{2}$

$c(t) = \mathcal{L}^{-1}\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$= \left(1 - e^{\frac{-\zeta\omega_d}{\sqrt{1-\zeta^2}}}\cdot\sin(\omega_d t + \cos^{-1}\zeta)\right)u(t)$

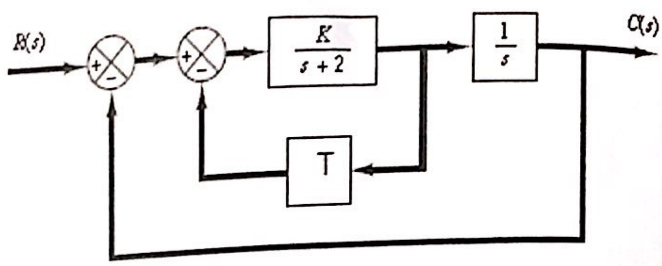
$\left(1 - e^{-\frac{7}{2}}\cdot\sin\left(\frac{\sqrt{31}}{2}t + \cos^{-1}\left(\frac{7\sqrt{5}}{20}\right)\right)\right)u(t)$

$$\boxed{= u(t) - e^{-7/2}\sin\left(\frac{\sqrt{31}}{2}t + 0.6719\right)u(t)}$$

1.5

Scanned by CamScanner

Q2) Given the block diagram shown.



a) Determin K, and T for the system to have an undamped natural frequency $\omega_n = 4$ rad s^{-1}, and damping ratio $\zeta = 0.5$.

$\omega_n = 4$ rad s^{-1}, $\zeta = 0.5$ ((SOS)) ✓

$$\frac{K/s+2}{1+KT/s+2} = \boxed{\frac{K}{s+2+KT}}$$

$$\frac{\frac{K}{s+2+KT}\cdot\frac{1}{s}}{\frac{K}{s+2+KT}\cdot\frac{1}{s}+1}$$ denominater poles

$$K + \frac{s+2+KT}{s} + s+2+KT$$

$Ks + s+2+KT + s^2 + 2s + KTs = 0$

$$\boxed{s^2 + s(K+3+KT) + (KT+2) = 0}$$

$$\boxed{\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}}$$

3

* coeficient of s^1 $= 2\zeta\omega_n = 4 = K+3+KT \rightarrow \boxed{K(1+T)=1}$ ①
* & $KT+2 = \omega_n^2 = \boxed{KT = 14}$ ② $\rightarrow$ $K + KT = 1$

$K + 14 = 1$

$\boxed{K = -13}$ $\boxed{T = -\frac{14}{13}}$

b) With K and T as calculated above

The value of the **maximum overshoot** is

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = \boxed{0.163}$$

1

The value of the **time to the first peak** is

$$T_p = \frac{\pi}{\omega_d} \quad \text{where } \omega_d = \omega_n\sqrt{1-\zeta^2} \quad \text{so } \boxed{T_p = 0.9069\ s}$$

1

*c) what values of **positive** ***K*** **and** ***T*** make the system **unstable**? **Justify**.

Unstable → change in signs.

so $K+3+KT < 0$ or $KT+2 < 0$ → $K(1+T) < -0.5$, $KT < -2$ range for unstable

$K(1+T) < -0.5$ or $KT < -2$

any positive K&T will make system stable

1

Q3 a) A particular system has the following characteristic equation:

$$s^4 + K s^3 + (6-4K)s^2 + 6K s + 2 = 0$$

By examining only the **first three rows** of the Routh's array, can you conclude the stability of the system? . If you can , what is it ?.

s^4	1	$(6-4k)$	2
s^3	K	$6K$	0
s^2	$\frac{(6k-4k^2)-6k}{k}$ $\boxed{-4k}$	$\frac{12K}{6k}$ $\boxed{2}$	0
s^1	$6k+\frac{1}{2}$	0	
s^0	2		

$\frac{-4k^2}{k} \Rightarrow \boxed{-4k}$ $\boxed{2}$

$\frac{-24k^2 - 2k}{-4k}$

$6K + \frac{1}{2}$

first colowmn

1
K
$-4K$
$6K+\frac{1}{2}$
2

$\frac{12k+1}{6k+\frac{1}{2}} = 2$

1

K= unknown can't know stabilitly BUT Can conclude range of "k" when system is stable $\boxed{k>0 \text{ and } -4k>0 \;\&\; 6k+\frac{1}{2}>0}$ must all follow the sign of 1 which is positive

b) Use Routh's criterion to determine the stability of a system whose characteristic polynomial is. CP

$$s^6 + 3s^5 + 8s^4 + 18s^3 + 37s^2 + 75s + 50 = 0$$

s^6	1	8	37	50
s^5	3	18	75	0
s^4	2	11/3	50	0
s^3	25/2	$\frac{-1875}{11}$	0	
s^2	$\frac{1021}{33}$	50	0	
s^1	-190.655	0		
s^0	50			

① ②
③ ④

$\frac{(2\times3)-(1\times4)}{3}$

2.5

1.5

Two changes in sign

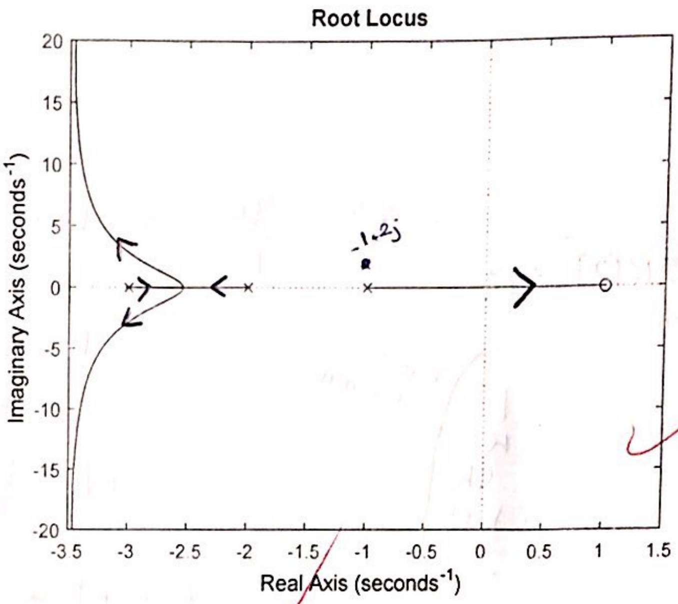
so system

unstable

Scanned by CamScanner

Q4) Given the root locus of a particular system is as shown

Note that the scale for the two axes is not the same.



2

i) Determine the open loop poles and zeros.

poles $(s+3)(s+2)(s+1)$

Pole(s) is (are) : -3, -2, -1

Zero(s) is (are) : 1

zeros $(s-1)$

1

ii) Draw the **appropriate arrows** on the root locus.

* iii) For what positive value of the root locus parameter ***K*** will the closed loop system be stable?

stable → no positive real part

so $s<0$ $\quad \frac{1}{K} = \frac{\ell_z}{\ell_{p_1}\ell_{p_2}\ell_{p_3}} \Rightarrow \frac{1}{K} = \frac{1}{1\times2\times3} \Rightarrow \boxed{K < 6}$

1

iv) Use the **angle condition** to affirm that the point $s = -1+2j$ cannot be a closed loop pole.

$180 + 90 + \theta \cancel{+\theta} \overset{??}{=}$ odd multiple of 180°

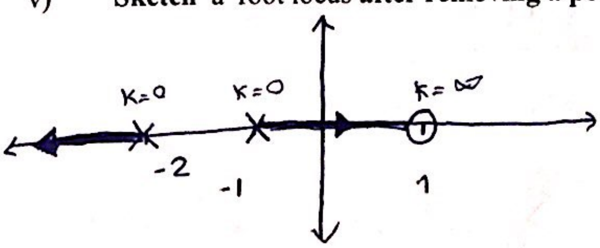
$\angle G(s)H(s) = (1+2h)180°, \; h = 0,1,2..$

odd multiples of 180°

$\Rightarrow 270 \neq 180$ so can't be closed loop pole.

0.5

v) **Sketch** a root locus **after removing a pole of your choice.**



1

Scanned by CamScanner