بسم الله الرحمن الرحيم

The University of Jordan
Department of Mathematics

* "اللهم لا سهل إلا ما جعلته سهلا" وأنت تجعل الحزن إذا شئت سهلا" *

Eng. Math. I (0301202)
2nd Exam 06/12/2014 (12:30-13:30)

* لا إله إلا أنت ربي عليك توكلت رب العرش العظيم *

Name: [redacted]

Number: [redacted]

8 - 9   ح ث خ

Lecturer: د. بيماء الزالق

Lecture Time: 8 - 9 أحد ثلاثاء خميس

Q1. (5 Points) Find the general solution of the nonhomogeneous 4th order ODE $y^{iv} - y''' - 2y'' = -12x$.

3

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Sol.: Homogenously

$\boxed{y_h}$

$$y^{(4)} - y''' - 2y'' = 0$$

$$r^4 - r^3 - 2r^2 = 0$$

$$\cancel{r^2(r^2 - r - 2) = 0} \qquad r^2(r^2 - r - 2) = 0$$

$$\to (r-2)(r+1)$$

$$\boxed{r_1 = r_2 = 0} \qquad \boxed{r_3 = 2, \; r_4 = -1}$$

$$y_h = C_1e^0 + C_2e^0x + C_3e^{2x} + C_4e^{-x}$$

$$\boxed{r = 0, 0, 2, -1} \to y_h = C_1 + C_2X + C_3e^{2x} + C_4e^{-x}$$

$\boxed{y_p}$ $\to$

مثل C_1 فاضرب بـ x

$$y_p = \boxed{Ax^2 + b} \Rightarrow \boxed{Ax^2 + bx^3}$$

x

From the table

إذا كان التشابه نضرب x عدة مرات

$$G.S \gg y = \underbrace{C_1 + C_2X + C_3e^{2x} + C_4e^{-x}}_{y_h} + \underbrace{Ax^2 + bx^3}_{y_p}$$

Q2. (5 Points) Solve the initial value problem $x^2y''' - 2y' = 0$, $y(1) = -1$, $y'(1) = 0$, $y''(1) = 0$.

3.5

~~Euler~~ ~~GS~~ MAKE it Euler

$\boxed{y = x^r}$

$(x^2y''' - 2y') * x = 0 * x$

$\boxed{x^3y''' - 2yx = 0}$

$r(r-1)(r-2) - 2r = 0$

$r(r^2 - 3r + 2) - 2r = 0$

$r^3 - 3r^2 + \cancel{2r - 2r} = 0$

$r^3 - 3r^2 = 0$

$r^2(r-3) = 0$

$\boxed{r_1 = r_2 = 0}$ $\boxed{r_3 = 3}$

$y_h = C_1x^0 + C_2x^0 \ln x + C_3x^3$

G.S $\rightarrow$ $\boxed{y_h = C_1 + C_2x + C_3x^3}$

$\boxed{y(1) = C_1 + C_2 + C_3 = -1}$ ... (1)

$y' = 0 + C_2 + 3C_3x^2 =$

$y'(1) = \boxed{0 + C_2 + 3C_3 = 0}$ --- (2)

$y'' = 6C_3x$

$y''(1) = 6C_3 = 0$

$\boxed{C_3 = 0}$ $\boxed{C_2 = 0}$ $\boxed{C_1 = -1}$

~~G.S~~

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Q3. (5 Points) Find the general solution of the homogeneous system $\begin{matrix} y_1' = 3y_1 - y_2 \\ y_2' = y_1 + y_2 \end{matrix}$

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$$A = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$$

$$(\lambda I - A) = \begin{bmatrix} \lambda - 3 & 1 \\ -1 & \lambda - 1 \end{bmatrix}$$

$$(\lambda - 3)(\lambda - 1) + 1 = 0$$

$$\lambda^2 - \lambda - 3\lambda + 3 + 1 = 0$$

$$\rightarrow \lambda^2 - 4\lambda + 4 = 0$$

$$(\lambda - 2)(\lambda - 2)$$

$(\lambda - 2)^2 \longrightarrow$ Only 1 eigenValue that equals $\boxed{\lambda = 2}$

$\boxed{\text{For } (\lambda = 2)}$

$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-X_1 + X_2 = 0$$

$$X_2 = X_1 \qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix} X_1$$

$\bar{K}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow$ eigenvector

the other eigenvector

$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$-1X_1 + X_2 = 1$$

$$X_2 = 1 + 1X_1$$

$$K^2 = \begin{bmatrix} 1X_1 \\ 1 + 1X_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1+1 \end{bmatrix} X_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} K_2$$

G.S $\gg C_1 \bar{K}_1 e^{\lambda t} + C_2 [K_1 t + K_2] e^{\lambda t}$

$$y_h = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + C_2 \left[\begin{bmatrix} 1 \\ 1 \end{bmatrix} t + \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right] e^{2t}$$

3.5

Q4. (5 Points) The general solution of the homogeneous system $\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ is $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$.
Using the variation of parameters method, find a particular solution of the nonhomogeneous system

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \end{bmatrix}. \rightarrow g(t)$$

$$\boxed{y_p = F(t)\, u} \quad ; \boxed{u = \int_0^t F^{-1}(t) \cdot g(t)}$$

$$F(t) = \begin{bmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{bmatrix} \quad ; \quad g(t) \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$|F(t)| = \cancel{-1} - e^0 = -2$$

$$F^{-1}(t) = \frac{1}{-2} \begin{bmatrix} -e^{-t} & -e^{-t} \\ -e^{t} & e^{t} \end{bmatrix} = \begin{bmatrix} \frac{-e^{-t}}{2} & \frac{-e^{-t}}{2} \\ \frac{-e^{t}}{2} & \frac{e^{t}}{2} \end{bmatrix} \rightarrow F^{-1}$$

$$\begin{bmatrix} \frac{-e^{-t}}{2} & \frac{-e^{-t}}{2} \\ \frac{-e^{t}}{2} & \frac{e^{t}}{2} \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix} = \begin{bmatrix} -e^{-t} + e^{-t} \\ -e^{t} - e^{t} \end{bmatrix}$$

$$u = \int_0^t \begin{bmatrix} 0 \\ -2e^t \end{bmatrix} = \begin{bmatrix} t \\ -2 + 2e^t \end{bmatrix}$$

$$y_p = F(t) \cdot u = \begin{bmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{bmatrix} \begin{bmatrix} t \\ -2 + 2e^t \end{bmatrix}$$

$$y_p = \begin{bmatrix} te^t + 2e^{-t} - 2 \\ te^t + 2e^{-t} + 2 \end{bmatrix}$$

Q5. (5 points) Find the inverse of the matrix $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.

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$$\left[\begin{array}{ccc|ccc} 2 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} R_1 \\ \checkmark R_2 \\ 2R_2 + R_3 \to R_2 \end{array} \left[\begin{array}{ccc|ccc} 2 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right]$$

$$2R_3 + R_1 \to R_1 \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 4 & 2 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right]$$

$$\begin{array}{r} R_1/2 \\ \checkmark R_2 \\ \checkmark R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 2 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right]$$

$I \qquad A^{-1}$

$$\boxed{A^{-1} = \begin{bmatrix} 1/2 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}}$$