Seat number → 15

Jordan University

Mathematics Department

Calculus I, First Exam, 5/4/2014

Student's Name: [redacted] . Student Number: 0132436 .

Lecture Time: [redacted]

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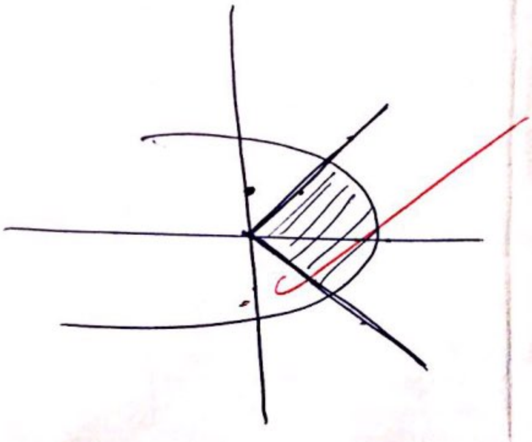
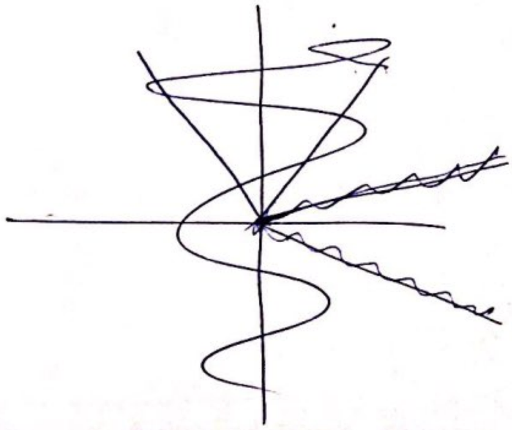
1) (4 points) Find the area of the region bounded by the two curves $x = 6 - y^2$ and $x = |y|$.





$X = X$

$|y| = 6 - y^2$

$y^2 + |y| - 6 = 0$

$(y+3)(y-2)$ $\quad$ $y = 3$

$y = 2 \;,\; y = -3$ $\quad$ $y = -3$

2

$$\int_{-3}^{2} 6 - y^2 - |y| = 6y - \frac{y^3}{3} - \frac{y^2}{2} \Big|_{-3}^{2}$$

$$\left(12 - \frac{8}{3} - 2\right) - \left(-18 + 9 - \frac{9}{2}\right)$$

10 $\quad$ 9

$$12 - \frac{8}{3} - 2 + 18 - 9 + \frac{9}{2}$$

$$\frac{19}{1} - \frac{8}{3} + \frac{9}{2}$$

2) (4 points each) Evaluate the following integrals:

a) $\int \frac{dx}{(x^2+2x+2)^2}$

$$\int \frac{dx}{((x+1)^2+1)^2}$$

$(x+1)^2+1$

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$u = x+1$

$du = dx$

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$$\Rightarrow \int \frac{du}{(u^2+1)^2}$$

$$\int \frac{\sec^2\theta\, d\theta}{(\tan^2\theta+1)^2}$$

$u = \tan\theta$

$du = \sec^2\theta\, d\theta$

$\sqrt{u^2+1}$, u, 1

$$\int \frac{\sec^2\theta\, d\theta}{(\sec^2\theta)^2}$$

$$\int \frac{d\theta}{\sec^2\theta} \Rightarrow \int \frac{d\theta}{\tan^2\theta+1} \Rightarrow \int \frac{d\theta}{\tan^2\theta} + \int 1\, d\theta$$

$+ \theta$

b) $\int \frac{x^3+x^2+x+2}{x^4+3x^2+2}\,dx$

$$\int \frac{x^3+x^2+x+2}{(x^2+2)(x^2+1)} = \int \frac{Ax+B}{(x^2+2)} + \frac{Cx+D}{(x^2+1)}$$

1

$$x^3+x^2+x+2 = (Ax+B)(x^2+1) + (Cx+D)(x^2+2)$$

$$\int \frac{dz}{\underbrace{(z^2+1)^2}_{u}}$$

$(z^2+1)^{-2}$

$-2(z^2+1)^{-1} \cdot 2z$

$u = \frac{1}{(z^2+1)^2} \quad \Rightarrow du = \frac{-4z}{(z^2+1)}$

$dv = dz \quad \Rightarrow v = z$

$z = x+1$

$dz = dx$

$$\frac{z}{(z^2+1)^2} + 4\int \frac{z}{(z^2+1)}$$

$$\frac{z}{(z^2+1)^2} + 2\ln(z^2+1)$$

$$\frac{(x+1)}{((x+1)^2+1)^2} + 2\ln((x+1)^2+1)$$

c) $\int \frac{dx}{\cos(x) - 3\sin(x) + 1}$

Special Sub

$u = \tan\left(\frac{x}{2}\right)$

$\cos = \frac{1-u^2}{1+u^2}$

$\sin = \frac{2u}{1+u^2}$

$du =$ ____

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3) (4 point) Evaluate the improper integral $\int_0^1 \sqrt{t}\ln(t)\, dt$

$\int \sqrt{t} \ln(t)$

$u = \ln t \longrightarrow du = \frac{1}{t} dt$

$dV = t^{\frac{1}{2}} \longrightarrow \frac{2\sqrt{t^3}}{3}$

$\frac{(2\sqrt{t^3})(\ln t)}{3} - \frac{2}{3}\int \frac{t^{\frac{3}{2}}}{t} \cdot dt$

$\frac{(2\sqrt{t^3})(\ln t)}{3} - \frac{2}{3} \cdot \frac{2}{3} \frac{\sqrt{t^3}}{3}$

$\frac{2\sqrt{t^3}(\ln t)}{3} - \frac{4}{9}\sqrt{t^3}$

2

$\int_0^1 \lim_{n \to 0^+} \frac{(2\sqrt{t^3})(\ln t)}{3} - \frac{4\sqrt{t^3}}{9} \Big|$

$\left(0 - \frac{4}{9}\right)\Big|_{t=1}$ conv

$\left(\frac{2\sqrt{n^3}\ln(n)}{3} - 0\right)\Big|_{\substack{t=n \\ n \to 0^+}}$

$\frac{2\sqrt{n}}{3} \cdot \frac{1}{n} = \infty$

Convergent