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17/30

Jordan University
Mathematics Department
Calculus III, Second Exam, 4/8/2014

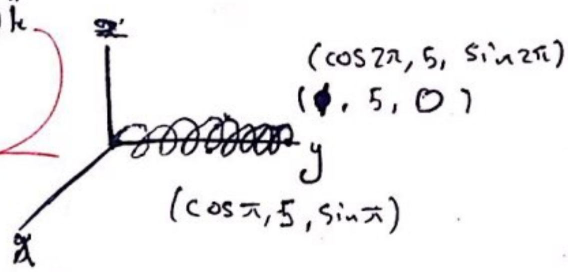
Seat num (12)

Student's Name: [redacted] . Student's Number: [redacted]
Lecture Time: [redacted]

1) a) (2 points) Sketch the graph of $r(t) = < \cos(t),\ 5,\ \sin(t) >, \quad 0 \le t \le 2\pi.$

$r(t) = \cos(t)\hat{i} + 5\hat{j} + \sin(t)\hat{k}$

2



b) (4 points) Find the length of the curve $r(t) = \frac{1}{3}t^3\, i + \frac{1}{2}t^2\, k, \quad 0 \le t \le 2$

$\int_0^2 \| r' \|$

1.5

$r'(t) = t^2\hat{i} + t\hat{k}$

$\int_0^2 t^2\hat{i} + \int_0^2 t\hat{k} = \frac{t^3}{3}\Big|_0^2 + \frac{t^2}{2}\Big|_0^2$

$\frac{8}{3} + \frac{4}{2} = \frac{16+12}{6} = \frac{28}{6} = \frac{14}{3}$

2) (4 points) If $f(x) = \cosh(x)$, then show that $\kappa(x) = sech^2(x)$.

$f(x) = \cosh(x)$

~~$f'(x) = \sinh(x)$~~

$\frac{1}{f(x)^2} = k(x) = \operatorname{Sech}^2(x)$

3) Find the limit, if it exists, or show that the limit does not exist. (Show your work).

a) (2 points) $\lim_{(x,y)\to(1,1)} \frac{x^4-y^4}{x^2-y^2}$ =

تحليل

① ~~Let y = 0~~

$\lim_{x\to 1} \frac{x^4}{x^2} - x^2 = \boxed{1}$

② ~~Let~~

$\lim_{(x,y)\to(1,1)} \frac{(x^2-y^2)(x^2+y^2)}{x^2-y^2} = \boxed{2}$

∴ Limit Exists

2

b) (4 points) $\lim_{(x,y)\to(1,0)} \frac{xy^4-y^4}{(x-1)^2+y^8}$ = $\frac{0}{0}$!!

(i) Let $\boxed{y = x}$

$\lim_{x\to 1} \frac{x^5 - x^4}{(x-1)^2+x^8} = \frac{1-1}{0+1} = \frac{0}{1} = \boxed{0}$

(ii) Let $y = -x$

$\lim_{x\to 1} \frac{-x^5 + x^4}{(x-1)^2+x^8} = \frac{-1+1}{(x-1)^2-x^2} = \frac{0}{-1}$

Nothing

0

4) (6 points) If $z = f(x,y)$, where $x = s^2e^t$ and $y = s^2 - t^3$, then find z_s and z_{st}

$\frac{dz}{ds} = \frac{df}{dx}\cdot\frac{dx}{ds} + \frac{df}{dy}\cdot\frac{dy}{ds}$

$= f_x(2se^t) + f_y(2s)$

z = f

x y

s t s t

6

$\frac{dz^2}{dt\,ds} = f_{xt}(2se^t) + f_x(2se^t) + f_{yt}(2s) +$

$= \left[f_{xx}(2e^t) + f_{xy}(2e^t) \right] (2se^t) + \left[f_{yy}(2s) + f_{yx}(-2s) \right](2s)$

$+ f_x(2se^t)$

$$\frac{y^5 - y^4}{(y-1)^2 + y^8}$$

$$\frac{x\frac{1}{x^4} - \frac{1}{x^4}}{(x-1)^2 + \frac{1}{x^8}}$$

$$\frac{0-0}{1} = 0$$

$$\frac{\frac{1}{x^5} - \frac{1}{x^4}}{(x-1)^2 + \frac{1}{x^8}}$$

$$\cancel{\frac{\frac{1}{1} - \frac{1}{1}}{(x-1)^2 + \frac{1}{x^8}}}$$

$$\cancel{0 + \frac{1}{}}$$

$(y-1)$

$$\frac{y^5 - y^4}{y^2 + 2y + 1 + y^8}$$

$-1 - 1$

$y = -x$

$$\frac{-x^5 - x^4}{(x-1)^2 - x^8} = \frac{-2}{}$$

-1

$$\frac{0}{1} = 0$$

$$\frac{-y^5 - y^4}{(-y-1)^2 - y^8}$$

$x = -y$

$$\cancel{\frac{-x^5 + x^4}{(x-1)^2 =}}$$

$(\quad)$

$$\cancel{\frac{-y^5 - y^4}{(-y-1)^2 - y^8}} = \frac{}{}$$

5) For $f(x, y, z) = x^2 - xyz$,

a) (4 points) Find the directional derivative of f at $(1, -1, 2)$ in the direction of
$a =$ $2i + j + 2k$.

$\nabla f(x,y,z) = 2x - yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}$

$\nabla f(1,-1,2) = 4\,\hat{i} + 2\,\hat{j} - 1\,\hat{k}$

$D_a f = \langle 4, 2, -1 \rangle \cdot \dfrac{\langle 2, 1, 2 \rangle}{\sqrt{4+1+4}}$

$\langle 4, 2, -1 \rangle \cdot \langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle$

$\frac{8}{3} + \frac{2}{3} - \frac{2}{3} = \boxed{\frac{8}{3}}$

4

2/2

b) (2 points) Find the maximum rate of change in f at $(1, -1, 2)$.

$\|\nabla f\| = \sqrt{4^2 + 2^2 + (-1)^2} = \sqrt{16 + 4 + 1} = \sqrt{21}$

6) (4 points) Find the point on the paraboloid $x^2 + y^2 - z = 0$ at which the tangent plane is parallel to the plane $2x + 2y + z = 7$.

الحل خلف الورقة

$f(x,y) = 2x + 2y + z = 7$

1.5

tangent plane $\rightarrow$ ~~$f(x,y)$~~ $= f(x_0, y_0) + f_x(x_0, y_0) + f_y(x_0, y_0)$

خلف الورقة

Plane equ $=$ $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$\langle \overset{a}{2}\hat{i} + \overset{b}{2}\hat{j} + \overset{c}{\hat{k}} \rangle \rightarrow$ normal ~~Direction~~ of Plane

~~$2(x - x_0) + b(y - y_0) + c(z - z_0) = 0$~~

~~$2x = 2a$~~

~~$2y = 2a$~~

~~$z = a$~~

~~$x = a$~~

~~$y = a$~~

~~$a^2 + a^2 - a = 0$~~

~~$2a + 2a + a = 7$~~

~~$5a = 7$~~

~~$a = \frac{5}{7}$~~

~~$x = \frac{5}{7}$~~

~~$y = \frac{5}{7}$~~

~~$z = \frac{5}{7}$~~

$\langle 2,2,1\rangle \to$ Normal of the Plane

$x^2 = 2\alpha$
$y^2 = 2\alpha$
$z = -\alpha$

$4\alpha + 4\alpha - \alpha = 7$

$\cancel{7}\alpha$

$\boxed{\alpha = 1}$

$x = \pm\sqrt{2}$
$y = \pm\sqrt{2}$
$z = -1$

$\therefore$ Point are

$\boxed{(+\sqrt{2}, +\sqrt{2}, -1) \text{ or } (-\sqrt{2}, -\sqrt{2}, -1)}$