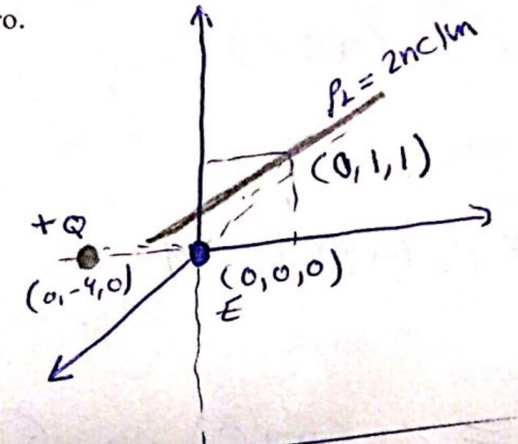
$$\varepsilon_o = \frac{10^{-9}}{36\pi}\ \text{F/m}, \quad \begin{bmatrix} \mathbf{a}_x \\ \mathbf{a}_y \\ \mathbf{a}_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{a}_\rho \\ \mathbf{a}_\phi \\ \mathbf{a}_z \end{bmatrix}$$

7

Problem 1 (12 points)

A line y = 1, z = 1 carry 2 nC/m and a point charge +Q located at (0, -4, 0). Determine +Q and the force on the line if the electric field at the origin is zero.

$$\boxed{E = E_{Line} + E_{\text{from point charge}}}\ \left(\frac{N}{C}\right)\ 1$$



$$\bar{E}_{Line} = \frac{\rho_L}{2\pi\varepsilon_o \rho}\hat{a}_\rho \quad \left(\frac{N}{C}\right)\ 1$$

$$= \frac{2 * 10^{-9}}{2\pi\varepsilon_o(\sqrt{2})}\ (0, -1, -1)\ 1$$

$$\bar{E}_{Line} = (0, -18, -18)\ N/C$$

$$\bar{R}_{1\,line} = (0,0,0) - (x,1,1)$$

$$\bar{\rho}\ \bar{R}_{1\,line} = (0,-1,-1)$$

$$|\bar{R}_1| = \sqrt{2}$$

$$\bar{E}_{from\,Q} = \frac{Q\ (\bar{r}_2)}{4\pi\varepsilon_o r^3}\ 1$$

$\hat{a}_y$ اتجاه

$$\boxed{E = \frac{4Q\ \hat{a}_y}{7.111 * 10^{-9}}\ \frac{N}{C}}$$

$$\bar{R}_2 = (0,0,0) - (0,-4,0)$$

$$\bar{R}_2 = (0,4,0)$$

$$|\bar{R}_2| = 4\ \ 1$$

$$\text{if } \bar{E}_{(0,0,0)} = 0$$

$$\therefore\ +18\hat{a}_y + 18a_z = \frac{4Q\ \hat{a}_y}{7.111\,10^{-9}}$$

($w\hat{a}_y$ = $\hat{a}_y$) أخذ بالتساوي يعني (لانه

1

$$18 = \frac{4Q}{7.111 * 10^{-9}}$$

$$\boxed{Q = 3.1995 * 10^{-8}\ C}\ 1$$

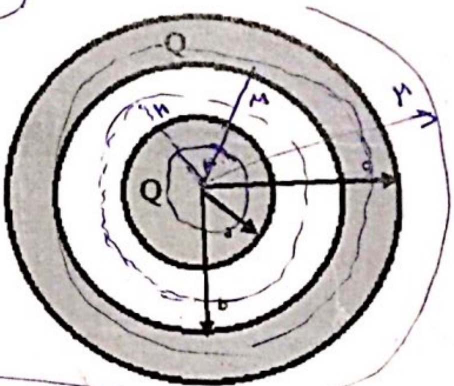
كولوم

8.5

Problem 2 (13 points)

A solid good conducting sphere with $\varepsilon = \varepsilon_o$ of radius "a" has a positive net charge Q is enclosed by a conducting spherical shell of inner radius "b" and external radius "c" (where c > b > a) has the same center with the solid sphere and a charge of -Q. If the free space region (a < r < b) has $\rho_v = \frac{6Q}{4\pi(b^3-a^3)}$ C/m³. Determine:

a) **E** everywhere
b) V everywhere
c) The charge distribution on each surface.
d) Energy density stored in the region a < r < b.



sol: (a) $\boxed{\oint_S \mathbf{D}\cdot d\mathbf{s} = Q_{enc} = \int_V \rho_v\, dv}$ قانون عام

① Let $(r<a) \rightarrow \boxed{\bar{E}=0}$ 0.5

② $\boxed{a<r<b}$ $\oint \mathbf{D}\cdot d\mathbf{s} = \left(\int \rho_v\, dv + Q\right)$

$\bar{E}\epsilon_0 (4\pi r^2) = \left(\int \rho_v\, dv + Q\right)$

$\bar{E}\epsilon_0\, 4\pi r^2 = \iiint \frac{6Q\, dv}{4\pi(b^3-a^3)} + Q$

$\bar{E}\epsilon_0\, 4\pi r^2 = \int_0^{2\pi}\int_0^{\pi}\int_a^{r} \frac{6Q\, r^2 \sin\theta\, dr\, d\theta\, d\phi}{4\pi(b^3-a^3)} + Q$

$\bar{E}\epsilon_0\, 4\pi r^2 = \frac{6Q}{4\pi(b^3-a^3)}\left(\int_0^{2\pi}\int_0^{\pi}\int_a^{r} \frac{r^2 \sin\theta\, dr\, d\theta\, d\phi}{1} + 1\right)$

$\bar{E}\epsilon_0\, 4\pi r^2 = \frac{6Q(4\pi)}{4\pi(b^3-a^3)}\left(\int_a^{r} r^2\, dr + 1\right)$

$dr,\ r\,d\theta,\ r\sin\theta\, d\phi$

$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$\int_0^{\pi} \sin\theta\, d\theta = 2$

$\int_0^{2\pi} d\phi = 2\pi$

Q جت كعامل مشترك بذبح لانو للمعلوم ان كل موجود كولوم.

$r^2 4\pi\epsilon_0 E = \frac{6Q}{(b^3-a^3)}\left[\frac{r^3}{3}\right]_a^r + 1$

$4\pi\epsilon_0 E r^2 = \frac{6Q}{(b^3-a^3)}\left[\frac{r^3}{3}\right.$

$\bar{E} = \frac{6Q}{(b^3-a^3)}\left[\frac{r^3}{3} - \frac{a^3}{3} + \right.$

$\bar{E} = \left(\frac{6Q r^3}{3(b^3-a^3)} + \frac{6Q}{(b^3-a^3)}\right.$

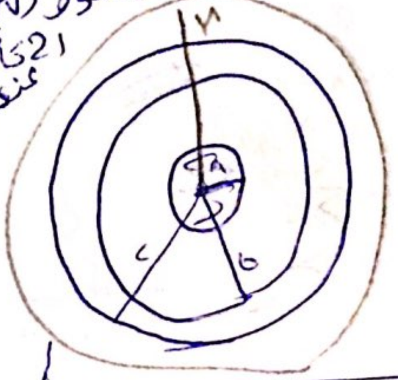
③ $\boxed{b < r < c}$ $\boxed{\bar{E} = 0}$ 0.5

(بزيد مجموع عشان (C + C) كولوم بكولوم.

④ $\boxed{r > c}$

ص وآنا برا بشوف (ρ_v) و Q

ا2 تأثير بالمجال عندي



$$\oint D \cdot ds = Q_{enc} = \int_v \rho_v \, dv + Q$$

مساحة سطح كروي

$$\bar{E}\,\epsilon_0\,(4\pi r^2) = \int_v \frac{6Q \quad dv}{4\pi(b^3-a^3)} + Q$$

$\boxed{dv = r^2 \sin\theta \, dr \, d\theta \, d\phi}$

$$\bar{E}\,\epsilon_0\,(4\pi r^2) = \int_0^{2\pi}\int_0^{\pi}\int_a^{b} \frac{6Q\, r^2 \sin\theta \, dr\, d\theta\, d\phi}{4\pi(b^3-a^3)} + Q$$

المنطقة اللي فيها (ρ_v) ولما أطلع من برا عليها r تكون من a لـ b بس

$$\int_0^{\pi} \sin\theta\, d\theta = 2, \quad \int_0^{2\pi} 1\, d\phi = 2\pi \Big\} \; 4\pi$$

ثابت برا أكا من

$$\bar{E}\,\epsilon_0\,(4\pi r^2) = \frac{\cancel{4\pi}\; 6Q}{\cancel{4\pi}(b^3-a^3)}\left[\int_a^b r^2\, dr + 1\right]$$

$$\bar{E}\,\epsilon_0\,(4\pi r^2) = \frac{6Q}{(b^3-a^3)}\left[\left.\frac{r^3}{3}\right]_a^b + 1\right]$$

$$\bar{E}\,\epsilon_0\,(4\pi r^2) = \frac{6Q}{(b^3-a^3)}\left[\frac{b^3}{3} - \frac{a^3}{3} + 1\right]$$

$$\bar{E}\,\epsilon_0\,(4\pi r^2) = \frac{6Q}{(b^3-a^3)}\left[\frac{1}{3}(b^3-a^3) + 1\right]$$

$$\bar{E}\,\epsilon_0\,(4\pi r^2) = \frac{6Q}{3} + \frac{6Q}{(b^3-a^3)}$$

$$\bar{E} = \frac{6Q\left(\frac{1}{3} + 1/(b^3-a^3)\right)}{4\pi\epsilon_0 r^2} \; \frac{N}{C} \; \hat{a}_r$$

4 0.5

(C) سؤال (C) فرع

$r < a \rightarrow \boxed{Q_{enc} = QC}$ كولوم

$a < r < b \rightarrow Q = \int_v \rho_v \, dv$

$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$

$$Q_{enc} = \int_0^{2\pi}\int_0^{\pi}\int_a^{b} \frac{6Q r^2 \sin\theta \, dr \, d\theta \, d\phi}{4\pi(b^3-a^3)}$$

4π

$$= \frac{6Q(4\pi)}{4\pi(b^3-a^3)} \int_a^b r^2 \, dr$$

(1)

$$= \frac{6Q}{(b^3-a^3)}\left[\frac{b^3}{3} - \frac{a^3}{3}\right]$$

$$= \frac{6Q}{(b^3-a^3)} \frac{(b^3-a^3)}{3}$$

$= 2Q$ (C) كولوم

مذكور بالسؤال ← $b < r < c$

$Q_{enc} = -Q$ (C) كولوم

(d) $W_E = \frac{1}{2}\int_v \epsilon_0 |E|^2 dv \rightarrow$ (J) جول

(0.5)

$$= \frac{\epsilon_0}{2}\int_v |E|^2 (dv) \rightarrow$$

بين a, b $E \rightarrow$

$$E = \frac{6Q\left(\frac{(r^3-a^3)}{3}+1\right)}{4\pi\epsilon_0 r^2} \; \frac{N}{C}$$

$$\boxed{E = \frac{6Q}{(3)4\pi\epsilon_0 r^2}\left[(r^3-a^3)+\frac{1}{3}\right] \; V/m}$$

(D) سؤال (C) فرع

$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$

$$= \frac{\epsilon_0}{2}\int_0^{2\pi}\int_0^{\pi}\int_a^{b} |E|^2 \, r^2 \sin\theta \, dr \, d\theta \, d\phi \rightarrow$$ جول (J)

طويل أنو أدعوهاي وأشكال لانه بطلع كثير

(B) سؤال (C) فرع طلوب (volt) لأنه

كثير طويل

$V = -\int_c E \, dl \rightarrow$

رح يكون براحة شوية $\int \rho_v dv + Q$

$$V = \frac{\int_v \rho_v \, dv}{4\pi\epsilon_0 r}$$ $\boxed{dv = r^2 \sin\theta \, dr \, d\theta \, d\phi}$

(1) أول نقطة رح آخذها من (∞) أعتبر $(r<\ell)$ $\boxed{V(\infty)=0}$ (C) نعوض

$$= \int_v \frac{6Q}{4\pi(b^3-a^3)} dv = \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{\pi}\int_\infty^{r} \frac{6Q \, r^2 \sin\theta \, dr \, d\theta \, d\phi}{4\pi(b^3-a^3)}$$

$$= \frac{4\pi \epsilon_0 6Q}{4\pi\epsilon_0 (b^3-a^3)} \int_\infty^r \frac{r^3 dr}{r^2} =$$

$$\boxed{V_{\infty \rightarrow c} = \frac{4\pi\epsilon_0 6Q \, r^4}{(b^3-a^3)4} \; (Volt)}$$ (1)

(e) رح أنتقل من (C) لـ (B)

$E = 0 \therefore V = 0 + \frac{\text{نفس الشيء}}{C}$

(B, C) بين $= \frac{4\pi\epsilon_0 Q \, r^4}{4(b^3-a^3)}$ (Volt) (C)

(3) رح أنتقل من (B) لـ (A)

$\boxed{dv = r^2 \sin\theta \, dr \, d\theta \, d\phi}$

$$V = \frac{\int_v \rho_v \, dv}{4\pi\epsilon_0 r} \; (volt)$$ ✗

$$= \int_v \frac{6Q}{4\pi(b^3-a^3)} dv \Big/ (4\pi\epsilon_0 r) = \int_v \frac{6Q \, 4\pi\epsilon_0 r}{4\pi(b^3-a^3)} dv$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \frac{6Q}{(b^3-a^3)} r^3 \sin\theta \, dr \, d\theta \, d\phi = \frac{4\pi 6Q}{(b^3-a^3)}\left[\frac{b^4-a^4}{4}\right]$$

(4) رح أروح من $A \leftarrow 0$ صفر

$\boxed{E = 0}$

جسم مصمت
$E = 0 +$ إلي رح يحافظ
على أتصال هو الطاقة

4

$$= V_{(B)} = \frac{4\pi 6Q}{(b^3-a^3)}\left(\frac{b^4-a^4}{4}\right) \; (volt)$$

(0.5)

Problem 3: (5 points)

3.5

Find the potential at point A(6,1,8) due to a line charge with 1 nC/m located on the y axis given that the potential at point B(4, 1, 3) is = V_o Volt.

$$\bar{E}_{line} = \frac{\rho_L \, \hat{a}_\rho}{2\pi\epsilon_0 \rho} \left(\frac{N}{C}\right)$$

$$\bar{E} = \frac{10^{-9} \, \hat{a}_y}{2\pi \frac{10^{-9}}{36\pi} 10\text{D}}$$

$$\boxed{\bar{E} = 0.18 \, \hat{a}_y \; \frac{N}{C}}$$

$$V = -\int_L E \, dl$$

1

$$V = \frac{Q}{4\pi\epsilon_0 r} = \frac{\int_L \rho_L dl}{4\pi\epsilon_0 r}$$

$$V_{BA} = V_A - V_B$$

$$\boxed{V_A = V_{BA} + V_o} \text{ (Volt)}$$

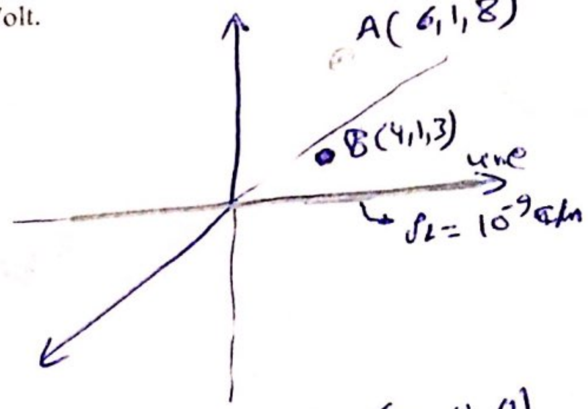
1

$$V = -\int_{a}^{b} 0.18 \, dy$$

$$\boxed{V = \#}$$

$$\boxed{V_A = \# + V_o} \text{ (Volt)}$$

1



$$\bar{R} = (6,1,8) - (0,y,0)$$

عين عن (A)

$$\bar{\rho} = (6,0,8)$$

$$|\bar{\rho}|^2 = 10\text{D}$$

0.5

$$-E = \nabla V$$

$$Q = \int_L \rho_L dl$$

$$= \int_{-\infty}^{\infty} 10^{-9} \, dy$$

$\frac{26}{30}$

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(34)

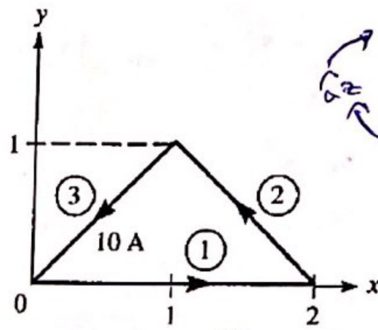
Note that bold letters are vectors

$$\nabla^2 A = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial A}{\partial r}\right) + \boxed{\frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial A}{\partial \theta}\right)} + \frac{1}{r^2 sin\theta^2}\frac{\partial^2 A}{\partial \phi^2} \qquad \varepsilon_o = \frac{10^{-9}}{36\pi}\ \text{F/m} \qquad \int \csc x\, dx = ln\left|tan\frac{x}{2}\right|$$

(5

Problem 1 (7 points)

The conducting triangular loop below carries a current of 10 A. Find **H** at (0, 0, 5) due to side 2 of the loop.



$$\bar{H} = \frac{I}{4\pi\rho}\left(\cos\alpha_2 - \cos\alpha_1\right)\hat{a}_\phi \quad \left(\frac{A}{m}\right)$$ (1) (0.5)

To find α_1 : $\bar{I}\cdot\bar{A} =$

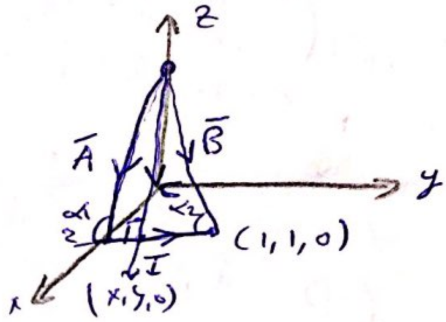
$= (-1,1,0)\cdot(2,0,5) = -2$

$\bar{I}\cdot\bar{A} = |I||A|\cos\alpha_1$

$$\boxed{\cos\alpha_1 = \frac{-2}{\sqrt{2}\sqrt{29}}}$$ (1)

To find α_2 : $\bar{I}\cdot\bar{B} = (-1,1,0)\cdot(1,1,-5) = -1+1+0=0$

$$\boxed{\cos\alpha_2 = 0}$$ (1)



$\bar{I} = (1,1,0)-(2,0,0) = (-1,1,0)$

$\bar{A} = (2,0,0)-(0,0,5) = (2,0,-5)$

$\bar{B} = (1,1,0)-(0,0,5) = (1,1,-5)$

To find (ρ) : عمودي على ضلع 2 يمر من (0,0,5) و (I) لأنه متعامدات .

$\therefore\ \bar{c}\cdot\bar{I} = 0$

$(x,y,-5)\cdot(-1,1,0) = 0$

$-x+y=0$ $\boxed{x=y}$

$\bar{C} = (x,y,0)-(0,0,5)$

$= (x,y,-5)$

من (x, y) بحيث متساويين كانت نفس (B)،

$\therefore\ \rho = \bar{B} = (1,1,-5) = |\bar{\rho}| = \sqrt{27}$

To find $\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho$ (0.5)

$= (\hat{a}x, \hat{a}y) \times (\hat{a}x, \hat{a}y, \hat{a}z)$

$= \hat{a}z - \hat{a}y - \hat{a}z + \hat{a}x$

$= \hat{a}x, -\hat{a}y$

$$\bar{H} = \frac{10}{4\pi\sqrt{27}}\left(0 - \frac{-2}{\sqrt{2}\sqrt{29}}\right)(\hat{a}x, -\hat{a}y) \quad A/m$$ (1)

تعويض أرقام .

$= \hat{a}z - \hat{a}y - \hat{a}z + \hat{a}x$

9

Problem 2 (7 points)

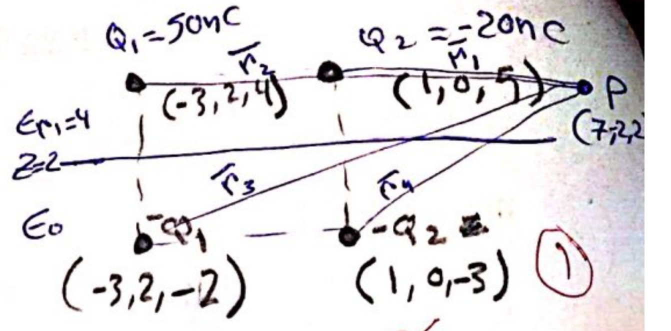
Two-point charges of 50 nC (Q_1) and -20 nC (Q_2) are located at (- 3, 2, 4) and (1, 0, 5) in region $z \geq 2$ with $\varepsilon_r = 4$ above the conducting ground plane z = 2. Calculate:

(a) the surface charge density at (7, -2, 2)

(b) **D** at (3, 4, 8)

(c) **D** at (1, 1, 1)

(method of images)



(a) $\bar{D} = \dfrac{Q}{4\pi r^2}\hat{a}_r$ $\rho_s = \bar{D}\cdot\hat{a}_n$

$D = D_1 + D_2 + D_3 + D_4$

$= \dfrac{1}{4\pi}\left(\dfrac{-20nC\,\bar{r}_1}{r_1^{3/2}} + \dfrac{50nC\,\bar{r}_2}{r_2^{3/2}} + \dfrac{-50nC\,\bar{r}_3}{r_3^{3/2}} + \dfrac{20nC\,\bar{r}_4}{r_4^{3/2}}\right)$

$= \dfrac{10^{-9}}{4\pi}\left(\dfrac{-20\,\bar{r}}{(6^2+4+9)^{3/2}} + \dfrac{50\,\bar{r}_2}{(100+16+4)^{3/2}} + \dfrac{-50}{r_2} \cdots\right)$

$= \dfrac{10^{-9}}{4\pi}\Big[\left(\dfrac{-20}{343}(6\hat{a}_x, -2\hat{a}_y, -3\hat{a}_z)\right)$
$+ \left(+0.03836\,(10\hat{a}_x, -4\hat{a}_y, -2\hat{a}_z)\right)$
$+ \left(-0.33\,(10\hat{a}_x, -4\hat{a}_y, 4\hat{a}_z)\right)$
$+ \left(0.0467\,(6\hat{a}_x, -2\hat{a}_y, 5\hat{a}_z)\right)\Big]$

$\boxed{D = \#}$ $(\hat{a}_x, \hat{a}_y, a_z)\left(\frac{C}{m^2}\right)$ بس ما في حل = 2

$\left(\frac{Vf}{m^2}\right)$

$\rho_s = \bar{D}\cdot\hat{a}_n$ (C/m^2) ويكون 2 باتجاه العمودي $\hat{a}_z$ أي عمودي

(كل العمل صح)

(a) $\bar{r}_1 = (7,-2,2) - (1,0,5) = (6,-2,-3)$

$\bar{r}_2 = (7,-2,2) - (-3,2,4) = (10,-4,-2)$

$\bar{r}_2 = (7,-2,2) - (-3,2,-2) = (10,-4,4)$

$\bar{r}_3 = (7,-2,2) - (1,0,-3) = (6,-2,5)$

$\bar{r}_4 = (7,-2,2) - (1,0,-3)$

$D = \varepsilon_0 E$

(b) $\bar{r}_5 = (3,4,8) - (1,0,5) = (2,4,3)$

$\bar{r}_6 = (3,4,8) - (-3,2,4) = (6,2,4)$

$\bar{r}_7 = (3,4,8) - (-3,2,-2) = (6,4,10)$

$\bar{r}_8 = (3,4,8) - (1,0,-3) = (2,4,11)$

(B) نفس ال

(b) $\bar{D} = \dfrac{10^{-9}}{4\pi}\left(\dfrac{20(2\hat{a}_x, 4\hat{a}_y, 3\hat{a}_z)}{(16+4+9)^{3/2}} + \dfrac{50(6,2,4)}{(36+4+16)^{3/2}} + \dfrac{-50(6,0,10)}{(36+100)^{3/2}} + \dfrac{20(2,4,11)}{(4+16+11^2)^{3/2}}\right)$

$D = \ldots \left(\dfrac{Vf}{m^2}\right)$

$\bar{r}_9 = (1,1,1) - (1,0,5) = (0,1,-4)$

$\bar{r}_{10} = (1,1,1) - (-3,2,4) = (4,-1,-3)$

$\bar{r}_{11} = (1,1,1) - (-3,2,-2) = (4,-1,3)$

$\bar{r}_{12} = (1,1,1) - (1,0,-3) = (0,1,4)$

(c) نفس ال

(c) $\bar{D} = \dfrac{Q}{4\pi r^2}\hat{a}_r$

$\dfrac{10^{-9}}{4\pi}\Big[\left(\dfrac{20(0,1,4)}{(16+1)^{3/2}}\right) + \left(\dfrac{50(4,-1,-3)}{(16+1+9)^{3/2}}\right)$
$\left(\dfrac{-50(4,-1,3)}{(16+1+9)^{3/2}}\right) + \left(\dfrac{20(0,1,4)}{(16+1)^{3/2}}\right)\Big]$

$\left(\dfrac{Vf}{m^2}\right)$

Problem 3: (7 points)

(4)

Two half-space dielectric regions, region 1 ($z \geq 0$) with $\varepsilon_{r1} = 3$ and region 2 ($z \leq 0$) with $\varepsilon_{r2} = 4.5$ F/m. In the first region $\mathbf{E_1} = 0.4\,\mathbf{a_x} + 0.3\,\mathbf{a_y} - 0.75\,\mathbf{a_z}$ V/m; Then find:

a) $\mathbf{E_2}$

b) the bounded surface charge densities at $z = 0^-$ and at $z = 0^+$; if they exist.

c) The angle which $\mathbf{E_2}$ makes with the positive z axis and show a graph of this

(a) $E_{1t} = E_{2t}$

$$\boxed{E_{2t} = 0.4\,\hat{a}_x + 0.3\,\hat{a}_y} \quad \frac{N}{C}$$

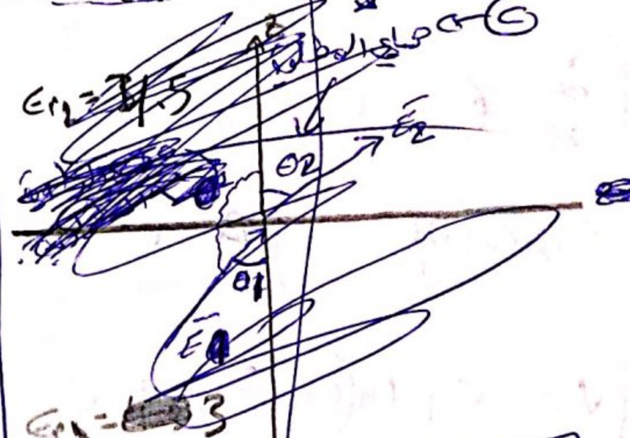
$$E_{1N} = -0.75\,\hat{a}_z\,\frac{N}{C}$$

$$\epsilon_{r2} \bar{E}_{2N} = \bar{E}_{1N}\,\epsilon_{r1} \qquad \bar{E}_2 = \frac{-0.75 \cdot 3}{4.5}$$

$$\boxed{\bar{E}_{2N} = -0.5\,\frac{N}{C}}$$

$$\boxed{\bar{E}_2 = 0.4\,\hat{a}_x + 0.3\,\hat{a}_y - 0.5\,\hat{a}_z \quad \frac{N}{C}}$$

(3)



~~$\theta_2 = \tan^{-1}\left(\frac{E_t}{E_N}\right) = \theta_2 = 45°$~~ $\boxed{\theta_2 = 45°}$

~~(c) $\sin^{-1}\left(\frac{E_t}{E_2}\right) = \theta_2$~~

(b) at $z = 0^-$ $\quad \epsilon_{r2} = 4.5$

$$\bar{D} = \epsilon_0 \epsilon_{r2} \bar{E}_2$$

$$\bar{D} = (4.5)\epsilon_0\,\bar{E}_2 = (\ \#\ )$$

بدنا بس مركبة عمودية !!

$$\rho_s = \bar{D}\cdot\hat{a}_n = 4.5\epsilon_0\,(-0.5)\,\hat{a}_z\cdot\hat{a}_z$$

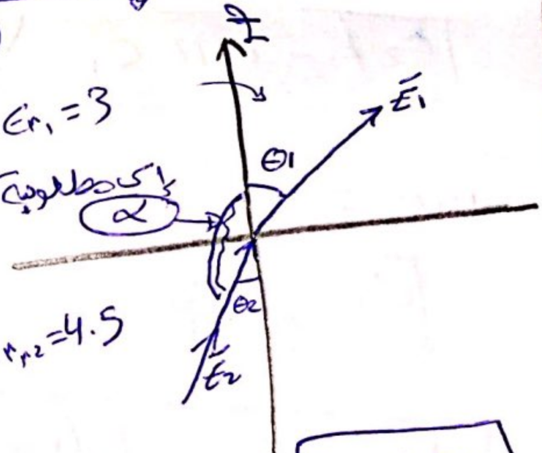
$$= 2.25\,\epsilon_0 \quad C/m^2$$

$z = 0^+$ $\quad \epsilon_{r1} = 3$

$$\rho_s = \bar{D}\cdot\hat{a}_n = 3\,\epsilon_0\,(-0.5)\,\hat{a}_z\cdot\hat{a}_z$$

$$= -1.5\,\epsilon_0 \quad C/m^2$$

(c)



$$\sin\theta_2 = \frac{E_T}{E_2} \qquad \boxed{\theta_2 = 45°}$$

$$\alpha = 180 - \theta_2$$

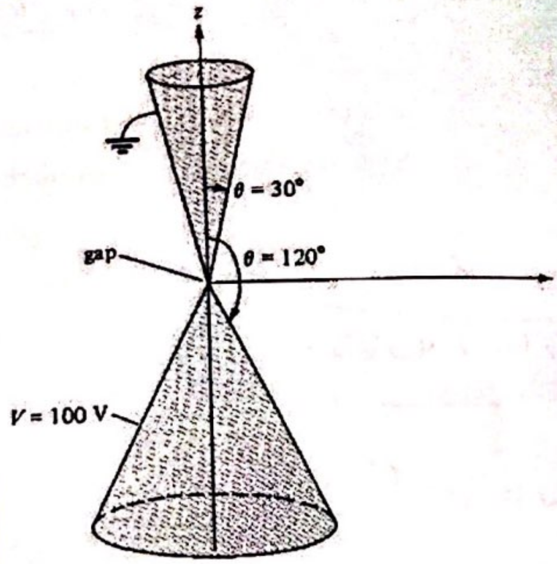
$$\boxed{\alpha = 135°}$$

(1)

6

Problem 4: (7 points)

Find V and E at (3, 0, 4) due to the two conducting cones of infinite extent below.



$\nabla^2 V = 0 \rightarrow$ (تغيير بالنسبة θ)

$\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial V}{\partial\theta}\right) = 0$ 1

$\int \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial V}{\partial\theta}\right) = \int 0$

$\cancel{\sin\theta} \int \frac{\partial V}{\partial\theta} = \int \frac{C_1}{\sin\theta}$

$\boxed{V = C_1 \ln\left(\tan\left(\frac{\theta}{2}\right)\right) + C_2}$ (Volt) 1

$\rightarrow V(\theta = 30) = 0, \quad V(\theta = 120) = 100 \rightarrow$ To find C_1, C_2 1

$0 = C_1 \ln\left(\tan\left(\frac{30}{2}\right)\right) + C_2$

$\boxed{C_2 = -1.317\, C_1}$ X 0.5

$100 = C_1 \ln\left(\tan\left(\frac{120}{2}\right)\right) + -1.317 C_1$

$\boxed{C_1 = -130.26}$ 0.5

$\boxed{V = -130.26 \ln\left(\tan\left(\frac{\theta}{2}\right)\right) + (1.317 \times 130.26)}$ $\hat{a}_\theta$ 0.5

$\bar{E} = -\nabla V$ $\left(\frac{N}{C}\right)$ 1

$= 130.26 \frac{1}{\tan\left(\frac{\theta}{2}\right)} \sec^2\left(\frac{\theta}{2}\right) \frac{1}{2}$ 0.5

$\bar{E}\Big|_{\theta = 36.87} = 205.958\ \frac{N}{C}\ \hat{a}_\theta$

$(3, 0, 4) \rightarrow (r, \theta, \phi)$

$\theta \Rightarrow \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$ $\tan\theta = \frac{\rho}{z}$

$\boxed{\theta = 36.87}$ 0.5

$\boxed{V\Big|_{(3,0,4)} = 314.65726 \text{ (Volt)}}$ 0.5

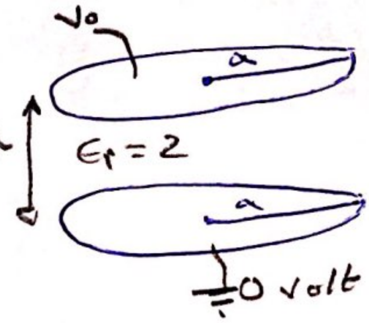
Problem 5: (3 points)

Two parallel disks of radius 'a' are separated by a distance 'd' connected to a 'v_o' battery filled with a dielectric material of $\varepsilon_r = 2$. If the stored energy in the capacitor is 'W';

a) show that the electric field between the plates is:

$$E = \frac{1}{a}\sqrt{\frac{W}{\pi\varepsilon_o d}}$$

b) find its capacitance



3

a) $w = \frac{1}{2} C V^2$

$w = \frac{1}{2}\int_v |\vec{E}|^2 \epsilon \, dv$

$2w = \int_v |\vec{E}|^2 \epsilon_o \, 2 \, dv$

$2w = |\vec{E}|^2 \int 2\epsilon_o \, dv$

$w = |\vec{E}|^2 \epsilon_o \int_0^a \int_0^{2\pi} \int_0^d \rho \, d\rho \, d\phi \, dz$

$w = |\vec{E}|^2 \epsilon_o \, a^2 \, \pi \, d$

$|\vec{E}| = \sqrt{\frac{w}{\epsilon_o a^2 \pi d}}$

$\boxed{|\vec{E}| = \sqrt{\frac{w}{\epsilon_o \pi d}} \left|\frac{1}{a}\right|} \left(\frac{v}{c}\right)$

1.5

$w = \frac{1}{2}\epsilon E^2$

$dv = \rho \, d\phi \, dz$

b) $C = \frac{Q}{V_o}$

$\bar{D} = \epsilon_o \epsilon_r \bar{E}$

$\rho_s = \bar{D} \cdot \hat{a}_n \quad (c/m^2)$

$= \frac{\epsilon_o 2}{a}\sqrt{\frac{w}{\pi \epsilon_o d}}$

$= \frac{2}{a}\sqrt{\frac{\epsilon_o w}{\pi d}}$

$Q = \int \rho_s \, ds$

$ds = \rho \, d\phi \, dz$

$Q = \int\int \frac{2}{a}\sqrt{\frac{\epsilon w}{\pi d}}$

$Q = \sqrt{\frac{\epsilon_o w}{\pi d}} \frac{2}{a} \int_0^{2\pi} \int_0^{d} \rho \, d\phi \, dz$

$= \sqrt{\frac{w \epsilon_o}{\pi d}} \frac{2\cancel{a}}{\cancel{a}} \, 2\pi d$

1.5