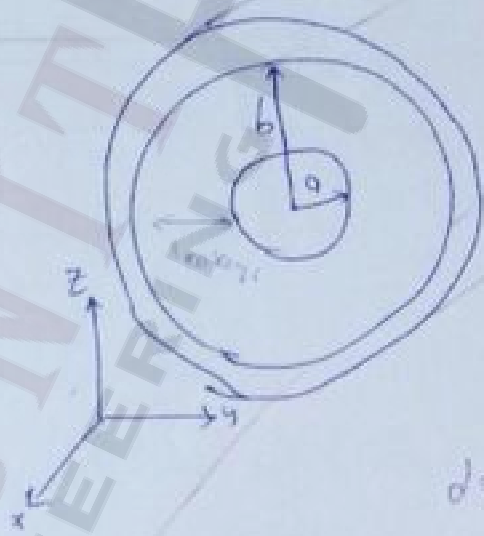
Prob. # 1: A coaxial cable whose inner (solid) and outer conductors are made out of copper with radii = a and b (with $b = a.e$) with e = 2.7183). The medium between the two conductors is a composite material whose $\sigma_2 = 2\pi\ 10^{-6}\ \Omega^{-1}/m$. Determine the leakage resistance between the two conductors for 1000 meters of this cable **[5 PTS]**.



$$dR = \frac{dl}{\sigma\, dA}$$

$$= \frac{dr}{\sigma r\, d\phi\, dz}$$

$$= \int \frac{dr}{r}$$

$$= \frac{1}{\sigma\, d\phi\, dz} \cdot \int_a^b \frac{dr}{r}$$

$$= \frac{1}{\sigma\, d\phi\, dz} \ln\left(\frac{b}{a}\right)$$

$ds = r\, d\phi\, dz\, a_r + dr\, dz\, a_\phi + r\, dr\, d\phi\, a_z$

$$R = \frac{\ln(b/a)}{2\pi r}$$

$$dG = \frac{\sigma\, d\phi\, dz}{\ln(b/a)} = \sigma \frac{\int_0^{1000}\int_0^{2\pi} d\phi\, dz}{\ln(b/a)} = \sigma \int_0^{1000} \frac{2\pi\, dz}{\ln(b/a)}$$

$$= \sigma \int_0^{1000} 2\pi\, z\Big|_0^{1000}$$

$$G = \frac{2\pi\sigma(1000)}{\ln(b/a)}$$

$$\Rightarrow R = \frac{\ln(b/a)}{2\pi\sigma(1000)} = \frac{\ln\left(\frac{a.e}{a}\right)}{2\pi(2\pi \times 10^{-6})(1000)}$$

$$= \frac{1}{4\pi^2 \times 10^{-3}}\ \Omega$$

$$= \boxed{\frac{10^3}{4\pi^2}}\ \Omega$$

Prob. # 2: Two half-space dielectric regions, region 1 ($z \geq 0$) with $\varepsilon_1 = 3\varepsilon_0$ F/m and region 2 ($z \leq 0$) with $\varepsilon_2 = 4.5\varepsilon_0$ F/m. In the first region $\mathbf{E_1} = 0.4\,\mathbf{a_x} + 0.3\,\mathbf{a_y} - 0.75\,\mathbf{a_z}$ V/m; Then find:

(i) $\mathbf{E_2}$ and the bounded surface charge densities at $z = 0^-$ and at $z = 0^+$; if they exist **[7 PTS]**.

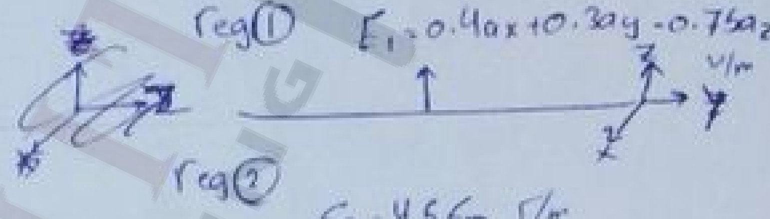
ρ_s

(1) $E_2 = a\vec{a_x} + b\vec{a_y} + c\vec{a_z}$

~~$E_{1t} = E_{2t}$~~

$\epsilon_1 = 3\epsilon_0$ C/m

reg(1) $E_1 = 0.4a_x + 0.3a_y - 0.75a_z$ V/m



reg(2) $\epsilon_2 = 4.5\epsilon_0$ F/m

~~E_{2t}~~ $= E_1$

$\boxed{E_2 = E_{2t} + E_{2n}}$

$\rho_s = Q$

$E_{2n} = E_{1n} = -0.75\vec{a_z}$

$E_{2t} - E_{1t} = \rho_s$

$E_{2t} = E_{1t} + \rho_s$

$= (0.3\,a_y) + (4.5\epsilon_0)$

$E_{2t} = 0.3a_y + 4.5(\epsilon_0)$

~~$E_2 = E$~~ $\boxed{E_2 = -0.75a_z + 0.3a_y}$

$\boxed{\rho_s = 0}$

(ii) The angle which $\mathbf{E_2}$ makes with the positive z axis and show a graph of this **[3 PTS]**.

$E_1 \sin\theta_1 = E_{1t} = E_{2t} = E_2 \sin\theta_2$

$(0.4a_x + 0.3a_y - 0.75a_z)\sin\theta_1 = 0.3a_y$

$\sin\theta_1 = \frac{1}{\sqrt{2}}$

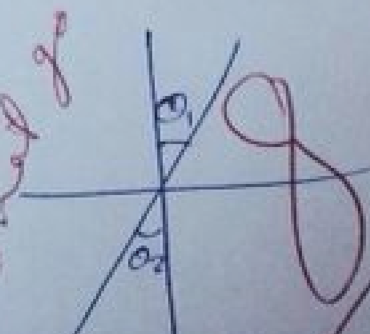
$\boxed{\theta_1 = 45^\circ}$

$\boxed{\theta_2 = 135^\circ}$

~~$E_1\sin\theta = E_2\sin\theta$~~

$E_1 \sin\theta = E_{1t} = E_{2t} = E$

$\theta = 45^\circ$



(correct ~~wrong~~ answer but it does not go along with

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Prob. # 3: A hollow infinite cylindrical shell whose radius $a \le r \le b$ and its conductivity = $10^6/(2\pi r)\ \Omega^{-1}$/m, If the electric field inside it $(a \le r \le b = 3a)$ is given by $\mathbf{E} = 0.5\ \mathbf{a}_z$ microVolt/m; Then: **(i)** Find the surface current density **J** inside this shell and the total current flowing inside it at the plane $z = 0$ **[5Pts]**.

$C = \frac{Q}{V}$, $E = \frac{V}{r}$ $E = 0.5\vec{a}_z$

$0.5\vec{a}_z = \frac{V}{3a - a} \Rightarrow V = 2a(0.5\vec{a}_z)\ V$

$\frac{10^6}{2\pi r} = \frac{Q}{}$ $\cancel{E = 0.5\vec{a}_z}$ $\frac{10^6}{2\pi r} = \frac{Q}{2\pi r L}$

$I = \frac{10^6 L}{\pi L^2} = \boxed{\frac{10^6}{L}}$ A $\boxed{\rho_L = L\,10^6} = J$

the presence of current leads to E and not E due to charge !!

(ii) Find the magnetic field **H** everywhere **[6 Pts]**.

(1) $\boxed{r < 0 \quad H = 0}$

(2) $H \rightarrow a \le r \le b$

$\oint \vec{H} \cdot d\vec{s} = I_{enc}$ $\oint H \cdot dl = I_{enc}$

$H = \frac{I}{2\pi r}$

$\cancel{2\pi r H = J}$

$\boxed{H = \frac{I r^2}{2(r^2 + a^2)^{3/2}} a_\phi}$

(3) $\cancel{r > b \quad \boxed{H = 0}}$

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(iii) Total magnetic flux crossing plane $0 \le r \le 2b$ and $0 \le z \le L$ located at $\phi = 0^\circ$ **[4 PTS]**.

$\psi = \oint B \cdot dS$

$\cancel{|R| = \sqrt{L^2 + 4b^2}}$

$R = -L a_z + r a_r$

$= \frac{\mu \oint I\, dl \times a_R}{2\pi R^2}$

$= \mu \int_0^L \int_0^{2b} \frac{I\, dr\, dz\, \vec{a}_r \times \vec{a}_R}{2\pi R^2} = \mu \int_0^L \int_0^{2b} \frac{I\, dr\, dz\,(-\vec{a}_z)}{2\pi R^2}$

$= \mu I \int_0^L \int_0^{2b} \frac{dr\, dz\,(-L a_z + 2b a_r)}{2\pi (L^2 + 4b^2)^{3/2}} = \mu I \int_0^L \frac{2b\, dz\,(-L a_z + 2b a_r)}{2\pi (L^2 + 4b^2)^{3/2}}$

$\boxed{z = 0}$

$= \frac{\mu I (-L a_z + 2b a_r)\, 2b (L)}{}$ $\boxed{\mu I (4b^2) L\ a_r}$