No. "Control"

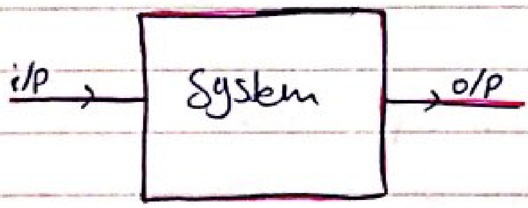
Definitions:

A system : a collection of objects joined together to perform a certain objective.

e.g A thermal system
i) wood (coal) + $O_2 \longrightarrow$ heat
ii) Voltage source or current source + resistance $\longrightarrow$ heat

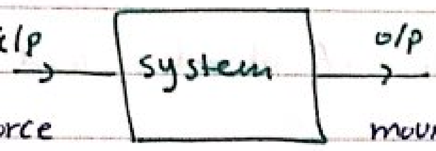
A commuting system : horse and carrige

$\Rightarrow$ In our course we utilize (use) feedback to achieve control



system without feedback
"open loop"

No. 4/6/2018

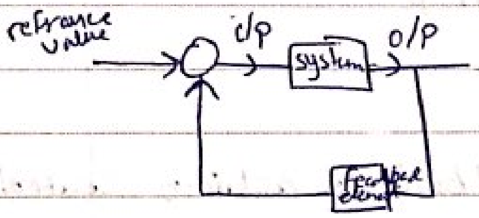


Force — movment

current — light, or, heat, movment

Instructions — degree of understanding

Feedback systems :- are systems where no o/p or part of it becomes the i/p or part of it.



Feedback system

Closed loop system

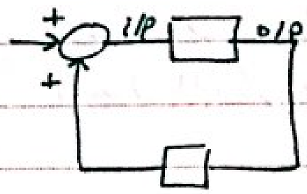
5/6/2018

Microwave is an open loop system.

washing machine " " " " .

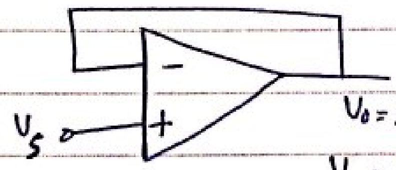
dishwasher machine " " " " .

positive and negative feedback :-



* with positive feedback o/p is returned to the input without inversion.

* with negative " " " " " "
" with inversion



$$V_o = A(V^+ - V^-)$$

$$V_o = A(V_s^+ - V_o)$$

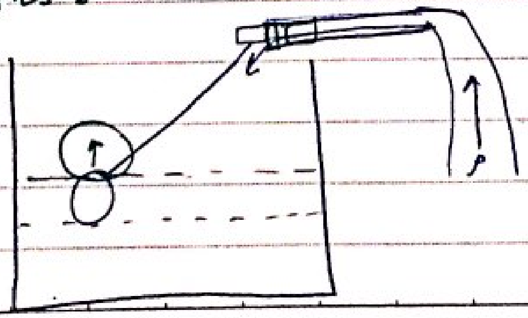
$$V_o = A(V_s - V_o)$$

$$(1+A)\, V_o = A\, V_s$$

$$\frac{V_o}{V_s} = \text{gain} = \frac{A}{1+A} \approx 1 \quad \text{Since } A \gg 1$$

* generally negative feedback systems are more useful than positive feedback systems (subjective according to application)

Examples :-

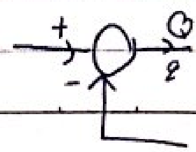


Negative Feedback

output ↑ ⇒ input ↓

الارتفاع

filled water



No.

الصورة الأكثر مشاهدة
المقال الأكثر قراءة ⟹ positive feedback
as the number of readers or watchers will increase when they read the title

electric iron ⟹ negative feedback

Antenna Control ⟹ negative feedback

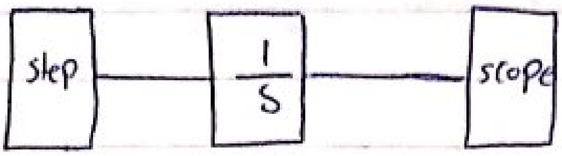
oscillator ⟹ Positive feedback

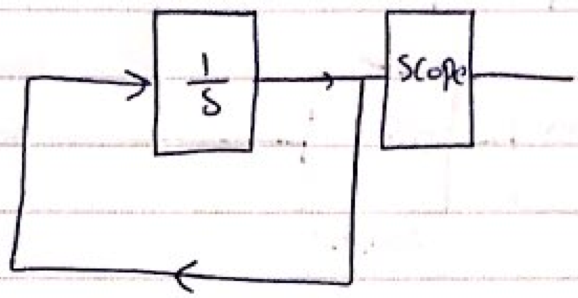
* parameter variations:- changes come up on the system

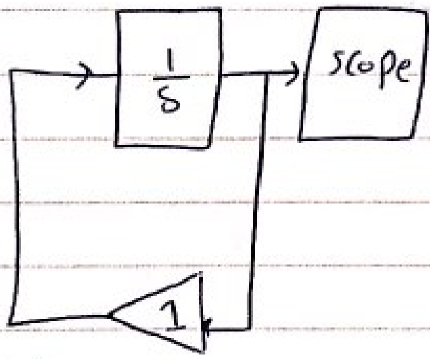
we say this system experinces parameter variation.

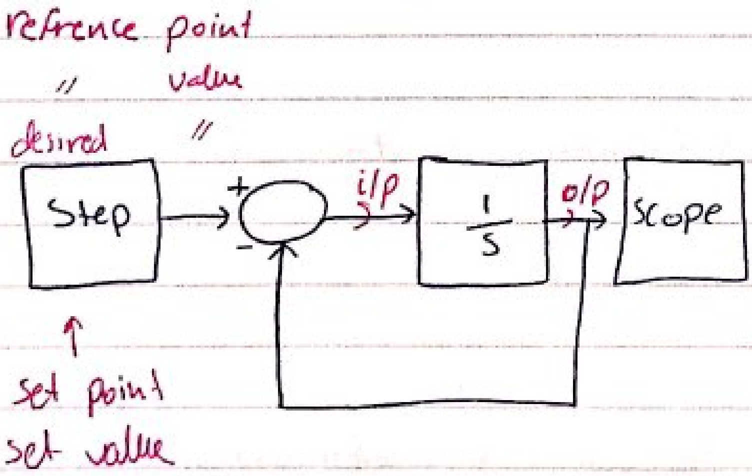
No. 10/6/2018

Positive and negative Feedback through Simulink :





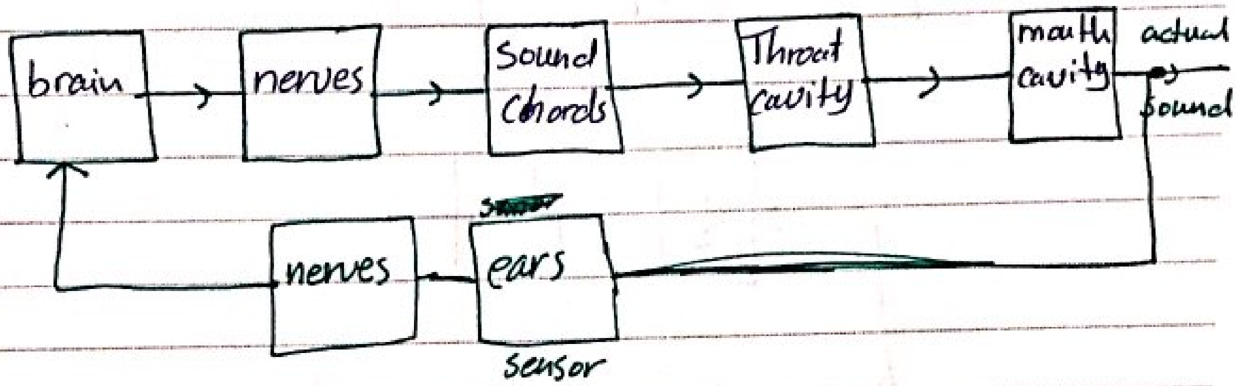




No. 11/6/2018

Schematic diagram of systems :-

Consider the process of producing a quality sound



$\boxed{ex}$: Draw a schematic diagram representing the process of driving a car or a bicycle.

13/6/2018

$\frac{1}{s}$ in the S-domain $\Leftrightarrow$ integration in time domain

Advantages of feedback :-

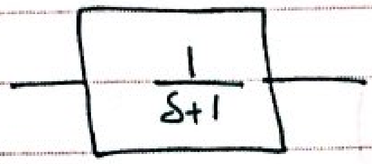
Generally :-

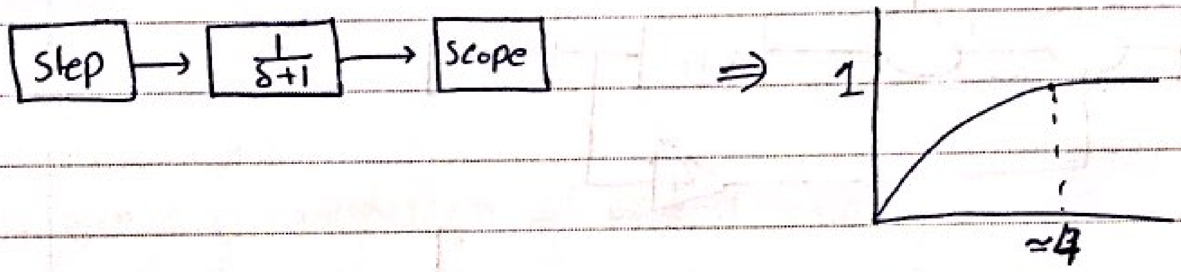
1) increased accuracy ↑ when wise ~~feedback~~ design | but not with careless design

2) increased speed, with wise design but not with careless.
careless ⟹ slow.
increased speed ⟹ increased bandwidth

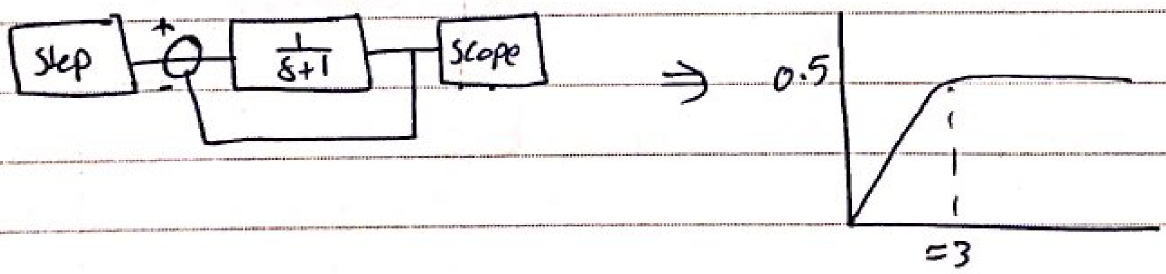
3) insensitivity to parameter variations.
4) insensitivity to external disturbances.

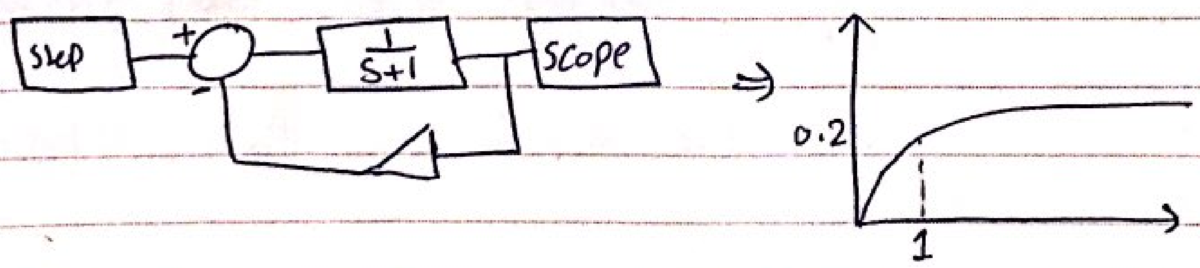
Example :- simulate





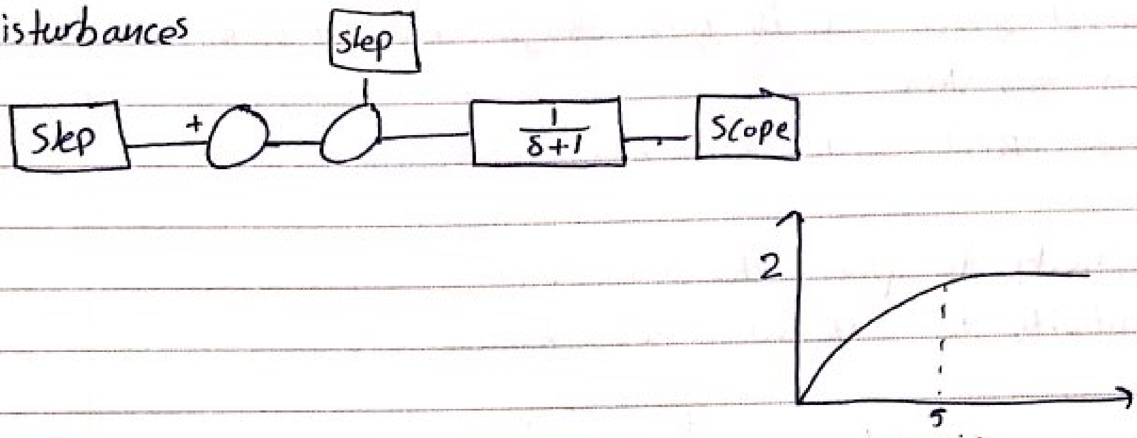
adding feedback



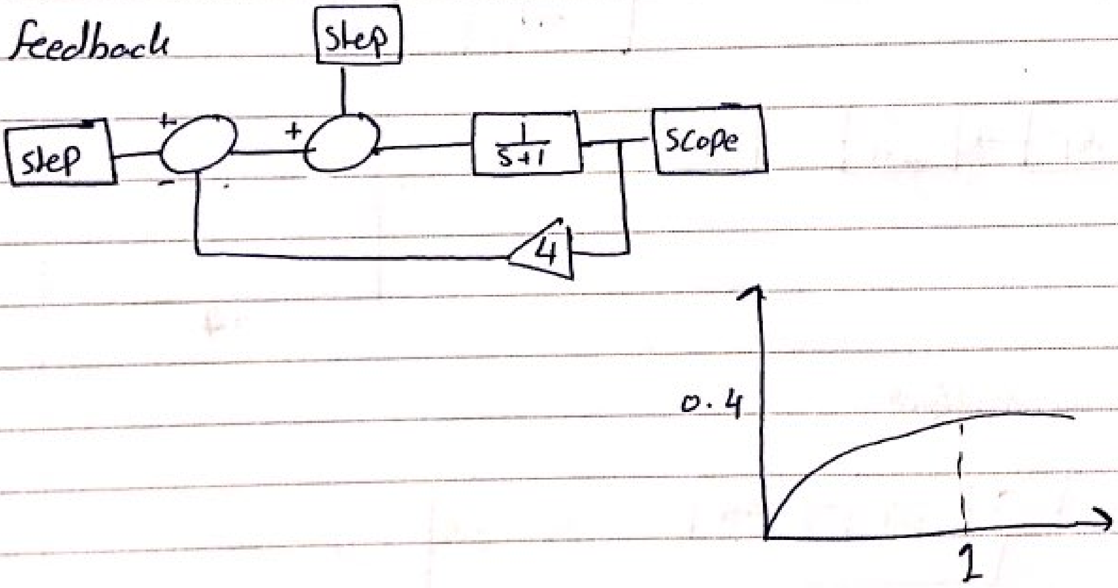


No.

Disturbances

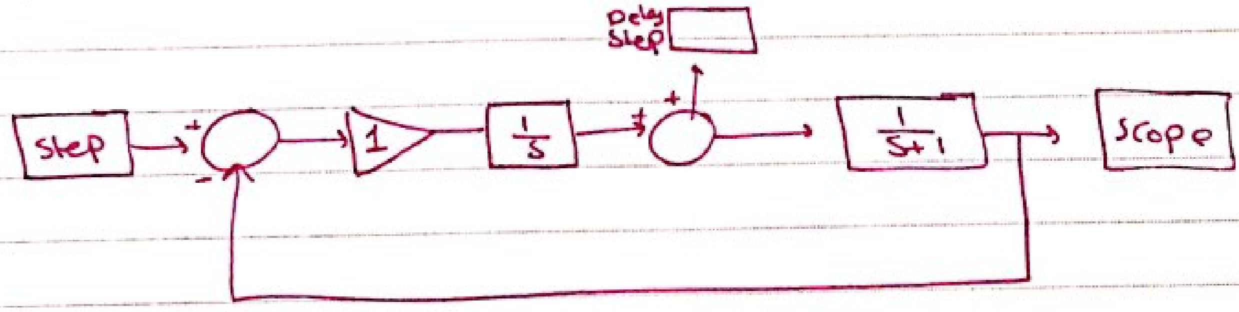


add feedback



No. 13/6/2018

example :- use simulink to simulate



* disadvantages of feedback
generally

① decreased gain
② possibility of unstability of careless design

unstabe → مفاعل نووي
→ جسم مهتز

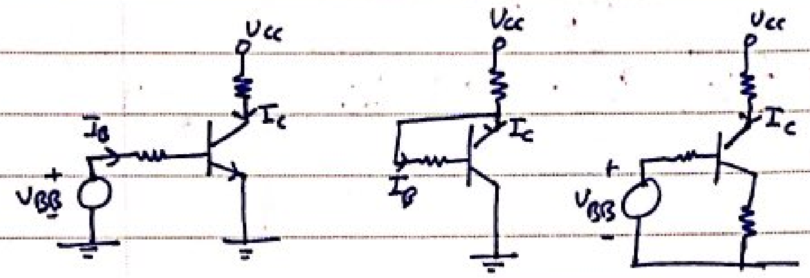
③ added complexity resulting extra cost, size, weight making trouble shooting more difficult

* washing machine is an open loop

Conclousion : Feedback has to be justified.

No. 19/6/2018

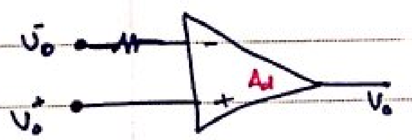
Positive and negative Feedback through electrical circuits

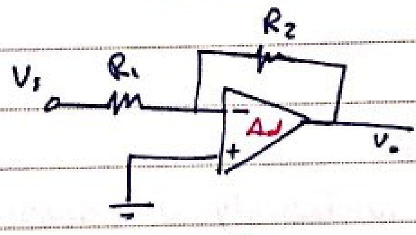


$\Downarrow I_c = \beta I_B$ open loop

$\Downarrow$ Closed loop

$\Downarrow$ Closed loop





$\Downarrow$

open loop

$V_o = A_d (V^+ - V^-)$

$\Downarrow$

Closed loop

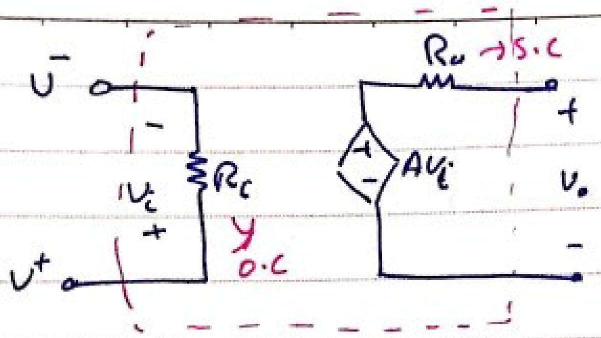
$V_o = f(V_s, V_o)$

~~Ex~~

Ex:- Review Dc analysis of npn transistor circuit.

Ex:- use the following op-amp model to verify op-amp circuit's relationship.

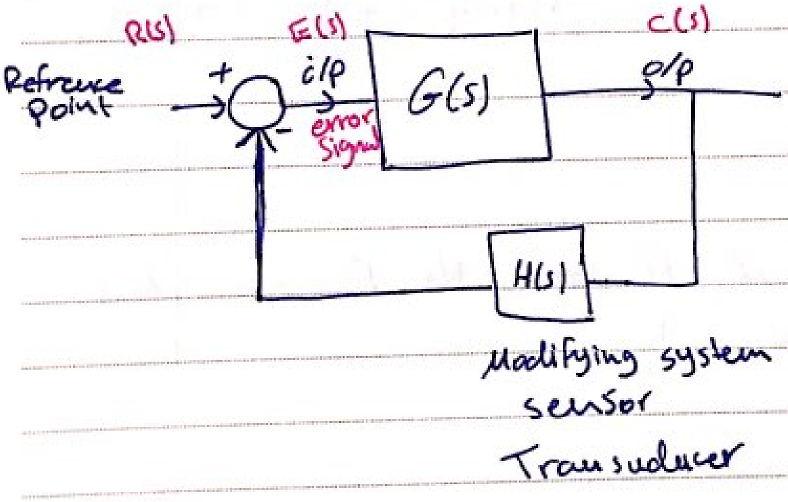
No.



practical model

* The idealized model is $\Rightarrow$ R_i o.c
R_o s.c

* Basic Feedback system



$\approx\Rightarrow$ physical system

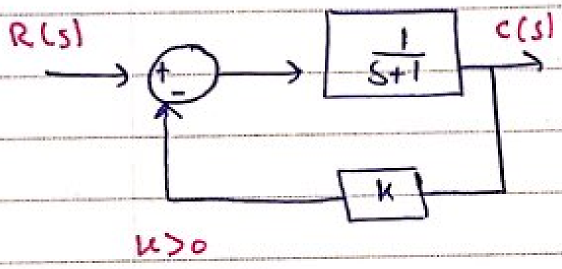
* it can be shown $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1 + G(s)\,H(s)}$

Ex:- prove

No. 20/6/2018

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{1+S}$$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{1+j\omega} \Rightarrow \text{bandwidth} = 1 \text{ rad} \cdot S^{-1}$$



$$\frac{C(s)}{R(s)} = \frac{\frac{1}{S+1}}{1+\frac{1}{S+1}\cdot k} = \frac{1}{S+1+k}$$

$$= \frac{1}{(1+k)\left(\frac{S}{1+k}+1\right)} = \frac{\frac{1}{1+k}}{1+j\frac{\omega}{1+k}}$$

bandwidth = $1+k$ >1

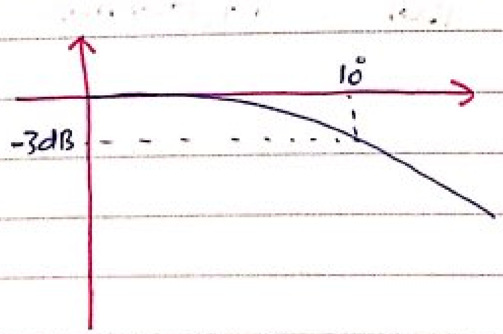
$\Rightarrow$ bandwidth has been increased, However, the DC gain has been decreased from 1 to, $\frac{1}{1+k}$

* to obtain gain set $S=0$

>> n=1 ; d=[1 1] ; sys = tf(n,d) , bode(sys) , grid

bandwidth=1

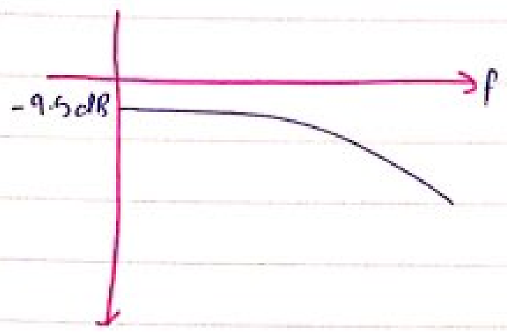
$$sys = \frac{1}{1+S}$$

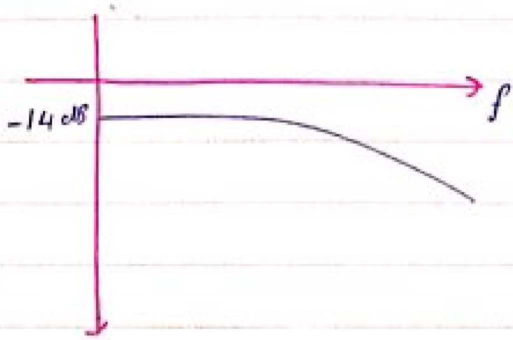


smile for life

No.

$$Sys = \frac{1}{s+1+k} \rightarrow \text{frequency} = 3 \quad , k = 2$$





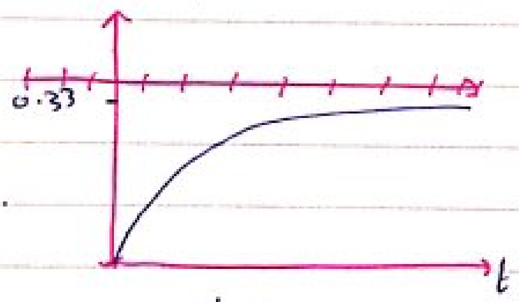
$k=2$

$20\log\left(\frac{1}{3}\right) = -9.5$

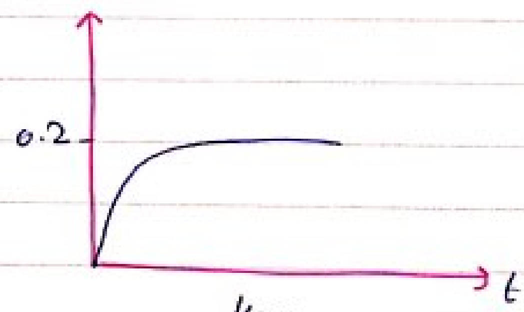
$k=4$

$20\log\left(\frac{1}{5}\right) = -14$

$k\uparrow$ $\Rightarrow$ entering the steady state condition faster $\Rightarrow$ steady state ~~magn~~ value decreases (magnitude)

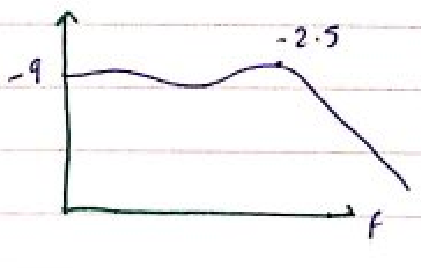


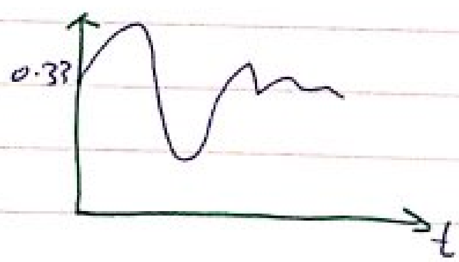
$k=2$



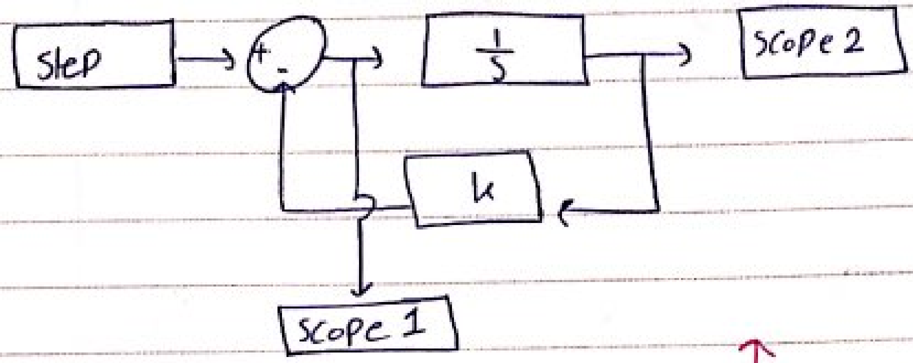
$k=4$

k=1; n=1; d=[1 1 1+k] , sys = tf(n,d) , bode(sys), grid





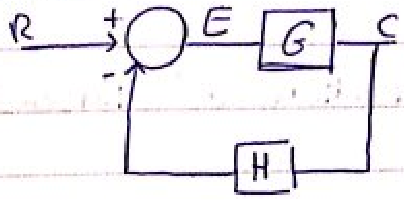
No.



No. 24/6/2018

Insensitivity to parameter variations :-

For simplicity consider the following system



$$\frac{C}{R} = \frac{G}{1+GH}$$

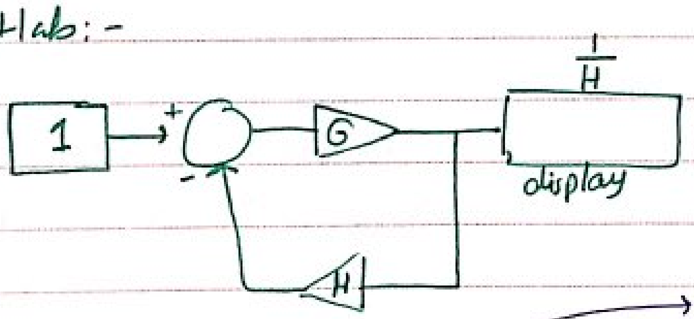
suppose G is variable, but $GH \gg 1$

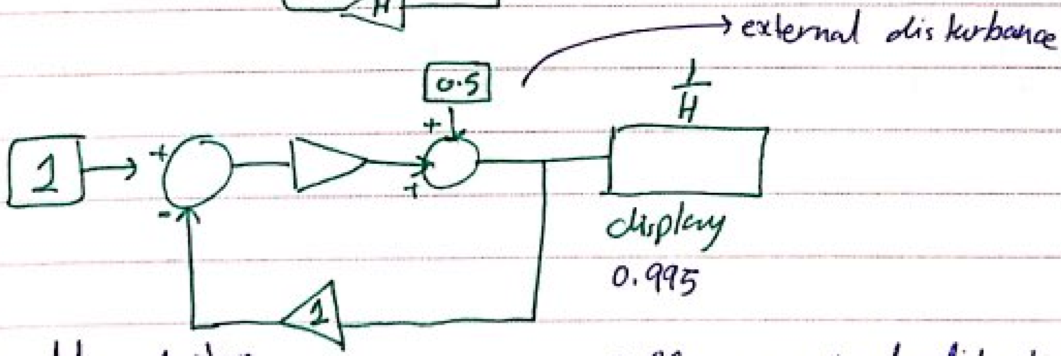
in which case

$$\frac{C}{R} = \frac{G}{GH} = \frac{1}{H}$$

i.e independent of the variable G

Matlab:-





G	H	display
100	1	0.99
1000	0.5	1.996
10^4	0.08	11.1597

0.990 → without disturbance $\boxed{0.5}$

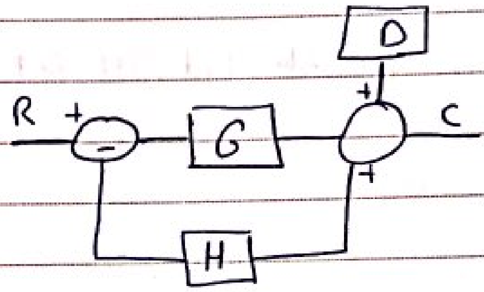
$\times$ $G\uparrow$ → better answer

No.

Feedback makes the system insensitive but to external disturbances.

26/6/2018

Justification of Insensitivity to external Disturbance.



$$C = D + Y = D + GE$$

$$C = D + G(R - Hc)$$

$$(1 + GH)C = D + GR$$

$$C = \frac{G}{1+GH} R + \frac{1}{1+GH} . D$$

* from the above Relationship, to reduce the effect of the disturbance we make (G.H) very much greater than 1, in which case

$$C = \frac{G}{1+GH} . R$$ almost dependent on our desired value.

No.

* Sensitivity of control systems :-

Systems undergo response variations due to parameter variations and external disturbances

Sensetivity :-

A Quantitive measure is a defined sensitivity measure

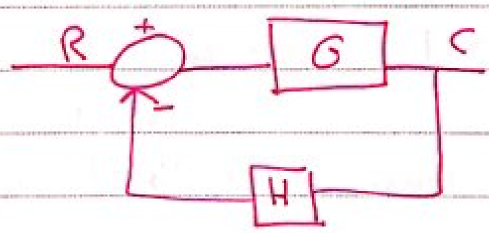
$$S^{T}_{k} = \lim_{\Delta k \to 0} \frac{\Delta T / T}{\Delta k / k}$$

sensitivity

example :- Consider the following system.

let $T = \frac{C}{R}$

$$= \frac{G}{1+GH}$$



$$\frac{k}{T} \lim_{\Delta k \to 0} \frac{\Delta T}{\Delta k} \Rightarrow \frac{k}{T} \frac{dT}{dk}$$

$$S^{T}_{G} = \frac{G}{T} \cdot \frac{1(1+GH) - GH}{(1+GH)^2} \Rightarrow \frac{G}{T} \cdot \frac{1}{(1+GH)^2}$$

$$T = \frac{G}{1+GH}$$

$$\frac{G}{\frac{G}{1+GH}} \cdot \frac{1}{(1+GH)^2} = \frac{1}{1+GH}$$

No.

required to be $\ll 1$ for insensitivity

i.e $GH \gg 1$ for the closed closed loop transfer function $\boxed{T}$

To be insensitivile to variations in $\boxed{G}$

$\Rightarrow$ Sensitivity of $T = \frac{C}{R}$ with respect to H

$$S_H^T = \frac{H}{T} \cdot \frac{dT}{dH} = \frac{H}{T} \cdot \frac{0 - G^2}{(1+GH)^2}$$

$$\frac{H(1+GH)}{G} \cdot \frac{-G^2}{(1+GH)^2} = \frac{-GH}{1+GH}$$

if $GH \gg 1$ sensitivity $S_H^T \approx -1$

is T sensitive with H or not? it's Sensitive

(*) since S_H^T not very less than 1, then T is sensitive to variations in H

$\Rightarrow$ الزيادة في H يقابلها نقصان في G

* The negative sign means an increase in H results a decrease in T and vice versa.

No.

* example :- given $G = 1000$, $H = 0.1$, calculate

1) T, when G, becomes 1200 and 800

2) calculate T when H becomes 0.2 and 0.5

use $S = \frac{\Delta T / T}{\Delta k / k}$

Cof with calculated $S = \frac{k\, dT}{T\, dk}$

$$T\Big|_{\substack{G=1000 \\ H=0.1}} = \frac{C}{R} = \frac{G}{1+G_1 H} = \frac{1000}{1+(1000 \times 0.1)} = 9.9009$$

$$T\Big|_{\substack{G=1000 \\ H=0.2}} = \frac{G}{1+GH} = \frac{1000}{1+(1000+0.2)} = 4.975$$

$$T\Big|_{\substack{G=1000 \\ H=0.05}} = 1.9960 \quad \Big/ \quad T\Big|_{\substack{G=1200 \\ H=0.1}} = 9.917 \quad \Big/ \quad T\Big|_{\substack{G=800 \\ H=0.1}} = 9.876$$

~~exer~~ Excersise:- prof for $\int_{k}^{\text{constant}} = 0$

$$\int_{k}^{\alpha G_1} = \int_{k}^{G}$$ α is scalar

$$\int_{k}^{G_1 G_2} = \int_{k}^{G_1} + \int_{k}^{G_2}$$

No.

$$\int_{k}^{\frac{1}{G}} = -\int_{k}^{G}$$

Hence $$\int_{k}^{\frac{G_1}{G_2}} = \int_{k}^{G_1} - \int_{k}^{G_2}$$

$$\int_{k}^{G_1(G_2(u))} = \int_{k}^{G_1(u)} \int_{k}^{G_2(u)}$$

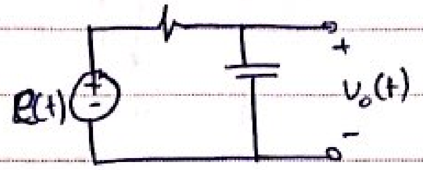
No.

* Modeling for Systems [electrical]

⇒ Numetic : systems that depends on air pressure
teeth drill use numetic

⊛ revise Laplace Transform.

Consider the following circuit



$E(t)$ = Set Value

$$V_o'(t) = \frac{1}{C} \int i_c(t) \, dt$$

$$i(t) = \frac{e(t) - V_o(t)}{R}$$

⊛ in order to change a derivation or an integration to Algebric Form. we have to change and transfer it to another domain

⇒ Laplace transform :-

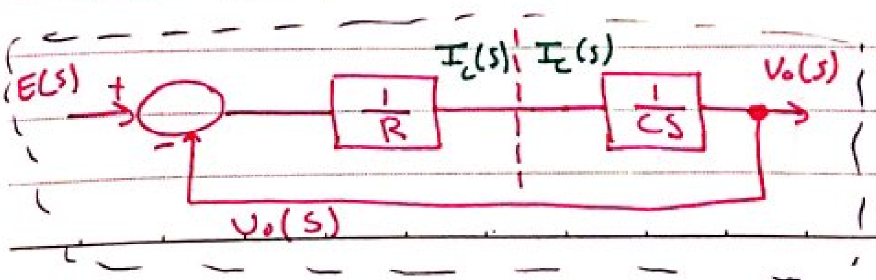
$$V_o(S) = \frac{1}{C} \cdot \frac{1}{S} \, I(S)$$

$$I(s) = \frac{E(s) - V_o(s)}{R}$$

Approxemating Solutions :-

block diagram ⇒ for each equation we determine input & output

Then Reduction



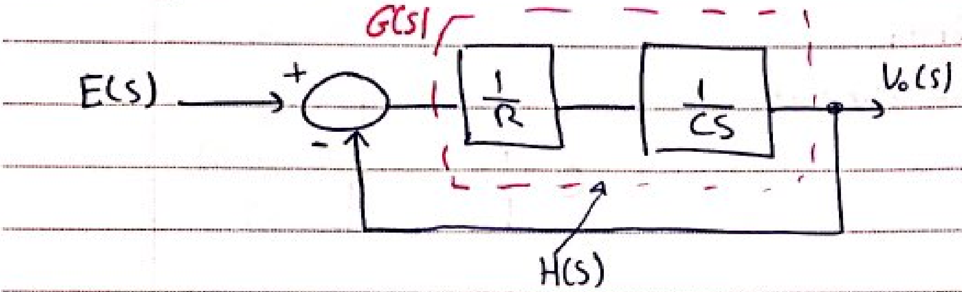
overall block diagram

second equation

sm;)e for life

⇒ to reduction the relation and block diagrames using the equation I know

Reducing the block diagram.



$$\frac{V_o(s)}{E(s)} = \frac{1/RCS}{1+\frac{1}{RCS}+1} = \frac{1}{1+RCS}$$

overall relation

⇒ laplace transform for a Constant = $\frac{\text{Const.}}{s}$

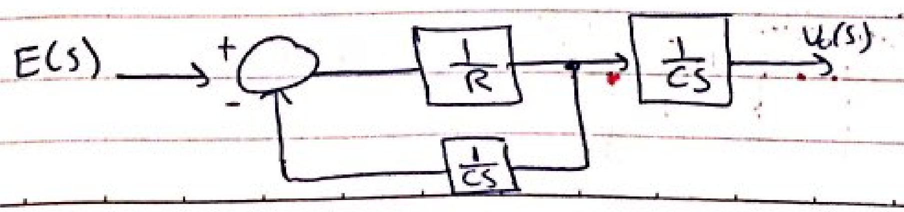
From the blocks not from equations and variables

- much easier than calculating it by hand

※ Differently, we can get the following block diagram, in which case

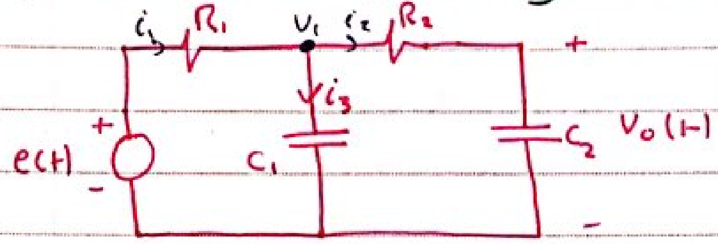
$$\frac{V_o(s)}{E(s)} = \frac{1}{CS} \cdot \frac{1/R}{1+1/R \cdot \frac{1}{CS}} = \frac{CS}{1+RCS} * \frac{1}{CS}$$

$$\frac{V_o(s)}{E(s)} = \frac{1}{1+RCS}$$ ※ same



No.

example:- Consider the following circuit :-



$$V_o(s) = \frac{1}{SC_2} \cdot I_2(s) \quad \ldots (1)$$ always start with the output

$$I_2(s) = \frac{V_1(s) - V_o(s)}{R_2} \quad \ldots (2)$$

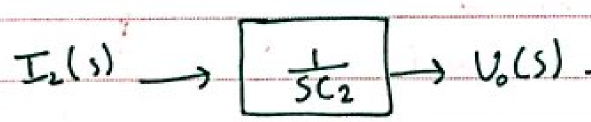
$$V_1(s) = \frac{1}{SC_1} \cdot I_3(s) \quad \ldots (3)$$

$$I_3(s) = I_1(s) - I_2(s) \quad \ldots (4)$$

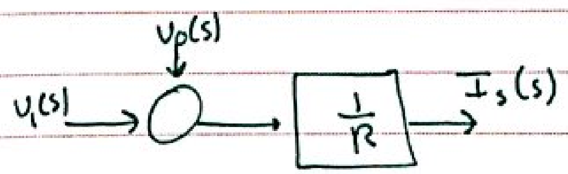
$$I_1(s) = \frac{E(s) - V_1(s)}{R_1} \quad \ldots (5)$$

(*) we make a block diagram for each equation. -

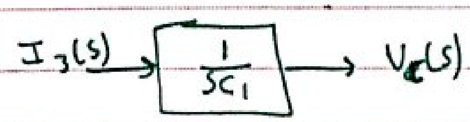
→ obtain a detailed block diagram involving integrator.
The output is $V_o(s)$ and the set value is $E(s)$



... (1)



... (2)



... (3)

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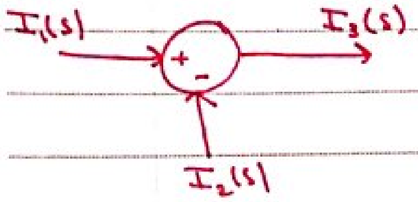
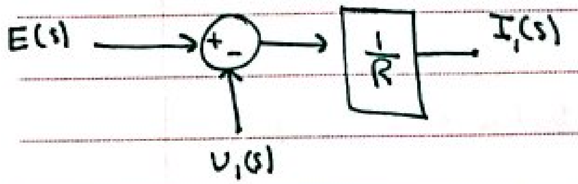
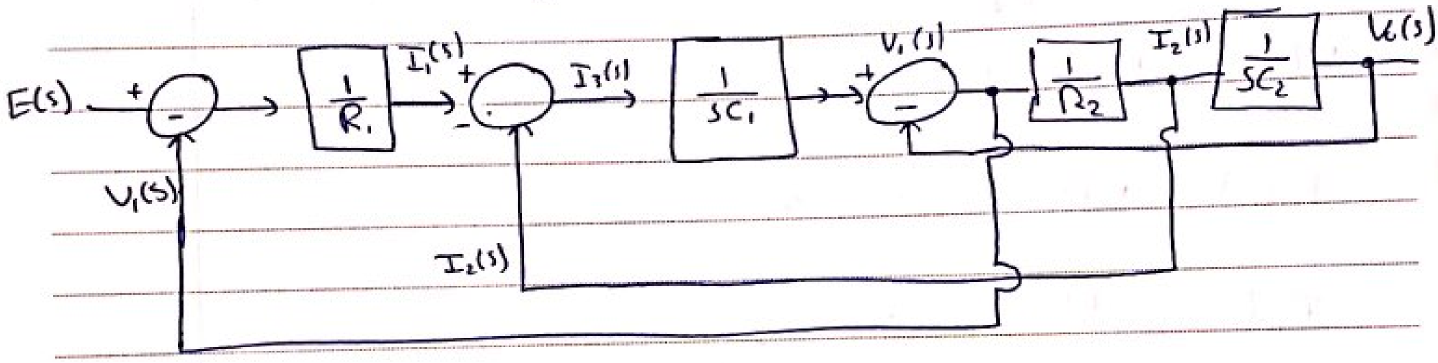


... (4)



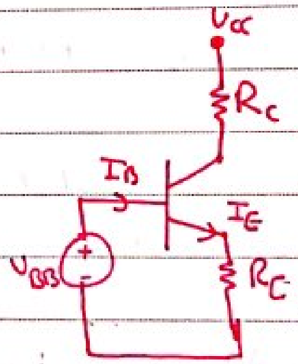
... (5)

Now combine these together.



Excersize:-

obtain a detailed block diagram with V_E as o/p, V_{BB} as set value showing where to measure I_B, I_C, I_E in the case wher V_{BE} doesn't depend on I_C and in the case where it depends somehow on I_C



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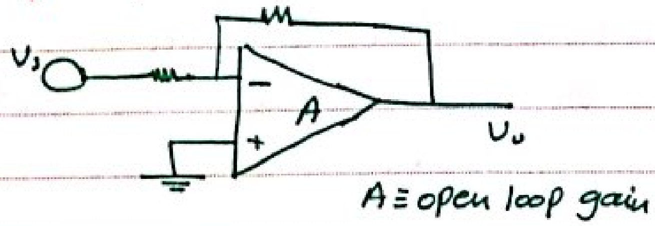
Exeersize:- given the following op-Amp circuit



⟹ Use the op-Amp model to obtain a block diagram with V_o as output V_s as set value

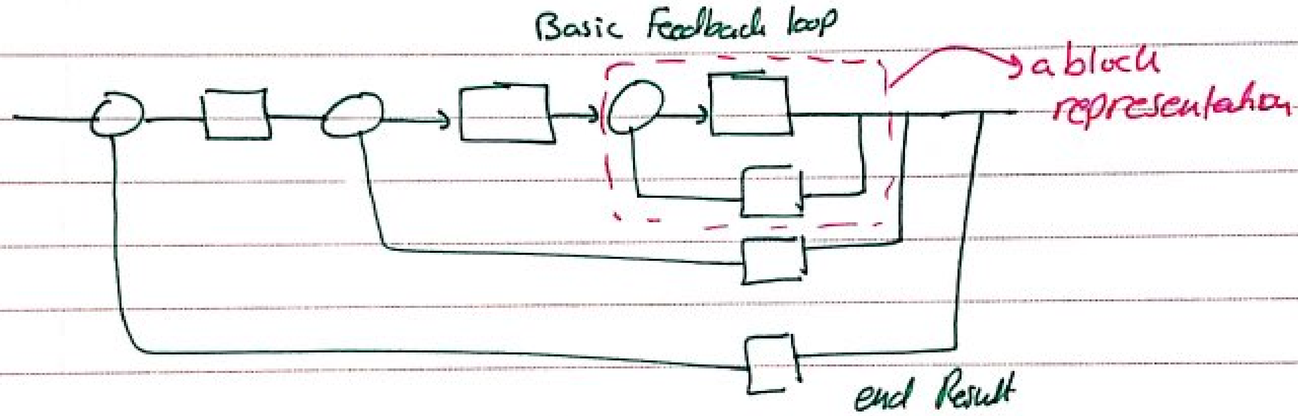
⟹ Reduce the block diagram to obtain $\frac{V_o}{V_s}$

Excersize:- model an armature control dc motor (see book)

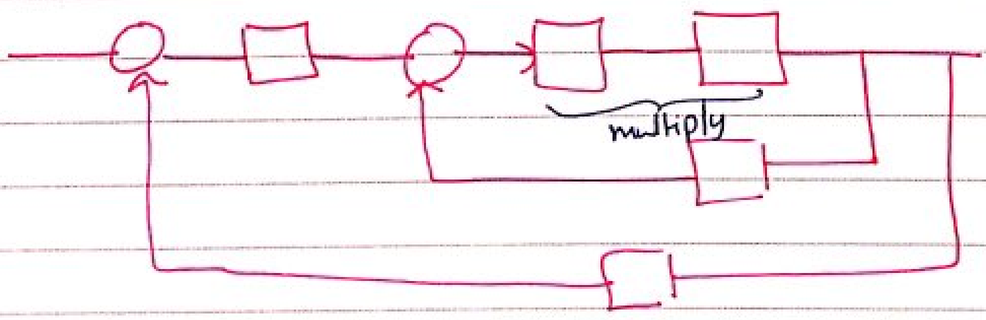
Block diagram Reduction :-

Starting with a Complecated block diagram, ending with a simple one. involving logic Feedback system within a bigger Feedback system in a

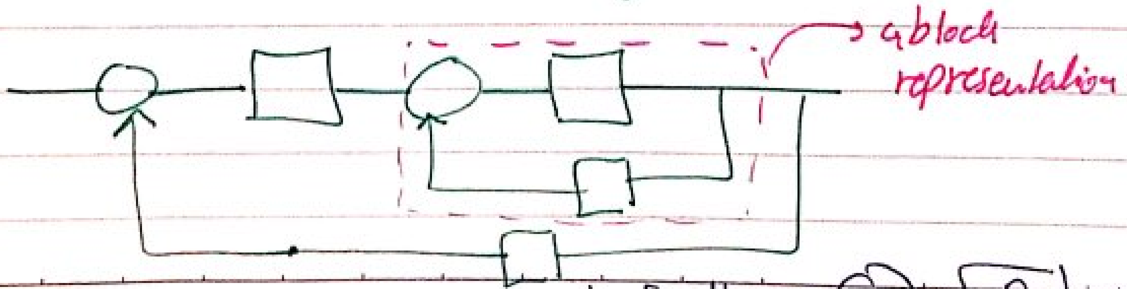
This is achieved by manipulating signals and blocks in an appropriate manner keeping the relationship between signals the same



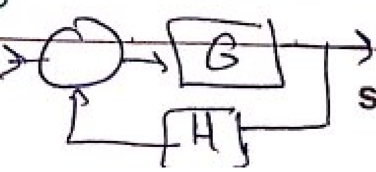
⇓



⇓



↳ Finally



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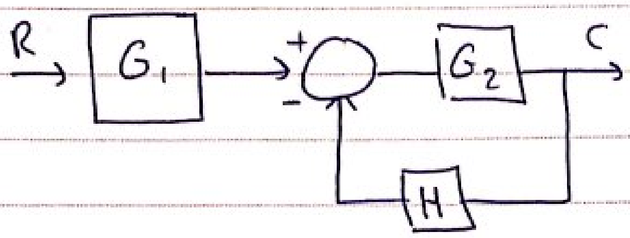
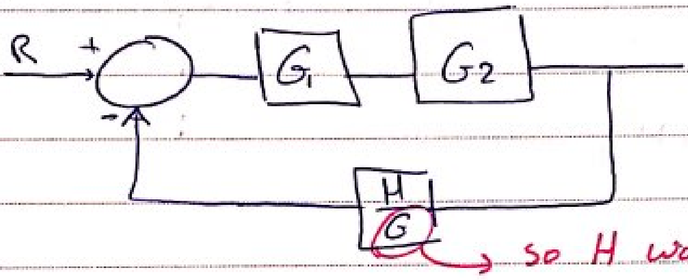
Example see the table in the book

Example



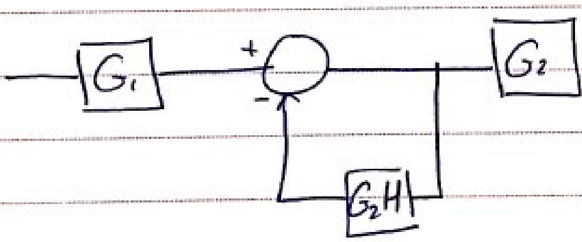
$$\frac{C}{R} = \frac{G_2}{1+G_2H} \cdot G_1$$

if we have changed the order
⇓



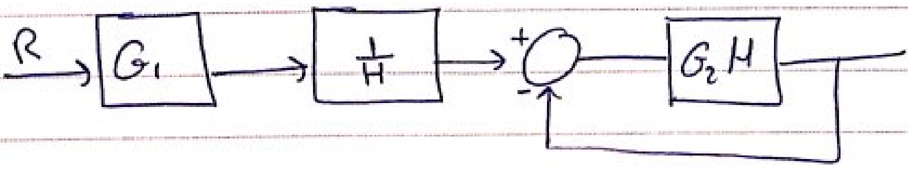
$$\frac{C}{R} = \frac{G_1G_2}{1+G_1G_2+\frac{1}{G_1}} = \frac{G_1G_2}{1+HG_2}$$

so H won't be multiplied with G_1



$$\frac{C}{R} = \frac{1}{1+G_2H} \cdot G_2G_1$$

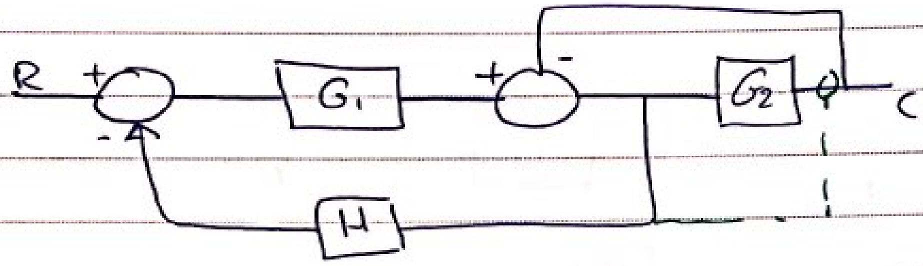
unity Feedback

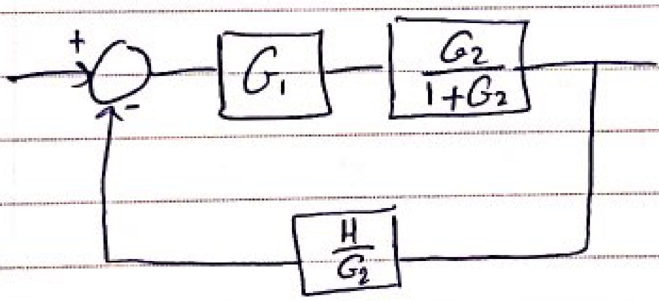


$$\frac{C}{R} = \frac{G_2H}{1+G_2H} \cdot \frac{1}{H} G_1$$

No.

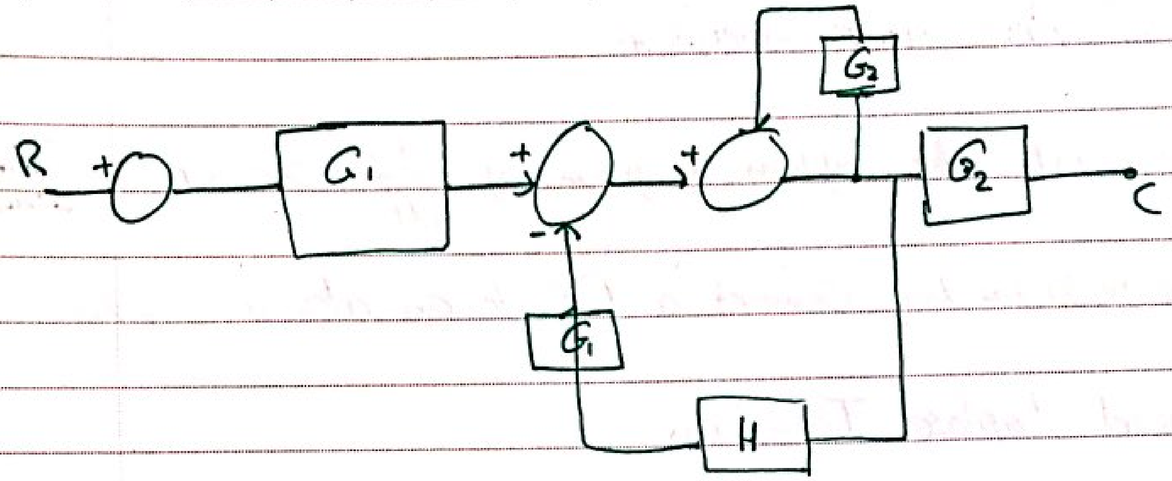
example using block diagram reduction techniques find

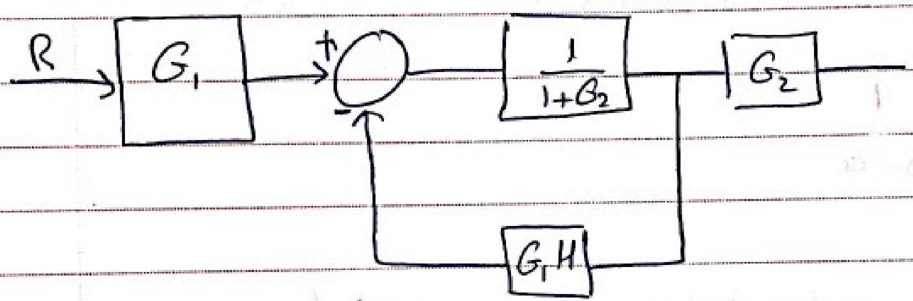




$$\frac{C}{R} = \frac{\frac{G_1 G_2}{1+G_2}}{1 + \frac{G_1 G_2}{1+G_2} \times \frac{H}{G_2}} = \frac{G_1 G_2}{1 + G_2 + G_1 H}$$

No.





$$\frac{C}{R} = G_1 \, \frac{\frac{1}{1+G_2}}{1+\frac{1}{1+G_2}\cdot G_1 H} \cdot G_2 = \frac{G_1 G_2}{1+G_2+G_1 H}$$

No. 2/7/2018

Analysis of first order systems :-

A linear first order system is given by $\frac{dc(t)}{dt} + ac = r(t)$. Time domain

⇒ To ease understanding convert a DE to an algebriac equation we need Laplace Transform.

Hence, $S\,C(s) + a\,C(s) = R(s)$; $C(t) = 0$

$$T(s) = \frac{C(s)}{R(s)} = \frac{1}{S+a}$$

Generally, a first order system is described by

$$\frac{C(s)}{R(s)} = \frac{k}{TS+1}$$

k is the gain
T is the time constant

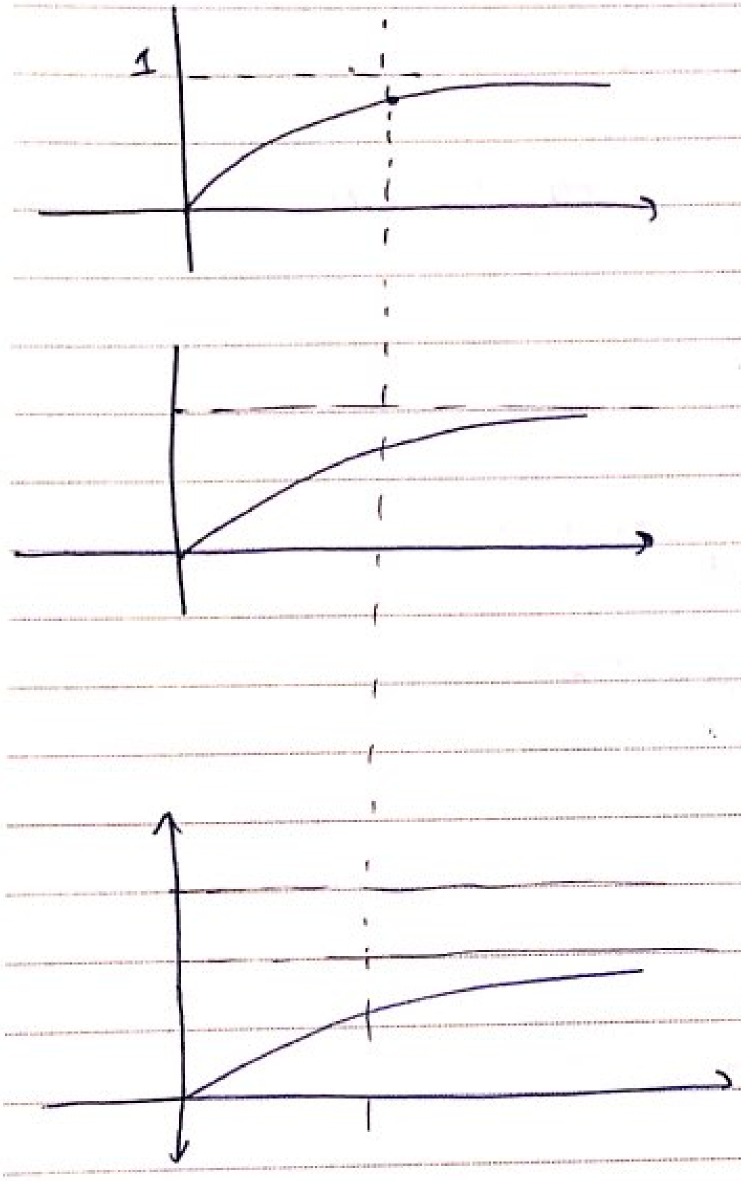
Example: consider the following system

$$\frac{C(s)}{R(s)} = \frac{1}{TS+1}$$

obtain responses for different $T = 1, 2, 4, 5$

No. 3/7/2018

The small the time constant, the faster the system



$T = 1$

$T = 2$

$T = 4$

$$c(t) = \left(1 - e^{-t/T}\right) \cdot u(t)$$

No.

Second order systems :- (linear)

characterized by :-

$$\frac{d^2c}{dt^2} + 2\underbrace{\zeta \omega_n \frac{dc}{dt} + \omega_n^2 c}_{\text{changes with system}} = f(t) = \omega_n^2 r(t)$$

⇒ Time inveriant system (coeffients are real numbers)
(ζ, c)
↳ damping coefficient

⇒ It can be shown that

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

* all initial conditions in Transfer functions = 0

* if $\zeta = 0$ then, $\frac{\omega_n^2}{s^2+\omega_n^2}$ $\frac{Const}{s + Cons}$ = Sinosoidal (sin, cos)

* The system will oscilate

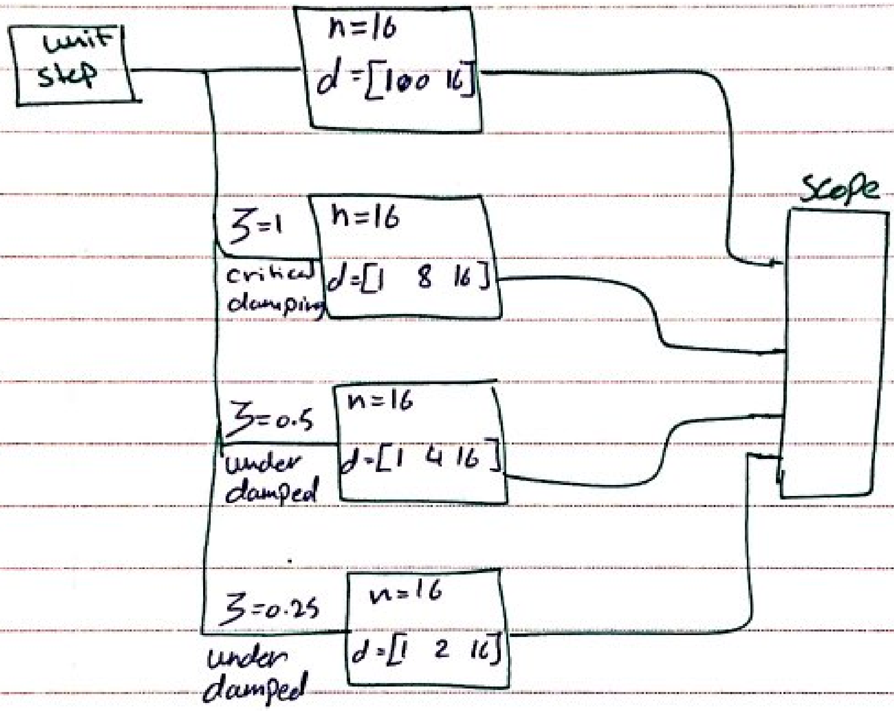
* ω_n is the undamped natural frequency
* ζ is the damping ratio

smile for life

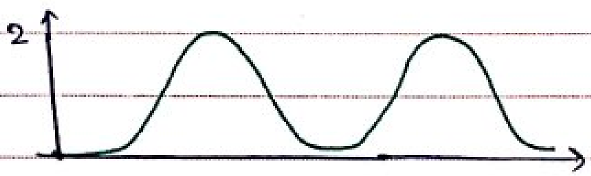
Response as a function of ζ :-

build systems of the form

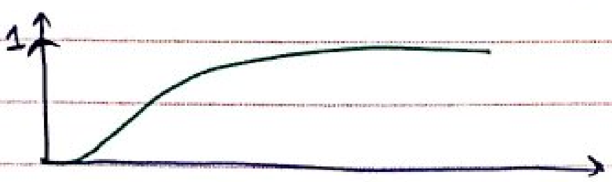
$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$, to find the steady state let $s=0$



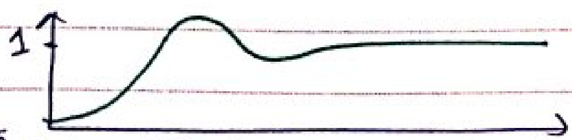
First case



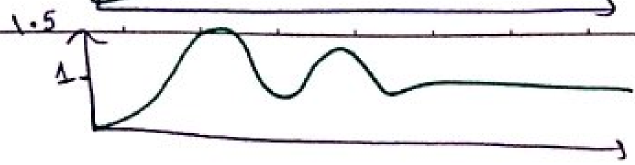
Second case



third case



Fourth case



No.

$$\zeta = \frac{\text{coefficient of } S}{2\omega_n}$$

$$\omega_n = \sqrt{\text{coeff. of } S^0}$$

↳ absolute term

General characterization of A particular second order system
Given by $G(s) = \frac{C(s)}{R(s)} = \frac{k\,\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

undamped	critical damping	over damping
$0 < \zeta < 1$	$\zeta = 1$	$1 < \zeta < \infty$
poles are complex	poles are identical	poles are different and real

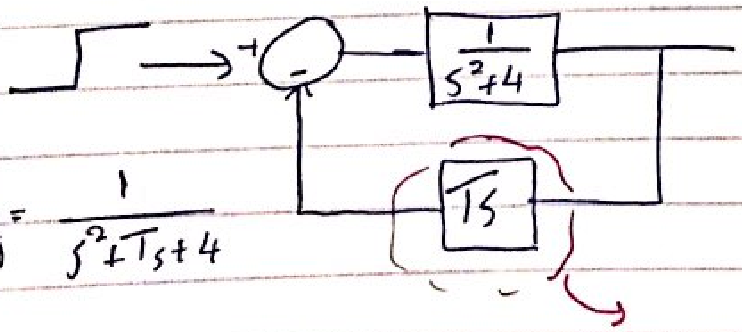
* k is the gain of the system

* smaller ζ → larger overshoot (it doesn't have an effect on the steady state)

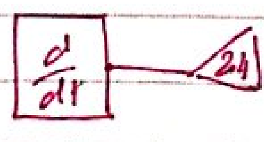
* 2nd order → can make oscillation
  1 st order → can't

given $G(s) = \frac{1}{s^2+4}$

Introduce a controller such that the system has $\zeta = 0.7$
$\omega_n = 2$ rad/s
$2\omega_n(0.7) + 2 = 2.8$



$\frac{C(s)}{R(s)} = \frac{1}{s^2 + Ts + 4}$



No. 9/7/2018

Time response of a second order system Due to a unit step

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad , \quad R(s) = \frac{1}{s}$$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$\zeta < 1 \Rightarrow$ poles are Complex

the response $\Rightarrow$ Sinusoidal

$$= \frac{A}{s} + \frac{Bs + C}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

where $\omega_d = \omega_n\sqrt{1-\zeta}$ known as the underdamped natural freq.

$\omega_n > \omega_d$

* it can be shown that

$$c(t) = \left\{ 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \cos^{-1}\zeta) \right\} u(t)$$

* steady state value of $c(t) = C_{ss} = \lim_{t\to\infty} c(t) = \lim_{s\to 0} s\,C(s)$

N.B: the final value theorm: $\lim_{t\to\infty} c(t) = \lim_{s\to 0} s\,C(s)$

No.

provided all Poles of $sC(s)$ are in LHS of the S-plane.

Ex:- for $G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

Determine the output time response ($c(t)$) due to a unit step

Where i) $\zeta = 1$ and $\omega_n = 2 \text{ rad s}^{-1}$

ii) $\zeta = 2$ and $\omega_n = 2 \text{ rad s}^{-1}$

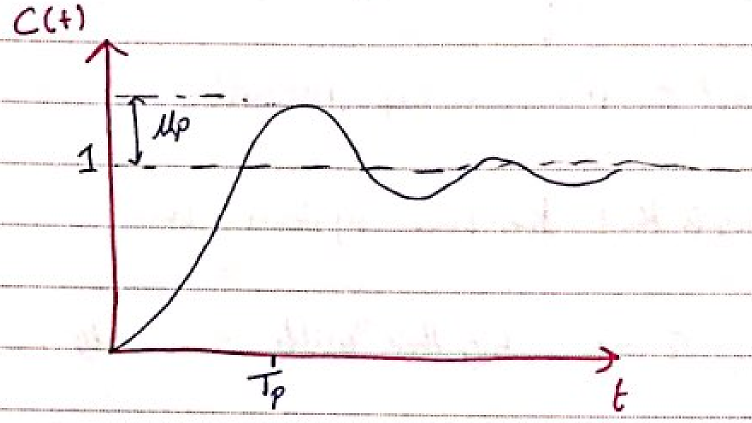
iii) use matlab to plot $c(t)$

* Simulate $\boxed{\text{ramp}} \longrightarrow \boxed{\dfrac{1}{s^2+s+100}} \longrightarrow \boxed{\text{scope}}$

No. 10/7/2018

Specifications based on a second order system.

given $G(s) = \dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ $\zeta < 1$



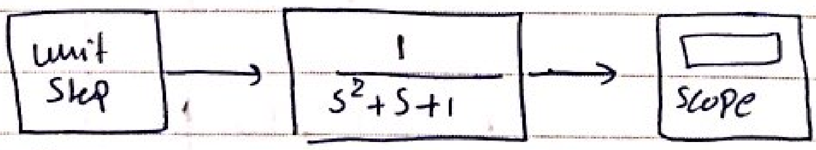
for 2% criterions and for $\zeta < 0.8$

Time to first peak $T_p = \dfrac{\pi}{\omega_d}$

$$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \cdot C_{ss}$$

$$T_s \approx \frac{4}{\zeta\omega_n}$$

Simulate :-



$T_p = \dfrac{\pi}{\sqrt{0.75}}$

No. 1/7/2018

Design using specifications :-

Consider the following system $\rightarrow \boxed{\frac{1}{s^2-4}} \rightarrow$

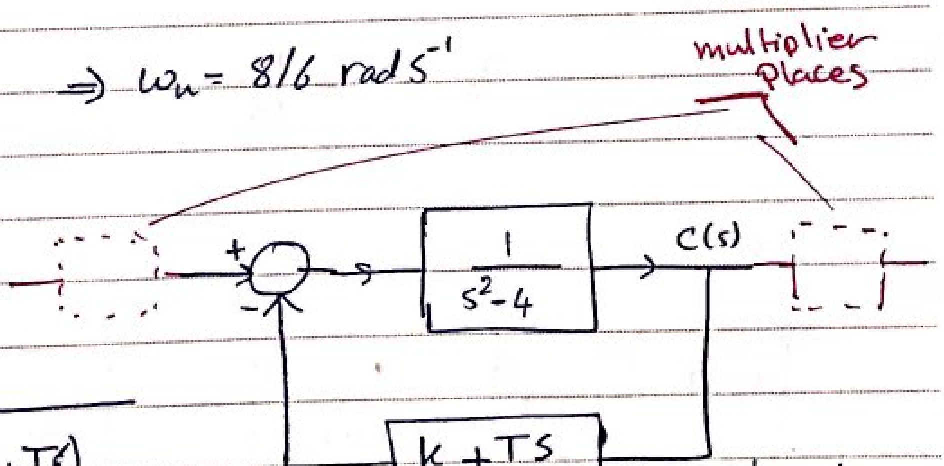
It's badly in need for control as it's open loop unstable.

$\Rightarrow$ Design a controller such that the new system has

$\zeta = 0.6$ and settling time $= 5$ sec. together with a steady state value of 2 due to a unit step excitation.

Using $T_s = \frac{4}{\zeta \omega_n}$

Sol :- $5 = \frac{4}{0.6\,\omega_n} \Rightarrow \omega_n = 8/6 \text{ rad s}^{-1}$



$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s^2-4}}{1 + \frac{1}{s^2-4}(k+Ts)}$$

$C_{ss} = \frac{1}{k-4} \neq 2$

$$= \frac{1}{s^2 - 4 + k + Ts}$$

* $k > 4$ to make a stable system

$$= \frac{1}{s^2 + Ts + (k-4)}$$

$k - 4 = \omega_n^2 = \frac{16}{9}$

$k = \frac{52}{9} \simeq 5.77$

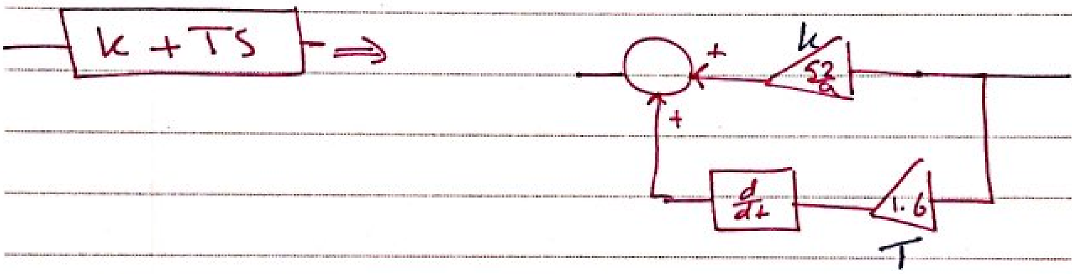
$T = 2\zeta\omega_n = 2 \times 0.6 \times \frac{4}{3} = \frac{4.8}{3} = 1.6$

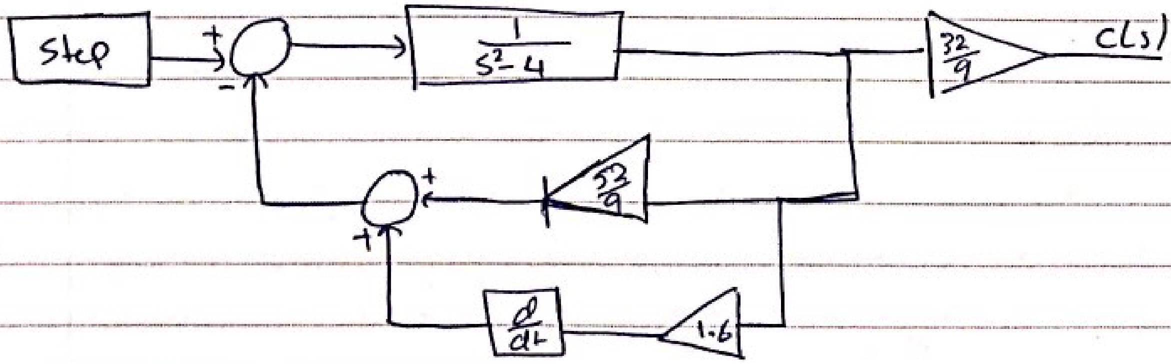
Therefor; use a multiplier (gain factor) outside the loop

$M = 2k - 8$

$= \frac{104}{9} - 8 = \frac{32}{9} = 3.\bar{5}$

$2 = M \frac{1}{k - \phi}$





No. 15/7/2018

Stability of linear systems:

* stability of linear systems is independent of the external excitation given to the system

* Impulse is the simplest form

a stable system exhibits a bounded output due to a bounded input.

i.e $\lim_{t \to \infty}$ output = finite value

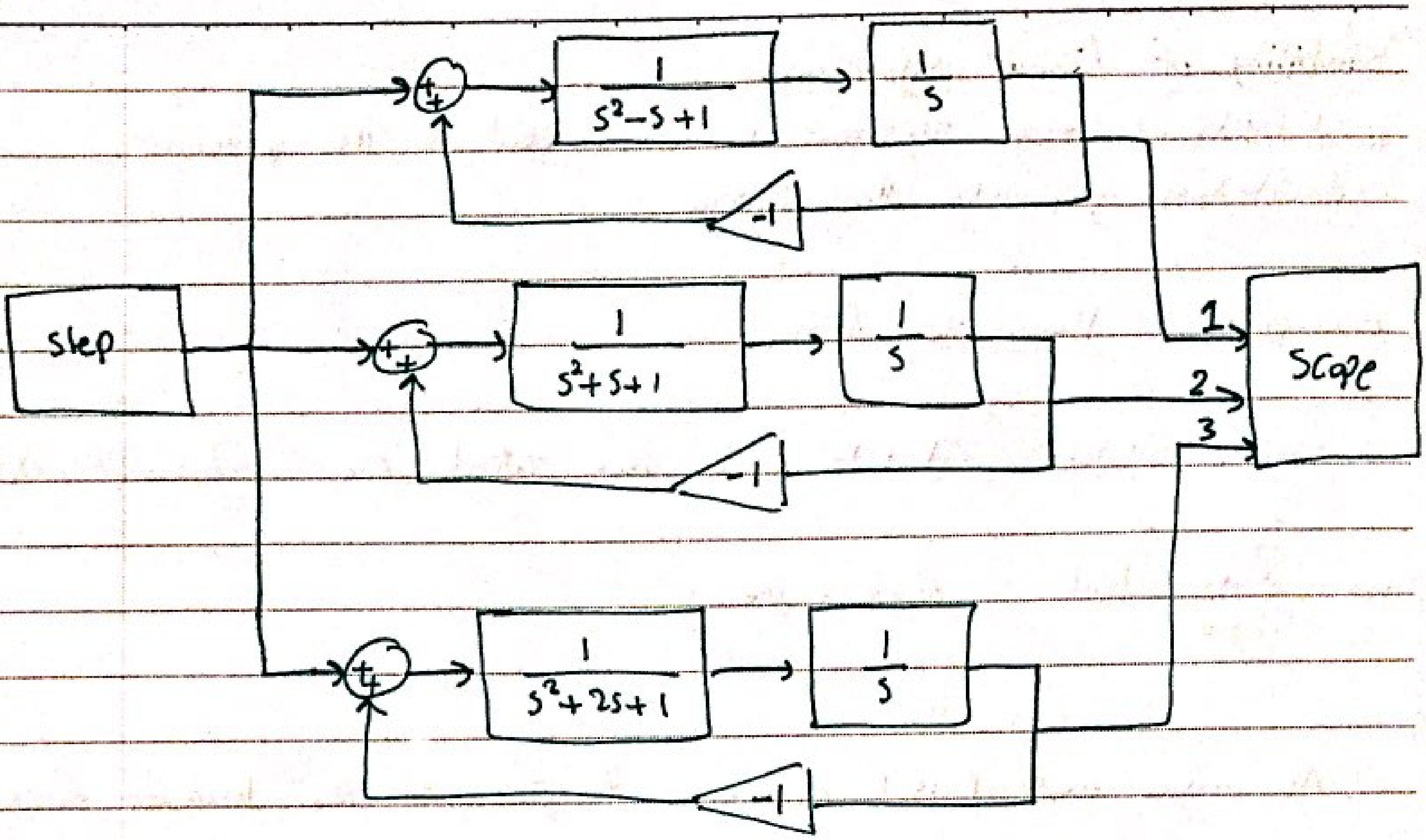
⟹ An easier method is to determine the poles of the transfer function.

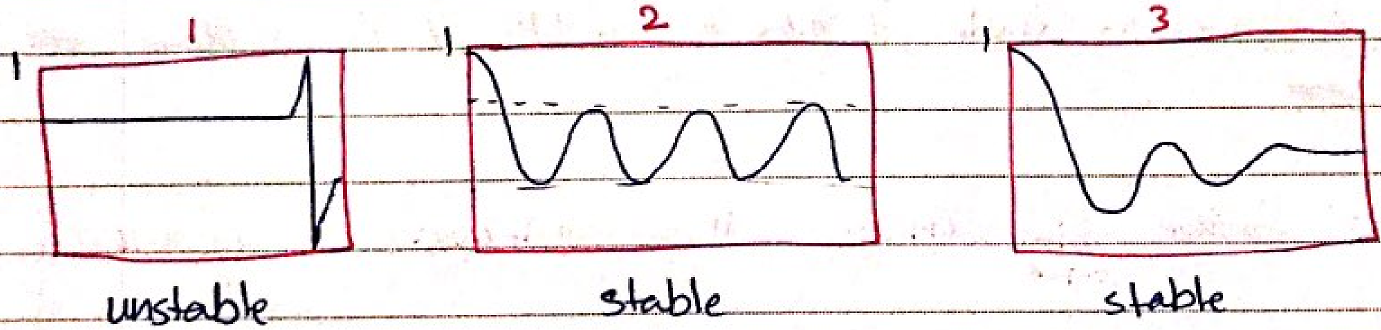
⟹ A system is stable, if poles are in LHS of the s-plane.

* if farther $\lim_{t \to \infty} c(t) = 0$ then system is asymptotically stable.

* if at last a single pole is in the right hand side of the s-plane then the system is unstable.

No. 16/7/2018





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iii) using Routh's stability Criterion (method)

⇒ stability is judged with respect to the characteristic equation (~~[illegible]~~) polynomial
(C.E)

C.E = $1 + G(s)H(s) = 0$ = denominator after simplification

Best illustrated by an example :-

* given C.E = $2s^4 + 4s^3 + 5s^2 + 5s +$

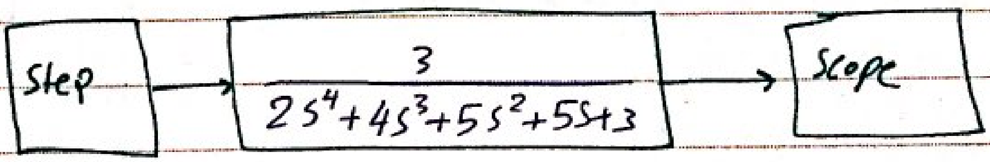
s^4	2	5	3
s^3	4	5	0
s^2	2.5	3	
s^1	0.2	0	
s^0	3		

first ⇒ $\frac{4\times5 - 2\times5}{4} = 2.5$

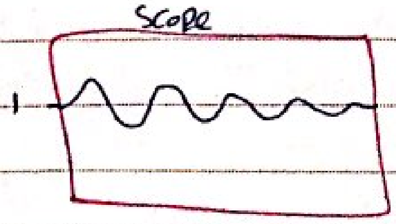
$\frac{4\times3 - 2\times0}{4} = 3$

$\frac{2.5\times5 - 4\times3}{2.5} = 0.2$

Stable since there is no change in sign in the first column.



Steady state value = 1
sub s=0



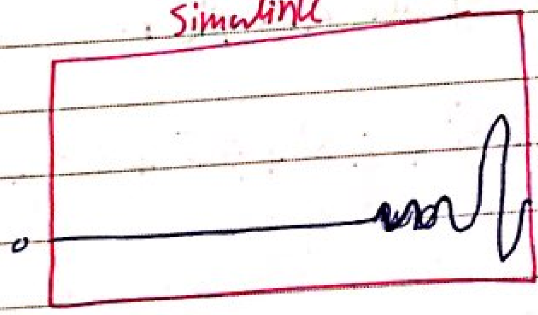
No.

$C.E = 2s^4 + 4s^3 + 5s^2 + 20s + 3$

s^4	2	5	3
s^3	4	20	0
s^2	-5	3	
s^1	$\frac{112}{5}$	0	
s^0	3		

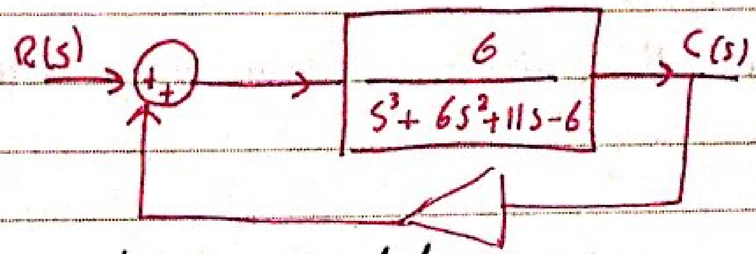
unstable

Simulink



Values of k which Ensure Stability

Consider the following system



* Cancellation methods are not recommended.

$$\frac{C(s)}{R(s)} = \frac{6}{s^3+6s^2+11s-6+6k}$$

s^3	1	11
s^2	6	$6k-6$
s^1	$\frac{72-6k}{6}$	0
s^0	$6k-6$	

$\longrightarrow 6k-6>0$

$k>1$

$\therefore$ for stability $1<k<12$

* $k\downarrow \rightarrow$ it enters the steady state faster

N.B :- if there is a change in sign in the C.E then the system is unstable

* إذا كانت معاملات المقام لها نفس الإشارة فالنظام Stable.
* " " " " اشارات مختلفة " unstable.

N.B: if there is a missing power in CE the system is unstable

No. 18/7/2018

Special cases :-

i) The case where the first element in the first column is 0 remedies

1) Replace the 0 by ϵ, Continue as used usual then take the limit as $\epsilon \to 0$

2) replace every S in the C.E by $\frac{1}{S}$, then apply to the C.E

3) multiply C.E by $(S+1)$, then apply to the new C.E

example:- let $C.E = 2S^4 + 3S^3 + 4S^2 + 6S + 2 = 0$

$$\begin{array}{c|ccc} S^3 & 2 & 4 & 2 \\ S^2 & 3 & 6 & 0 \\ \hline S^1 & \cancel{0}\ \epsilon & 2 & \\ S^0 & \frac{6\epsilon - 6}{\epsilon} & 0 & \\ & 2 & & \end{array}$$

$\lim_{\epsilon \to 0} \frac{6\epsilon - 6}{\epsilon} = -\infty$ system is unstable, with two poles of positive real part.

Matlab :- roots ([2 3 4 6 2])

ans =

0.1074 + j1.3442,

0.1074 − j1.3442

Ex:- Verify instability using the remain two remedies.

II) the case where the numbers in a row are Zero, this is best illustrated by an example.

let C.E $2s^5 + 3s^4 + 4s^3 + 6s^2 + 4s + 6 = 0$

s^5	2	4	4
s^4	3	6	6
s^3	$\cancel{0}$ 12	$\cancel{0}$ 12	0
s^2	3	6	
s^1	-12		
s^0	6		

→ generate the ~~aux~~ auxilliary polynomials obtained using the coefficient ~~of~~ if the row above that row of zero values

$$A(s) = 3s^4 + 6s^2 + 6$$

$$\frac{d\,A(s)}{ds} = 12s^3 + 12s$$

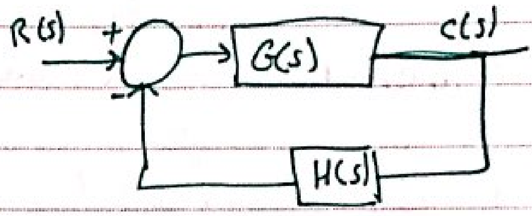
* when there is a row of zeros → the roots are symmetric around the origion

No. 22/7/2018

Static Error Coefficient

enable the calculation of the steady state Error due to different excitations without finding the time response then evaluating limits

$$E(s) = \frac{1}{1 + G(s)H(s)} \cdot R(s)$$



* choose the controller that gives zero errors

$e_{ss} \Rightarrow$ a number

$$e_{ss} = \lim_{t \to \infty} \underbrace{e(t)}_{\text{time response}}$$

* find value theorem

$$\lim_{s \to 0} S\,E(s)$$

* e_{ss} Can be zero, finite value or ∞ depending on the type of the system. Defined as the number of integrators in $G(s)H(s)$

* type: defined as number of integrators in ~~G(s)~~ (H(s) مضروب ب G(s))

* $G(s)H(s) \Rightarrow$ open loop transformation

No.

* $\dfrac{G(s)}{1+G(s)H(s)} \Rightarrow$ Closed loop transformation

* $G(s)\,H(s) = \dfrac{k(s-z_1)(s-z_2)\cdots(s-z_n)}{S^{N}(s-P_1)(s-P_2)\cdots(s-P_n)}$ → zeros, → poles

Integratorsال عدد

using Matlab $\Rightarrow$

due to a unit step & define k_p (position error Coef.)
as $k_p = \lim\limits_{s\to 0} G(s)H(s) \Rightarrow e_{ss} = \dfrac{1}{1+k_p}$

* find $k_p \Rightarrow$ [zero, ∞, finite]

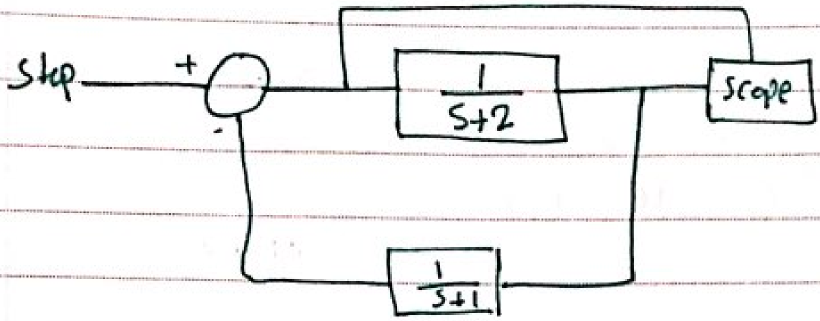
* what is the steady state error $\Rightarrow \dfrac{1}{1+k_p}$

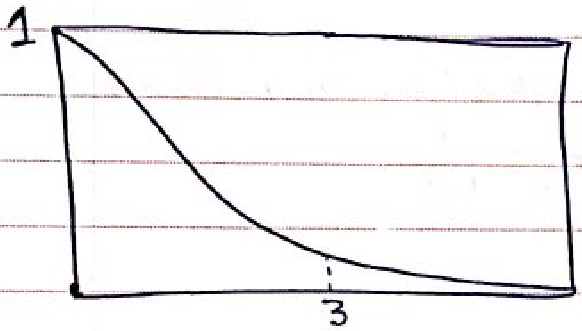
Since we don't have an ite integrator

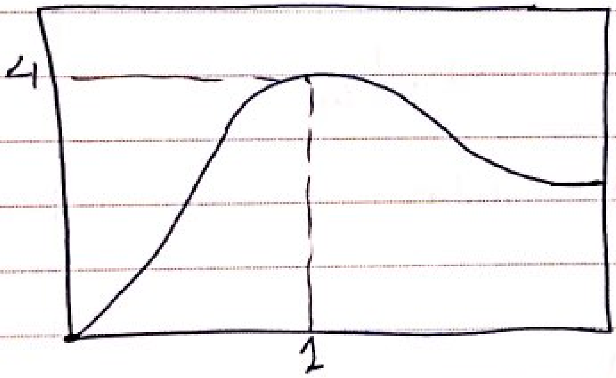
$$G(s)\,H(s) = \frac{k(s-z_1)(s\cdots}{\boxed{}(S-P_1)(s-P_2)\cdots}$$

S^N does not exist

$\Rightarrow$ if - on the other side - there was $S, S^2, \ldots$
ther error is zero.


Step
+
-
1
S+2
Scope
1
S+1


1
3


4
1

No. 23/7/2018

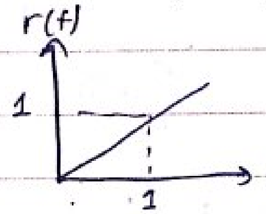
* generally integrators slowdown system response.
 diffrintiators speed up " " .

Concolusion :- $N \geq 1$ results in zero steady state error due to unit step.

Velocity error Coefficient (k_v)
needed when $r(t) = t\,u(t)$

i.e a unit ramp input

$$k_v = \lim_{s \to 0} s\,G(s)\,H(s)$$

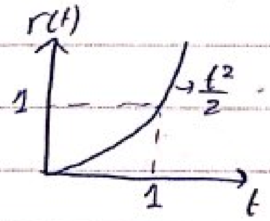


Accelration error Coefficient (k_a)
needed when $r(t) = \frac{t^2}{2} u(t)$

$$k_a = \lim_{s \to 0} s^2\,G(s)\,H(s)$$

i.e a unit parapolic input

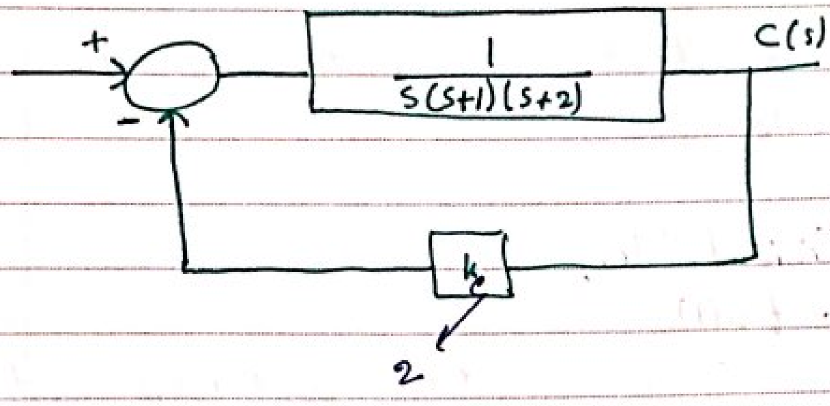
$$e_{ss} = \frac{1}{k_a}$$



N.B :- all error Coefficients are Valid provided the closed loop system is asymptocially stable.

No.

* Consider the following system



$$\frac{C(s)}{R(s)} = \frac{1}{s^3+3s^2+2s+k}$$

s^3	1	2
s^2	3	k
s^1	$\frac{6-k}{3}$	0
s^0	k	

$0 < k < 6$

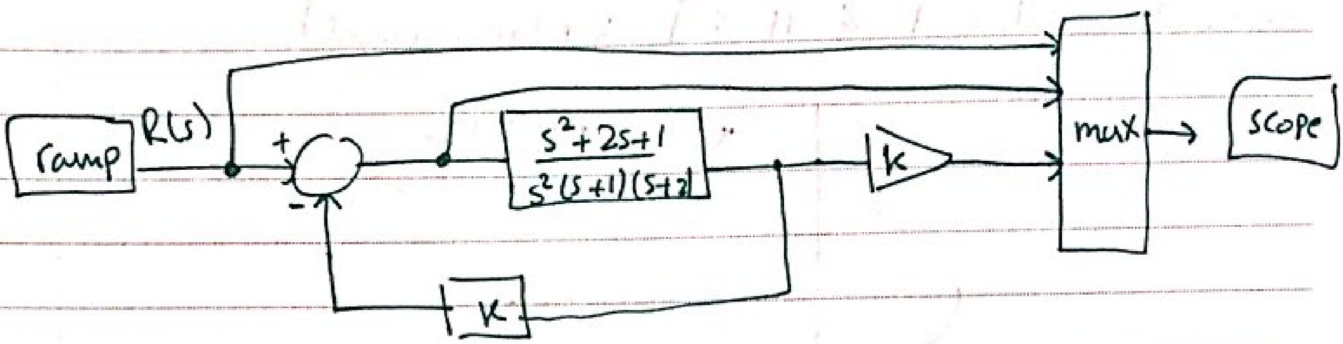
* if the function is now $\frac{C(s)}{R(s)} = \frac{1}{s^4+3s^3+2s^2+ks+k}$

s^4	1	2	k
s^3	3	k	0
s^2	$6-k$	$3k$	
s^1	$\frac{-3k-k^2}{6-k}$	0	
s^0	k		

$$\left.\begin{array}{l} k>0 \\ 6-k>0 \\ \frac{3k-k^2}{6-k}>0 \end{array}\right\} \rightarrow \quad < k <$$

No. 24/7/2018

Ex:- for the system shown, determine values for k which ensure system stability



25/7/2018

* The root locus (RL)

$\Rightarrow$ is a depiction of all roots of a poly-nomial in terms of a certain parameter.

* In control we are intrested of variation of roots of the characteristic equation.

$$1 + G(s)H(s) = 0$$

as a certian parameter (like the gain k) varies

$$1 + k\frac{1}{(s+2)(s+4)} = 0$$

$$\Rightarrow s^2 + 6s + 8 + k = 0 \quad , k > 0$$

Mattab

$\Rightarrow$ n=1 ; d=[1 6 8] ; rlocus(n,d)

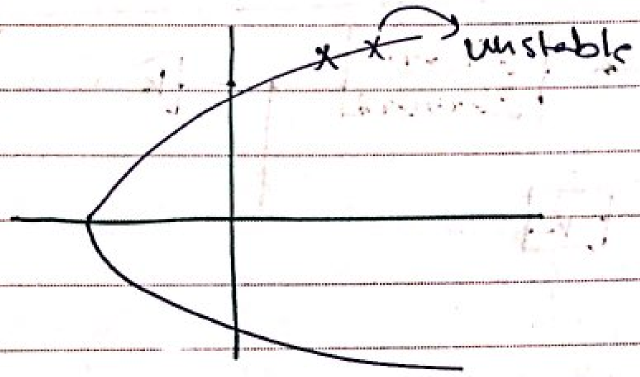
No.

ex2

$$1 + k \frac{1}{(s+1)(s+2)(s+3)}$$

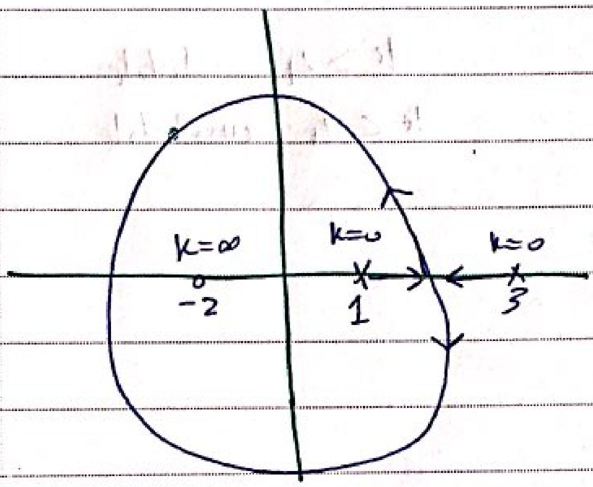
Matlab

```
>> n=1 ; d=[1 6 11 6] ; rlocus(n,d)
```



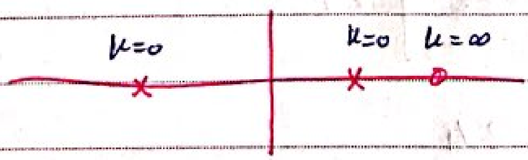
example :- Consider

$$C.E = 1 + k \frac{s+2}{(s-1)(s-3)} = 0$$



→ Root loci start at ~~poles~~ open loop poles $(k=0)$, ends at zeros $(k=\infty)$

→ we have a root locus on a line segment if the sum of the number of poles and zeros is odd



to calculate the intersection of the RL with the imaginary axis use Routh criteria.

for $1 + k \dfrac{s+2}{s^2-4s+3} = 0$

$s^2 - 4s + 3 + ks + 2k = 0$

s^2	1	$2k+3$
s^1	$k-4$	0
s^0	$2k+3$	

$k > 4$ stable
$k < 4$ unstable

to determine values of k on the RL use the magnitude condition

ie $\left| k \dfrac{Z(s)}{P(s)} \right| = 1$

s = closed loop pole

$k \dfrac{s+2}{s^2-4s+3} = 0 \Rightarrow k \left| \dfrac{-3+2}{9+12+3} \right| = 1$

$k = 24$

No. 31/7/2018

Sketching The RL

* starting with $1+G(s)H(s)=0$ = CE = Polynomial = $p(s,k)$

The first step represent the CE in the form

$$1+k\frac{Z(s)}{P(s)}=0$$

$$\frac{Z(s)}{P(s)}=\frac{-1}{k}$$ $\Longrightarrow$ in our case $k>0$

→ this equation lead to the magnitude

$$\left|\frac{Z(s)}{P(s)}\right|=\frac{1}{k}$$

* known as the negative condition

↳ and we use it to calculate R.L

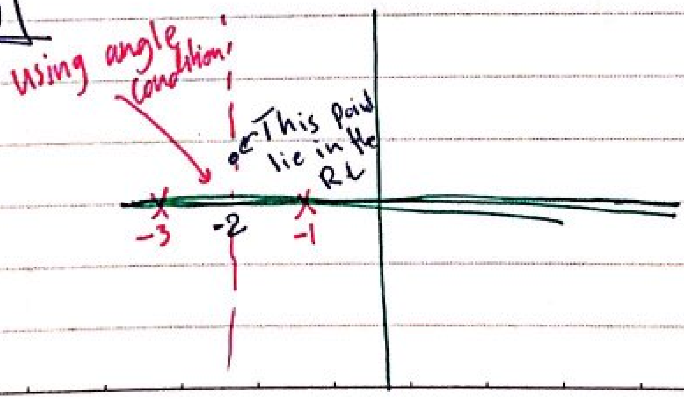
→ and we can conclude the phase from the minus sign

$$\angle\frac{Z(s)}{P(s)}=(1\pm2h)180°$$

* known as the angle condition

↳ and we use it to draw the RL

* $\boxed{e.g}$



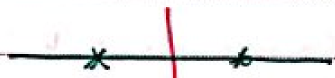
* additional roots for plotting the R.L

- Angle of asymplotes

$$\theta = \frac{(1 \pm 2h)180^\circ}{\#\text{of poles} - \#\text{of zeros}}$$

$$\delta = \frac{\text{sum of poles} - \text{sum of zeros}}{n_p - n_z}$$

$\boxed{e.x}$



↳ the angle of asymplote is 90

↳ the angle of asymplote is 60°

↳ the angle of asymplote is 180°

No.

* Break away points:

let $k = -\frac{P(s)}{Z(s)}$

let $\frac{dk}{ds} = 0$

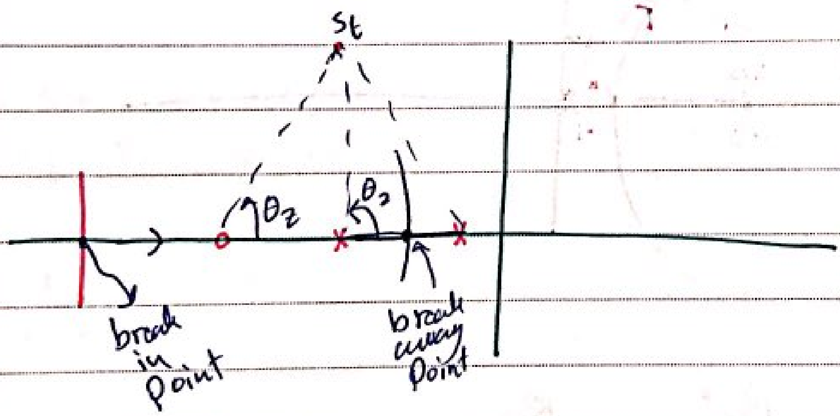
* Routh we know the intersection with j-axis
* Break away points we know the intersection with real-axis

* Solve for S

any S lying on the RL is a breakaway/In point

* To finish the RL, we use the angle condition at point on certinity

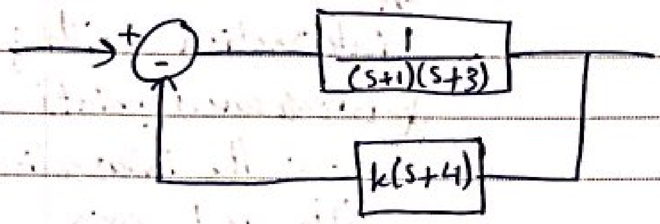
* ex :-



if S_t is in RL then

$$\theta_z - (\theta_1 + \theta_2) = (1 \pm 2k)\, 180^\circ$$

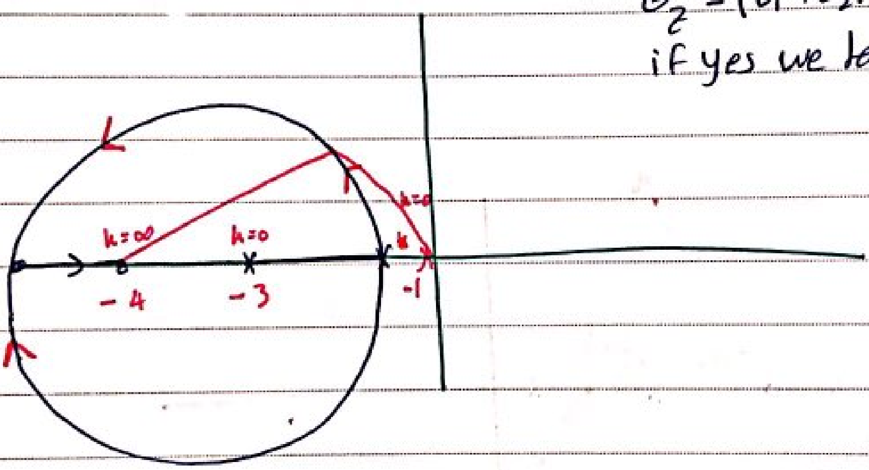
No. 1/8/2018



$1 + G(s)H(s) = 0$

$$1 + k\frac{s+4}{(s+1)(s+3)} = 0$$

$\theta_z - (\theta_1 + \theta_2) \overset{?}{=} 180\,(1 \mp 2h)$
if yes we take it



$$\circledast\ \theta = \frac{(1 \pm 2h)}{2-1} = 180^\circ, -180$$

$$\circledast\ \sigma = \frac{-1-3-(-4)}{2-1} = 0$$

* To compute break away point.

$$-k = \frac{s^2 + 4s + 3}{s+4}$$

No.

$$-\frac{dk}{ds} = \frac{(s+4)(2s+4)-(s^3+4s+3)(1)}{(s+4)^2} = 0$$

$$2s^2+4s+8s+16 - s^2 - 4s - 3$$

$$s^2+8s+13=0 \qquad \Delta = 64 - 4*1*13$$
$$= 12$$

$$s = \frac{-8 \mp \sqrt{12}}{2}$$

$$s = -4 \pm \sqrt{3} = -4 \pm 1.73$$
$$= -5.73, \ -2.27$$

* we use Routh to see if there is an intersection with the imaginary axis.

$$s^2+4s+3+ks+4k=0$$
$$s^2+(k+4)s+3+4k=0$$

s^2	1	$3+4k$
s^1	$k+4$	0
s^0	$3+4k$	

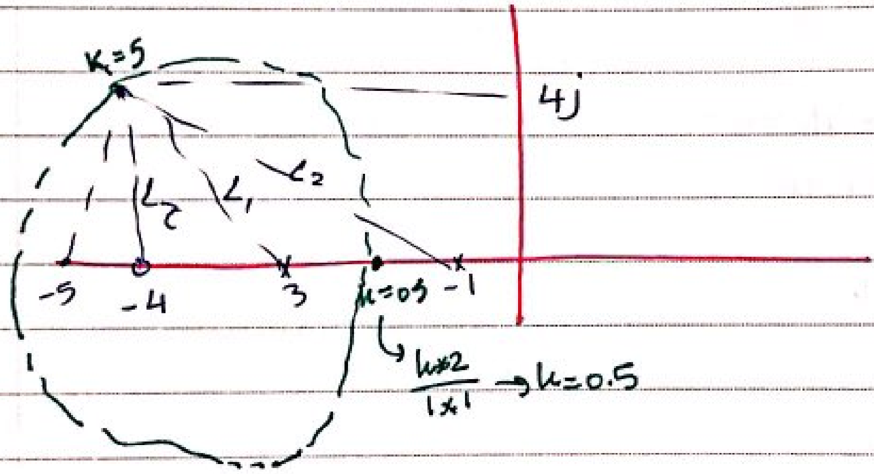
$$\left.\begin{array}{l} k+4>0 \longrightarrow k>-4 \\ 3+4k>0 \longrightarrow k>-\frac{3}{4} \end{array}\right\} \text{for stability}$$

No.

* But if we want to know if there is an intersection with imageuary axis we must have a row of zero ~~(k=-4) but~~ so we must force (k+4) be zero (k=-4) but we want a positive k not negative.

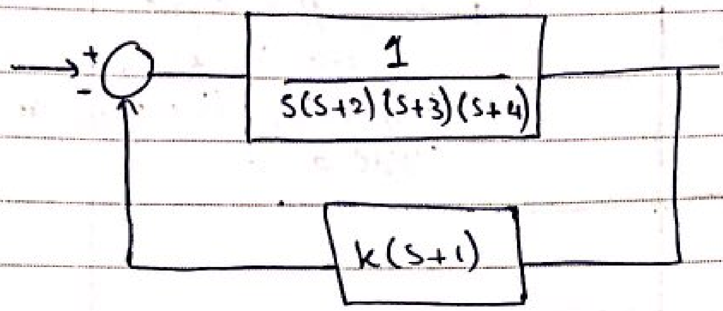
So there is no values of k to make this
=> no intersection with the imagerary axis.

* magnitude Condition to Calibrate RL.



$$\frac{k_1 L_2}{L_1 L_2} = 1 \longrightarrow k=5$$

>>n=[1 4]; d=[1 4 3]; rlocus(n,d)



>> d = conv([1 2 0] , [1 7 12])

$s(s+2)$ $(s+3)(s+4)$

* angle of asymptote :-

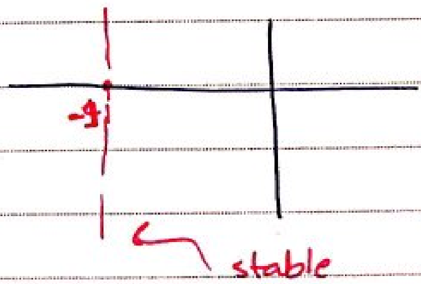
$$\frac{180}{\text{\# of poles} - \text{\# of zeros}} = \frac{180}{4-1} = 60$$

→ stop to the right so we have intersection with imajenary axis for high value of k , so it's possible to be unstable

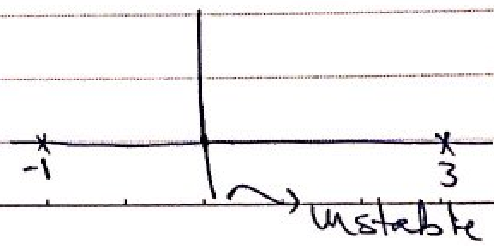
→ if stable , the steady state error due to u(t) is zero because we have an integrator

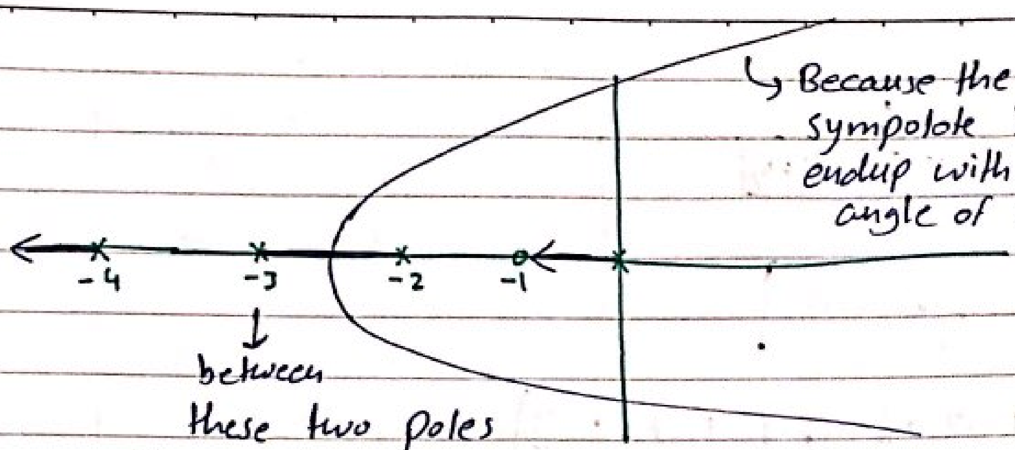
$\boxed{e.x}$

(a) $1 + \frac{k}{(s+1)(s+2)}$



(b) $1 + \frac{k}{(s+1)(s-3)}$





↳ Because the angle a symptote is 60°. So we endup with a line has an angle of 60°

between these two poles there is breakaway point

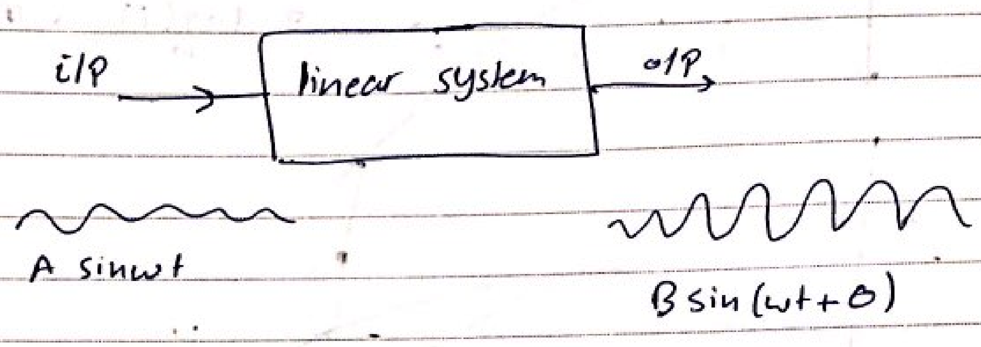
* any point at root locus is a closed loop pole at the value of k

* if # of poles − # of zeros ≥ 2
→ The sum of closed loop poles = # of open loop poles.

No. 5/8/2018

* Bode diagram

The FR response of L.S when the excitation is sinusoidal.



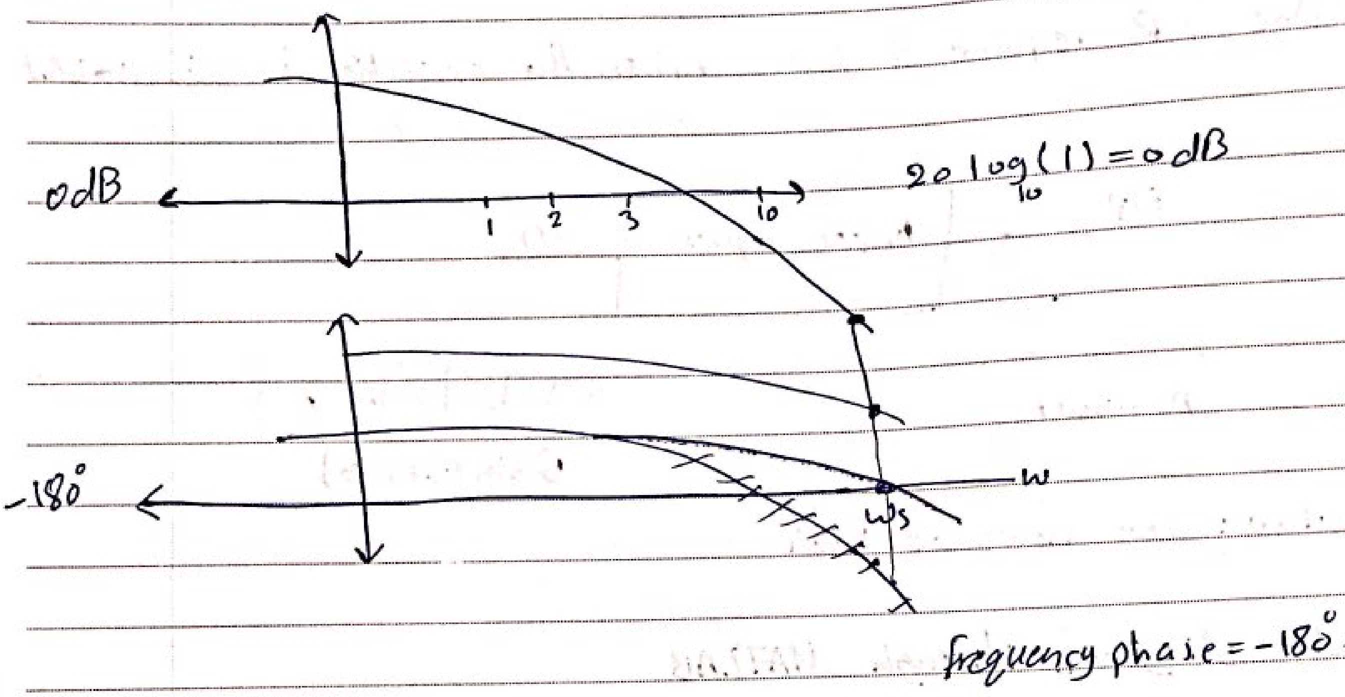
Review: see notes of Book

Bode diagrams Through MATLAB

```
>> n=1 ; d=[1 1] ; sys=tf(n,d) , bode(sys)
```

No. 6/8/2018

Stability using the bode diagram



* look for ω_s is $-180°$

* if magnitude is negative (under zero dB) = stable system

$20 \log |G|_{w=w_s} < 0$. The closed loop system is stable

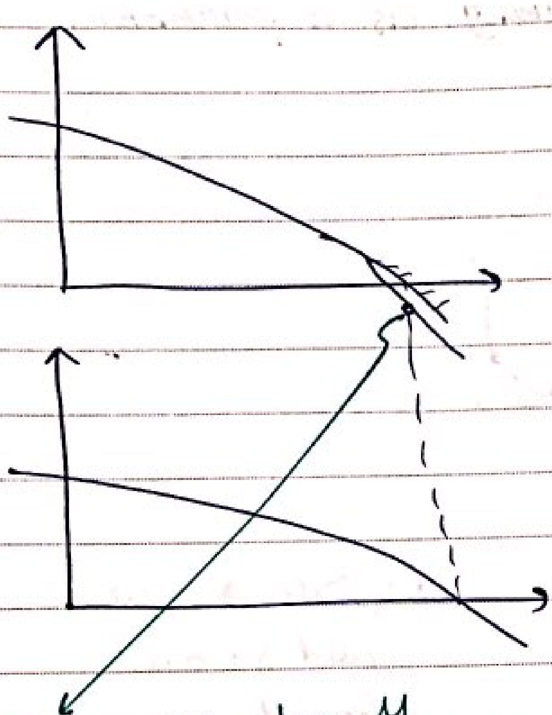
stable

if @ zero (magnitude) margenally stable

given margin ⇒ bode diagram

marginallly → sustained oscillation

symplotically stable $\Rightarrow$ 8o lots of oscillations



$$9.9 = 20 \log_{10} M$$

$$M = 10^{\frac{9.9}{20}}$$

$$M = 3.12$$

⊛ Nyquist Diagram

frequency domain method

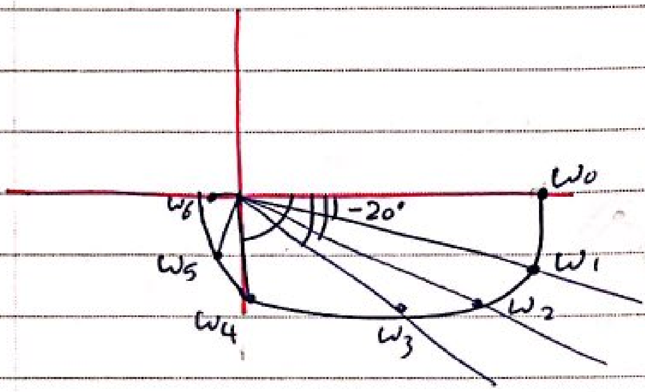
– Nyquist one figures / – Bode two figures

↳ not logarethmitc

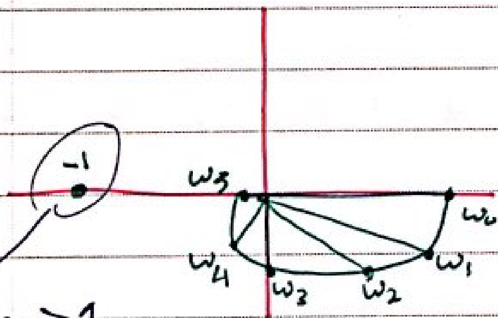
⊛ polar plots → phase / magnitude and frequency as a parameter

No.

Def:- it's a polar plot sharing magnitude of gain and phase taking the frequency as a parameter.



$\omega_3 > \omega_2$, $\omega_1 > \omega_0$
and so on
from the arrow direction



magnitude $\Rightarrow 1$

phase $\Rightarrow -180°$

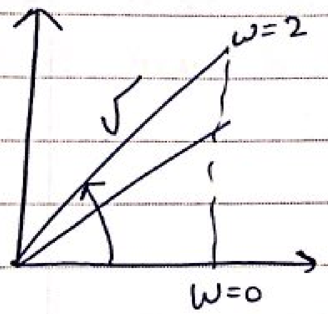
No.

(*) (ND) Nyquist diagram for certain transfer functions

$G(s)\,H(s) = 1 + S$

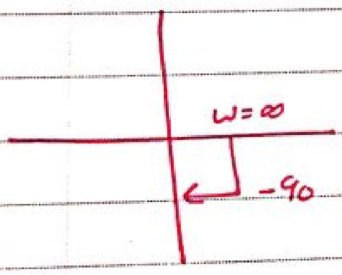
n = [1 1] ; d = [1] ; nyquist (n,d)

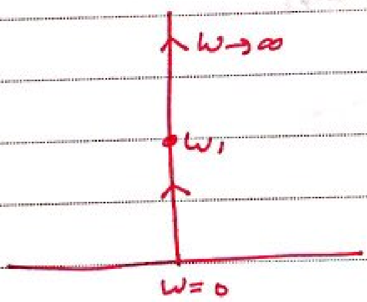
$G(j\omega)\,H(j\omega) = 1 + j\omega$



$| = \sqrt{1+\omega^2}$

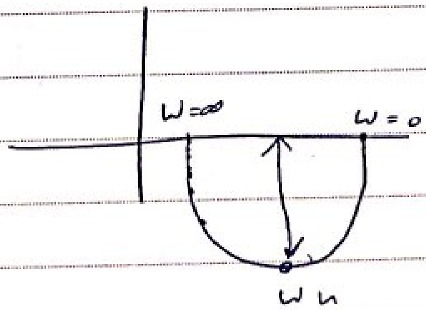
$\angle = \tan^{-1}\frac{\omega}{1}$





$G(s)\,H(s) = \frac{1}{S+1}$

$\frac{16}{S^2 + 2S + \underset{\omega n^2}{\underline{16}}}$



$\frac{0}{(j\omega)(j\omega)+90+0}$

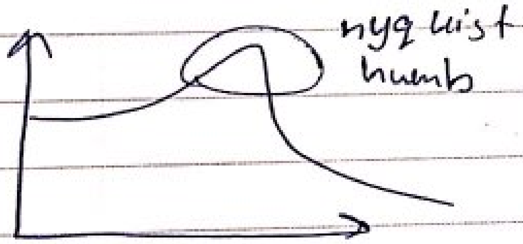
smile for life

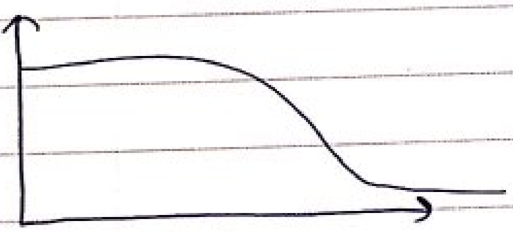
⊛ if expromantly optained the frequency -90 is the value of Wn

⊛ the distance between origion and ⊛

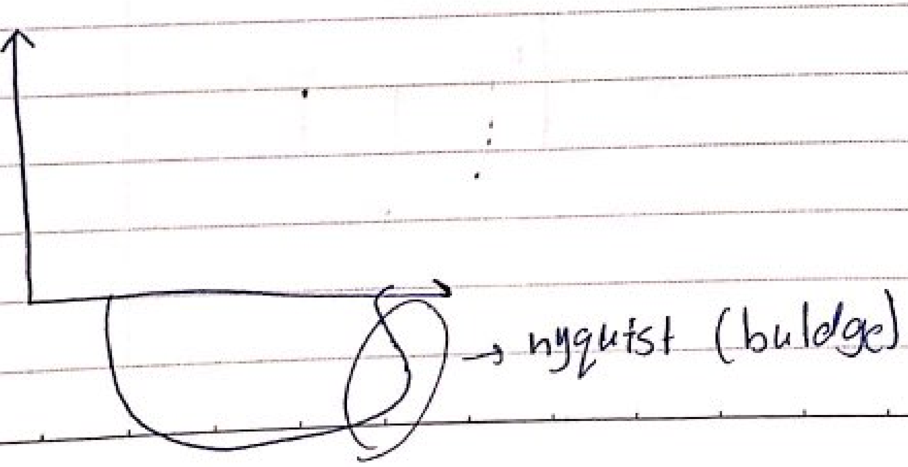
$\zeta < 1$ if there was bulging

⊛ humb
Bode
⊛ buldge





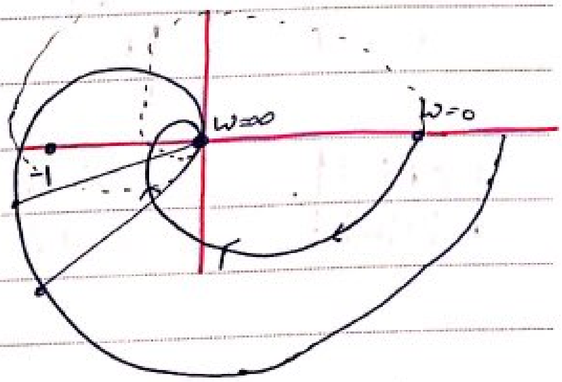
Bode



No. 7/8/2017

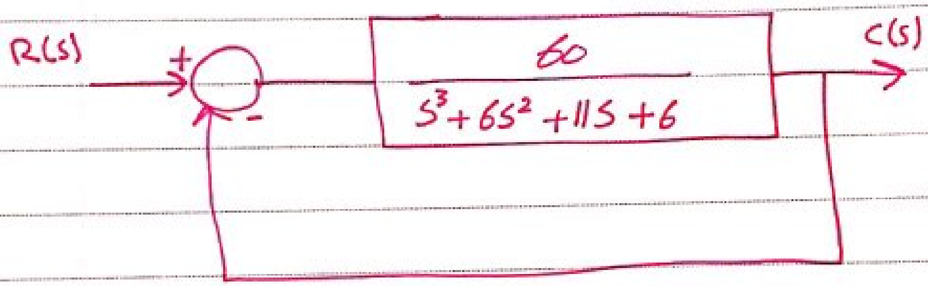
Stability using the ND :-

(*) Draw the ND for $-\infty < \omega < \infty$ resulting in a closed Contour.



if the contour encircles (enclosed) the -1 point then the system is unstable

Example :-

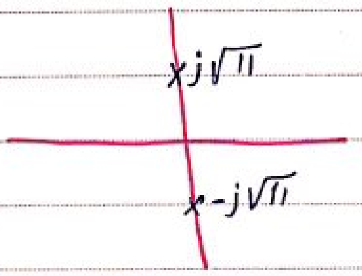


$$\frac{C(s)}{R(s)} = \frac{60}{s^3+6s^2+11s+66}$$

s^3	1	11
s^2	6	66
s^1	0	
s^0		

$A_s = 6s^2 + 66$

$s = \pm j\sqrt{11}$

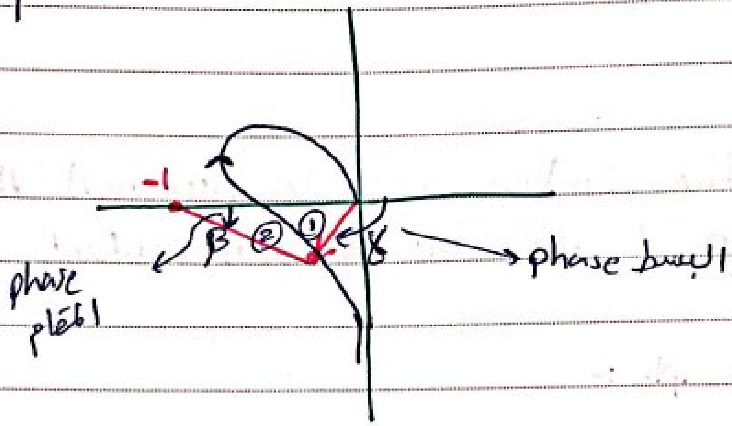


Closed loop response using the ND (graphically)

Applies to unity negative Feedback

$$\frac{C(j\omega)}{R(j\omega)} = \frac{|G(j\omega)|}{|1 + G(j\omega)|}$$

$$\therefore \left|\frac{C(j\omega)}{R(j\omega)}\right| = \frac{\text{① طول خط}}{\text{② طول خط}}$$



$$\angle \frac{C(j\omega)}{R(j\omega)} = \frac{\gamma}{\beta}$$

8/8)

N.B Closed loop gain $= \dfrac{①}{②}$

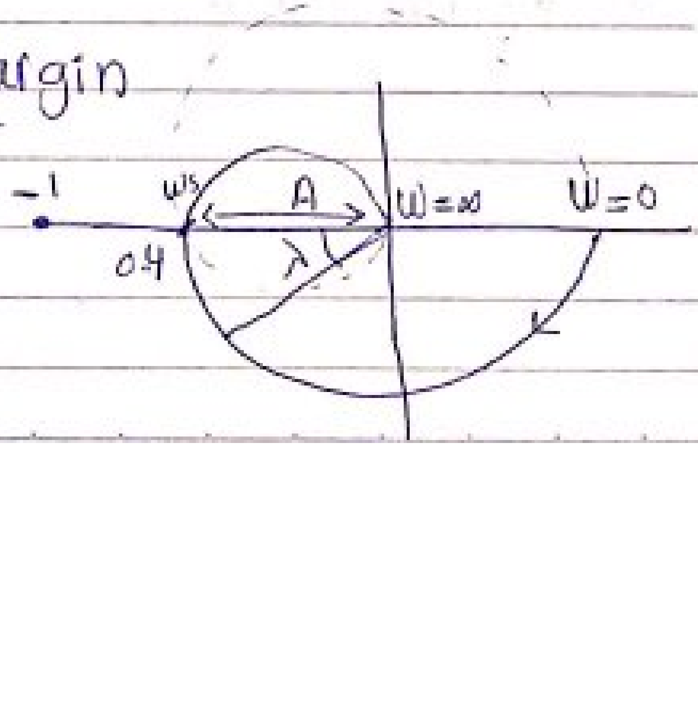
closed loop phase $= \psi$

N.B The above calculations are adopted providing the closed loop system is stable.

gain and phase margin

$G.M = \dfrac{1}{A} = \dfrac{1}{0.4} = 2.5$

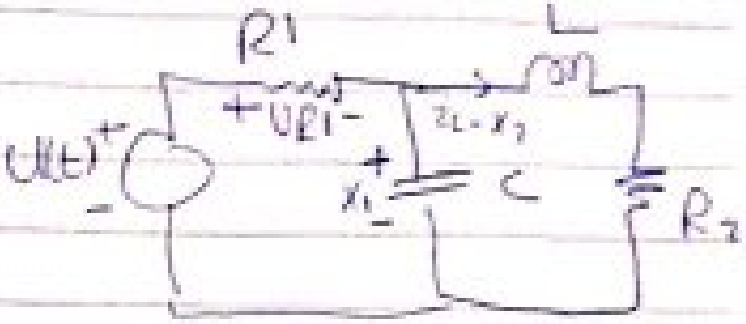
$P.M = \lambda$



State space approach to systems

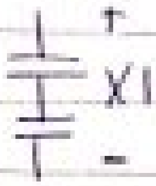
Modeling of systems

$\dot{X} = \frac{dX}{dt}$



$C\dot{X}_1 = \frac{U - X_1}{R_1} - X_2$

* We need state for every independent storage element (L, C)



— no need for two states.

$\dot{X}_1 = \frac{-1}{R_1C}X_1 - \frac{1}{C}X_2 + \frac{1}{R_1C}U$

$L\dot{X}_2 = X_1 - R_2X_2$

Output y is VR1

$y = U - X_1$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \dot{X} = \begin{bmatrix} \frac{-1}{R_1C} & \frac{-1}{C} \\ \frac{1}{L} & \frac{-R_2}{L} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1C} \\ 0 \end{bmatrix} u(t)$$

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$y = [-1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 1.u$

$\dot{x} = Ax + Bu$ state equation

$x \in R^n, u \in R^m$ — DE to be solved

$y = Cx + Du$ output equation

$y \in R^p$ → algebraic equation

$p \leq n$

State ~~resp~~ time responce

It can be shown that $x(t) = e^{A(t-t_0)} [x(t_0)]$ ~~th~~

$$e^{A(t-t_0)} x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)} Bu(\tau) d\tau$$

c.f $\frac{dx}{dt} = ax + f(t)$

$$ax = e^{a(t-t_0)} x(t_0) + \int^{t} e^{a(t-\tau)} f(\tau) d\tau$$

$$x = e^{at} x(0) + e^{at} \int_{0}^{t_0 t} e^{-a\tau} f(\tau) d\tau \text{ ; for } t_0 = 0$$

Where $e^{At} = \mathcal{L}^{-1} [SI - A]^{-1}$ ⇒ closed form solution or $e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$

$e^{At} = I_n + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \dots$

>> expm(A*0.1)

Ex: Compute i) e^{At} when $A = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$ using $\mathcal{L}$

ii) e^{At} when $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ using the 2 methods.

check: >> syms t
>> expm(A*t);
>> collect(I); or simplify(I);

Example: Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

$$[SI - A]^{-1} = \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}^{-1} = \frac{1}{\underbrace{S(S+3)+2}_{(S+1)(S+2)}} \begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}$$

$$e^{At} = \mathcal{L}^{-1} \begin{bmatrix} \frac{S+3}{(S+1)(S+2)} & \frac{1}{(S+1)(S+2)} \\ \frac{-2}{(S+1)(S+2)} & \frac{S+3}{(S+1)(S+2)} \end{bmatrix}$$

$$= \mathcal{L}^{-1} \begin{bmatrix} \frac{2}{(S+1)} - \frac{1}{S+2} & \frac{1}{S+1} - \frac{1}{S+2} \\ \frac{-2}{S+1} + \frac{2}{S+2} & \frac{-1}{S+1} + \frac{2}{S+2} \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

Properties of e^{At}

① $e^{At}\big|_{t=0} = I_n$

② $e^{A(t_1+t_2)} = e^{At_1}\, e^{At_2}$

$e^{(A+B)t} = e^{At} \cdot e^{Bt}$, only if $AB = BA$

$[e^{At}]^{-1} = e^{A(-t)}$

the time response when $u = 0$

$x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$x(t) = e^{At}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \cancelto{0}{e^{At}\int_0^t \cdots x o \, \partial\tau}$$

~~$x(t) =$~~

$$e^{At} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} 3e^{-t} - 2e^{-2t} \\ -3e^{-t} + 4e^{-2t} \end{bmatrix} \; ; x_1(t) = 3e^{-t} - 2e^{-2t} \; ; x_2(t) = -3e^{-t} + 4e^{-2t}$$

— State

Example: For the previous system compute
$x(t)$ when $u(t)$ is a unit step.

- Determind the eign values of A if an eignvalue has +ve real part then the system is unstable.

Example: $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} x$

$|A - \lambda I| = 0 \Rightarrow \lambda^2 + 2\lambda + 2 = 0$

$\lambda = -1 \mp j$ $\longrightarrow$ stable

Ex:

$A1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$ $A2 = \begin{bmatrix} -5 & 3 & 3 \\ -6 & 3 & 6 \\ 0 & 1 & 0 \end{bmatrix}$

$A3 = \begin{bmatrix} 4 & 5 \\ -5 & 4 \end{bmatrix}$

Transfer function $G(s)$

It can be shown

$$G(s) = C\,[sI_n - A]^{-1} B + D$$

i.e from state space approach to input/output approach.

Steady State values due to a Unit step

It can be shown that for asymtotically stable system the steady state

$$C_{ss} = \lim_{t \to \infty} x(t) = -A^{-1}B$$