“EM”

FIRST EXAM

FALL-2013

POWER UNIT

ELECTRICAL ENGINEERING

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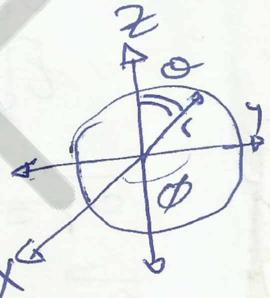
$\frac{10.5}{20}$

UNIVERSITY OF JORDAN
Electrical Engineering Dept.

Electromagnetics I: EE251 $\varepsilon_o = (1/36\,\pi)\ 10^{-9}$ F/m **First Mid-Term Exam:22/10/2013**

Prob. # 1[5 PTS]: A certain electrical source produces an electric field **E** = - 10 $\mathbf{a}_x$ + 10 $\mathbf{a}_y$ V/m at P(5, 45°, 135°); Then write down **E** in spherical coordinate.

$x = -10 \quad y = 10 \quad \therefore r = \sqrt{200}$



$\cancel{E = -10(\vec{a_r}\sin\theta\cos\phi + a_\theta\cos\theta\cos\phi - a_\phi\sin\phi)}$
$\cancel{+10(\vec{a_r}\sin\theta\sin\phi + a_\theta\cos\theta\sin\phi + a_\phi\cos\phi)}$

$\cancel{= -10\left(\frac{1}{\sqrt2}\cdot\frac{1}{\sqrt2}\vec{a_r} + a_\theta\frac{1}{\sqrt2}\cdot\frac{-1}{\sqrt2} - a_\phi\cdot\frac{1}{\sqrt2}\right)}$
$\cancel{+10\left(\vec{a_r}\frac{1}{\sqrt2}\cdot\frac{+1}{\sqrt2} + a_\theta\frac{1}{\sqrt2}\cdot\frac{+1}{\sqrt2} - a_\phi\cdot\frac{1}{\sqrt2}\right)}$

$\cancel{= -5a_r - 5a_\theta - \frac{10}{\sqrt2}a_\phi - 5}$

$\Rightarrow \vec{a_x} = a_r\sin\theta\cos\phi + a_\theta\cos\theta\cos\phi - a_\phi\sin\phi$

$= \frac{-1}{2}\vec{a_r} - \frac{1}{2}a_\theta - \frac{1}{\sqrt2}a_\phi$

الحل في الخلف ⇒

$\vec{a_y} = +\frac{1}{2}\vec{a_r} + \frac{1}{2}a_\theta - \frac{1}{\sqrt2}a_\phi$

$x = -10 \quad y = 10 \quad \therefore r = \sqrt{(10)^2+(10)^2} = \sqrt{200}$

$\theta = \cos^{-1} 0 = \frac{\pi}{2}$

$\phi = \tan^{-1}\frac{10}{10} = \tan^{-1}(-1)$

$= \frac{3\pi}{4}$

$\therefore E = \sqrt{200}$

في الخلف ⇒

$(5, 45^\circ, 135^\circ)$
$\theta \quad \phi$

$$E = E_r \vec{a_r} + E_\theta \vec{a_\theta} + E_\phi a_\phi$$

$$E_r = \sqrt{(10)^2 + (10)^2} = \sqrt{200}$$

$$E_\theta = \cos^{-1}(0) = \frac{\pi}{2}$$

$$E_\phi = \tan^{-1}(-1) = \frac{3\pi}{4}$$

$$\cancel{\vec{a_r} = a_x \cos\phi \sin\theta + a_y \sin\phi \sin\theta + a_z \cos\theta}$$

$$\cancel{\vec{a_\theta} = a_x \cos\theta \cos\phi + a_y \sin\phi \cos\theta - a_z \sin\theta}$$

$$\cancel{a_\phi = -a_x \sin\phi + a_y \cos\phi}$$

$$\cancel{\therefore \vec{a_r} = a_x \frac{1}{\sqrt2} * \frac{1}{\sqrt2} + a_y \frac{1}{\sqrt2} * \frac{1}{\sqrt2} + a_z \frac{1}{\sqrt2}}$$

$$\cancel{\vec{a_\theta} = a_x \frac{1}{\sqrt2} * \frac{1}{\sqrt2} + a_y}$$

$$a_x = -\frac{1}{2}\vec{a_r} - \frac{1}{2} a_\theta - \frac{1}{\sqrt2}\vec{a_\phi}$$

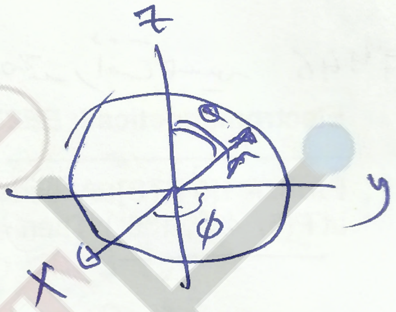
$$a_y = \frac{1}{2}\vec{a_r} + \frac{1}{2} a_\theta - \frac{1}{\sqrt2}\vec{a_\phi}$$

$$E = -10\left(-\frac{1}{2}\vec{a_r} - \frac{1}{2} a_\theta - \frac{1}{\sqrt2} a_\phi\right) + 10\left(\frac{1}{2}\vec{a_r} + \frac{1}{2} a_\theta - \frac{1}{\sqrt2} a_\phi\right)$$

$$\therefore E = 5a_r + 5a_\theta + \frac{10}{\sqrt2} a_\phi + 5a_r + 5a_\theta - \frac{10}{\sqrt2} a_\phi$$

$$= 10\vec{a_r} + 10\vec{a_\theta}$$

$$\therefore E = \sqrt{200}\,\vec{a_r} + \frac{\pi}{2}\vec{a_\theta}$$



1.5

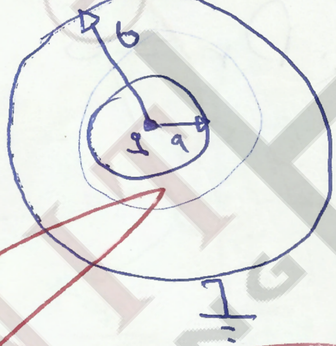
Next time give one answer!!

V/m

two answers one is ✓ the other is ✗ will cancel each other!!

Prob. # 2 [8 PTS]: A good conducting spherical shell whose inner and outer radii are "a" and "b" respectively ($a < b$). A positive charge q is located at the center of this shell; Then find the **electric field**, **electric flux density** and the **potential** for this arrangement everywhere except at r = 0.

$\bar{E}$ D V



8

$\cancel{E = \frac{q}{4\pi\epsilon_0 r^2} a_r}$

$\cancel{Q = \iint \rho_s \, ds}$

for $r < a$?

$$\oint\!\!\oint D \cdot ds = \iint \rho_s \, ds = q$$

$$\int_0^{2\pi}\!\!\int_0^{\pi} D_r r^2 \sin\theta \, d\theta \, d\phi = \iint \rho_s r^2 \sin\theta \, d\theta \, d\phi$$

$$D_r 4\pi r^2 = 0$$

$$\therefore D_r = 0 \qquad E_r = 0 \qquad V = 0 \, V$$

$a \leq r \leq b$

$$D_r 4\pi r^2 = \int_0^{2\pi}\!\!\int_0^{\pi} \rho_s a^2 \sin\theta \, d\theta \, d\phi$$

تابع $\Longrightarrow$

$$D_r 4\pi r^2 = \rho_s a^2 \int_0^{2\pi} -\cos\theta \Big|_0^{\pi} d\phi$$

$$= \rho_s a^2 \int_0^{2\pi} 2 \, d\phi = \rho_s a^2 4\pi$$

$$D_r \cancel{4\pi} r^2 = \rho_s a^2 \cancel{(4\pi)} \qquad D = \epsilon_0 E$$

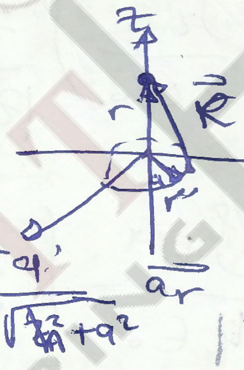
$$D_r = \frac{\rho_s a^2}{r^2} \, C/m^2 \qquad E_r = \frac{\rho_s a^2}{\epsilon_0 r^2} \, V/m$$

Prob. # 3 [7 PTS]: A ring which is made out of very thin conducting wire. The ring carry a linear charge density ρ_o C/m. The ring radius = a m and it is located in the x-y plane with its center at the origin; Then: **(i)** Find **E** at point (0, 0, h). **(ii)** Find V and **E** at the origin.

$\therefore \vec{R} = r - r'$

$= h\vec{a_z} - a\vec{a_r}$

$|\vec{R}| = \sqrt{h^2+a^2}$



$\therefore \vec{a_r} = \frac{h}{\sqrt{h^2+a^2}} a_z - \frac{a}{\sqrt{h^2+a^2}} a_r$

(i) $E = \int \frac{\rho_L \, dL}{4\pi\epsilon_0 R^2} \vec{a_r}$

$= \int_0^{2\pi} \frac{\rho_L a \, d\phi}{4\pi\epsilon_0 (h^2+a^2)^{3/2}} \left(h\vec{a_z} - a\vec{a_r} \right)$

~~by sim~~ using simetry $\therefore$

$= \int_0^{2\pi} \frac{\rho_L a \, d\phi}{4\pi\epsilon_0 (h^2+a^2)^{3/2}} \left(h\vec{a_z} \right)$

$E_z = \frac{\rho_L h a}{4\pi\epsilon_0 (h^2+a^2)^{3/2}} \int_0^{2\pi} d\phi$

$= \frac{\rho_L h a}{4\pi\epsilon_0 (h^2+a^2)^{3/2}} \phi \Big|_0^{2\pi}$

$= \frac{\rho_L h a}{\cancel{4\pi}_{2}\, \epsilon_0 (h^2+a^2)^{3/2}} * \cancel{2\pi} = \frac{\rho_L h a}{2\epsilon_0 (h^2+a^2)^{3/2}}$ V/m

تابع $\Rightarrow$

(ii) $V = -\int E \cdot dL$

$= -\int \frac{\rho_L z a \, \vec{a_z}}{2\epsilon_0 (z^2+a^2)^{3/2}} dL = -\int \frac{\rho_L z a}{2\epsilon_0 (z^2+a^2)^{3/2}} dz$ $\Rightarrow$

$$V = -\int \frac{\rho_L z a}{2\epsilon_0 (z^2+a^2)^{3/2}}\, dz$$

$$= -\frac{\rho_L a}{2\epsilon_0}\int \frac{z}{(z^2+a^2)^{3/2}}\, dz \qquad u = z^2+a^2$$

$$du = 2z\, dz$$

$$dz = \frac{du}{2z}$$

$$= -\frac{\rho_L a}{4\epsilon_0}\int \frac{1}{u^{3/2}}\, du \qquad u^{-3/2}$$

$$\frac{u^{-1/2}}{-1/2} = -2u^{-1/2}$$

$$= +\frac{\rho_L a}{2\,\cancel{4}\epsilon_0}\left(\cancel{+2}\, u^{-1/2}\right)$$

$$= \frac{\rho_L a}{2\epsilon_0}\,\frac{1}{\sqrt{u}} = \frac{\rho_L a}{2\epsilon_0\sqrt{z^2+a^2}}$$

$$\boxed{V(0,0,0) = \frac{\rho_L \cancel{a}}{2\epsilon_0 \cancel{a}} = \frac{\rho_L}{2\epsilon_0}\ V}$$

$$E_z(0,0,0) = 0\ V/m$$