بسم الله الرحمن الرحيم

(لا إله إلا الله)

لا إله إلا أنت سبحانك إني كنت من الظالمين

اللهم لا سهل إلا ما جعلته سهلا

وأنت تجعل الحزن إذا شئت سهلا

$\boxed{8:00 - 9:00}$

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The University of Jordan

Mathematics Department

Math 202 / *First Exam*

Date: 01/011/2014

$\boxed{8 : 9}$ $\boxed{\text{شعبة ١}}$

Name:......[redacted]...... Student Number:..0132436.......Section:...1.....

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(1) If $\underbrace{(ax^3 + by^2)}_{}\overset{M}{} dx + \overset{N}{\underbrace{(4xy - cy^3)}} dy = 0$ is exact, then find the value of b.

0

$M = \Psi_x = ax^3 + by^2 dx$

$\Psi = \int ax^3 + by^2 dx$

$\Psi = \frac{aX^4}{4} + by^2 + g(x)$

$\Psi_x = \frac{\cancel{4}aX^3}{\cancel{4}} + 1 + \cancel{g'(x)} \to 0$

~~$\int \Psi_x = \int aX^3 + 1$~~ $\Rightarrow \Psi = \frac{aX^4}{4} + X$

~~$\boxed{b = x}$~~ $\boxed{a = 1}$

(2) Solve the initial value problem: $\frac{dy}{dx} = \frac{y^3}{1 - 2xy^2}$, y(0)=1.

IVP

0

~~$y' = \frac{y^3}{1 - 2xy^2}$~~

~~$y'(1 - 2xy^2) = y^3$~~

~~$y' - 2xy^2 y' = y^3$~~

~~$\frac{dy}{dx} =$~~

$\frac{dy}{dx} = \frac{y^3 / X^3}{\frac{1}{X^3} - \frac{2y^2}{X^2}}$

$= \frac{(\frac{y}{x})^3}{\frac{1}{X^3} - 2(\frac{y}{X})^2}$

Non-Homo

1.5

$y_1 \quad y_2$

(3) If $W(\sin x, f(x)) = \sin^2 x$ for $0 < x < \frac{\pi}{2}$, then find $f(x)$ given that $f(\frac{\pi}{4}) = \frac{\sqrt{2}\pi}{8}$.

$$\begin{vmatrix} \sin x & f(x) \\ \cos x & f'(x) \end{vmatrix} = \sin^2 x$$

$$\sin x\, f'(x) - \cos(x)\, f(x) = \sin^2 x$$

$$\frac{\sqrt{2}}{2} \cdot \underset{f'(\frac{\pi}{4})}{\cancel{f'(x)}} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}\pi}{8} = \frac{1}{2}$$

$$\frac{\sqrt{2}}{2} f'(\tfrac{\pi}{4}) - \frac{2\pi}{8} = \frac{1}{2}$$

$$f'(\tfrac{\pi}{4}) =$$

$$y_2 = y_1 \int \frac{w\, g(x)}{y_1^2}\, e^{\int \boxed{P(x)}\, dx} \, dx$$

1

(4) Find a solution of the O.D.E: $y'' + (y')^2 = 2e^{-y}$.

$\cancel{u = y'}$

$\cancel{u' = y''}$

~~$u' + u^2 = 2e^{y}$~~

~~y missing~~

X - missing

$$\boxed{\begin{aligned} u &= y' \\ y'' &= u\frac{du}{dx} \end{aligned}}$$

$$\boxed{\frac{u^2}{2} = y}$$

$$u\frac{du}{dx} + u^2 = 2e^{-(\frac{u^2}{2})}$$

$$\int u(1+u-2e^{(-\frac{u^2}{2})})\, du = \int dx$$

$$= x + C$$

$$\boxed{\int u + u^2 - 2e^{(\frac{u^2}{2})} \cdot du = x + C}$$

$x \longrightarrow A$

$e^{3x} \longrightarrow Be^{3x}$

$\cos^2 x = (E_1\cos x + E_2 \sin x) + x(E_3 \cos x + E_4 \sin x)$

2.5

(5) Let $y'' - 10y' + 21y = xe^{3x}\cos^2 x$. Find a suitable form for the particular solution Y_p if the method of undetermined coefficients is to be used.

$(y = e^{rx})$

$r^2 - 10y' + 21 = 0$

$\Delta = 100 - 4(1)(21) = 100 - 84 = +ve \quad \pm\sqrt{16} = 4$

$r = \frac{-10 \pm 4}{2} = \frac{-10+4}{2} = \frac{-6}{2} = -3 = r_1$

$= \frac{-10-4}{2} = \frac{-14}{2} = -7 = r_2$

$(r+3)(r+7)$

$y_h = C_1 e^{+3x} + C_2 e^{+7x}$

$y_p = A\left[Be^{3x}\left[E_1 \cos x + E_2 \sin x\right) + (E_3 x\cos x + E_4 x \sin x)\right]\right]$

4

(6) Find the general solution of the differential equation: $y'' + 4y' + 4y = e^{-2x}\ln x$. $\longrightarrow g(x)$

$r^2 + 4r + 4 = 0$

$\Delta = 16 - 4(1)(4) = 0$ − ~~unique~~ one Solution

$r_2 = r_1 = \frac{-4}{2} = -2$

$(r+2)^2 = 0$

$y_h = C_1 \overset{y_1}{e^{-2x}} + C_2 \overset{y_2}{e^{-2x}\cdot x}$

$y_p = -y_1 \int \frac{y_2\, g(x)}{W(y_1, y_2)}\,dx + y_2 \int \frac{y_1 \quad g(x)}{W(y_1, y_2)}\,dx$

$W = \begin{vmatrix} e^{-2x} & x\cdot e^{-2x} \\ -2e^{-2x} & e^{-2x} - 2xe^{-2x} \end{vmatrix}$

$= e^{-2x}(e^{-2x} - 2xe^{-2x}) + 2x e^{-4x}$

$e^{-4x} ~~- 2xe^{-4x} + 2xe^{-4x}~~$

$\boxed{W = e^{-4x}}$

$$\boxed{y_p = -e^{-2x}\int \frac{x\cdot e^{-2x} \cdot e^{-2x}\ln x}{e^{-4x}} + x\cdot e^{-2x}\int \frac{-e^{-2x}\cdot e^{-2x}\ln x}{e^{-4x}}}$$

← ??

~~[illegible]~~ c