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EE 251: Electromagnetics 1 Second Exam (Fall 2017) Dec. 3rd, 2017 Duration: 75 Min.

$\frac{21.5}{30}$

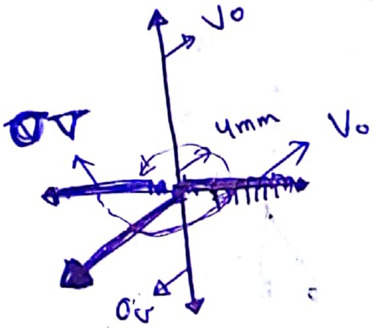
Note that bold letters are vectors

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \qquad \varepsilon_o = \frac{10^{-9}}{36\pi} \text{ F/m}$$

5.5

Problem 1 (6 points)

Consider two semi-infinite lines are placed on the positive y-axis which is connected to 'V_o' and on the negative y-axis that is connected to ground. If the separation between the two lines is 4 mm, find the electric field intensity at (3, 4, -2)



$\boxed{\nabla^2 V = 0}$

$\frac{d^2 V}{d\phi^2} = 0$

at $\rho = 0.44 = V_o$
at $\rho = -0.44 = 0V$

$\frac{dV}{d\phi} = A$

$\boxed{\cancel{V = Ay + B}}$

$\cancel{\frac{1}{\rho^2}} \frac{d^2 V}{d\phi^2} = 0$

$\boxed{A\phi + B = V}$

at $\phi = 90^\circ$ $V = V_o$
at $\phi = 270^\circ$ $V = 0$

$A * \left(\frac{\pi}{2}\right) + B = V_o$

$A * \left(\frac{3\pi}{2}\right) + B = 0$

$\boxed{\cancel{B = -\frac{2A}{3\pi}}}$

أو العكس

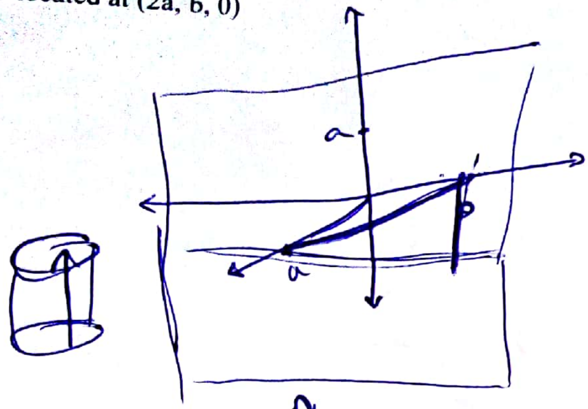
$\frac{\pi}{2} A - \frac{2A}{3\pi} = V_o$

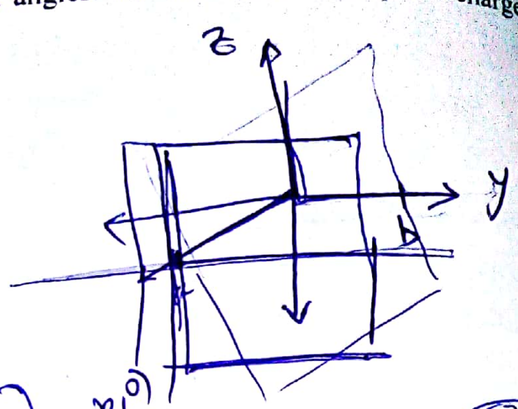
$A\left(\frac{\pi}{2} - \frac{2}{3\pi}\right) = V_o$

$\boxed{A = \frac{V_o}{\left(\frac{\pi}{2} - \frac{2}{3\pi}\right)}}$

Problem 2 (6 points)

A line x = a, y = b (where a > 0 and b > 0) carry a line charge of (ρ_L) C/m is located between two semi-infinite grounded conducting planes intersecting at right angles. Find the force on a point charge 'Q' located at (2a, b, 0)

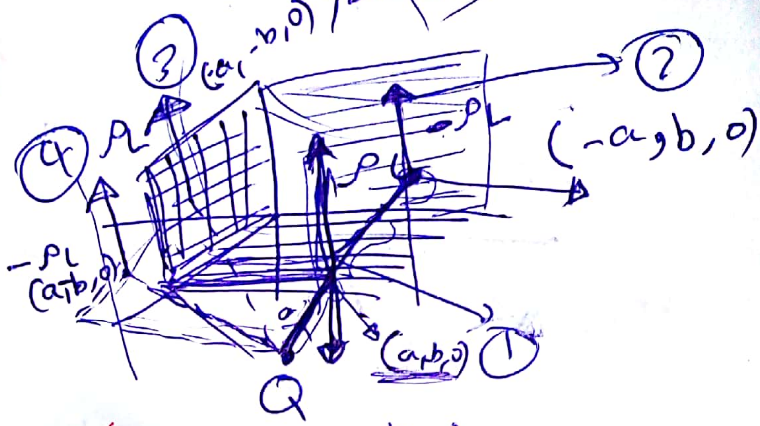




$$D \cdot ds = \frac{\rho_L}{\cancel{4}\pi\epsilon_0 \rho}$$

$$\frac{\rho_L}{2\pi\epsilon_0 \rho}$$

0.5



$$F = Q\vec{E}$$ 0.5

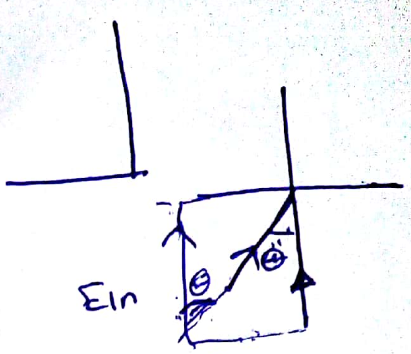
$$= Q\left(\frac{\rho_L}{2\pi\epsilon_0 \rho_1}\hat{a}_{\rho_1} - \frac{\rho_L}{2\pi\epsilon_0 \rho_2}\hat{a}_{\rho_2} - \frac{\rho_L}{2\pi\epsilon_0 \rho_3}\hat{a}_{\rho_3} + \frac{\rho_L}{2\pi\epsilon_0 \rho_4}\hat{a}_{\rho_4}\right)$$

$$= Q\left(\frac{\rho_L}{2\pi\epsilon_0}\right)\left(\frac{1}{a}\hat{a}_x - \frac{1}{3a}\hat{a}_x - \frac{(a\,\hat{a}_x + 2b\,\hat{a}_y)}{\sqrt{a^2+4b^2}} + \frac{3a\,\hat{a}_x + 2b\,\hat{a}_y}{\sqrt{9a^2+4b^2}}\right).$$

2

For the boundary between two ~~magnetic~~ dielectric media. If the boundary having a surface charge distribution according to $D_{2n} - D_{1n} = \rho_s$, show that:

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}\left[1 + \frac{\rho_s}{D_1 \cos\theta_1}\right]$$



$$D_{2n} - D_{1n} = \rho_s .$$

$$\varepsilon_{r1} E_{1n} - \varepsilon_{r2} E_{2n} = \rho_s .$$

(1) $\varepsilon_{r1}(E_1 \cos\theta_1) - \varepsilon_{r2}(E_1 \cos\theta_2) = \rho_s$

$\boxed{E_{1t} = E_{2t}}$ (1)

$E_{1t} = E \sin\theta_1$ (1)

$$\varepsilon_{r1}\left[\frac{E_{1t}\cos\theta_1}{\sin\theta_1}\right] - \varepsilon_{r2}\left(\frac{E_{2t}\cos\theta_2}{\sin_2}\right) = \rho_s$$

α

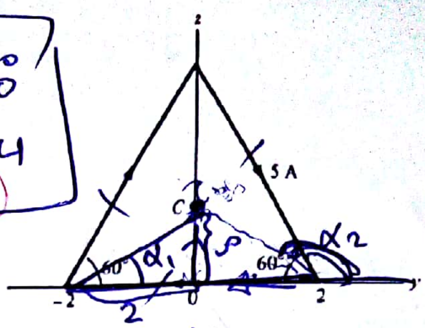
$$E_{1t}\left(\frac{\varepsilon_{r1}}{\tan\theta_1} - \frac{\varepsilon_{r2}}{\tan\theta_2}\right) = \rho_s$$

Problem 4: (6 points)

6

Find **H** at the center C of an equilateral triangular loop of side 4 m carrying 5 A of current as in the figure shown.

$$\boxed{\rho = 2\tan 30^\circ = 1.154}$$



$$\vec{H} = 3 * \vec{H}_1$$

$$\vec{H} = 3 * \left(\frac{I}{4\pi\rho} (\cos \underset{30}{\alpha_2} - \cos \underset{150^\circ}{\alpha_1})\, a_\phi \right) \quad H_1$$

$$2\cos 30$$

$$= 3 \left(\frac{I}{4\pi (1.154)} \right) * (1.732)$$

$$\hat{a_\phi} = \hat{a_l} \times \hat{a_\rho}$$

$$-\hat{a_y} \times \hat{a_z}$$

$$= -\hat{a_x}$$

$$= 3 * \frac{5}{4\pi (1.154)} * 1.732\, (-\hat{a_x})$$

$$= 1.792 \text{ A/m}^2 \cdot (-\hat{a_x})$$

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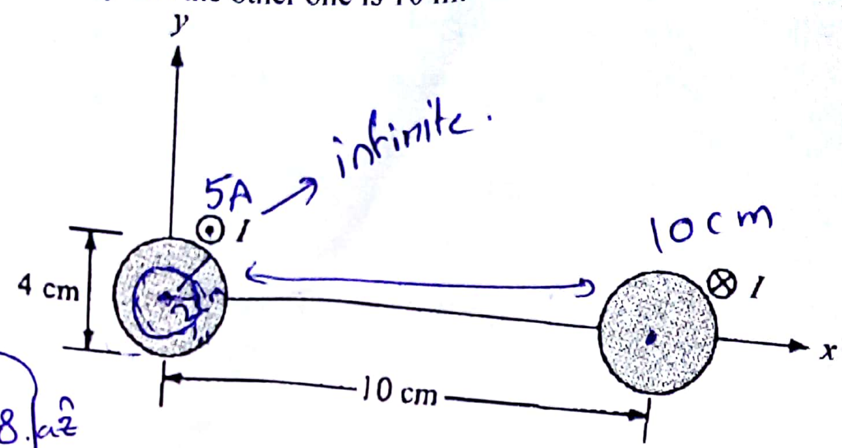
Problem 5: (7 points)

Consider the two-wire transmission line whose cross section is illustrated in the figure below. Each wi of radius 2 cm and the wires are separated 10 cm. The wire centered at (0, 0) carries current 5 A while other centered at (10 cm, 0) carries the return current. If all regions have $\mu = \mu_o$, find:

a) **H** at (1 cm, 0), if the two conductors are infinitely long.

b) The force between them, if the two conductors became of zero radius, and the length of t conductor located at the origin is infinity and the other one is 10 m.

2



$J = \frac{5}{\pi (2)^2} = 0.398\, a_z$

$\oint H \cdot dL = I_{enc}$

$\rho H_\phi (2\pi) = \iint_S J \cdot ds$

$\rho H_\phi (2\pi) = \int_0^{2\pi} \int_0^{1} 0.398 \rho \, d\rho \, d\phi$

$= \rho H_\phi (2\pi) = 0.398 \frac{\rho^2}{2} (2\pi) \Big]_0^1$

$\boxed{\rho = 1}$

$= \vec{H} = 0.398 \left(\frac{1}{2}\right) = .199 \frac{A/m^2}{1} a_\phi$

$\vec{F} =$ ~~$\frac{\mu I_1 I_2}{4\pi (\ )} \oint \oint (dl \times dl \times R)$~~

$\frac{\mu I_1 I_2}{2\pi} \oint_{L_1} \oint_{L_2} \frac{dl_1 \times (dl_2 \times R_{21})}{(R_{21})^2}$

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$\nabla^2 V = 0 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right)$

$\frac{dV}{d\rho} = \frac{A}{\rho}$

$\boxed{V = A\ln\rho + B}$

V at ~~C~~ 2mm =

2

$\boxed{A = \frac{V_0}{-\pi}}$

$V = A\phi - \frac{3\pi}{2}A$

$\frac{\pi}{2}$ $-\pi$

$E = -\nabla V$

$= -A\,\hat{a_\phi}$

$\vec{E} = -\frac{V_0}{\rho(-\pi)}$ $= \boxed{\frac{V_0}{\pi}\,a_\phi}$

1

$\nabla^2 V = 0$

$\frac{dV}{d\phi} = A$

$V = A\phi + B$

$\left(\phi = \frac{\pi}{2}\right) \Rightarrow V = V_0$

$\phi = \frac{3\pi}{2} \Rightarrow V = 0$

$\frac{\pi}{2}A + B = V_0$

$A\left(\frac{3\pi}{2}\right) + B = 0$

$\boxed{B = -\frac{3\pi}{2}A}$

$\frac{\pi}{2}A - \frac{3\pi}{2}A = V_0$

$\boxed{A = \frac{V_0}{(-\pi)}}$