The University of Jordan
Electrical Eng. Dept.
First Exam

Sun. 29/10/2017

Advanced Control Systems EE549

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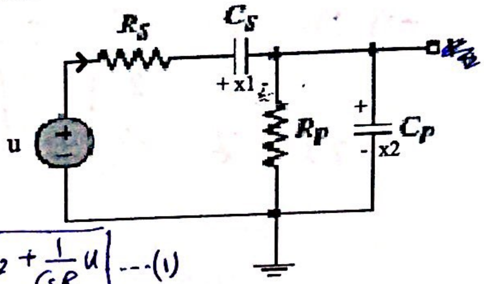
Answers should be written in ink.

Exam Duration:70 min

Mark out of 30: 27

Q1 Given the circuit shown

i) With the two states **x1** and **x2** as shown, and the **output** as the current in **R_s**, **Model** the circuit .Get theA,B,C, D matrices.



the current through $R_s = C_s \frac{dx_1}{dt}$

so By KVL: $-u + C_s R_s \frac{dx_1}{dt} + x_1 + x_2 = 0$

$\Rightarrow C_s R_s \dot{x}_1 = -x_1 - x_2 + u \Rightarrow \boxed{\dot{x}_1 = -\frac{1}{C_s R_s} x_1 - \frac{1}{C_s R} x_2 + \frac{1}{C_s R} u}$ ---(1)

* the current through $R_p = C_s \frac{dx_1}{dt} - C_p \frac{dx_2}{dt}$

By KVL $-x_2 + C_s R_p \frac{dx_1}{dt} - C_p R_p \frac{dx_2}{dt} = 0 \Rightarrow \dot{x}_2 = \frac{1}{C_p R_p}\left(C_s R_p \dot{x}_1 - \frac{x_2}{R_p}\right)$

$\Rightarrow \boxed{\dot{x}_2 = \frac{-1}{C_p R_s} x_1 - \frac{1}{C_p R_s} x_2 + \frac{1}{C_p R_s} u - \frac{1}{C_p R_p} x_2}$ --(2)

$\boxed{y = C_s \dot{x}_1 = -\frac{1}{R_s} x_1 - \frac{1}{R_s} x_2 + \frac{1}{R_s} u}$

(3)

The state space is given By:

$$\dot{X} = \begin{bmatrix} \frac{-1}{C_s R_s} & \frac{-1}{C_s R_s} \\ \frac{-1}{C_p R_s} & -\left(\frac{1}{C_p R_s} + \frac{1}{C_p R_p}\right) \end{bmatrix} X + \begin{bmatrix} \frac{1}{C_s R_s} \\ \frac{1}{C_p R_p} \end{bmatrix} u \quad , \quad y = \begin{bmatrix} -\frac{1}{R_s} & -\frac{1}{R_s} \end{bmatrix} X + \frac{1}{R_s} \cdot u$$

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ii) Obtain the transfer function **using any method**.

$G(s) = C[sI - A]^{-1} B + D$

$$[sI - A] = \begin{bmatrix} s + \frac{1}{C_s R_s} & \frac{1}{C_s R_s} \\ \frac{1}{C_p R_s} & s + \frac{1}{C_p R_s} + \frac{1}{C_p R_p} \end{bmatrix}$$

$$|sI - A| = s^2 + \left(\frac{1}{C_p R_s} + \frac{1}{C_p R_p}\right) - \frac{1}{C_s C_p R_s^2}$$

$\frac{R_p + R_s}{R_p R_s C_p}$

$$C \cdot [sI - A]^{-1} = \begin{bmatrix} -\frac{1}{R_s} & -\frac{1}{R_s} \end{bmatrix} \cdot \begin{bmatrix} s + \frac{R_p + R_s}{R_p R_s C_p} & \frac{-1}{C_s R_s} \\ \frac{-1}{C_p R_s} & s + \frac{1}{C_s R_s} \end{bmatrix}$$

$\Rightarrow$ answer $\begin{bmatrix} \frac{1}{C_s R_s} \\ \frac{1}{C_p R_p} \end{bmatrix} + \frac{1}{R_s} \rightarrow \underline{\underline{D}}$

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$$G(s) = \frac{-s}{C_s R_s^2} - \frac{R_p + R_s}{R_s^3 R_p C_s C_p} + \frac{1}{C_p C_s R_s^3} + \frac{1}{C_s C_p R_p R_s^2} - \frac{s}{R_s R_p C_p} - \frac{1}{C_s C_p R_p R_s^2} + \frac{1}{R_s}$$

iii) Determine the **steady state value** of the output **by any method** when **u = 7** volts.

$X_{ss} = -A^{-1} B$

since $u = 7$volt (it means due to) a <u>unit step</u>

$$X_{ss} = \frac{-1}{|A|} \begin{bmatrix} -\left(\frac{R_p + R_s}{R_p R_s C_p}\right) & \frac{1}{C_s R_s} \\ \frac{1}{C_p R_s} & \frac{-1}{C_s R_s} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{C_s R_s} \\ \frac{1}{C_p R_p} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-(R_p + R_s)}{R_p R_s^2 C_p C_s} + \frac{1}{C_p C_s R_p R_s} \\ \frac{1}{C_p C_s R_s^2} - \frac{1}{C_p C_s R_p R_s} \end{bmatrix} \cdot \frac{-1}{|A|}$$

where $|A|$ given by:

$$|A| = \frac{R_p + R_s}{R_p R_s^2 C_p C_s} - \frac{1}{C_s C_p R_s^2}$$

after simplify:

$$X_{ss} = \begin{bmatrix} R_p / R_s \\ 1 - R_p / R_s \end{bmatrix} ; Y_{ss} = 0$$

Q2 Given matrix A as shown.

a) Use **any method** ,what is the eigenvector associated with the eigenvalue **-12?**

$$A = \begin{bmatrix} 13 & 5 & -5 \\ -2 & -10 & 2 \\ 23 & 7 & -15 \end{bmatrix}$$

$adj[(A-\lambda I)] = \text{eigenvector} = adj(A-(-12)I)$

$$= adj \begin{bmatrix} 25 & 5 & -5 \\ -2 & 2 & 2 \\ 23 & 7 & -3 \end{bmatrix} = \begin{bmatrix} -20 & \cdots \\ 40 & \cdots \\ -60 & \cdots \end{bmatrix} \quad \text{so eigenvector} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

i) What is the eigenvalue associated with the eigenvector $[1 \quad 0 \quad 1]^T$?

By the Rule $Ax = \lambda x$

$$\Rightarrow \begin{bmatrix} 13 & 5 & -5 \\ -2 & -10 & 2 \\ 23 & 7 & -15 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 8 \end{bmatrix} = 8\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \text{so eigenvalue} = 8$$

ii) Use **a matrix property** to determine the third eigenvalue .

We Know that $trace[A] = \sum \lambda[A]$:

so ~~13-10-15~~ $13-10-15 = -12 = -12+8+\lambda_3$

so $\lambda_3 = -8$

iii) Use a matrix property **different** from that in part ii above , to confirm the validity of the calculated eigenvalues . Don't use $|A-\lambda I_n|=0$

$|A| = 13[+150-14]-5[30-46]-5[-14+230] =$ By Calculator $= \underline{768}$

$\prod \lambda[A] = |A| = (-8)(-12)(8) = +768$ ##

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b) Use matrix properties to obtain the **determinant of the inverse of a matrix** in terms of the matrix determinant .

$|A^{-1}| = \dfrac{1}{|A|}$ so $|A^{-1}| = \dfrac{1}{768} = 1.302083333 \times 10^{-3}$

0.5

c) Use matrix properties to obtain the **eigenvalue of the inverse of a matrix** in terms of the matrix eigenvalue .

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$\lambda[A^{-1}] = \dfrac{1}{\lambda[A]}$

so eigenvalues for A^{-1}: $\dfrac{-1}{12}, \dfrac{1}{8}, \dfrac{-1}{8}$

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Q3 a) Given $A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$, calculate e^{At}, using an **infinite power series method**.

$$e^{At} = I_n + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \cdots$$

$$A^2 = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix},\quad A^3 = \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix},\quad A^4 = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ \cdots$$

$$e^{At} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \begin{bmatrix}0 & -t\\ t & 0\end{bmatrix} + \begin{bmatrix}-t^2 & 0\\ 0 & -t^2\end{bmatrix}\frac{1}{2!} + \begin{bmatrix}0 & t^3\\ -t^3 & 0\end{bmatrix}\frac{1}{3!} + \cdots$$

$$= \begin{bmatrix}1-\frac{t^2}{2!}+\frac{t^4}{4!}-\cdots & -t+\frac{t^3}{3!}-\frac{t^5}{5!}+\cdots\\ t-\frac{t^3}{3!}+\frac{t^5}{5!}-\cdots & 1-\frac{t^2}{2!}+\frac{t^4}{4!}-\cdots\end{bmatrix} = \begin{bmatrix}\sum_{n=0}^{\infty}\frac{(-1)^n t^{2n}}{(2n)!} & \sum_{n=0}^{\infty}\frac{(-1)^{n+1} t^{2n+1}}{(2n+1)!}\\ \sum_{n=0}^{\infty}\frac{(-1)^n t^{2n+1}}{(2n+1)!} & \sum_{n=0}^{\infty}\frac{(-1)^n t^{2n}}{(2n)!}\end{bmatrix}$$

$$\equiv e^{At}.$$

$*$ check $e^{At}\Big|_{t=0} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = I_n$ ✓

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b) A system is given by $\dot{x} = \begin{bmatrix}-1 & 2\\ 0 & -4\end{bmatrix}x + \begin{bmatrix}5\\ 6\end{bmatrix}u$

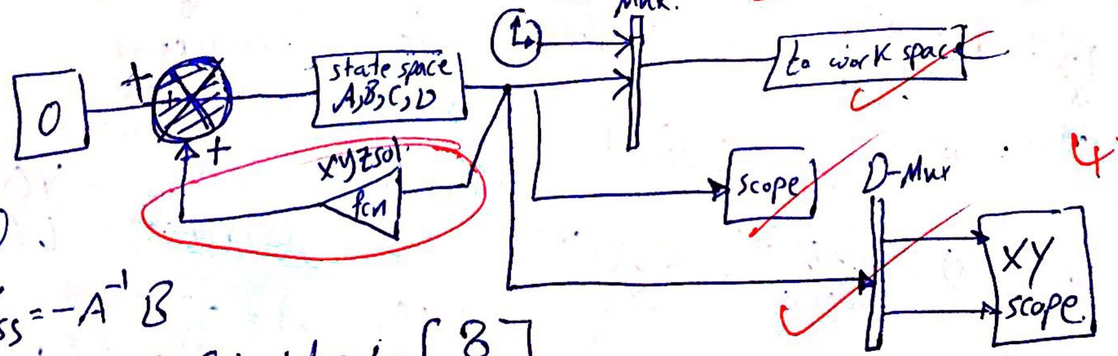
together with the information that it has **two outputs**.

Draw an appropriate Simulink diagram to answer the following

i) Solving and showing the **two states** on a **scope**. What are the A,B,C, and D matrices entered?.
ii) Obtaining a data file named **xyzsol** involving all states **together with a time record**.
iii) Using the **xy scope** to plot x_1 against x_2.
iv) Implement a matlab command used to plot the **three states against each other**.

(i) A=[-1 0;0 -4]; B=[5;6]; C=[1 0;0 1]; D=[0;0];

(ii)



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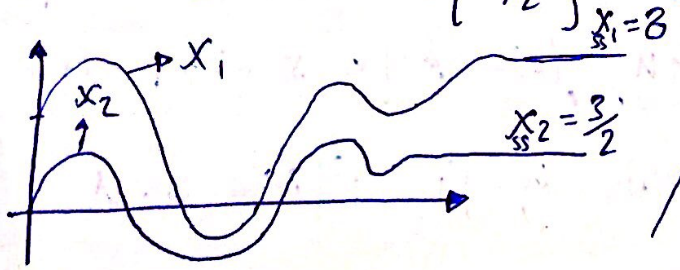
(iii)

$X_{ss} = -A^{-1}B$

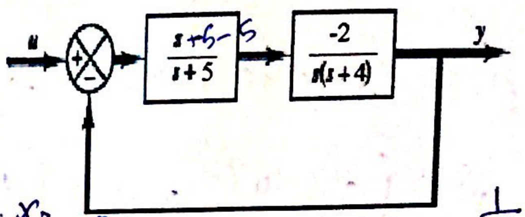
$\Rightarrow X_s \Rightarrow$ By Calculator $X_{ss} = \begin{bmatrix}8\\ 3/2\end{bmatrix}$



(iv)

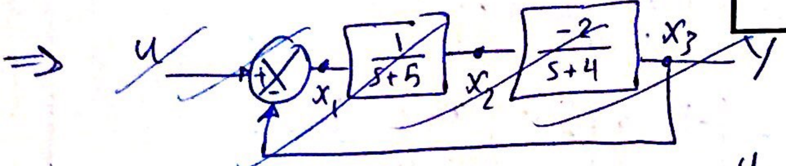
>>plot3(x1:t, x2:t, x:t)

Q4 Given the following block diagram

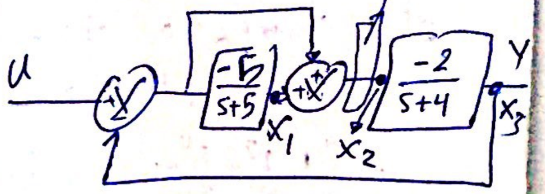


a) Choose **three states** (not less than three), with **u**, and **y** as the input and output respectively , to model the system shown in the block diagram .

$\Rightarrow$



$\Rightarrow$



$$x_1 = \frac{-5}{s+5}\left[U - x_3\right]$$

$$\Rightarrow \boxed{\dot{x}_1 = -5x_1 + 5x_3 - 5u} \quad \text{---}(1)$$

$$x_2 = \frac{1}{s}\left[U - x_3 + x_1\right].$$

$$\Rightarrow \boxed{\dot{x}_2 = x_1 - x_3 + u} \quad \text{--}(2)$$

$$x_3 = \frac{-2}{s+4}x_2$$

$$\Rightarrow \boxed{\dot{x}_3 = -2x_2 - 4x_3} \quad (3)$$

$$\boxed{y = x_3} \quad \text{-}(4)$$

State space is:

$$\dot{x} = \begin{bmatrix} -5 & 0 & 5 \\ 1 & 0 & -1 \\ 0 & -2 & -4 \end{bmatrix} x + \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix} u \quad \& \quad y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x + 0 \cdot u.$$

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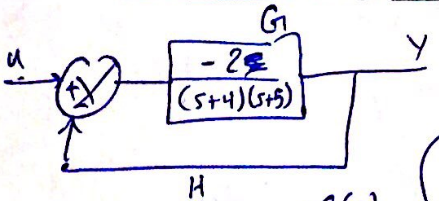
b) Reduce the block diagram to obtain the transfer function, then use it to obtain a **state space representation** involving a **diagonal A** matrix. What are the A,B,C, and D matrices in this case.



$$\Rightarrow G(s) = \frac{G_1}{1+HG_1} = \frac{\frac{-2}{(s+4)(s+5)}}{1 + \frac{-2}{(s+4)(s+5)}}$$

$$\boxed{G(s) = \frac{-2}{s^2+9s+18}} = \frac{-2}{(s+6)(s+3)} = \frac{Y(s)}{U(s)}.$$

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$$\text{so} \quad Y(s) = \underbrace{\frac{A\,U(s)}{s+6}}_{X_1(s)} + \underbrace{\frac{B\,U(s)}{s+3}}_{X_2(s)}$$

where $A = \frac{-2}{-3} = \boxed{\frac{2}{3}}$

$B = \frac{-2}{3} = \boxed{-\frac{2}{3}}$

$$x_1 = \frac{\frac{2}{3}U}{s+6} \Rightarrow \boxed{\dot{x}_1 = -6x_1 + \frac{2}{3}u} \quad \text{--}(1)$$

$$x_2 = \frac{-\frac{2}{3}U}{s+3} \Rightarrow \boxed{\dot{x}_2 = -3x_2 - \frac{2}{3}u} \quad \text{-}(2) \quad \& \quad \boxed{y = x_1 + x_2} \quad \text{--}(3)$$

$$\dot{x} = \begin{bmatrix} -6 & 0 \\ 0 & -3 \end{bmatrix} x + \begin{bmatrix} 2/3 \\ -2/3 \end{bmatrix} u \quad \& \quad y = \begin{bmatrix} 1 & 1 \end{bmatrix} x + 0 \cdot u.