بسم الله الرحمن الرحيم

اللهم لا سهل إلا ما جعلته سهلاً ، وأنت تجعل الحزن إذا شئت سهلاً

الجامعة الأردنية	الاختبار الثاني : تفاضل وتكامل 2	السبت: 2014/5/3
اسم الطالب: [redacted]		مدرس المادة: د. محمد محيلان
الرقم الجامعي: [redacted]	14	وقت المحاضرة: 1 - 2

Q1: The base of the solid is the region bounded by $y=\ln x$, $x=1$ and $y=1$. Cross – sections perpendicular to the y-axis are semicircles with diameter is on the base . Find the volume of this solid

$e^y = x$

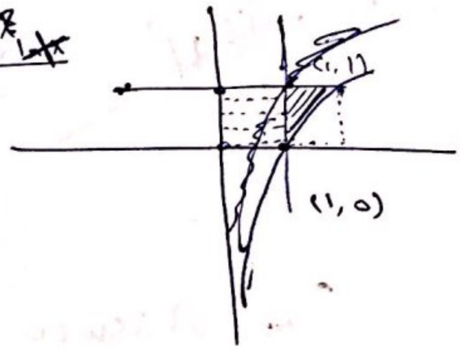
$$\int A(x)\,dy = \int \frac{\pi}{2}\left(\frac{r}{2}\right)^2$$

$$\int_0^1 \frac{\pi}{8}\left(e^{2y}\right)$$

$$\pi/8 \left[\frac{e^{3y}}{3}\right]_0^1$$

$$\pi/8 \left[\frac{e^3}{3} - \frac{1}{3}\right]$$

2



Q2 : Use the shell method to find the volume of the solid generated by rotating the region bounded by $y=5-x^2$ and $y=1$ about the line $x=3$

$$\int 2\pi\, r_{ap}(h)_{ap}\, dx$$

$r = (3 - y)$

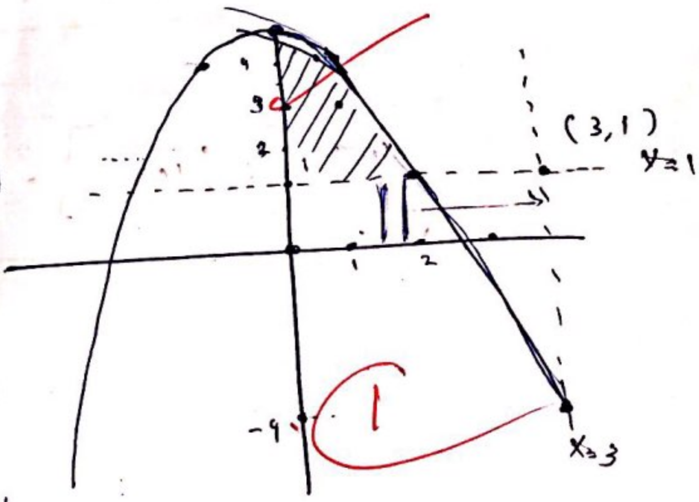
$h = 5 - x^2$

$y = 5 - x^2$

$x^2 = 5 - y$

$x = \sqrt{5-y}$

$$\int_1^5 2\pi\,(3-y)\left(\sqrt{5-y}\right) dy$$



1

$$2\pi \int_0^1 \left(3\sqrt{5-y} - y\sqrt{5-y}\right) dy$$

$$6\pi \int_1^5 (\sqrt{5-y})^{1/2} - 2\pi \int_1^5 y\sqrt{5-y}\, dy$$

$u = \sqrt{5-y}$ $\longrightarrow du = \frac{1}{2\sqrt{5-y}}\, dy$

$dv = y\, dy$ $\longrightarrow v = \frac{y^2}{2}$

$2\sqrt{5-y}$

$$6\pi \frac{2}{3} \cdot (5-y)^{3/2} \Big|_1^5$$

$$\boxed{4\pi\,(5-y)^{3/2}\Big|_1^5}$$

$$\boxed{\frac{y^2}{2}\sqrt{5-y}} + \frac{1}{4}\int_1^5 \frac{y^2}{\sqrt{5-y}} \longrightarrow$$

$$4\pi\,(5-y)^{3/2}\Big|_4^5 - \frac{y^2}{2}\sqrt{5-y} + \frac{1}{2}\cdot\sqrt{5-y}\Big|_4^5$$

$$\cancel{4\pi\left[\sqrt[3]{(5-4)^2} - \sqrt[3]{5^2}\right] - 2\pi\left[\left(\frac{1}{2}\sqrt{4} + \frac{1}{2}\sqrt{4}\right) - \frac{\sqrt{5}}{2}\right]}$$

$$\cancel{4\pi\left[\sqrt[3]{(4)^2} - \sqrt[3]{5^2}\right] - 2\pi\left[\left(\frac{\sqrt{4}}{2} + \frac{\sqrt{4}}{2}\right) - \left(\frac{\sqrt{5}}{2}\right)\right]}$$

~~Final Answer~~

Final Answer

$$4\pi\left[\sqrt[3]{(5-5)^2} - \sqrt[3]{4^2}\right] - 2\pi\left[\frac{25}{2}\cdot\sqrt{0} + \frac{1}{2}\cdot\cancel{0}\right]$$

$$\left(\frac{\sqrt{4}}{2} + \frac{\sqrt{4}}{2}\right)$$

$$4\pi\left[-\sqrt[3]{(4)^2}\right] - 2\pi\left[\frac{\sqrt{4}}{2} + \frac{\sqrt{4}}{2}\right]$$

Q3: Find the length of the curve $y = 2\ln(\sin\frac{1}{2}x)$, $\frac{\pi}{6} \le x \le \frac{\pi}{3}$

$$\int_{\pi/3}^{\pi/6} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$y = 2\ln\left|\sin\left(\frac{y}{2}\right)\right|$$

$$y' = 2\,\frac{\cos(x/2)}{\sin\frac{y}{2}}$$

$$y'^2 = 4\left(\cot(x/2)\right)^2$$

$$\int_{\pi/3}^{\pi/6} \sqrt{1 + 4\left(\cot(x/2)\right)^2}$$

3

Q4: Find the limit of the sequence $\left\{\left(\frac{2n+4}{2n-6}\right)^n + n\sin\frac{1}{n}\right\}_{n=1}^{\infty}$

n^n

$$\lim_{n\to\infty}\left(\frac{2n+4}{2n-6}\right)^n + \lim_{n\to\infty} n\sin\frac{1}{n}$$

∞

$$\lim_{n\to\infty}\frac{(2n+4)^n}{(2n-6)^n} + \boxed{\frac{\sin\frac{1}{n}}{1/n} = 1}$$

$$\frac{2n\cdot 2}{2n\cdot 2 - 6} \cdot \frac{2n-6}{2n+4} = \frac{2}{2} = 1$$

$$\boxed{1 + 1 = 2}$$

2

Q5: Find the sum $\sum_{n=1}^{\infty}\left(3^{n-1}4^{-n} + \frac{1}{n^2+3n+2}\right)$

$$\frac{3^n \cdot 3^{-1}}{4^n}$$

$$\frac{1}{3}\left(\frac{1}{4}\right)^n$$

By Geometric

$$\Sigma = \frac{1/4}{1 - 1/4} = \frac{1/4}{3/4}$$

$$\boxed{= 1/3}$$

$$\frac{1}{n^2 + 3n + 3}$$

$$(n+1)(n+2)$$

2

$$\frac{1}{(n+1)(n+2)} = \frac{A}{(n+1)} + \frac{B}{(n+2)}$$

$$A(n+2) + B(n+1)$$

$$\boxed{A = -1} \quad \boxed{B = +1}$$

$$\frac{+1}{n+2} - \frac{1}{n+1}$$

by telescop.

0

0

$$\frac{1}{n+2} - \frac{1}{n+1}$$

$$= \left(\cancel{\frac{1}{3}} - \frac{1}{2}\right)_{a_1} + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{3}}\right)_{a_2}$$

$$+ \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{4}}\right)_{a_3} + \dots + \left(\cancel{\frac{1}{6}} - \cancel{\frac{1}{5}}\right)_{a_{n+1}}$$

$$\lim \left(\frac{1}{n+2} - \frac{1}{2}\right)$$

$$1/3 - 1/2 =$$

$$\frac{2-3}{6} = \boxed{\frac{-1}{6}}$$

Scanned by CamScanner

Q5: Test for convergence:

(a) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n\sqrt{n}}$ $=$ $\cancel{\frac{\sin^2 n}{n \cdot n^{1/2}}}$ b C.T

$0 \le \sin^2 n \le 1$

$0 \le \frac{\sin^2 n}{n\sqrt{n}} \le \frac{1}{n\sqrt{n}}$

3

it Converg b C.T $\frac{\sin^2 n}{n\sqrt{n}} \le \frac{1}{n\sqrt{n}}$

(b) $\sum_{n=1}^{\infty} \frac{\sqrt{n^3+n^2+1}}{n^2+4}$

$3/2 - 4/2$

Limit. Compare test

$-1/2$

$a_n = \frac{n^{3/2}}{n^{4/2}}$ $\sum b_n = \cancel{\frac{1}{n^{1/2}}}$ 1

$\boxed{p \le 1}$

it Diverges by P-Series

rays of Inter Section

(c) $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$

BASEL KHAMIS is Always Back mother FUCKER

C.T

BASEL KHAMIS

~~Diverg tes~~

~~$\lim_{n\to\infty}$~~

0

$\cancel{\ln n \le n^3}$

$\therefore$ $\frac{\ln n}{[illegible]} \le \frac{1}{n^2}$

$\boxed{\text{it Diverges}}$ nn

by C.T