University of Jordan

Faculty of Engineering & Technology

Department of Electrical Engineering

Course Title:	Exam:	Date:
Electromagnetics I	2nd Exam	Dec/09/2015
Course No.: 0903251	**Semester:** 1st Term 2015-2016	**Time Period:** 1:30 Hr.
Instructor: Dr. Ahmad Atieh & Dr. Yanal Faouri		

Q.1	Q.2	Q.3	Q.4	Total /30
4	1.5	1	0	6.5

اللهم لا سهل إلا ما جعلته سهلاً وأنت تجعل الحزن إذا شئت سهلاً

Student Name: [redacted]

Student Number: [redacted] Seat no. 20

Section: 2 Dr. Ahmad Atieh

$$d\mathbf{S} = \rho\, d\phi\, dz\, \mathbf{a}_\rho \quad d\rho\, dz\, \mathbf{a}_\phi \quad \rho\, d\phi\, d\rho\, \mathbf{a}_z$$

$$d\mathbf{l} = d\rho\, \mathbf{a}_\rho + \rho\, d\phi\, \mathbf{a}_\phi + dz\, \mathbf{a}_z$$

$$\varepsilon_o = \frac{10^{-9}}{36\pi} F/m$$

$$\mathbf{E} = \int \frac{\rho_L\, dl}{4\pi\varepsilon_o R^2}\mathbf{a}_R$$

$$\mathbf{E} = \int \frac{\rho_S\, dS}{4\pi\varepsilon_o R^2}\mathbf{a}_R$$

$$+ \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)$$

$$\nabla^2 \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \cancel{\frac{1}{r^2\sin^2\phi}\frac{\partial^2}{\partial\theta^2}} + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial}{\partial\phi}\right)$$

$$\nabla = \frac{\partial}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial}{\partial\theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\mathbf{a}_\phi$$

$$\int \frac{1}{\sin x}dx = \ln(\tan x/2)$$

$$\int \frac{x}{a^2+x^2}dx = \frac{1}{2}\ln|a^2+x^2|$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$$

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Note that bold letters are vectors

Q1) (7 marks)

a) Find the charge enclosed within an object formed by the planes x=0 and x=1, y=0 and y=2, z=0 and z=3 when an electric flux density $\mathbf{D} = 2xy\,\mathbf{a}_x + x^2\,\mathbf{a}_y$ applied to it?

b) What is the energy density in the object?

c) Calculate the energy stored within the object?

(a) $\Psi = Q_{enc} = \int_S D \cdot dS = \int_v \rho_v \, dv$

$\nabla \cdot D = \rho_v$

$\boxed{\rho_v = 2y}$

$Q = \int_0^3\int_0^2\int_0^1 2y \, dx\, dy\, dz = \int_0^3 dz \int_0^2 2y\, dy \int_0^1 dx$

$Q = (3) * (y^2)\Big|_{y=0}^{2} * (1)$

$Q_{enc} = (3) * (4) * (1) = \underline{\underline{12\ C}}$

4

4/7

(c) $W = \frac{1}{2} \int \vec{D} \cdot \vec{E} \, dS \;\boxed{= \frac{1}{2}\int \varepsilon_0 |E|^2}$

$\vec{D} = \varepsilon_0 E$

$\vec{E} = \frac{\vec{D}}{\varepsilon_0} \Rightarrow \vec{E} = \frac{2xy}{\varepsilon_0}\vec{a}_x + \frac{x^2}{\varepsilon_0} a_y$

2

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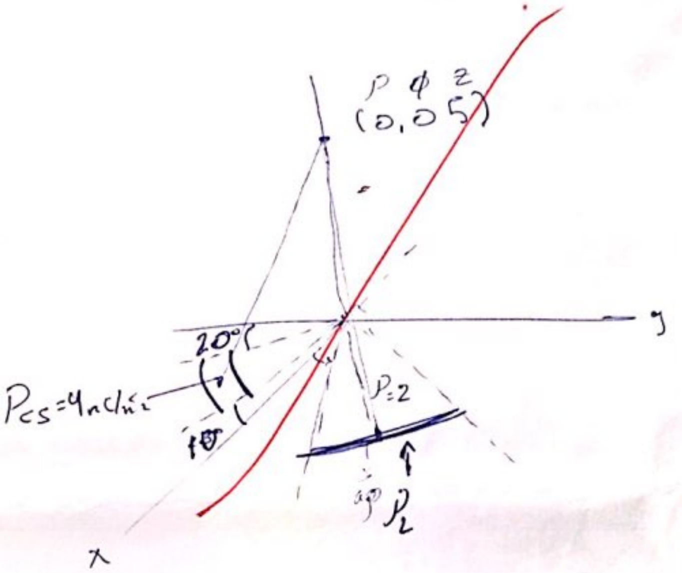
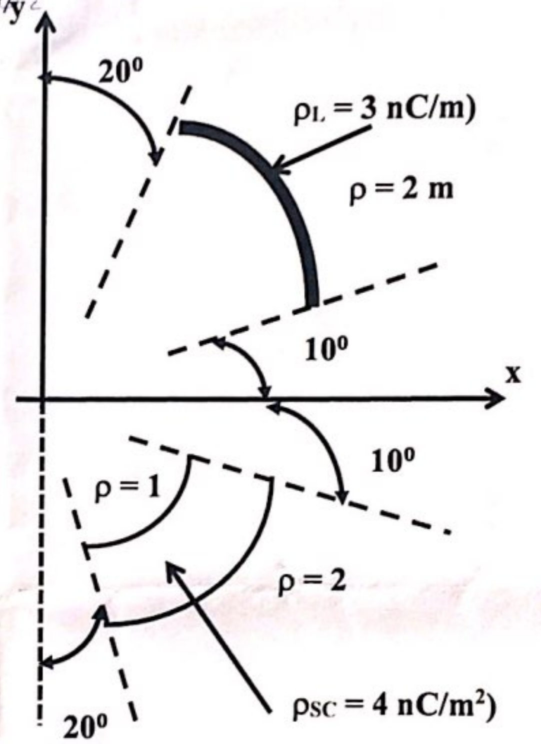
Q2) (8 marks)

For the free charge distribution shown below in free space, determine the electric field intensity at point P (0, 0, 5) m?





$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$E_1 = \int_L \frac{\rho_L \, dl}{4\pi \varepsilon_0 R^2} \vec{a_\rho}$$

$$E_2 = \int_S \frac{\rho_s \, ds}{4\pi \varepsilon_0 R^2} a_\rho$$

$h = 5$

$\cancel{\vec{R} = -a \, a_\rho + h \, \vec{a_z}}$

$h = 5$

$\vec{R} = -a\vec{a_\rho} + h \, \vec{a_z}$

$\vec{dl} = \rho \, d\phi \, a_\phi$, $z = 0$, $\rho = 2$

0.5

$ds = \rho \, d\phi \, d\rho$ $\vec{a_z}$

0

$z = 0$

$1 \leftarrow \rho \rightarrow 2$

$290° \le \phi \le 350°$

1.5/8

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$5.38 = \sqrt{2^2 + 5^2}$

$|R| = 7.68\ m$

$$E_1 = \int_{10^\circ}^{70^\circ} \frac{(3*10^{-7})(2)}{4\pi\varepsilon_0 (7.68)^3}\left[-5.38\vec{a_\rho} + 5\vec{a_z}\right] = \int_{10^\circ}^{70^\circ} 119*10^{-3}\left[-5.38\vec{a_\rho} + 5\vec{a_z}\right] d\phi$$

$$= (60)*(119*10^{-3})\left[-5.38\vec{a_\rho} + 5a_z\right]$$

$$= -(7.14*5.38\vec{a_\rho}) + (7.14*5\vec{a_z})$$

$$\vec{E_1} = -38.413\vec{a_\rho} + 35.7\vec{a_z}\ V/m$$

$\rho_s |E_1|$

$$E_2 = \int_s \frac{\rho_s\, ds}{4\pi\varepsilon_0 R^2} \qquad ds = \rho\, d\phi\, d\rho$$

0.5

$$\vec{E_2} = \iint \frac{\rho_s}{4\pi\varepsilon_0 |R|^2}\rho\, d\phi\, d\rho$$

$$|E_2| = \frac{(4*10^{-9})}{4\pi\varepsilon_0}\int_{\phi=290^\circ}^{350^\circ}\int_{\rho=1}^{2}\frac{1}{\rho}\, d\phi\, d\rho = \frac{4*10^{-9}}{4\pi\varepsilon_0}\int_{290^\circ}^{350^\circ} d\phi \int_1^2 \frac{1}{\rho}$$

$$|E_2| = (35.95)*(60^\circ)*(0.693) = 1494.801\ V/m$$

$$E_{tot} = |E_1| + |E_2|$$

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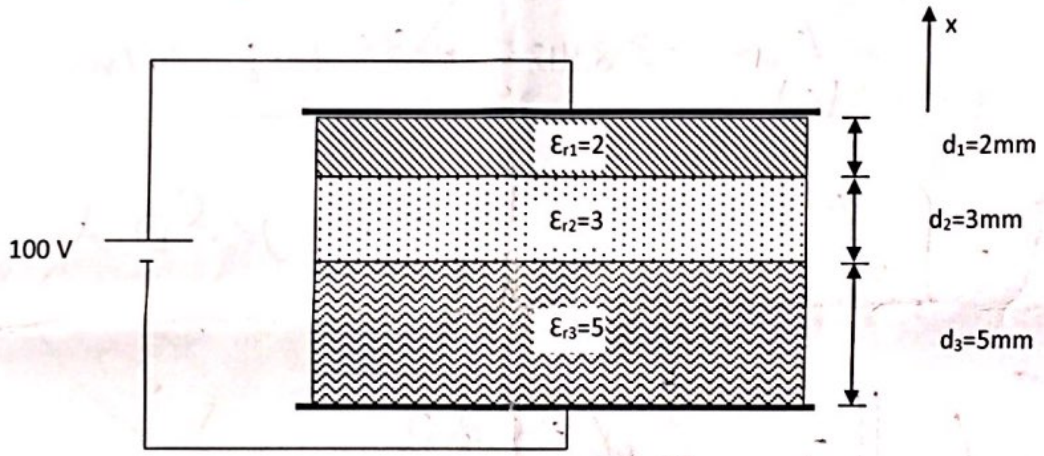
Q3) (8 marks)

5

The Figure below represents a parallel plate capacitor with area 0.5 m^2 and separation 10 mm contains three dielectrics with interfaces normal to **E** and **D** as shown. Assume the voltage difference between the plates is 100 V. Find:

a) Voltage drop across each dielectric.
b) Electric flux density D_n for each region.
c) Total capacitance.
d) Total energy stored over all capacitor.



$D_{n1} = D_{n2} = D_{n3}$

$E_{n1} = E_{n2} = E_{n3}$

$D_n = \varepsilon_r \varepsilon_0 E$

$E = \frac{D_n}{\varepsilon_r \varepsilon_0}$

$D_{n1} = D_{n2}$

$\varepsilon_{r1} \varepsilon_0 E_{1n} = \varepsilon_{r2} \varepsilon_0 E_{n2}$

$V = -\nabla E$

$V = -\nabla \frac{D_n}{\varepsilon_r \varepsilon_0}$

c) $C = \frac{\cancel{\varepsilon S}}{d} = \frac{Q}{V} = \frac{\varepsilon d}{S} =$

$C = \frac{(\varepsilon_{r_1}+\varepsilon_{r_2}+\varepsilon_{r_3})\varepsilon_0(10*10^{-3})}{(0.5)}$

$C = 1.7708\times10^{-12}\ F$ ✗

d) $W_E = \frac{1}{2} V C^2 = \frac{1}{2}(100)(1.7708\times10^{-12})^2$ ✗

$$\boxed{W_{EC} = 1.567\times10^{-22}\ J}$$

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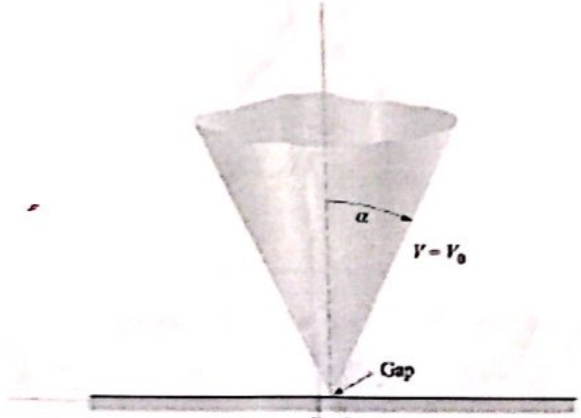
Q4) (7 marks) 0

For the conducting cone shown below, if the surrounding medium is free space; calculate V and **E** at P (r, 2α, 0°) and ρ_s at the cone surface.



→ Using Poisson's Equ.

$$\nabla^2 V = \frac{-\rho_s}{\varepsilon_0}$$

$$\frac{d^2 V}{d\cancel{x}^2} = \frac{-\rho_s}{\varepsilon_0}$$

Zero

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \theta}\right) + \cancel{\frac{1}{r^2 \sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi \frac{\partial}{\partial \phi}\right)}$$

$$\frac{1}{r^2}\frac{dr}{dr}\left(r^2 \frac{dV}{dr}\right) + \frac{1}{r^2 \sin 2\alpha}\frac{\partial}{d\theta}\left(\sin 2\theta \frac{\partial V}{d\theta}\right) = -\frac{\rho_s}{\varepsilon_0}$$