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EE 251: Electromagnetics 1   First Exam (Fall 2017)   October 29th, 2017

Duration: 75 minutes

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Note that bold letters are vectors

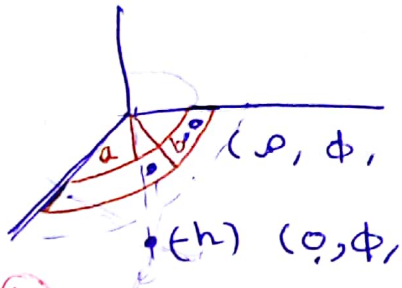
$$\varepsilon_o = \frac{10^{-9}}{36\pi}\text{ F/m},\ \nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z},\ \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix}\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

8.5

Problem 1 (10 points)

A quarter hollow disk defined by a < ρ < b, 0 < ϕ < 90° located at z = 0 plane carries a uniform charge of ρ_s C/m^2.

A. **E** at (0, 0, -h).
B. V at (0, 0, -h).
C. The total charge on the disk.



$(\rho, \phi, 0)$

$(-h)\quad (0, \phi, -h)$

$R = (-\rho, 0, -h)$

$$E = \int \frac{\rho_s \bar{R}\, ds}{4\pi\epsilon_o R^3} = \frac{\rho_s}{4\pi\epsilon_o}\int \frac{-\rho a_\rho - h a_z}{(\sqrt{\rho^2+h^2})^3}\rho\, d\rho\, d\phi$$

(1) (1) (0.5) (1)

canceled due to symmetry.

$$= \frac{\rho_s}{(2)4\pi\epsilon_o}\int_0^{\frac{\pi}{2}}\int_a^{\rho} \frac{-2\rho a_\rho - \cancel{h a_z}}{(\rho^2+h^2)^{3/2}}\rho\, d\rho\, d\phi \quad = \frac{\rho_s}{4\pi\epsilon_o}$$

(0.5)

$$= \frac{\rho_s}{8\pi\epsilon_o}\left(\frac{\pi}{2}\right)*\left[(\rho^2+h^2)^{-\frac{1}{2}}\right]_a^{\rho} * 2$$

$$\boxed{E = \frac{\rho_s}{8\epsilon_o}\left((\rho^2+h^2)^{-\frac{1}{2}} - (\rho^2+a^2)^{-\frac{1}{2}}\right)a_\rho}$$

(C) $\int \rho_s\, ds$

$$\int_0^{\frac{\pi}{2}}\int_a^b \rho_s\, \rho\, d\rho\, d\phi$$

$$= \rho_s\left(\frac{\pi}{2}\right) * \frac{(b^2-a^2)}{2}\ C$$

(2)

(b)

$$V = \int \frac{\rho_s\, ds}{4\pi\epsilon_o r} = \rho_s\int \frac{\rho\, d\rho\, d\phi}{4\pi\epsilon_o\sqrt{\rho^2+h^2}}$$

(1)

$$= \frac{\rho_s}{(2)4\pi\epsilon_o}\int_a^{\rho}\int_0^{\frac{\pi}{2}} \frac{2\rho\, d\rho\, d\phi}{\sqrt{\rho^2+h^2}}$$

$$= \frac{\rho_s}{\ } \left(\frac{\pi}{2}\right)\left[(\rho^2+h^2)^{\frac{1}{2}} * 2\right]^{\rho}$$

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Problem 2 (15 points)

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An infinite cylinder of radius 'a' made of copper is encountered by another infinite copper cylinder of inner radius 'b' and outer radius 'c' where a < b < c and the region between the two conductors is made of Teflon with $\boxed{\varepsilon = 2\varepsilon_o}$. If the charge distribution when the geometry at balance was distributed as follow:

$$\rho_v = \begin{cases} \dfrac{4Q}{a^2} & \rho < a \\ 0 & a < \rho < b \\ \dfrac{-2Q}{(c^2 - b^2)} & b < \rho < c \\ 0 & \rho > c \end{cases}$$

If the copper has $\varepsilon_r = 1$. Find:

a) E everywhere

b) V everywhere

c) The charge distribution on each surface.

d) Energy density stored in the region a < ρ < b.

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① if $\rho < a$

$\oint D \cdot ds = Q_{enc} = 0$

$E = 0$ 0.5

② if $a < \rho < b$

$\int D \cdot ds = Q_{enc}$ 1

$\bar{D} = D_\rho a_\rho$

$= \int \frac{4Q}{a^2} dv$

$= \int_0^L \int_0^{2\pi} D_\rho \rho \, d\phi \, dz$

$D_\rho \rho (2\pi) L = \int_0^L \int_0^{2\pi} \int_0^a \frac{4Q}{a^2} \rho \, d\rho \, d\phi \, dz$

$\bar{D} = \frac{2Q}{\rho} a_\rho$

$\boxed{\bar{E} = \frac{2Q}{\varepsilon \rho} a_\rho}$ 1

$\frac{4Q}{a^2} \left(\frac{a^2}{2}\right)(2\pi)L$

③ $b < \rho < c$

$\int D \cdot ds = Q_{encl} = 0$

④ $\rho > c$

$D_\rho \rho (2\pi) L = \int_0^{\rho} \rho_v \, dv + \int_a^b \rho_v \, dv + \int_b^c + \int_c^{\rho}$

$a < \rho < b$

$D_\rho \rho (2\pi) L = \int_0^L \int_0^{2\pi} \int_0^a \frac{4Q \rho \, d\rho \, d\phi}{a^2} + \int_0^L \int_0^{2\pi} \int_b^c - \frac{2Q \rho \, d\rho \, d\phi}{c^2 - b^2}$

$D \rho (2\pi) L = 4\pi Q L + -2Q \frac{(c^2 - b^2)}{2} (2\pi) L$

$\bar{D} = 2Q + -Q \, a_\rho$

$\bar{D} = Q$

$\boxed{\bar{E} = \frac{Q}{\rho \varepsilon_0}}$

1

$E = \begin{cases} 0, & \rho < a \\ \frac{2Q}{\varepsilon \rho} a_\rho, & a < \rho < b \\ 0, & b < \rho < c \\ Q \, a_\rho, & \rho < c \end{cases}$

0.5

② $\rho > c$

$V = -\int E \cdot dL = \frac{1}{\epsilon_0}\int_{\infty}^{\rho} \frac{Q\,d\rho}{\rho} = \frac{-1}{\epsilon_0} Q(\ln\rho - \ln\infty) =$

$= \frac{Q}{\epsilon_0}\left(\ln\rho - \ln\infty\right) = \boxed{\frac{Q \ln\rho}{\epsilon_0}}$

$b < \rho < c$

$\frac{1}{\epsilon_0}\int_{\infty}^{c} Q\,d\rho + \int_{c}^{\rho} 0\,d\rho = \boxed{\frac{Q \ln c}{\epsilon_0}}$

$a \leq \rho < b$

$\int_{\infty}^{c} + \int_{c}^{b} + \int_{b}^{\rho} \frac{2Q}{\epsilon\rho}$

a

$$\boxed{V = \frac{Q \ln c}{\epsilon_0} + 0 + \frac{2Q}{\epsilon}\ln\left(\frac{\rho}{b}\right)}$$

$\rho < a$

$= \int_{\infty}^{c} + \int_{c}^{b} + \int_{b}^{a} + \int_{a}^{\rho}$

$$\boxed{V = \frac{Q \ln c}{\epsilon_0} + 0 + \frac{2Q}{\epsilon}\ln\left(\frac{a}{b}\right)} + 0$$

3.5

$w_E = \frac{\epsilon}{2} E^2 = \frac{2\epsilon_0}{2} E^2 = \epsilon_0 * E^2$

$= \frac{4Q^2}{\epsilon^2 \rho^2} * \epsilon_0$ jaul

$= \frac{4Q^2\,\epsilon_0}{\epsilon^2 (b^2 - a^2)}$

ε E.E

Problem 3: (5 points)

(4.5)

If V = $\rho^2 z \sin\phi$, calculate the energy within the region defined by $1 < \rho < 4$, $-2 < z < 2$ and $0 < \phi < 60^o$.

$E = -\nabla . V$

$E = -2\rho z \sin\phi \, a_\rho - \rho z \cos\phi \, a_\phi - \rho^2 \sin\phi \, a_z$ (2.5)

$E . E = 4\rho^2 z^2 \sin^2\phi + \rho^2 z^2 \cos^2\phi + \rho^4 \sin^2\phi$

$= \int_{-2}^{2}\int_{0}^{60^o}\int_{1}^{4} 4\rho \; \rho \, d\rho \, d\phi \, dz$ (1)

$\frac{1}{2}(1-\cos 2\phi)$

$\frac{1}{2} * \epsilon \int\int\int 4\rho^3 z^2 \sin^2\phi + \rho^3 z^2 \cos^2\phi + \rho^5 \sin^2\phi$

$= \frac{1}{2}\epsilon * 4\frac{\rho^4}{4}\Big]_1^4 \frac{z^3}{3}\Big]_{-2}^{2} \frac{1}{2}\left(1 - \frac{\sin 2\phi}{2}\right)\Big]_0^{60^o} + \frac{\rho^4}{4}\Big]_1^4 * \frac{z^3}{3}\Big]_{-2}^{2}$

$* \frac{1}{2}\left(1 + \frac{\sin 2\phi}{2}\right)\Big]_0^{60^o} + \frac{\rho^5}{5}\Big]_0^4 * \left(\frac{1}{2} - \frac{1}{2}\frac{\sin 2\phi}{2}\right)_0^{60}$

$= \cancel{\frac{1}{2}\epsilon_0} \Big(\qquad \Big)$