اللهم لا سهل إلا ما جعلته سهلاً وأنت تجعل الحزن إذا شئت سهلاً

14.5

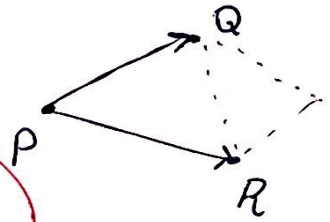
12/20

بسم الله الرحمن الرحيم

Jordan University
Mathematics Department
Calculus III, First Exam, 14/7/2014

Student's Name: [redacted] . Student's Number: [redacted] .
Lecture Time: د. أنوار البستنجي

1) a) (3 points) Find the area of the triangle whose vertices are $P(1,-1,2)$, $Q(2,1,2)$, and $R(1,2,3)$.



$PQ\langle 1, 2, 0\rangle$

$PR\langle 0, 3, 1\rangle$

3

$3-0$

$$(1/2)\times\|PQ\times PR\| = \begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 0 & 3 & 1\end{vmatrix} = \langle 2, -1, 3\rangle$$

$$\frac{1}{2}\times\sqrt{(2)^2+(-1)^2+(3)^2} = \boxed{\frac{\sqrt{14}}{2}}\ (\text{unit})^2$$

$4 + 1 + 9$

b) (3 points) Find cosine the angle between a diagonal of a cube and one of its edges.

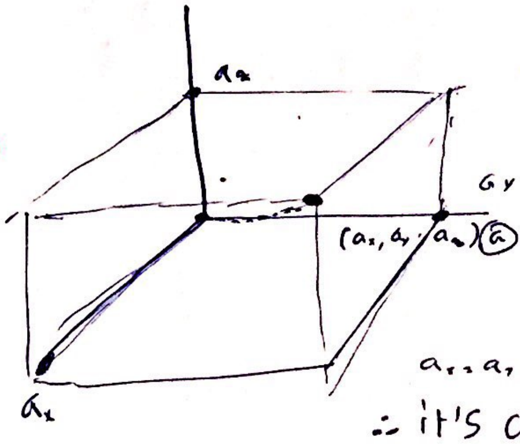
$$\cos\theta = \frac{a^2}{\sqrt{a^2 \ldots a^2}}$$

2.5

$$\frac{a^2}{\ldots}$$

$$\cos\omega = \frac{a^2}{a^2\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$



$a_x = a_y = a_z$

$\therefore$ it's cube

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2) Sketch the graph of the following:

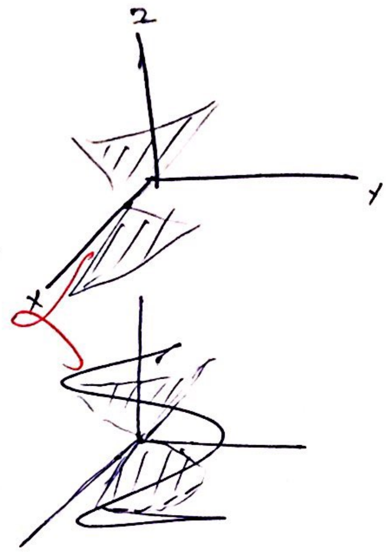
a) (2 points) $x^2 - 2x + y^2 + z - 9 = 0$

$(x-1)^2 + y^2 + z = 9 + 1$

$z = -(x-1)^2 + y^2 + 8$

center

Hyperboloid $(1,0,0)$

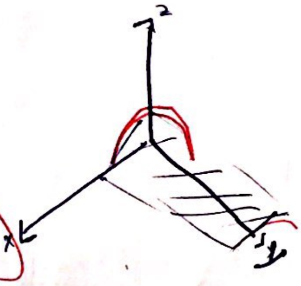


b) (2 points) $z = 1 - x^2$

~~Cylinder~~ = Plane

$z + x^2 = 1$

~~Center~~

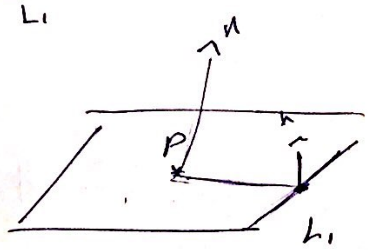


3) (4 points) Find equation of the plane that passes through the point $\boxed{P(1,1,0)}$ and contains the line with symmetric equations $x = y - 1 = \frac{z+1}{2}$.

L_1

$$\frac{x-0}{1} = \frac{y-1}{1} = \frac{z+1}{2}$$

$d_1 \quad d_2 \quad d_3$



$Q:(0,1,-1)$

$V_{L_1} = \langle 1, 1, 2 \rangle$

$V_{PQ} = \langle -1, 0, -1 \rangle$

$$V_{PQ} \times V_{L_1} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & -1 \\ 1 & 1 & 2 \end{vmatrix} = \langle 1, 1, -1 \rangle$$

$\langle a, b, c \rangle$

π:

$1(x-1) + 1(y-1) - 1(z+1) = 0$

Final Answer

4) (2 points) Find equation of the line that passes through the point $P(2,0,1)$ and is orthogonal to the plane $x = 2z + 10$

$x_0 \; y_0 \; z_0$

$\overset{a}{-}x + \overset{b}{0}y + \overset{c}{2}z + 10 = 0$

$\langle -1, 0, 2 \rangle \parallel L_1$

$\cancel{x = 2 - 1t}$
$\cancel{y = 0}$
$\cancel{z = 1 + 2t}$

$$\boxed{\begin{aligned} x &= 2 - 1t \\ y &= 0 \\ z &= 1 + 2t \end{aligned}}$$

2

5) a) (2 points) Show that the distance between the parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is $D = \frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$

$\pi_1 : ax + by + cz + d_1 = 0$

$\pi_2 : ax + by + cz + d_2 = 0$

$V_0 \langle a, b, c \rangle$

$V_1 \langle a, b, c \rangle$

$P_1(x_0, y_0, z_0)$

$P_2 \; (x, y, z)$

$$\boxed{\frac{|ax_0 + by_0 + cz_0 + d_1|}{\sqrt{a^2 + b^2 + c^2}}}$$

$$D = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

$P_1(0,0,0)$

b) (2 points) Use part a) to find equations of the planes that are parallel to the plane $2x - y + 2z = 1$ and 5 units away from it.

$\langle 2, -1, 2 \rangle$

$$(5) \times \frac{|-1 - d_2|}{\sqrt{4+1+4}} = \frac{-1-d_1}{\sqrt{9}} = \boxed{\frac{-1-d_2}{3} \times 5}$$

0.5

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