# Chapter 1: Introduction to Engineering Economy

**Engineering economy:**

Systematic evaluation of the economic merits of proposed solutions to engineering problems.

➔ Answers basic economic questions:

- Do benefits exceed costs?
- How to conduct a certain activity?
- How to utilize the staff?
- Which alternative to choose?

## Successful engineering proposal/design:

- Organization benefit.
- Innovative technology.
- Clear outcomes.
- Valid measure of economic merit.

## Engineering economy principles

- Develop alternatives.
- Focus on the differences.
- Use a consistent viewpoint.
- Use a common unit of measure.
- Consider all relevant criteria.
- Make uncertainty explicit.
- Revisit your decisions.

# Economic analysis procedure

- Problem definition.
- Development of alternatives.
- Development of prospective outcomes.
- Selection of a decision criterion.
- Analysis and comparison of alternatives.
- Selection of the preferred alternative.
- Performance monitoring and post-evaluation of results.

# Problem definition example

Example:

A small furniture-manufacturing company wants to increase their profits to get a loan from the bank to purchase a more modern pattern-cutting machine. One proposed solution is to sell waste wood chips and shavings to a local charcoal manufacturer instead of using them to fuel space heaters for the company's office and factory areas.

- Define the company's problem.

  *Not enough revenue to cover costs or to produce significant profit.*

- *Alternatives?*

# Economic analysis example

Someone bought a small apartment building for $100,000. He spent $10,000 of his own money for the building and obtained a mortgage from a local bank for the remaining $90,000. The annual mortgage payment to the bank is $10,500. He also estimates that annual maintenance on the building and grounds will be $15,000. There are four apartments in the building that can each be rented for $360 per month.

- Problem?

  *Yes → money spent ($10,500 + $15,000 = $25,500) every year exceeds revenue ($360×4×12 = $17,280) → $8,220 loss per year.*

  *Maybe rent is too low?*

- Alternatives?
  - *Raise the rent.*
  - *Decrease maintenance cost.*
  - *Sell the building.*
  - *Abandon the building.*
- Development of prospective outcomes?

  *Option 1: raise rent so the net balance is zero.*

  *$8,220/4 apts/ 12 months = $171.25 increase per apartment per month (48% increase).*

  *Option 2: lower monthly expenses.*

  *$10,500 + X = $17,280 → X = $6,780 per year (maintenance) → $565 per month.*

  *Option 3: Selling the building.*

  *Option 4: Abandoning the building.*

- Selection of a decision criterion?

  *Minimization of losses.*

- Analysis and comparison of alternatives?

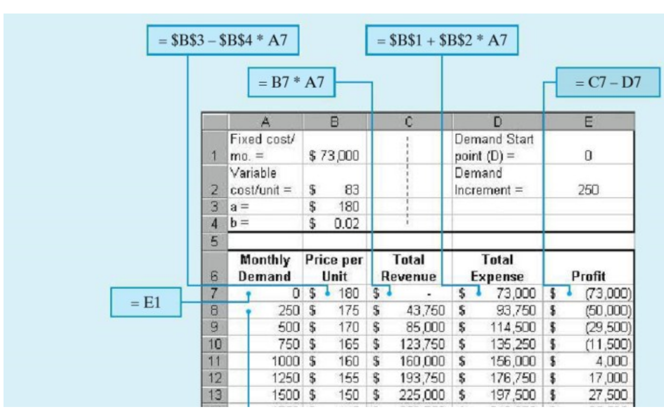
  *Based on the selected criterion.*

- Selection of the preferred alternative?

  *Select the best achievable option.*

# Spreadsheets

- ➢ Excellent for large and repetitive problems.
- ➢ Graphical output is easily generated.




| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Fixed cost/ mo. = | $ 73,000 | | Demand Start point (D) = | 0 |
| 2 | Variable cost/unit = | $ 83 | | Demand Increment = | 250 |
| 3 | a = | $ 180 | | | |
| 4 | b = | $ 0.02 | | | |
| 5 | | | | | |
| 6 | Monthly Demand | Price per Unit | Total Revenue | Total Expense | Profit |
| 7 | 0 | $ 180 | $ - | $ 73,000 | $ (73,000) |
| 8 | 250 | $ 175 | $ 43,750 | $ 93,750 | $ (50,000) |
| 9 | 500 | $ 170 | $ 85,000 | $ 114,500 | $ (29,500) |
| 10 | 750 | $ 165 | $ 123,750 | $ 135,250 | $ (11,500) |
| 11 | 1000 | $ 160 | $ 160,000 | $ 156,000 | $ 4,000 |
| 12 | 1250 | $ 155 | $ 193,750 | $ 176,750 | $ 17,000 |
| 13 | 1500 | $ 150 | $ 225,000 | $ 197,500 | $ 27,500 |

# Chapter 2: Cost Concepts and Design Economics

## Objective

Analyze short term-alternatives when the time-value of money is not a factor.

# Cost categories

1) Fixed costs (costs unaffected by changes in a specific range of operating conditions).

   Examples: insurance on facilities, taxes on facilities, general management and administrative salaries, license fees, interest costs on borrowed capital.

2) Variable costs (costs that vary with the quantity of output).

   Examples: materials and labor costs.

3) Incremental cost (additional cost resulting from increasing the output by one or more units).

   Examples: mileage cost, production of 1 vs. 2 units.

# Cost categories example

Classify each of the following cost items into fixed or variable costs:

| | |
|---|---|
| Raw materials | Variable |
| Direct labor wages | Variable |
| Supplies | Variable |
| Property taxes | Fixed |
| Utilities (electricity bill) | Fixed and Variable |
| Administrative salaries | Fixed |
| Sales commission | Variable |
| Rent | Fixed |
| Shipping charges | Variable |

# Example: highway paving

A new highway is to be paved and the contractor has two locations to set up their asphalt mixing equipment. The job requires 50,000 $yd^3$ of asphaltic material and the project duration is estimated to be 4 months (17 weeks of 5 working days). Which option is better?

| Cost Factor | Site *A* | Site *B* |
|---|---|---|
| Average hauling distance | 4 miles | 3 miles |
| Monthly rental of site | $2,000 | $7,000 |
| Cost to set up and remove equipment | $15,000 | $50,000 |
| Hauling expense | $2.75/yd$^3$-mile | $2.75/yd$^3$-mile |
| Flagperson | Not required | $150/day |

| Cost | Fixed | Variable | Site *A* | Site *B* |
|---|---|---|---|---|
| Rent | √ | | = $8,000 | = $28,000 |
| Setup/removal | √ | | = 15,000 | = 50,000 |
| Flagperson | √ | | = 0 | 5(17)($150) = 12,750 |
| Hauling | | √ | 4(50,000)($2.75) = 550,000 | 3(50,000)($2.75) = 412,500 |
| | | | Total: $573,000 | $503,250 |

➔ Site B is better.

Assume the contractor is paid $12/$yd^3$ asphalt delivered to the site and assume the cost of material is $1.5/$yd^3$ . At what point does he break-even and begin to make a profit?

Break-even means: Total revenue = Total expenses

$90,750 + [$2.75 × 3 X] + [$1.5 × X]  = $12 X
X = 40,333 $yd^3$

# Other categories of cost

- Direct costs (directly measured and allocated to a specific outcome or work activity).
  - Examples: labor and material costs associated with a certain construction activity are direct costs for that activity.
- Indirect costs (overhead or burden): difficult to allocate to a specific output or work activity. A specific formula can be used (proportions).
  - Examples: plant operating costs, common tools, general supplies, general maintenance.
- Standard costs (established ahead of production or service delivery: anticipated labor and material costs + overhead cost per unit).
  - Useful for bidding, cost estimation, comparison, and evaluation.

# Cost terminology

Cash cost: involves payment of cash and results in cash flow.

- Example: estimates for the cost of travel, labor, material, etc.

Book cost: does not involve a cash transaction and is normally reflected as a noncash cost.

- Example: depreciation due to the use of assets such as equipment (not a cash flow).

Sunk cost: payment occurred in the past with no relevance to future cost and revenue estimates (not a part of future cash flows and is typically disregarded in engineering economy problems).

- Example: non-refundable down payment on a car.

Opportunity costs: monetary advantage foregone due to limited resources or the cost of the best rejected opportunity.

- Example: working and getting paid for one year or going to college and paying tuition.

Life-cycle cost: summation of all costs related to a product, a system, a structure, or a service during its lifespan.

- Acquisition phase (need, alternatives, design) … greatest potential for savings occurs here.
- Operation phase (production or delivery until product/service is retired or disposed).
  - Example: buying a modern hybrid car vs. an old SUV.

# General price-demand relationship

$$p = a - b \times D$$

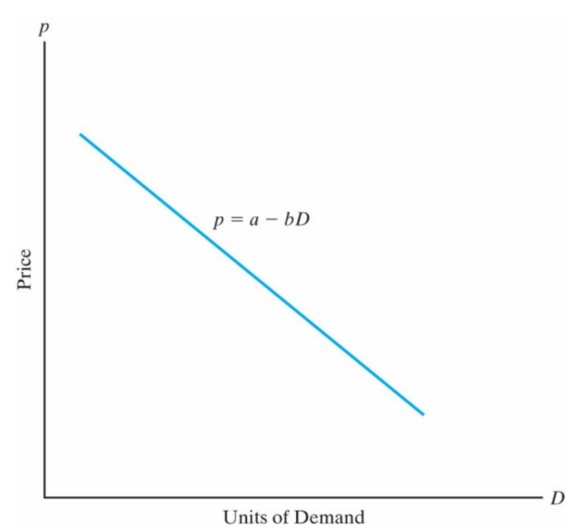
Where:

$p$ = price

$D$ = demand

$a$ and $b$ = constants that depend on the product or service.

$0 < D < a/b \qquad a > 0;\ b > 0$



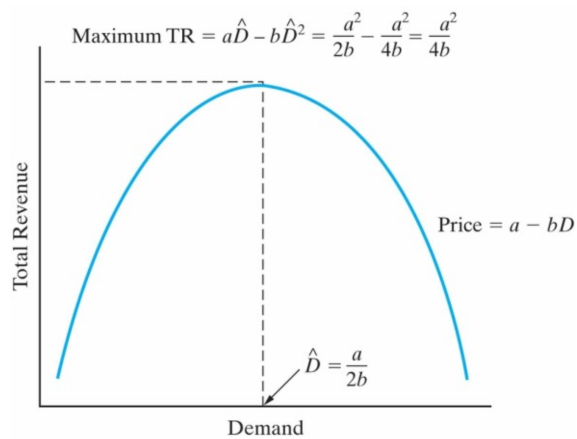
# Total revenue

Total revenue (TR) from selling a product or a service is:

$$TR = price \times demand = p \times D$$

Recall that $p = a - bD$

$$TR = aD - bD^2$$




# Cost, volume, and breakeven point

Total costs ($C_T$) = Fixed costs ($C_F$) + Variable costs ($C_V$)

Assuming a linear relationship between variable costs and demand,

$C_V = cv \times D$, where $c_v$ is the variable cost per unit demand.

$$C_T = C_F + c_V \times D$$

- Scenario 1: demand is a function of price.
- Scenario 2: demand and price are independent from each other.

# Profit – scenario 1

Profit = Total revenue – Total costs

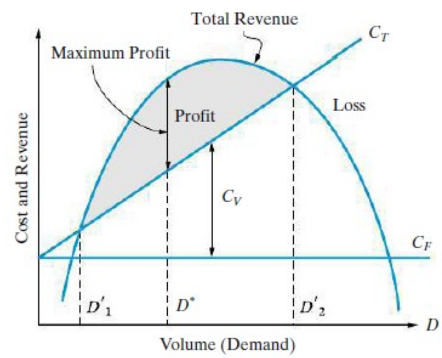
$$Profit = (aD - bD^2) - (C_F + c_V D)$$

$$Profit = -bD^2 + (a - c_V)D - C_F$$

To maximize profit,

$$\frac{d(Profit)}{dD} = a - c_v - 2bD = 0$$

Optimal $D$ → $D^* = \frac{a - c_v}{2b}$




Breakeven point (profit = 0) is found by:

Total revenue = Total cost

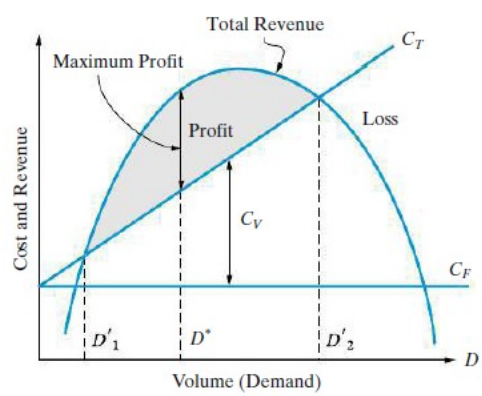
$$aD - bD^2 = C_F + c_V D$$

$$-bD^2 + (a - c_V)D - C_F = 0$$

Solve for $D$ to get $D'$ (breakeven demand)

(2 breakeven demands $D'1$ and $D'2$): range of profitable demand

$$D' = \frac{-(a - c_v) \pm \sqrt{(a - c_v)^2 - 4(-b)(-C_F)}}{2(-b)}$$



# Example

A company produces an electronic timing switch that is used in consumer and commercial products. The fixed cost ($C_F$) is $73,000 per month, and the variable cost ($c_v$) is $83 per unit. The selling price per unit is $p = \$180 - 0.02(D)$.

1) Determine the optimal volume for this product and confirm that a profit occurs (instead of a loss) at this demand.

2) Find the volumes at which breakeven occurs; that is, what is the range of profitable demand.

(1)

$D^* = \frac{a-c_v}{2b} = \frac{\$180-\$83}{2*0.02} = 2{,}425\ units\ per\ month\ (maximum\ profit).$

Or write down the equation of profit, derive, and equate to zero.

$$P = 180\,D - 0.02\,D^2 - (73{,}000 + 83\,D) = -0.02\,D^2 + 97\,D - 73{,}000$$

For a profit to occur, the 2nd derivative should be negative (-0.04).

Also, substitute the optimal demand ($D^*$) in the profit equation:
*Profit*= [\$180 × 2,425 – 0.02 × 2,425$^2$] – [\$73,000 + \$83 × 2,425] = \$44,612 (+ve profit).

(2) $a$ = 180 $b$ = 0.02 $c_v$ = 83 $C_F$ = 73000

$$D' = \frac{-(180-83) \pm \sqrt{(180-83)^2 - 4(-0.02)(-73{,}000)}}{2(-0.02)}$$

$D'_1$ = 932 units and $D'_2$ = 3,918 units.

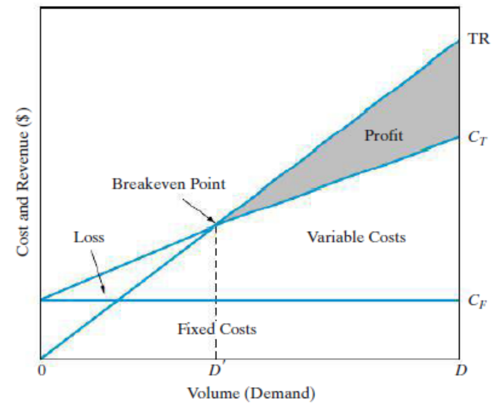
# Profit – scenario 2

Price per unit ($p$) and demand ($D$) are independent from each other.

Profit = Total revenue – Total costs

$$Profit = p\,D\ - (C_F + c_V D)$$

➔ Only one breakeven point.




# Example

An engineering consulting firm measures its output in a standard service hour unit. The variable cost ($c_v$) is $62 per standard service hour and the charge-out rate [i.e., selling price ($p$)] is $85.56 per hour. The maximum output of the firm is 160,000 hours per year, and its fixed cost ($C_F$) is $2,024,000 per year. What is the breakeven point in standard service hours and in percentage of total capacity?

$$\text{Total revenue} = \text{Total costs}$$

$$D' \times \$85.56/h = \$2{,}024{,}000 + \$62 \times D'$$

$$D' = 85{,}908\ h.$$

$$\%\ of\ total\ capacity = \frac{85{,}908}{160{,}000} \times 100\% = 54\%$$

# Present economy studies

Duration < one year → time influence on money is ignored (present economy).

Comparing multiple alternatives:

(1) For variable known revenue and benefits, select the alternative with maximum profit.

(2) For constant or unknown revenues and benefits, select the alternative with minimum total cost per defect-free product or service.

# Example

The demand for a certain part is 100,000 units. The part is produced on a high-speed turret lathe, using screw-machine steel costing $0.30 per pound. A study was conducted to determine whether it might be cheaper to use brass screw stock, costing $1.40 per pound. Because the weight of steel required per piece was 0.0353 pounds and that of brass was 0.0384 pounds, the material cost per piece was $0.0106 for steel and $0.0538 for brass. However, when the manufacturing engineering department was consulted, it was found that, although 57.1 defect-free parts per hour were being produced by using steel, the output would be 102.9 defect-free parts per hour if brass were used. Assuming the machine attendant is paid $15.00 per hour, and the variable (i.e., traceable) overhead costs for the turret lathe are estimated to be $10.00 per hour. Which material should be used for this part?

Unknown or constant revenue (demand is constant) → compare the cost per defect-free unit

| | Steel | Brass |
|---|---|---|
| Material | $0.30 × 0.0353 = $0.0106 | $1.40 × 0.0384 = $0.0538 |
| Labor | $15.00/57.1 = 0.2627 | $15.00/102.9 = 0.1458 |
| Variable overhead | $10.00/57.1 = 0.1751 | $10.00/102.9 = 0.0972 |
| Total cost per piece | $0.4484 | $0.2968 |
| Saving per piece by use of brass = $0.4484 − $0.2968 = $0.1516 | | |

➔ Select brass

# Example

Two machines with approximately the same capital investment are being considered for the production of a part. The important differences between the machines are their production capacities (production rate × available production hours) and their reject rates (percentage of parts produced that cannot be sold). Consider the following table:

| | Machine *A* | Machine *B* |
|---|---|---|
| Production rate | 100 parts/hour | 130 parts/hour |
| Hours available for production | 7 hours/day | 6 hours/day |
| Percent parts rejected | 3% | 10% |

The material cost is $6.00 per part, and all defect-free parts produced can be sold for $12 each (rejected parts have negligible scrap value.) For either machine, the operator cost is $15.00 per hour and the variable overhead rate for traceable costs is $5.00 per hour.

Assume that the daily demand for this part is large enough that all defect-free parts can be sold. Which machine should be selected?

Variable total revenue → Rule #1 → Profit maximization

Profit per day = Total revenue per day – Total costs per day

$$= \left[\text{production rate} \times \text{production hours} \times \frac{\$12}{\text{part}} \times \left(1 - \frac{\text{rejected\%}}{100}\right)\right] - \left[\text{production rate} \times \text{production hours} \times \frac{\$6}{\text{hour}}\right] - \left[\text{production hours} \times \left(\frac{\$15}{\text{hour}} + \frac{\$5}{\text{hour}}\right)\right]$$

For machine A:

$$\text{Profit per day} = \left[\frac{100\text{ parts}}{\text{hour}} \times \frac{7\text{ hours}}{\text{day}} \times \frac{\$12}{\text{part}} \times (1 - 0.03)\right] - \left[\frac{100\text{ parts}}{\text{hour}} \times \frac{7\text{ hours}}{\text{day}} \times \frac{\$6}{\text{part}}\right] - \left[\frac{7\text{ hours}}{\text{day}}\right]$$

# Energy efficiency studies

Two pumps delivering 100 hp (1 hp = 0.746 kW) will be operated for one year (4,000 h) for agricultural purposes. Assuming the electricity costs $0.1 per kWh. Which pump would you select?

| | Pump A | Pump B |
|---|---|---|
| Purchase price | $2,900 | $6,200 |
| Maintenance cost | $170 | $510 |
| Efficiency | 80% | 90% |

$$\text{Elect consumption(\$)} = \frac{\text{Power delivered}}{\text{efficiency}} \times \text{\#hours} \times \text{price}$$

For pump A:

$\text{Consumption} = 100\text{ hp} \times \frac{0.746\text{ kW}}{\text{hp}} \times \frac{1}{0.8} \times 4{,}000\text{ h} \times \frac{\$0.10}{\text{kWh}}$ = $37,300

➢ Total owning and operating cost = $37,300 + $2,900 + $170 = $40,370

For pump B:

$\text{Consumption} = 100\text{ hp} \times \frac{0.746\text{ kW}}{\text{hp}} \times \frac{1}{0.9} \times 4{,}000\text{ h} \times \frac{\$0.10}{\text{kWh}}$ = $33,156

➢ Total owning and operating cost = $33,156 + $6,200 + $510 = $39,866

➔ Select pump B

# Making vs. outsourcing

- In-house production vs. purchasing (outsourcing).
- Indirect and overhead costs could be shared among other activities.
- Accurate analysis is needed in decision-making.

# Chapter 3: Cost Estimation Techniques

## Objective

To present various methods for estimating important factors (costs, revenue, useful lives, residual values, etc.) in an engineering economy study.

Cost estimation is useful for:

1) Setting up a selling price for a quote or a bid.
2) Determining if a product will be profitable.
3) Justifying capital for process changes or improvements.
4) Setting benchmarks for productivity improvements.

## Two fundamental approaches for cost estimation

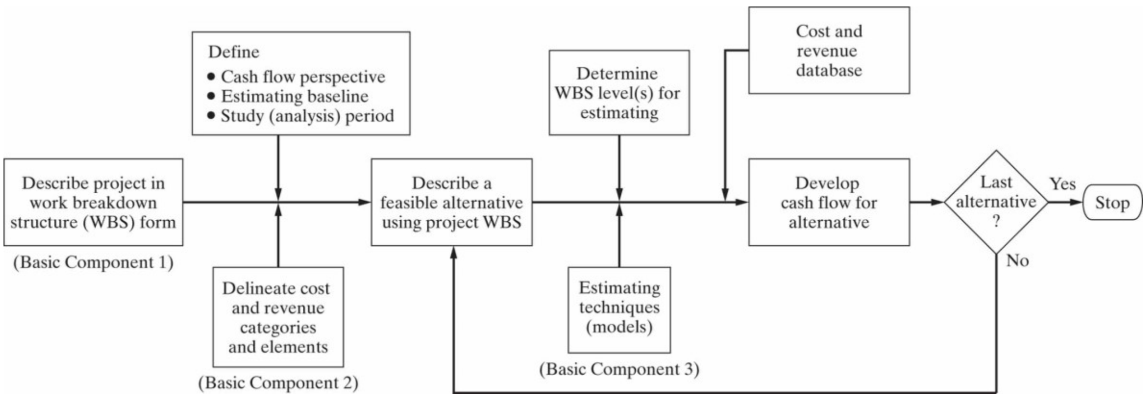
- Top-down approach
  - Good for early estimates when developing alternatives.
  - Uses historical data from similar projects with adjustments to account for inflation, deflation, and other factors.
- Bottom-up approach
  - More detailed approach.
  - Project is broken down into small units.
  - The estimated overall cost is the sum of the units' costs + other costs (e.g. overhead).

## Integrated cost estimation approach

Components

1. Work breakdown structure (WBS):
   Successive levels of the work elements and their interrelationships.
2. Cost and revenue structure (classification):
   Delineation of cost and revenue categories and elements for different WBS levels.
3. Estimating techniques (models):
   Selected mathematical models to estimate future costs and revenues.

# Integrated approach

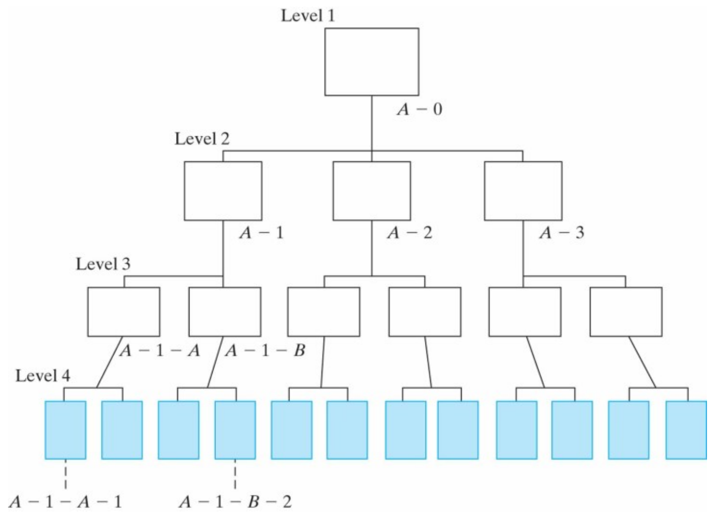



# Work breakdown structure (WBS)

- WBS is a basic tool in project management.
- WBS defines all project elements and their interrelationships, collecting and organizing information, and developing relevant cost and revenue data and management activities. WBS includes recurring (maintenance) and non-recurring (initial construction) work elements.
- Each WBS level further details the work elements. The resources required for a work element are the sum of resources of sub-elements below it.
- WBS Includes functional and physical work elements.
  - Functional (logistic support, project management, and marketing).
  - Physical (labor, materials, and resources required for the making of a product).

Example:

Develop the first 3 WBS levels for the construction of a small commercial building.




# Cost and revenue structure

- In this structure, costs and revenue to be included in the analysis are identified and categorized.

- Examples of cost and revenue categories:
  - Capital investment.
  - Labor costs.
  - Material costs.
  - Maintenance costs.
  - Overhead.
  - Disposal costs.
  - Sales revenue.
  - Market (or salvage) value.

# Estimating techniques (models)

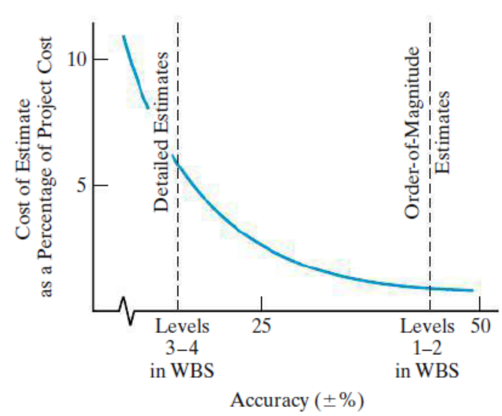
The goal is to develop cash flow projections, not <u>exact</u> future data (which is almost impossible).

Cost and revenue estimates can be classified to:

- <u>Order-of-magnitude estimates</u>
  - Planning and initial evaluation of a project to select feasible alternatives (±30 – 50% accuracy).
  - Level 1 or 2 of the WBS.
- <u>Semi-detailed (or budget) estimates</u>
  - Preliminary or conceptual design stage of a project (±15% accuracy).
  - Level 2 or 3 of the WBS.
- <u>Definitive (detailed) estimates</u>
  - Detailed design estimates from drawings, specs, quotations, … used for bidding (±5% accuracy).
  - Level 3 and beyond.

# Sources of estimating data

- Accounting records.
- Other sources inside the firm.
- Sources outside the firm.
- Research and development (R&D).



# Selected models

Model 1: Indexes (ratio technique)

- An index is a dimensionless number used to estimate present and future costs from historical data.

$$C_n = C_k \times \frac{\bar{I}_n}{\bar{I}_k}$$

Where:

*k*: reference year.

*n*: year to be estimated at.

$C_n$ and $C_k$: cost or price in years *n* and *k*, respectively.

$\bar{I}_n$ and $\bar{I}_k$ are the index values for the years *n* and *k*, respectively.

# Example

A company wants to install a new boiler. The price of the boiler in the year 2000 was $525,000 when the index was 468. What is the price of the boiler in 2014 given that the index value is 542 in the year 2014?

$$C_{2014} = C_{2000} \times \frac{I_{2014}}{I_{2000}}$$

$$C_{2014} = 525{,}000 \times \frac{542}{468} = \$608{,}013$$

# Selected models

Model 2: Unit technique

- Widely used and understood.
- Good for preliminary estimates.

Examples:

- Cost per $m^2$ of construction × area of construction.
- Operating cost per mile × number of miles.

# Selected models

Model 3: Factor technique

- Extension of the unit technique.
- Good for preliminary estimates.

$$Cost = \sum_d C_d + \sum_m C_m U_m$$

Where:

$C_d$: cost of a component d that is estimated directly.

$f_m$: cost per unit of component m.

$U_m$: number of units of component m.

# Parametric cost estimating

- Utilizing historical cost data and statistical techniques to predict future costs.
- These models are used in early design stages to get an estimate of a product or project cost based on few physical characteristics (e.g. weight, volume, power).
- Common techniques (parametric models):
  - Power sizing technique.
  - Learning curve.

# Power sizing technique

- Referred to as the exponential model.
- Used for industrial plants and equipment.

$$\frac{C_A}{C_B} = \left(\frac{S_A}{S_B}\right)^X$$

Where:

$C_A$ and $C_B$: costs for plants A and B, respectively ($ as of the time for which the estimate is desired).

$S_A$ and $S_B$: sizes of plants A and B, respectively (same physical units).

$X$: cost-capacity factor which depends on the type of plant.

# Example

The purchase price of a commercial boiler (capacity S) was $181,000 eight years ago. Another boiler of the same basic design, except with capacity 1.42 S, is currently being considered for purchase. If the cost index was 162 for this type of equipment when the capacity S boiler was purchased and is 221 now, and the applicable cost capacity factor is 0.8, what is your estimate of the purchase price for the new boiler?

Let $C_A$ = cost of new boiler ($S_A = 1.42\,S$) today and $C_B$ = cost of old boiler today ($S_B = S$).

$$C_B = \$181{,}000 \times \frac{221}{162} = \$246{,}920.$$

$$C_A = \$246{,}920 \times \left(\frac{1.42S}{S}\right)^{0.8} = \$326{,}879$$

# Learning curve

- Also called experience curve or manufacturing progress function.
- Reflects increased efficiency and performance with repetitive production.

$$Z_u = K(u^n)$$

Where:

$u$ = output unit number.

$Z_u$ = number of input resource units needed to produce output unit $u$.

$K$ = number of input resource units needed to produce the first output unit.

$n$ = learning curve exponent = $\frac{\log s}{\log 2}$

$s$ = learning curve slope parameter expressed as a decimal ($s$ = 0.9 for a 90% learning curve).

# Example

The time required to assemble the first car is 100 hours and the learning rate is 80%. What is the time required to assemble the $10^{th}$ car?

$S = 0.8$ $K = 100$ hours $u = 10$ cars

$$Z_{10} = 100 \times 10^{\frac{\log 0.8}{\log 2}} = 47.65 \text{ hours}$$

*** This is not the total time to produce 10 units … it's the time to produce the $10^{th}$ unit ***

# Example

You have been asked to estimate the cost of 100 prefabricated structures, each structure provides 1,000 sq.ft of floor space, with 8-ft ceilings. In 2003, you produced 70 similar structures consisting of the same materials and having the same ceiling height, but each provided only 800 sq.ft of floor space. The material cost for each structure was $25,000 in 2003, and the cost capacity factor is 0.65. The cost index values for 2003 and 2014 are 200 and 289, respectively. The estimated manufacturing cost for the first 1,000 sq.ft structure is $12,000. Assume a learning curve of 88% and use the cost of the $50^{th}$ structure as your standard time for estimating manufacturing cost. Estimate the total material cost and the total manufacturing cost for the 100 prefabricated structures.

Material cost

$I_{2003} = 200$ $\quad I_{2014} = 289$
$S_{2003} = 800$ $\quad S_{2014} = 1000$ $\quad X = 0.65$
$C_{2003} = \$25{,}000$

$$C_{2014} = \$25{,}000 \times \left(\frac{289}{200}\right) \times \left(\frac{1000}{800}\right)^{0.65} = \$41{,}764$$

Manufacturing cost

$s = 0.88$ $\quad K = \$12{,}000$

$$Z_{50} = \$12{,}000 \times 50^{\left(\frac{\log 0.88}{\log 2}\right)} = \$5{,}832/\text{unit}$$

Total cost = (\$41,764+\$5,832)×100 = \$4,759,600.

# Chapter 4: The Time Value of Money

## Objective

Explain the time value of money calculations and economic equivalence.

Time value of money ➔ because money can earn more money over time (interest on capital).

# Interest

- <u>Simple interest</u>
  - Not commonly used.
  - Total interest is linearly proportional to the initial loan amount (principal).
- <u>Compound interest</u>
  - More common in personal and professional financing.
  - Interest is based on the remaining principal + any accumulated interest.

# Simple interest

$$I = P \times N \times i$$

$I$: Total simple interest paid or earned.
$P$: Principal amount lent or borrowed.
$N$: Number of interest periods (e.g., years).
$i$: Interest rate per interest period.

<u>Example</u>: $1,000 loan for 3 years at a simple interest rate of 10% per year.

$P$ = Principal = $1,000.
$N$ = Number of interest periods = 3 years.
$i$: Interest rate per interest period = 10% per year.

The total interest paid = $I$ = $1000 × 10% × 3 years = $300.
The total amount repaid at the end of the loan period = principal ($P$) + interest ($I$)
= $1000 + $300 = $1300.

# Example

You borrowed \$5,000 at a simple interest rate = 0.5% per month to be repaid after 4 years. How much will you pay back?

<u>or</u> what is the future equivalent of the borrowed \$5,000?

$P$ = Principal = \$5,000.

$N$ = Number of interest periods = 4 years.

$i$: Interest rate per interest period = 0.5% per month × 12 months/year = 6% per year.

The total interest paid = $I$ = \$5,000 × 6% × 4 years = \$1,200.

The total amount repaid (or future equivalent) = \$5,000 + \$1,200 = \$6,200.

# Compound interest

- Interest is based on the remaining principal + accumulated interest.

<u>Example</u>: \$1,000 loan for 3 years at a compound interest rate of 10% per year.

= amount owed at beginning of period × interest rate = \$1,000 × 10%

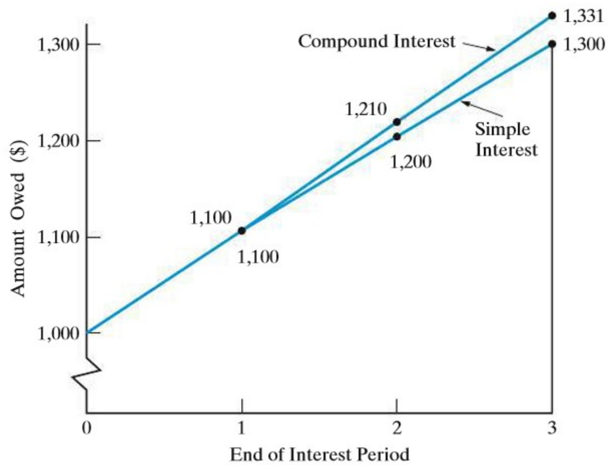
= amount owed at beginning of period + interest for period = \$1,000 + \$100

Compounding period (e.g., years, months)

| Period | Amount owed at beginning of period | Interest amount for period | Amount owed at end of period |
|---|---|---|---|
| 1 | \$1,000 | \$100 | \$1,100 |
| 2 | \$1,100 | \$110 | \$1,210 |
| 3 | \$1,210 | \$121 | \$1,331 |

Amount owed at end of previous period

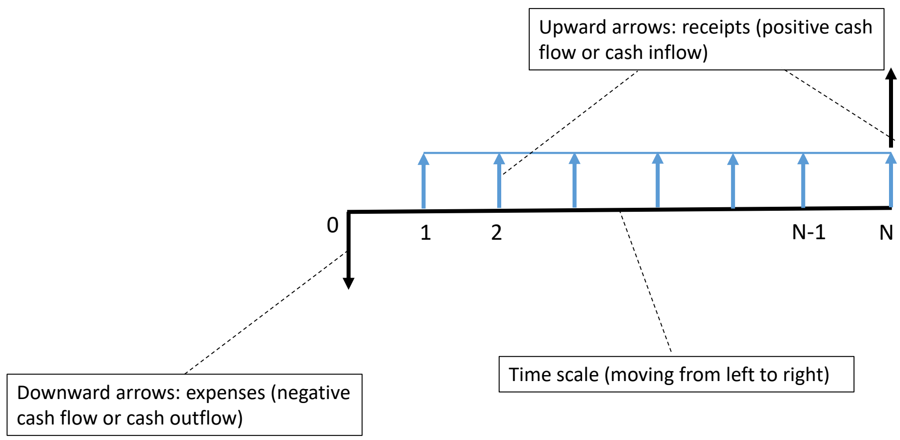
# Simple vs. compound interest




# The concept of economic equivalence

- For comparing alternatives when time value of money is a factor (compound interest is involved).
- Alternatives are reduced to an equivalent basis.
- Cash-flow diagram is an essential tool in economic equivalence.

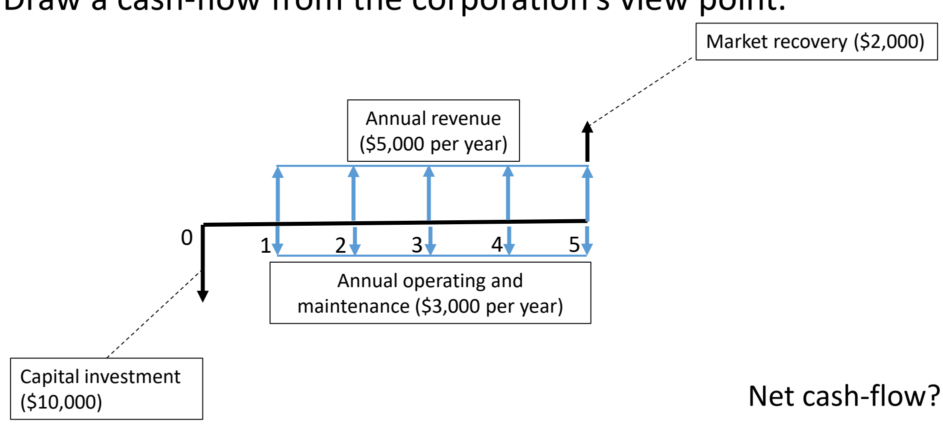
# Cash-flow diagrams




Direction of the arrows depends on the point of view (lender vs. borrower)

# Example

An investment of $10,000 will produce a uniform annual revenue of $5,000 for 5 years and have a market (recovery) value of $2,000 at the end of year (EOY) five. Annual operating and maintenance expenses are estimated at $3,000 at the end of each year. Draw a cash-flow from the corporation's view point.




Net cash-flow?

# Cash flow diagrams

Notation

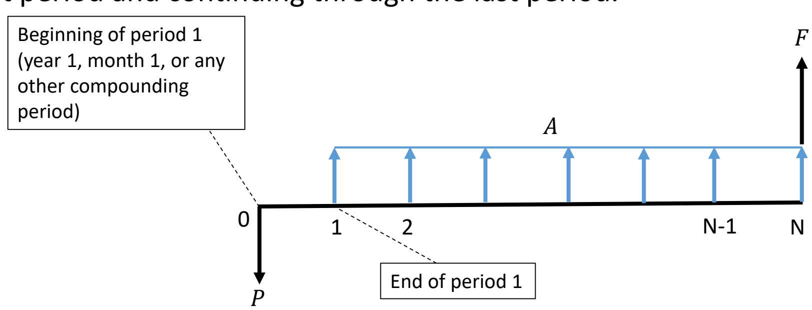
$i$: effective interest rate per interest period.

$N$: number of compounding interest periods (e.g., years).

$P$: present sum of money (or the equivalent sum of one or more cash flows at present time).

$F$: future sum of money (or the equivalent sum of one or more cash flows at future time).

$A$: end-of-period cash flow (or equivalent end-of-period value) in a uniform series starting at the end of first period and continuing through the last period.




# Relating present and future equivalent values

- For a single cash flow and using the compound interest rate formula.

$$F = P\,(1+i)^N$$

or

$$F = P\,(F/P\,, i\%, N)\ from\ tables\ in\ Appendix\ C$$

$$P = F\,(1+i)^{-N}$$

or

$$P = F\,(P/F\,, i\%, N)\ from\ tables\ in\ Appendix\ C$$

# Example

$1,000 loan for 3 years at a compound interest rate of 10% per year. How much will be repaid?

$P$ = $1,000 $N$ = 3 years $i$ = 10%

$F$ = ?

$$F = 1{,}000\ (1 + 0.1)^3 = \$1{,}331$$

or

Go to Appendix C, $i$ = 10% page to find $(F/P\ , 10\%, 3)$

TABLE C-13 Discrete Compounding; $i = 10\%$

| | Single Payment | | Uniform Series | | | | Uniform Gradient | |
|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor |
| N | To Find F Given P F/P | To Find P Given F P/F | To Find F Given A F/A | To Find P Given A P/A | To Find A Given F A/F | To Find A Given P A/P | To Find P Given G P/G | To Find A Given G A/G |
| 1 | 1.1000 | 0.9091 | 1.0000 | 0.9091 | 1.0000 | 1.1000 | 0.000 | 0.0000 |
| 2 | 1.2100 | 0.8264 | 2.1000 | 1.7355 | 0.4762 | 0.5762 | 0.826 | 0.4762 |
| 3 | 1.3310 | 0.7513 | 3.3100 | 2.4869 | 0.3021 | 0.4021 | 2.329 | 0.9366 |
| 4 | 1.4641 | 0.6830 | 4.6410 | 3.1699 | 0.2155 | 0.3155 | 4.378 | 1.3812 |
| 5 | 1.6105 | 0.6209 | 6.1051 | 3.7908 | 0.1638 | 0.2638 | 6.862 | 1.8101 |
| 6 | 1.7716 | 0.5645 | 7.7156 | 4.3553 | 0.1296 | 0.2296 | 9.684 | 2.2236 |

$F = 1{,}000 \times 1.331 = \$1{,}331$

# Example

You need $10,000 after five years so you decided to save money now. How much do you need to deposit now in the bank given that the interest rate is 5% per year?

$F$ = $10,000 $N$ = 5 years $i$ = 5%

$P$ = ?

$P = 10{,}000\ (1 + 0.05)^{-5} = \$7{,}835.26$

or

Go to Appendix C, $i$ = 5% page to find $(P/F\ , 5\%, 5)$

TABLE C-8 Discrete Compounding; $i = 5\%$

| | Single Payment | | Uniform Series | | | | Uniform Gradient | |
|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor |
| N | To Find F Given P F/P | To Find P Given F P/F | To Find F Given A F/A | To Find P Given A P/A | To Find A Given F A/F | To Find A Given P A/P | To Find P Given G P/G | To Find A Given G A/G |
| 1 | 1.0500 | 0.9524 | 1.0000 | 0.9524 | 1.0000 | 1.0500 | 0.000 | 0.0000 |
| 2 | 1.1025 | 0.9070 | 2.0500 | 1.8594 | 0.4878 | 0.5378 | 0.907 | 0.4878 |
| 3 | 1.1576 | 0.8638 | 3.1525 | 2.7232 | 0.3172 | 0.3672 | 2.635 | 0.9675 |
| 4 | 1.2155 | 0.8227 | 4.3101 | 3.5460 | 0.2320 | 0.2820 | 5.103 | 1.4391 |
| 5 | 1.2763 | 0.7835 | 5.5256 | 4.3295 | 0.1810 | 0.2310 | 8.237 | 1.9025 |

$P = 10{,}000 \times 0.7835$
$= \$7{,}835$

# Finding $\boldsymbol{i}$ given $F$, $P$, and $\boldsymbol{N}$

$$i = \sqrt[N]{F/P} - 1$$

Example:

What is the interest rate that will double an investment of $50,000 in 10 years?

$P$ = $50,000 $F$ = $100,000 $N$ = 10 years

$i$ = ?

$$i = \sqrt[10]{100{,}000/50{,}000} - 1 = 0.0718 = 7.18\%$$

(to use Appendix C tables, you need interpolation).

# Finding $N$ given $F$, $P$, and $i$

$$N = \frac{\log(F/P)}{\log(1+i)}$$

Example:

How many years does it take to double my money at an interest rate of 5% per year?

$F/P$ = 2 $i$ = 5%

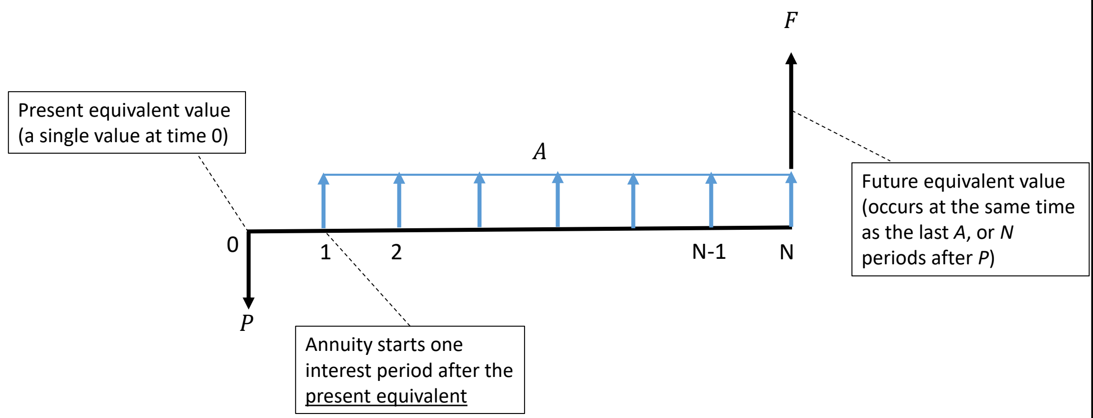
$N$ = ?

$$N = \frac{\log(2)}{\log(1+0.05)} = 14.2 \text{ years}$$

(to use Appendix C tables, you need interpolation).

# Uniform series (annuity) to present and future

$A$: series of uniform (equal) payments occurring at the end of each period for $N$ periods … also called annuity.




Example: repaying a loan in uniform monthly payments.

# Finding *F* given *A*

$$F = A\left[\frac{(1+i)^N - 1}{i}\right]$$

Or

$$F = A\,(F/A\,, i\%, N)\ from\ tables\ in\ Appendix\ C$$

Example:

How much will you have in 40 years if you invest $3,000 of your income each year in a project that earns 8% per year?

$A$= $3,000 $i$ = 8% $N$ = 40 years

$F$ = ?

$$F = 3{,}000\left[\frac{(1+0.08)^{40} - 1}{0.08}\right] = \$777{,}169.6$$

Or from Appendix C tables, $(F/A\,, 8\%, 40)$ = 259.0565 → $F$ = $3,000 × 259.0565 = $777,169.5

# Finding $\boldsymbol{P}$ given $A$

$$P = A\left[\frac{(1+i)^N - 1}{i\,(1+i)^N}\right]$$

Or

$$P = A\,(P/A\,, i\%, N)\ from\ tables\ in\ Appendix\ C$$

Example:

You took a loan which is to be repaid in uniform payments over 4 years. Assuming the interest rate is 1% per month, and your monthly payment is \$300. What is the principal amount (the amount of money borrowed)?

$A$= \$300 $i$ = 1% per month $N$ = 4 years × 12 months/year = 48 months

*** the period $N$ should be consistent with the interest rate (interest per month → period in months)

$P$ = ?

$$P = 300\left[\frac{(1+0.01)^{48} - 1}{0.01\,(1+0.01)^{48}}\right] = \$11{,}392.2$$

Or from Appendix C tables, $(P/A\,, 1\%, 48)$ = 37.9740 → $F$ = \$300 × 37.9740 = \$11,392.2

# Finding $\boldsymbol{A}$ given $\boldsymbol{P}$ or $\boldsymbol{F}$

- A given F

$$A = F\left[\frac{i}{(1+i)^N - 1}\right]$$

Or

$$A = F\,(A/F\,, i\%, N)\ from\ tables\ in\ Appendix\ C$$

- A given P

$$A = P\left[\frac{i\,(1+i)^N}{(1+i)^N - 1}\right]$$

Or

$$A = P\,(A/P\,, i\%, N)\ from\ tables\ in\ Appendix\ C$$

# Example

Calculate the compounded future value of 20 annual payments of \$5,000 each into a savings account that earns 6% per year. All 20 payments are made at the beginning of each year.

- Definition of annuity: occurs at the end of each compounding period.
- In the example, payments are made at the beginning of each period.

$$F = \$5{,}000\ (F/A\ , 6\%, 20)\ (F/P\ , 6\%, 1)$$

$$= \$5{,}000 \times 38.7856 \times 1.06 = \$194{,}963.68$$

Payments start at the beginning of each year, so the first annuity is at time 0. Hence, the present equivalent is at year -1 and the future equivalent is at year 19. We first use the (F/A) relationship to determine the future equivalent at year 19 and then we determine the future equivalent at year 20 using the (F/P) relationship.

Another way to solve:

$$F = \$5{,}000\ (F/P\ , 6\%, 20) + \$5000\ (F/A\ , 6\%, 19)(F/P\ , 6\%, 1)$$
$$F = \$5{,}000 \times 3.2071 + \$5000 \times 33.7600 \times 1.06$$

# Example

A loan of \$10,000 is to be repaid in 4 equal payments (over 4 years) and the interest rate is 10% per year. Determine the interest paid and principal repayment every year.

- Find the annual payment (annuity)

$$A = P\ (A/P\ , 10\%, 4) = \$10{,}000 \times 0.3155 = \$3{,}155 \text{ per year.}$$

- Fill out a table

| Year | Amount owed at beginning of period | Interest | Annual payment | Principal repayment |
|---|---|---|---|---|
| 1 | \$10,000 | \$1,000 | \$3,155 | \$2,155 |
| 2 | \$7,845 | \$785 | \$3,155 | \$2,371 |
| 3 | \$5,475 | \$547 | \$3,155 | \$2,608 |
| 4 | \$2,867 | \$287 | \$3,155 | \$2,868 |

Loan amount

= amount owed at beginning of previous year – principal repayment in previous year

= amount owed at beginning of period × interest rate

= annual payment – interest

# Solving for $\boldsymbol{N}$

- You borrowed \$100,000 at an interest rate of 7% per year. If the annual payment is \$8,000, how many years does it take to repay the loan?

$$\$100{,}000 = \$8{,}000\ (P/A\,, 7\%, N)$$
$$12.5 = \frac{1.07^N - 1}{0.07\ (1.07)^N} \Rightarrow 0.125\ (1.07)^N = 1$$

$$N = 30.73 \text{ years}$$

- You invested \$20,000 in a project and you are expected to gain \$4,000 annually. At a 10% interest rate, when will you recover your investment?

$$\$20{,}000 = \$4{,}000\ (P/A\,, 10\%, N)$$
$$5 = \frac{1.1^N - 1}{0.1\ (1.1)^N} \Rightarrow 0.5\ (1.1)^N = 1$$

$$N = 7.27 \text{ years}$$

# Solving for $\boldsymbol{i}$

- You wanted to start saving so that you will have \$60,000 in your bank account eight years from now. Each year, you deposit \$6,000 in your bank account. What should be the interest rate so you can achieve your goal?

$A = \$6{,}000$ $\quad$ $F = \$60{,}000$ $\quad$ $N = 8$ years

$i = ?$

$$\$60{,}000 = \$6{,}000\left[\frac{(1+i)^8 - 1}{i}\right]$$

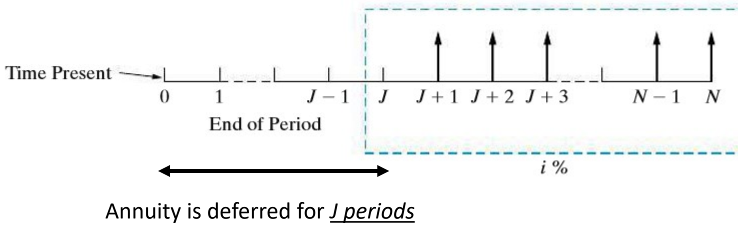
To solve:

- Trial and error.
- Interpolation.
- Calculators with solver.
- Spreadsheets (Excel function: Rate).

$$i = 6.29\%$$

# Deferred annuities

- Ordinary annuity (uniform series) appears at the end of the first period.
- Deferred annuity (also uniform series) begins at later time.

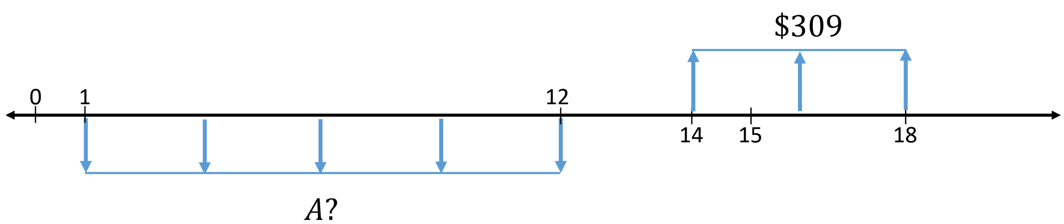



- To find the present equivalent ($P$) at time zero:

$$P = A(P/A\,, i\%, N - J)\ (P/F\,, i\%, J)$$

# Deferred annuities - example

How much money should be deposited each year for 12 years if you wish to withdraw \$309 each year for five years, beginning at the end of the 14$^{th}$ year? Assume the interest rate is 8% per year.



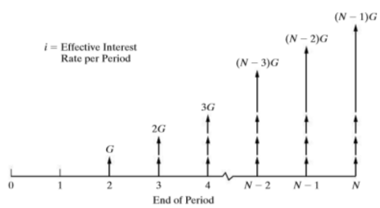

Find the present worth of both annuities and equate.

$$A\ (P/A\,, 8\%, 12) = \$309\ (P/A\,, 8\%, 5)\ (P/F\,, 8\%, 13)$$
$$A \times 7.5361 = \$309 \times 3.9927 \times 0.3677$$
$$A = \$60.2$$

# Uniform (arithmetic) gradient of cash flows

- Cash flow that changes by a constant amount (G) each period.

| End of Period | Cash Flows |
|---|---|
| 1 | (0)G |
| 2 | (1)G |
| 3 | (2)G |
| . | . |
| . | . |
| . | . |
| N − 1 | (N − 2)G |
| N | (N − 1)G |




Present equivalent

$$P = G \times \left\{\frac{1}{i}\left[\frac{(1+i)^N-1}{i(1+i)^N} - \frac{N}{(1+i)^N}\right]\right\}$$

Or $P = G \times (P/G, i\%, N)$ … tables in Appendix C

Annuity equivalent

$$A = G \times \left[\frac{1}{i} - \frac{N}{(1+i)^N-1}\right]$$

Future equivalent

$$F = \frac{G}{i} \times (F/A, i\%, N) - \frac{N \times G}{i}$$

# Examples

At a 15% interest rate, determine the present equivalent for the following cash flow.

| End of Year | Cash Flows ($) |
|---|---|
| 1 | 5,000 |
| 2 | 6,000 |
| 3 | 7,000 |
| 4 | 8,000 |

$$P = A\ (P/A, 15\%, 4) + G\ (P/G, 15\%, 4)$$

$$P = \$5{,}000 \times 2.855 + \$1{,}000 \times 3.79 = \$18{,}065$$

At a 15% interest rate, determine the present equivalent for the following cash flow.

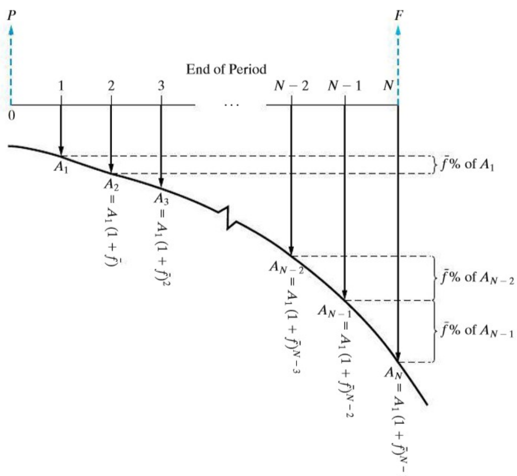
| End of Year | Cash Flows ($) |
|---|---|
| 1 | 8,000 |
| 2 | 7,000 |
| 3 | 6,000 |
| 4 | 5,000 |

$$P = A\ (P/A, 15\%, 4) - G\ (P/G, 15\%, 4)$$

$$P = \$8{,}000 \times 2.855 - \$1{,}000 \times 3.79 = \$19{,}050$$

# Geometric sequence of cash flows

- Cash flow that changes by a constant rate ($\bar{f}$) each period.
- First payment at EOY 1.




$$P = \begin{cases} \dfrac{A_1[1-(P/F,i\%,N)(F/P,\bar{f}\%,N)]}{i-\bar{f}} & \bar{f} \neq i \\ A_1 N (P/F,i\%,1) & \bar{f} = i. \end{cases}$$

Example: Assume that a payment of $1,000 is made at EOY 1 and decreases by 20% per year after the first year for 4 years. At a 25% interest rate, Determine the present equivalent.

$$\bar{f} = -20\%$$

$$P = \frac{\$1{,}000\,[1-(P/F\,,25\%,4)(F/P\,,-20\%,4)]}{0.25\ -(-0.2)}$$

$$P = \$1{,}849.38$$

**If interest rate is 20%?**

# Nominal and effective interest rates

- If compounding period is less than a year.
  - Annual rate is called nominal interest rate or annual percentage rate (APR).
  - Actual or exact rate is called effective interest rate.

- Example: if annual interest rate is 10% compounded annually, then the effective rate = nominal rate = 10%.

$$i = \left(1 + \frac{r}{M}\right)^M - 1$$

Where:

$i$: effective interest rate per year.
$r$: nominal interest rate per year.
$M$: number of compounding periods per year.

# Example

A credit card company charges 1.375% per month on the unpaid balance. They claim that the annual interest rate is (12 × 1.375% = 16.5%).

- What is the effective interest rate per month?

Since compounding is monthly, effective monthly rate = nominal monthly rate = 1.375%.

- What is the effective interest rate per year?

$r = 16.5\%$ $M = 12$ compounding periods per year

$$i = \left(1 + \frac{0.165}{12}\right)^{12} - 1 = 17.81\%$$

- Does this card provide a better deal than another card which charges 16.8% annual rate compounded bimonthly?

$M = 6$ compounding periods per year ⇒ $i$ for the other card $= \left(1 + \frac{0.168}{6}\right)^{6} - 1 = 18.02\%$

17.81% < 18.02% ⇒ the first card (16.5% per year compounded monthly) is better.

# Example

A loan of $15,000 requires monthly payments of $477 over a 36-month period.

- What is the nominal interest rate (APR)?

$P$ = $15,000 $A$ = $477 $N$ = 36 months

$$\$477 = \$15{,}000\ (A/P\,, i_{monthly}\%, 36)$$

By trial and error (or using solver) ⇒ $i$ = 0.75% per month

Nominal rate ($r$) = 0.75% × 12 = 9% per year.

- What is the effective interest rate per year?

$$i = \left(1 + \frac{0.09}{12}\right)^{12} - 1 = 9.38\% \text{ per year}$$

- What is the amount of unpaid loan principal after 20 months?

$$P_{20} = \$477\ (P/A\,, 0.75\%, 16) = \$7{,}166.59$$

# Examples

- A loan of \$2,000 at 10% annual interest rate for 8 years is to be repaid in two payments, @ EOY 4 and EOY 8. What is the value of the payments?

Consider every 4 years as one payment.

$$\Rightarrow r = 40\% \text{ per 4 years compounded annually.}$$

$$i = \left(1 + \frac{0.4}{4}\right)^4 - 1 = 46.41\% \text{ per 4 years}$$

Using A/P relationship:

$$A = \$2{,}000 \times \frac{(0.4641 \times 1.4641^2)}{1.4641^2 - 1} = \$1{,}739.9 \text{ every 4 years}$$

- If the monthly interest rate is 1%, what is the effective semi-annual rate?

⇒ Monthly rate = 1% = effective monthly = nominal monthly (no additional info on compounding is provided).

Nominal semi-annual rate = 6 × 1% = 6%

Effective semi-annual rate $i = \left(1 + \frac{0.06}{6}\right)^6 - 1 = 6.15\%$

# Continuous compounding

- Allowing interest to compound continuously throughout the period ⇒ $M$ approaches ∞.

$$i = e^r - 1$$

Where $i$ is the effective rate and $r$ is the nominal rate.

Continuous compounding factors

$$(P/F\,, r\%, N) = e^{-rN}$$

$$(F/A\,, r\%, N) = \frac{e^{rN} - 1}{e^r - 1}$$

$$(P/A\,, r\%, N) = \frac{e^{rN} - 1}{e^{rN}(e^r - 1)}$$

Notice: $r$ is substituted (not $i$) So we can use these formulas or we can substitute the effective interest rate ($i$) in *P/F. F/A, and P/A* equations presented earlier in the chapter

# Example

- A bank offers loans at an annual interest rate of 12% compounded continuously,
  - What is the effective annual interest rate?

$$r = 0.12 \text{ (nominal annual)}$$

$$i = e^{0.12} - 1 = 0.1275 = 12.75\%$$

  - What is the effective monthly interest rate?

$$r = \frac{0.12}{12} = 0.01 \text{ (nominal monthly)}$$

$$i = e^{0.01} - 1 = 0.01005 = 1.005\%$$

  - If you borrowed \$10,000 on these terms, what is the future equivalent of this loan after 5 years?

$(F/P, r\%, N) = e^{rN} = e^{0.12*5} = 1.8221 \Rightarrow F = \$10{,}000 \times 1.8221 = \$18{,}221$

Or $F = P \times (1+i)^N = \$10{,}000 \times (1 + 0.1275)^5 = \$18{,}221$

Or using the monthly interest: $F = \$10{,}000 \times (1 + 0.01005)^{60} = \$18{,}221$

# Example

- A nominal interest rate of 8% is compounded continuously.
  - What is the uniform EOY amount for 10 years that is equivalent to \$8,000 at EOY 10?

$F$= \$8,000 $A$ = ?

$$A = \$8{,}000\ (A/F, 8\%\ nominal, 10) = \$8{,}000 \times \frac{e^{0.08} - 1}{e^{0.08\times 10} - 1} = \$543.68$$

  - What is the present equivalent value of \$1,000 per year for 12 years?

$A$= \$1,000 $P$ = ?

$$P = \$1{,}000\ (P/A, 8\%\ nominal, 12) = \$1{,}000 \times \frac{e^{0.08\times 12} - 1}{e^{0.08\times 12}(e^{0.08} - 1)} = \$7{,}409.4$$

  - What is the future equivalent at the end of the 6th year of \$243 payments made every 6 months during the 6 years (first payment occurs 6 months from the present and the last occurs at EOY 6)?

$A$= \$1,000 $F$ = ?

$$F = \$243\ (F/A, 4\%, 12) = \$243 \times \frac{e^{0.04\times 12} - 1}{e^{0.04} - 1} = \$3{,}668.3$$

# APPENDIX C

# Interest and Annuity Tables for Discrete Compounding

For various values of $i$ from 1/4% to 25%,

$i$ = effective interest rate per period (usually one year);
$N$ = number of compounding periods;

$$(F/P, i\%, N) = (1+i)^N; \qquad (A/F, i\%, N) = \frac{i}{(1+i)^N - 1};$$

$$(P/F, i\%, N) = \frac{1}{(1+i)^N}; \qquad (A/P, i\%, N) = \frac{i(1+i)^N}{(1+i)^N - 1};$$

$$(F/A, i\%, N) = \frac{(1+i)^N - 1}{i}; \qquad (P/G, i\%, N) = \frac{1}{i}\left[\frac{(1+i)^N - 1}{i(1+i)^N} - \frac{N}{(1+i)^N}\right];$$

$$(P/A, i\%, N) = \frac{(1+i)^N - 1}{i(1+i)^N}; \qquad (A/G, i\%, N) = \frac{1}{i} - \frac{N}{(1+i)^N - 1}.$$

**TABLE C-1 Discrete Compounding; $i = 1/4\%$**

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| *N* | To Find *F* Given *P* *F/P* | To Find *P* Given *F* *P/F* | To Find *F* Given *A* *F/A* | To Find *P* Given *A* *P/A* | To Find *A* Given *F* *A/F* | To Find *A* Given *P* *A/P* | To Find *P* Given *G* *P/G* | To Find *A* Given *G* *A/G* | *N* |
| 1 | 1.0025 | 0.9975 | 1.0000 | 0.9975 | 1.0000 | 1.0025 | 0.000 | 0.0000 | 1 |
| 2 | 1.0050 | 0.9950 | 2.0025 | 1.9925 | 0.4994 | 0.5019 | 0.995 | 0.4994 | 2 |
| 3 | 1.0075 | 0.9925 | 3.0075 | 2.9851 | 0.3325 | 0.3350 | 2.980 | 0.9983 | 3 |
| 4 | 1.0100 | 0.9901 | 4.0150 | 3.9751 | 0.2491 | 0.2516 | 5.950 | 1.4969 | 4 |
| 5 | 1.0126 | 0.9876 | 5.0251 | 4.9627 | 0.1990 | 0.2015 | 9.901 | 1.9950 | 5 |
| 6 | 1.0151 | 0.9851 | 6.0376 | 5.9478 | 0.1656 | 0.1681 | 14.826 | 2.4927 | 6 |
| 7 | 1.0176 | 0.9827 | 7.0527 | 6.9305 | 0.1418 | 0.1443 | 20.722 | 2.9900 | 7 |
| 8 | 1.0202 | 0.9802 | 8.0704 | 7.9107 | 0.1239 | 0.1264 | 27.584 | 3.4869 | 8 |
| 9 | 1.0227 | 0.9778 | 9.0905 | 8.8885 | 0.1100 | 0.1125 | 35.406 | 3.9834 | 9 |
| 10 | 1.0253 | 0.9753 | 10.1133 | 9.8639 | 0.0989 | 0.1014 | 44.184 | 4.4794 | 10 |
| 11 | 1.0278 | 0.9729 | 11.1385 | 10.8368 | 0.0898 | 0.0923 | 53.913 | 4.9750 | 11 |
| 12 | 1.0304 | 0.9705 | 12.1664 | 11.8073 | 0.0822 | 0.0847 | 64.589 | 5.4702 | 12 |
| 13 | 1.0330 | 0.9681 | 13.1968 | 12.7753 | 0.0758 | 0.0783 | 76.205 | 5.9650 | 13 |
| 14 | 1.0356 | 0.9656 | 14.2298 | 13.7410 | 0.0703 | 0.0728 | 88.759 | 6.4594 | 14 |
| 15 | 1.0382 | 0.9632 | 15.2654 | 14.7042 | 0.0655 | 0.0680 | 102.244 | 6.9534 | 15 |
| 16 | 1.0408 | 0.9608 | 16.3035 | 15.6650 | 0.0613 | 0.0638 | 116.657 | 7.4469 | 16 |
| 17 | 1.0434 | 0.9584 | 17.3443 | 16.6235 | 0.0577 | 0.0602 | 131.992 | 7.9401 | 17 |
| 18 | 1.0460 | 0.9561 | 18.3876 | 17.5795 | 0.0544 | 0.0569 | 148.245 | 8.4328 | 18 |
| 19 | 1.0486 | 0.9537 | 19.4336 | 18.5332 | 0.0515 | 0.0540 | 165.411 | 8.9251 | 19 |
| 20 | 1.0512 | 0.9513 | 20.4822 | 19.4845 | 0.0488 | 0.0513 | 183.485 | 9.4170 | 20 |
| 21 | 1.0538 | 0.9489 | 21.5334 | 20.4334 | 0.0464 | 0.0489 | 202.463 | 9.9085 | 21 |
| 22 | 1.0565 | 0.9466 | 22.5872 | 21.3800 | 0.0443 | 0.0468 | 222.341 | 10.3995 | 22 |
| 23 | 1.0591 | 0.9442 | 23.6437 | 22.3241 | 0.0423 | 0.0448 | 243.113 | 10.8901 | 23 |
| 24 | 1.0618 | 0.9418 | 24.7028 | 23.2660 | 0.0405 | 0.0430 | 264.775 | 11.3804 | 24 |
| 25 | 1.0644 | 0.9395 | 25.7646 | 24.2055 | 0.0388 | 0.0413 | 287.323 | 11.8702 | 25 |
| 30 | 1.0778 | 0.9278 | 31.1133 | 28.8679 | 0.0321 | 0.0346 | 413.185 | 14.3130 | 30 |
| 36 | 1.0941 | 0.9140 | 37.6206 | 34.3865 | 0.0266 | 0.0291 | 592.499 | 17.2306 | 36 |
| 40 | 1.1050 | 0.9050 | 42.0132 | 38.0199 | 0.0238 | 0.0263 | 728.740 | 19.1673 | 40 |
| 48 | 1.1273 | 0.8871 | 50.9312 | 45.1787 | 0.0196 | 0.0221 | 1040.055 | 23.0209 | 48 |
| 60 | 1.1616 | 0.8609 | 64.6467 | 55.6524 | 0.0155 | 0.0180 | 1600.085 | 28.7514 | 60 |
| 72 | 1.1969 | 0.8355 | 78.7794 | 65.8169 | 0.0127 | 0.0152 | 2265.557 | 34.4221 | 72 |
| 84 | 1.2334 | 0.8108 | 93.3419 | 75.6813 | 0.0107 | 0.0132 | 3029.759 | 40.0331 | 84 |
| 100 | 1.2836 | 0.7790 | 113.4500 | 88.3825 | 0.0088 | 0.0113 | 4191.242 | 47.4216 | 100 |
| ∞ | | | | 400.0000 | | 0.0025 | | | ∞ |

**TABLE C-2 Discrete Compounding; $i = 1/2\%$**

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| $N$ | To Find $F$ Given $P$ $F/P$ | To Find $P$ Given $F$ $P/F$ | To Find $F$ Given $A$ $F/A$ | To Find $P$ Given $A$ $P/A$ | To Find $A$ Given $F$ $A/F$ | To Find $A$ Given $P$ $A/P$ | To Find $P$ Given $G$ $P/G$ | To Find $A$ Given $G$ $A/G$ | $N$ |
| 1 | 1.0050 | 0.9950 | 1.0000 | 0.9950 | 1.0000 | 1.0050 | 0.000 | 0.0000 | 1 |
| 2 | 1.0100 | 0.9901 | 2.0050 | 1.9851 | 0.4988 | 0.5038 | 0.990 | 0.4988 | 2 |
| 3 | 1.0151 | 0.9851 | 3.0150 | 2.9702 | 0.3317 | 0.3367 | 2.960 | 0.9967 | 3 |
| 4 | 1.0202 | 0.9802 | 4.0301 | 3.9505 | 0.2481 | 0.2531 | 5.901 | 1.4938 | 4 |
| 5 | 1.0253 | 0.9754 | 5.0503 | 4.9259 | 0.1980 | 0.2030 | 9.803 | 1.9900 | 5 |
| 6 | 1.0304 | 0.9705 | 6.0755 | 5.8964 | 0.1646 | 0.1696 | 14.655 | 2.4855 | 6 |
| 7 | 1.0355 | 0.9657 | 7.1059 | 6.8621 | 0.1407 | 0.1457 | 20.449 | 2.9801 | 7 |
| 8 | 1.0407 | 0.9609 | 8.1414 | 7.8230 | 0.1228 | 0.1278 | 27.176 | 3.4738 | 8 |
| 9 | 1.0459 | 0.9561 | 9.1821 | 8.7791 | 0.1089 | 0.1139 | 34.824 | 3.9668 | 9 |
| 10 | 1.0511 | 0.9513 | 10.2280 | 9.7304 | 0.0978 | 0.1028 | 43.387 | 4.4589 | 10 |
| 11 | 1.0564 | 0.9466 | 11.2792 | 10.6770 | 0.0887 | 0.0937 | 52.853 | 4.9501 | 11 |
| 12 | 1.0617 | 0.9419 | 12.3356 | 11.6189 | 0.0811 | 0.0861 | 63.214 | 5.4406 | 12 |
| 13 | 1.0670 | 0.9372 | 13.3972 | 12.5562 | 0.0746 | 0.0796 | 74.460 | 5.9302 | 13 |
| 14 | 1.0723 | 0.9326 | 14.4642 | 13.4887 | 0.0691 | 0.0741 | 86.584 | 6.4190 | 14 |
| 15 | 1.0777 | 0.9279 | 15.5365 | 14.4166 | 0.0644 | 0.0694 | 99.574 | 6.9069 | 15 |
| 16 | 1.0831 | 0.9233 | 16.6142 | 15.3399 | 0.0602 | 0.0652 | 113.424 | 7.3940 | 16 |
| 17 | 1.0885 | 0.9187 | 17.6973 | 16.2586 | 0.0565 | 0.0615 | 128.123 | 7.8803 | 17 |
| 18 | 1.0939 | 0.9141 | 18.7858 | 17.1728 | 0.0532 | 0.0582 | 143.663 | 8.3658 | 18 |
| 19 | 1.0994 | 0.9096 | 19.8797 | 18.0824 | 0.0503 | 0.0553 | 160.036 | 8.8504 | 19 |
| 20 | 1.1049 | 0.9051 | 20.9791 | 18.9874 | 0.0477 | 0.0527 | 177.232 | 9.3342 | 20 |
| 21 | 1.1104 | 0.9006 | 22.0840 | 19.8880 | 0.0453 | 0.0503 | 195.243 | 9.8172 | 21 |
| 22 | 1.1160 | 0.8961 | 23.1944 | 20.7841 | 0.0431 | 0.0481 | 214.061 | 10.2993 | 22 |
| 23 | 1.1216 | 0.8916 | 24.3104 | 21.6757 | 0.0411 | 0.0461 | 233.677 | 10.7806 | 23 |
| 24 | 1.1272 | 0.8872 | 25.4320 | 22.5629 | 0.0393 | 0.0443 | 254.082 | 11.2611 | 24 |
| 25 | 1.1328 | 0.8828 | 26.5591 | 23.4456 | 0.0377 | 0.0427 | 275.269 | 11.7407 | 25 |
| 30 | 1.1614 | 0.8610 | 32.2800 | 27.7941 | 0.0310 | 0.0360 | 392.632 | 14.1265 | 30 |
| 36 | 1.1967 | 0.8356 | 39.3361 | 32.8710 | 0.0254 | 0.0304 | 557.560 | 16.9621 | 36 |
| 40 | 1.2208 | 0.8191 | 44.1588 | 36.1722 | 0.0226 | 0.0276 | 681.335 | 18.8359 | 40 |
| 48 | 1.2705 | 0.7871 | 54.0978 | 42.5803 | 0.0185 | 0.0235 | 959.919 | 22.5437 | 48 |
| 60 | 1.3489 | 0.7414 | 69.7700 | 51.7256 | 0.0143 | 0.0193 | 1448.646 | 28.0064 | 60 |
| 72 | 1.4320 | 0.6983 | 86.4089 | 60.3395 | 0.0116 | 0.0166 | 2012.348 | 33.3504 | 72 |
| 84 | 1.5204 | 0.6577 | 104.0739 | 68.4530 | 0.0096 | 0.0146 | 2640.664 | 38.5763 | 84 |
| 100 | 1.6467 | 0.6073 | 129.3337 | 78.5426 | 0.0077 | 0.0127 | 3562.793 | 45.3613 | 100 |
| ∞ | | | | 200.0000 | | 0.0050 | | | ∞ |

## TABLE C-3 Discrete Compounding; $i = 3/4\%$

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| $N$ | To Find $F$ Given $P$ $F/P$ | To Find $P$ Given $F$ $P/F$ | To Find $F$ Given $A$ $F/A$ | To Find $P$ Given $A$ $P/A$ | To Find $A$ Given $F$ $A/F$ | To Find $A$ Given $P$ $A/P$ | To Find $P$ Given $G$ $P/G$ | To Find $A$ Given $G$ $A/G$ | $N$ |
| 1 | 1.0075 | 0.9926 | 1.0000 | 0.9926 | 1.0000 | 1.0075 | 0.000 | 0.0000 | 1 |
| 2 | 1.0151 | 0.9852 | 2.0075 | 1.9777 | 0.4981 | 0.5056 | 0.985 | 0.4981 | 2 |
| 3 | 1.0227 | 0.9778 | 3.0226 | 2.9556 | 0.3308 | 0.3383 | 2.941 | 0.9950 | 3 |
| 4 | 1.0303 | 0.9706 | 4.0452 | 3.9261 | 0.2472 | 0.2547 | 5.853 | 1.4907 | 4 |
| 5 | 1.0381 | 0.9633 | 5.0756 | 4.8894 | 0.1970 | 0.2045 | 9.706 | 1.9851 | 5 |
| 6 | 1.0459 | 0.9562 | 6.1136 | 5.8456 | 0.1636 | 0.1711 | 14.487 | 2.4782 | 6 |
| 7 | 1.0537 | 0.9490 | 7.1595 | 6.7946 | 0.1397 | 0.1472 | 20.181 | 2.9701 | 7 |
| 8 | 1.0616 | 0.9420 | 8.2132 | 7.7366 | 0.1218 | 0.1293 | 26.775 | 3.4608 | 8 |
| 9 | 1.0696 | 0.9350 | 9.2748 | 8.6716 | 0.1078 | 0.1153 | 34.254 | 3.9502 | 9 |
| 10 | 1.0776 | 0.9280 | 10.3443 | 9.5996 | 0.0967 | 0.1042 | 42.606 | 4.4384 | 10 |
| 11 | 1.0857 | 0.9211 | 11.4219 | 10.5207 | 0.0876 | 0.0951 | 51.817 | 4.9253 | 11 |
| 12 | 1.0938 | 0.9142 | 12.5076 | 11.4349 | 0.0800 | 0.0875 | 61.874 | 5.4110 | 12 |
| 13 | 1.1020 | 0.9074 | 13.6014 | 12.3423 | 0.0735 | 0.0810 | 72.763 | 5.8954 | 13 |
| 14 | 1.1103 | 0.9007 | 14.7034 | 13.2430 | 0.0680 | 0.0755 | 84.472 | 6.3786 | 14 |
| 15 | 1.1186 | 0.8940 | 15.8137 | 14.1370 | 0.0632 | 0.0707 | 96.988 | 6.8606 | 15 |
| 16 | 1.1270 | 0.8873 | 16.9323 | 15.0243 | 0.0591 | 0.0666 | 110.297 | 7.3413 | 16 |
| 17 | 1.1354 | 0.8807 | 18.0593 | 15.9050 | 0.0554 | 0.0629 | 124.389 | 7.8207 | 17 |
| 18 | 1.1440 | 0.8742 | 19.1947 | 16.7792 | 0.0521 | 0.0596 | 139.249 | 8.2989 | 18 |
| 19 | 1.1525 | 0.8676 | 20.3387 | 17.6468 | 0.0492 | 0.0567 | 154.867 | 8.7759 | 19 |
| 20 | 1.1612 | 0.8612 | 21.4912 | 18.5080 | 0.0465 | 0.0540 | 171.230 | 9.2516 | 20 |
| 21 | 1.1699 | 0.8548 | 22.6524 | 19.3628 | 0.0441 | 0.0516 | 188.325 | 9.7261 | 21 |
| 22 | 1.1787 | 0.8484 | 23.8223 | 20.2112 | 0.0420 | 0.0495 | 206.142 | 10.1994 | 22 |
| 23 | 1.1875 | 0.8421 | 25.0010 | 21.0533 | 0.0400 | 0.0475 | 224.668 | 10.6714 | 23 |
| 24 | 1.1964 | 0.8358 | 26.1885 | 21.8891 | 0.0382 | 0.0457 | 243.892 | 11.1422 | 24 |
| 25 | 1.2054 | 0.8296 | 27.3849 | 22.7188 | 0.0365 | 0.0440 | 263.803 | 11.6117 | 25 |
| 30 | 1.2513 | 0.7992 | 33.5029 | 26.7751 | 0.0298 | 0.0373 | 373.263 | 13.9407 | 30 |
| 36 | 1.3086 | 0.7641 | 41.1527 | 34.4468 | 0.0243 | 0.0318 | 524.992 | 16.6946 | 36 |
| 40 | 1.3483 | 0.7416 | 46.4464 | 34.4469 | 0.0215 | 0.0290 | 637.469 | 18.5058 | 40 |
| 48 | 1.4314 | 0.6986 | 57.5207 | 40.1848 | 0.0174 | 0.0249 | 886.840 | 22.0691 | 48 |
| 60 | 1.5657 | 0.6387 | 75.4241 | 48.1734 | 0.0133 | 0.0208 | 1313.519 | 27.2665 | 60 |
| 72 | 1.7126 | 0.5839 | 95.0070 | 55.4768 | 0.0105 | 0.0180 | 1791.246 | 32.2882 | 72 |
| 84 | 1.8732 | 0.5338 | 116.4269 | 62.1540 | 0.0086 | 0.0161 | 2308.128 | 37.1357 | 84 |
| 100 | 2.1111 | 0.4737 | 148.1445 | 70.1746 | 0.0068 | 0.0143 | 3040.745 | 43.3311 | 100 |
| ∞ | | | | 133.3333 | | 0.0075 | | | ∞ |

**TABLE C-4 Discrete Compounding; $i = 1\%$**

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| $N$ | To Find $F$ Given $P$ $F/P$ | To Find $P$ Given $F$ $P/F$ | To Find $F$ Given $A$ $F/A$ | To Find $P$ Given $A$ $P/A$ | To Find $A$ Given $F$ $A/F$ | To Find $A$ Given $P$ $A/P$ | To Find $P$ Given $G$ $P/G$ | To Find $A$ Given $G$ $A/G$ | $N$ |
| 1 | 1.0100 | 0.9901 | 1.0000 | 0.9901 | 1.0000 | 1.0100 | 0.000 | 0.0000 | 1 |
| 2 | 1.0201 | 0.9803 | 2.0100 | 1.9704 | 0.4975 | 0.5075 | 0.980 | 0.4975 | 2 |
| 3 | 1.0303 | 0.9706 | 3.0301 | 2.9410 | 0.3300 | 0.3400 | 2.922 | 0.9934 | 3 |
| 4 | 1.0406 | 0.9610 | 4.0604 | 3.9020 | 0.2463 | 0.2563 | 5.804 | 1.4876 | 4 |
| 5 | 1.0510 | 0.9515 | 5.1010 | 4.8534 | 0.1960 | 0.2060 | 9.610 | 1.9801 | 5 |
| 6 | 1.0615 | 0.9420 | 6.1520 | 5.7955 | 0.1625 | 0.1725 | 14.321 | 2.4710 | 6 |
| 7 | 1.0721 | 0.9327 | 7.2135 | 6.7282 | 0.1386 | 0.1486 | 19.917 | 2.9602 | 7 |
| 8 | 1.0829 | 0.9235 | 8.2857 | 7.6517 | 0.1207 | 0.1307 | 26.381 | 3.4478 | 8 |
| 9 | 1.0937 | 0.9143 | 9.3685 | 8.5660 | 0.1067 | 0.1167 | 33.696 | 3.9337 | 9 |
| 10 | 1.1046 | 0.9053 | 10.4622 | 9.4713 | 0.0956 | 0.1056 | 41.844 | 4.4179 | 10 |
| 11 | 1.1157 | 0.8963 | 11.5668 | 10.3676 | 0.0865 | 0.0965 | 50.807 | 4.9005 | 11 |
| 12 | 1.1268 | 0.8874 | 12.6825 | 11.2551 | 0.0788 | 0.0888 | 60.569 | 5.3815 | 12 |
| 13 | 1.1381 | 0.8787 | 13.8093 | 12.1337 | 0.0724 | 0.0824 | 71.113 | 5.8607 | 13 |
| 14 | 1.1495 | 0.8700 | 14.9474 | 13.0037 | 0.0669 | 0.0769 | 82.422 | 6.3384 | 14 |
| 15 | 1.1610 | 0.8613 | 16.0969 | 13.8651 | 0.0621 | 0.0721 | 94.481 | 6.8143 | 15 |
| 16 | 1.1726 | 0.8528 | 17.2579 | 14.7179 | 0.0579 | 0.0679 | 107.273 | 7.2886 | 16 |
| 17 | 1.1843 | 0.8444 | 18.4304 | 15.5623 | 0.0543 | 0.0643 | 120.783 | 7.7613 | 17 |
| 18 | 1.1961 | 0.8360 | 19.6147 | 16.3983 | 0.0510 | 0.0610 | 134.996 | 8.2323 | 18 |
| 19 | 1.2081 | 0.8277 | 20.8109 | 17.2260 | 0.0481 | 0.0581 | 149.895 | 8.7017 | 19 |
| 20 | 1.2202 | 0.8195 | 22.0190 | 18.0456 | 0.0454 | 0.0554 | 165.466 | 9.1694 | 20 |
| 21 | 1.2324 | 0.8114 | 23.2392 | 18.8570 | 0.0430 | 0.0530 | 181.695 | 9.6354 | 21 |
| 22 | 1.2447 | 0.8034 | 24.4716 | 19.6604 | 0.0409 | 0.0509 | 198.566 | 10.0998 | 22 |
| 23 | 1.2572 | 0.7954 | 25.7163 | 20.4558 | 0.0389 | 0.0489 | 216.066 | 10.5626 | 23 |
| 24 | 1.2697 | 0.7876 | 26.9734 | 21.2434 | 0.0371 | 0.0471 | 234.180 | 11.0237 | 24 |
| 25 | 1.2824 | 0.7798 | 28.2432 | 22.0232 | 0.0354 | 0.0454 | 252.895 | 11.4831 | 25 |
| 30 | 1.3478 | 0.7419 | 34.7849 | 25.8077 | 0.0287 | 0.0387 | 355.002 | 13.7557 | 30 |
| 36 | 1.4308 | 0.6989 | 43.0769 | 30.1075 | 0.0232 | 0.0332 | 494.621 | 16.4285 | 36 |
| 40 | 1.4889 | 0.6717 | 48.8863 | 32.8346 | 0.0205 | 0.0305 | 596.856 | 18.1776 | 40 |
| 48 | 1.6122 | 0.6203 | 61.2226 | 37.9740 | 0.0163 | 0.0263 | 820.146 | 21.5976 | 48 |
| 60 | 1.8167 | 0.5504 | 81.6697 | 44.9550 | 0.0122 | 0.0222 | 1192.806 | 26.5333 | 60 |
| 72 | 2.0471 | 0.4885 | 104.7099 | 51.1504 | 0.0096 | 0.0196 | 1597.867 | 31.2386 | 72 |
| 84 | 2.3067 | 0.4335 | 130.6723 | 56.6485 | 0.0077 | 0.0177 | 2023.315 | 35.7170 | 84 |
| 100 | 2.7048 | 0.3697 | 170.4814 | 63.0289 | 0.0059 | 0.0159 | 2605.776 | 41.3426 | 100 |
| ∞ | | | | 100.0000 | | 0.0100 | | | ∞ |

**TABLE C-5 Discrete Compounding; $i = 2\%$**

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| *N* | To Find *F* Given *P* *F/P* | To Find *P* Given *F* *P/F* | To Find *F* Given *A* *F/A* | To Find *P* Given *A* *P/A* | To Find *A* Given *F* *A/F* | To Find *A* Given *P* *A/P* | To Find *P* Given *G* *P/G* | To Find *A* Given *G* *A/G* | *N* |
| 1 | 1.0200 | 0.9804 | 1.0000 | 0.9804 | 1.0000 | 1.0200 | 0.000 | 0.0000 | 1 |
| 2 | 1.0404 | 0.9612 | 2.0200 | 1.9416 | 0.4950 | 0.5150 | 0.961 | 0.4950 | 2 |
| 3 | 1.0612 | 0.9423 | 3.0604 | 2.8839 | 0.3268 | 0.3468 | 2.846 | 0.9868 | 3 |
| 4 | 1.0824 | 0.9238 | 4.1216 | 3.8077 | 0.2426 | 0.2626 | 5.617 | 1.4752 | 4 |
| 5 | 1.1041 | 0.9057 | 5.2040 | 4.7135 | 0.1922 | 0.2122 | 9.240 | 1.9604 | 5 |
| 6 | 1.1262 | 0.8880 | 6.3081 | 5.6014 | 0.1585 | 0.1785 | 13.680 | 2.4423 | 6 |
| 7 | 1.1487 | 0.8706 | 7.4343 | 6.4720 | 0.1345 | 0.1545 | 18.904 | 2.9208 | 7 |
| 8 | 1.1717 | 0.8535 | 8.5830 | 7.3255 | 0.1165 | 0.1365 | 24.878 | 3.3961 | 8 |
| 9 | 1.1951 | 0.8368 | 9.7546 | 8.1622 | 0.1025 | 0.1225 | 31.572 | 3.8681 | 9 |
| 10 | 1.2190 | 0.8203 | 10.9497 | 8.9826 | 0.0913 | 0.1113 | 38.955 | 4.3367 | 10 |
| 11 | 1.2434 | 0.8043 | 12.1687 | 9.7868 | 0.0822 | 0.1022 | 46.998 | 4.8021 | 11 |
| 12 | 1.2682 | 0.7885 | 13.4121 | 10.5753 | 0.0746 | 0.0946 | 55.671 | 5.2642 | 12 |
| 13 | 1.2936 | 0.7730 | 14.6803 | 11.3484 | 0.0681 | 0.0881 | 64.948 | 5.7231 | 13 |
| 14 | 1.3195 | 0.7579 | 15.9739 | 12.1062 | 0.0626 | 0.0826 | 74.800 | 6.1786 | 14 |
| 15 | 1.3459 | 0.7430 | 17.2934 | 12.8493 | 0.0578 | 0.0778 | 85.202 | 6.6309 | 15 |
| 16 | 1.3728 | 0.7284 | 18.6393 | 13.5777 | 0.0537 | 0.0737 | 96.129 | 7.0799 | 16 |
| 17 | 1.4002 | 0.7142 | 20.0121 | 14.2919 | 0.0500 | 0.0700 | 107.555 | 7.5256 | 17 |
| 18 | 1.4282 | 0.7002 | 21.4123 | 14.9920 | 0.0467 | 0.0667 | 119.458 | 7.9681 | 18 |
| 19 | 1.4568 | 0.6864 | 22.8406 | 15.6785 | 0.0438 | 0.0638 | 131.814 | 8.4073 | 19 |
| 20 | 1.4859 | 0.6730 | 24.2974 | 16.3514 | 0.0412 | 0.0612 | 144.600 | 8.8433 | 20 |
| 21 | 1.5157 | 0.6598 | 25.7833 | 17.0112 | 0.0388 | 0.0588 | 157.796 | 9.2760 | 21 |
| 22 | 1.5460 | 0.6468 | 27.2990 | 17.6580 | 0.0366 | 0.0566 | 171.380 | 9.7055 | 22 |
| 23 | 1.5769 | 0.6342 | 28.8450 | 18.2922 | 0.0347 | 0.0547 | 185.331 | 10.1317 | 23 |
| 24 | 1.6084 | 0.6217 | 30.4219 | 18.9139 | 0.0329 | 0.0529 | 199.631 | 10.5547 | 24 |
| 25 | 1.6406 | 0.6095 | 32.0303 | 19.5235 | 0.0312 | 0.0512 | 214.259 | 10.9745 | 25 |
| 30 | 1.8114 | 0.5521 | 40.5681 | 22.3965 | 0.0246 | 0.0446 | 291.716 | 13.0251 | 30 |
| 36 | 2.0399 | 0.4902 | 51.9944 | 25.4888 | 0.0192 | 0.0392 | 392.041 | 15.3809 | 36 |
| 40 | 2.2080 | 0.4529 | 60.4020 | 27.3555 | 0.0166 | 0.0366 | 461.993 | 16.8885 | 40 |
| 48 | 2.5871 | 0.3865 | 79.3535 | 30.6731 | 0.0126 | 0.0326 | 605.966 | 19.7556 | 48 |
| 60 | 3.2810 | 0.3048 | 114.0515 | 34.7609 | 0.0088 | 0.0288 | 823.698 | 23.6961 | 60 |
| 72 | 4.1611 | 0.2403 | 158.0570 | 37.9841 | 0.0063 | 0.0263 | 1034.056 | 27.2234 | 72 |
| 84 | 5.2773 | 0.1895 | 213.8666 | 40.5255 | 0.0047 | 0.0247 | 1230.419 | 30.3616 | 84 |
| 100 | 7.2446 | 0.1380 | 312.2323 | 43.0984 | 0.0032 | 0.0232 | 1464.753 | 33.9863 | 100 |
| ∞ | | | | 50.0000 | | 0.0200 | | | ∞ |

## TABLE C-6 Discrete Compounding; $i = 3\%$

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| $N$ | To Find $F$ Given $P$ $F/P$ | To Find $P$ Given $F$ $P/F$ | To Find $F$ Given $A$ $F/A$ | To Find $P$ Given $A$ $P/A$ | To Find $A$ Given $F$ $A/F$ | To Find $A$ Given $P$ $A/P$ | To Find $P$ Given $G$ $P/G$ | To Find $A$ Given $G$ $A/G$ | $N$ |
| 1 | 1.0300 | 0.9709 | 1.0000 | 0.9709 | 1.0000 | 1.0300 | 0.000 | 0.0000 | 1 |
| 2 | 1.0609 | 0.9426 | 2.0300 | 1.9135 | 0.4926 | 0.5226 | 0.943 | 0.4926 | 2 |
| 3 | 1.0927 | 0.9151 | 3.0909 | 2.8286 | 0.3235 | 0.3535 | 2.773 | 0.9803 | 3 |
| 4 | 1.1255 | 0.8885 | 4.1836 | 3.7171 | 0.2390 | 0.2690 | 5.438 | 1.4631 | 4 |
| 5 | 1.1593 | 0.8626 | 5.3091 | 4.5797 | 0.1884 | 0.2184 | 8.889 | 1.9409 | 5 |
| 6 | 1.1941 | 0.8375 | 6.4684 | 5.4172 | 0.1546 | 0.1846 | 13.076 | 2.4138 | 6 |
| 7 | 1.2299 | 0.8131 | 7.6625 | 6.2303 | 0.1305 | 0.1605 | 17.955 | 2.8819 | 7 |
| 8 | 1.2668 | 0.7894 | 8.8923 | 7.0197 | 0.1125 | 0.1425 | 23.481 | 3.3450 | 8 |
| 9 | 1.3048 | 0.7664 | 10.1591 | 7.7861 | 0.0984 | 0.1284 | 29.612 | 3.8032 | 9 |
| 10 | 1.3439 | 0.7441 | 11.4639 | 8.5302 | 0.0872 | 0.1172 | 36.309 | 4.2565 | 10 |
| 11 | 1.3842 | 0.7224 | 12.8078 | 9.2526 | 0.0781 | 0.1081 | 43.533 | 4.7049 | 11 |
| 12 | 1.4258 | 0.7014 | 14.1920 | 9.9540 | 0.0705 | 0.1005 | 51.248 | 5.1485 | 12 |
| 13 | 1.4685 | 0.6810 | 15.6178 | 10.6350 | 0.0640 | 0.0940 | 59.420 | 5.5872 | 13 |
| 14 | 1.5126 | 0.6611 | 17.0863 | 11.2961 | 0.0585 | 0.0885 | 68.014 | 6.0210 | 14 |
| 15 | 1.5580 | 0.6419 | 18.5989 | 11.9379 | 0.0538 | 0.0838 | 77.000 | 6.4500 | 15 |
| 16 | 1.6047 | 0.6232 | 20.1569 | 12.5611 | 0.0496 | 0.0796 | 86.348 | 6.8742 | 16 |
| 17 | 1.6528 | 0.6050 | 21.7616 | 13.1661 | 0.0460 | 0.0760 | 96.028 | 7.2936 | 17 |
| 18 | 1.7024 | 0.5874 | 23.4144 | 13.7535 | 0.0427 | 0.0727 | 106.014 | 7.7081 | 18 |
| 19 | 1.7535 | 0.5703 | 25.1169 | 14.3238 | 0.0398 | 0.0698 | 116.279 | 8.1179 | 19 |
| 20 | 1.8061 | 0.5537 | 26.8704 | 14.8775 | 0.0372 | 0.0672 | 126.799 | 8.5229 | 20 |
| 21 | 1.8603 | 0.5375 | 28.6765 | 15.4150 | 0.0349 | 0.0649 | 137.550 | 8.9231 | 21 |
| 22 | 1.9161 | 0.5219 | 30.5368 | 15.9369 | 0.0327 | 0.0627 | 148.509 | 7.3186 | 22 |
| 23 | 1.9736 | 0.5067 | 32.4529 | 16.4436 | 0.0308 | 0.0608 | 159.657 | 9.7093 | 23 |
| 24 | 2.0328 | 0.4919 | 34.4265 | 16.9355 | 0.0290 | 0.0590 | 170.971 | 10.0954 | 24 |
| 25 | 2.0938 | 0.4776 | 36.4593 | 17.4131 | 0.0274 | 0.0574 | 182.434 | 10.4768 | 25 |
| 30 | 2.4273 | 0.4120 | 47.5754 | 19.6004 | 0.0210 | 0.0510 | 241.361 | 12.3141 | 30 |
| 35 | 2.8139 | 0.3554 | 60.4621 | 21.4872 | 0.0165 | 0.0465 | 301.627 | 14.0375 | 35 |
| 40 | 3.2620 | 0.3066 | 75.4012 | 23.1148 | 0.0133 | 0.0433 | 361.750 | 15.6502 | 40 |
| 45 | 3.7816 | 0.2644 | 92.7199 | 24.5187 | 0.0108 | 0.0408 | 420.633 | 17.1556 | 45 |
| 50 | 4.3839 | 0.2281 | 112.7969 | 25.7298 | 0.0089 | 0.0389 | 477.480 | 18.5575 | 50 |
| 60 | 5.8916 | 0.1697 | 163.0534 | 27.6756 | 0.0061 | 0.0361 | 583.053 | 21.0674 | 60 |
| 80 | 10.6409 | 0.0940 | 321.3630 | 30.2008 | 0.0031 | 0.0331 | 756.087 | 25.0353 | 80 |
| 100 | 19.2186 | 0.0520 | 607.2877 | 31.5989 | 0.0016 | 0.0316 | 879.854 | 27.8444 | 100 |
| ∞ | | | | 33.3333 | | 0.0300 | | | ∞ |

**TABLE C-7 Discrete Compounding; $i = 4\%$**

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| *N* | To Find *F* Given *P* *F/P* | To Find *P* Given *F* *P/F* | To Find *F* Given *A* *F/A* | To Find *P* Given *A* *P/A* | To Find *A* Given *F* *A/F* | To Find *A* Given *P* *A/P* | To Find *P* Given *G* *P/G* | To Find *A* Given *G* *A/G* | *N* |
| 1 | 1.0400 | 0.9615 | 1.0000 | 0.9615 | 1.0000 | 1.0400 | 0.000 | 0.0000 | 1 |
| 2 | 1.0816 | 0.9246 | 2.0400 | 1.8861 | 0.4902 | 0.5302 | 0.925 | 0.4902 | 2 |
| 3 | 1.1249 | 0.8890 | 3.1216 | 2.7751 | 0.3203 | 0.3603 | 2.703 | 0.9739 | 3 |
| 4 | 1.1699 | 0.8548 | 4.2465 | 3.6299 | 0.2355 | 0.2755 | 5.267 | 1.4510 | 4 |
| 5 | 1.2167 | 0.8219 | 5.4163 | 4.4518 | 0.1846 | 0.2246 | 8.555 | 1.9216 | 5 |
| 6 | 1.2653 | 0.7903 | 6.6330 | 5.2421 | 0.1508 | 0.1908 | 12.506 | 2.3857 | 6 |
| 7 | 1.3159 | 0.7599 | 7.8983 | 6.0021 | 0.1266 | 0.1666 | 17.066 | 2.8433 | 7 |
| 8 | 1.3686 | 0.7307 | 9.2142 | 6.7327 | 0.1085 | 0.1485 | 22.181 | 3.2944 | 8 |
| 9 | 1.4233 | 0.7026 | 10.5828 | 7.4353 | 0.0945 | 0.1345 | 27.801 | 3.7391 | 9 |
| 10 | 1.4802 | 0.6756 | 12.0061 | 8.1109 | 0.0833 | 0.1233 | 33.881 | 4.1773 | 10 |
| 11 | 1.5395 | 0.6496 | 13.4864 | 8.7605 | 0.0741 | 0.1141 | 40.377 | 4.6090 | 11 |
| 12 | 1.6010 | 0.6246 | 15.0258 | 9.3851 | 0.0666 | 0.1066 | 47.248 | 5.0343 | 12 |
| 13 | 1.6651 | 0.6006 | 16.6268 | 9.9856 | 0.0601 | 0.1001 | 54.455 | 5.4533 | 13 |
| 14 | 1.7317 | 0.5775 | 18.2919 | 10.5631 | 0.0547 | 0.0947 | 61.962 | 5.8659 | 14 |
| 15 | 1.8009 | 0.5553 | 20.0236 | 11.1184 | 0.0499 | 0.0899 | 69.736 | 6.2721 | 15 |
| 16 | 1.8730 | 0.5339 | 21.8245 | 11.6523 | 0.0458 | 0.0858 | 77.744 | 6.6720 | 16 |
| 17 | 1.9479 | 0.5134 | 23.6975 | 12.1657 | 0.0422 | 0.0822 | 85.958 | 7.0656 | 17 |
| 18 | 2.0258 | 0.4936 | 25.6454 | 12.6593 | 0.0390 | 0.0790 | 94.350 | 7.4530 | 18 |
| 19 | 2.1068 | 0.4746 | 27.6712 | 13.1339 | 0.0361 | 0.0761 | 102.893 | 7.8342 | 19 |
| 20 | 2.1911 | 0.4564 | 29.7781 | 13.5903 | 0.0336 | 0.0736 | 111.565 | 8.2091 | 20 |
| 21 | 2.2788 | 0.4388 | 31.9692 | 14.0292 | 0.0313 | 0.0713 | 120.341 | 8.5779 | 21 |
| 22 | 2.3699 | 0.4220 | 34.2480 | 14.4511 | 0.0292 | 0.0692 | 129.202 | 8.9407 | 22 |
| 23 | 2.4647 | 0.4057 | 36.6179 | 14.8568 | 0.0273 | 0.0673 | 138.128 | 9.2973 | 23 |
| 24 | 2.5633 | 0.3901 | 39.0826 | 15.2470 | 0.0256 | 0.0656 | 147.101 | 9.6479 | 24 |
| 25 | 2.6658 | 0.3751 | 41.6459 | 15.6221 | 0.0240 | 0.0640 | 156.104 | 9.9925 | 25 |
| 30 | 3.2434 | 0.3083 | 56.0849 | 17.2920 | 0.0178 | 0.0578 | 201.062 | 11.6274 | 30 |
| 35 | 3.9461 | 0.2534 | 73.6522 | 18.6646 | 0.0136 | 0.0536 | 244.877 | 13.1198 | 35 |
| 40 | 4.8010 | 0.2083 | 95.0255 | 19.7928 | 0.0105 | 0.0505 | 286.530 | 14.4765 | 40 |
| 45 | 5.8412 | 0.1712 | 121.0294 | 20.7200 | 0.0083 | 0.0483 | 325.403 | 15.7047 | 45 |
| 50 | 7.1067 | 0.1407 | 152.6671 | 21.4822 | 0.0066 | 0.0466 | 361.164 | 16.8122 | 50 |
| 60 | 10.5196 | 0.0951 | 237.9907 | 22.6235 | 0.0042 | 0.0442 | 422.997 | 18.6972 | 60 |
| 80 | 23.0498 | 0.0434 | 551.2450 | 23.9154 | 0.0018 | 0.0418 | 511.116 | 21.3718 | 80 |
| 100 | 50.5049 | 0.0198 | 1237.6237 | 24.5050 | 0.0008 | 0.0408 | 563.125 | 22.9800 | 100 |
| ∞ | | | | 25.0000 | | 0.0400 | | | ∞ |

## TABLE C-8 Discrete Compounding; $i = 5\%$

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| *N* | To Find *F* Given *P* *F/P* | To Find *P* Given *F* *P/F* | To Find *F* Given *A* *F/A* | To Find *P* Given *A* *P/A* | To Find *A* Given *F* *A/F* | To Find *A* Given *P* *A/P* | To Find *P* Given *G* *P/G* | To Find *A* Given *G* *A/G* | *N* |
| 1 | 1.0500 | 0.9524 | 1.0000 | 0.9524 | 1.0000 | 1.0500 | 0.000 | 0.0000 | 1 |
| 2 | 1.1025 | 0.9070 | 2.0500 | 1.8594 | 0.4878 | 0.5378 | 0.907 | 0.4878 | 2 |
| 3 | 1.1576 | 0.8638 | 3.1525 | 2.7232 | 0.3172 | 0.3672 | 2.635 | 0.9675 | 3 |
| 4 | 1.2155 | 0.8227 | 4.3101 | 3.5460 | 0.2320 | 0.2820 | 5.103 | 1.4391 | 4 |
| 5 | 1.2763 | 0.7835 | 5.5256 | 4.3295 | 0.1810 | 0.2310 | 8.237 | 1.9025 | 5 |
| 6 | 1.3401 | 0.7462 | 6.8019 | 5.0757 | 0.1470 | 0.1970 | 11.968 | 2.3579 | 6 |
| 7 | 1.4071 | 0.7107 | 8.1420 | 5.7864 | 0.1228 | 0.1728 | 16.232 | 2.8052 | 7 |
| 8 | 1.4775 | 0.6768 | 9.5491 | 6.4632 | 0.1047 | 0.1547 | 20.970 | 3.2445 | 8 |
| 9 | 1.5513 | 0.6446 | 11.0266 | 7.1078 | 0.0907 | 0.1407 | 26.127 | 3.6758 | 9 |
| 10 | 1.6289 | 0.6139 | 12.5779 | 7.7217 | 0.0795 | 0.1295 | 31.652 | 4.0991 | 10 |
| 11 | 1.7103 | 0.5847 | 14.2068 | 8.3064 | 0.0704 | 0.1204 | 37.499 | 4.5144 | 11 |
| 12 | 1.7959 | 0.5568 | 15.9171 | 8.8633 | 0.0628 | 0.1128 | 43.624 | 4.9219 | 12 |
| 13 | 1.8856 | 0.5303 | 17.7130 | 9.3936 | 0.0565 | 0.1065 | 49.988 | 5.3215 | 13 |
| 14 | 1.9799 | 0.5051 | 19.5986 | 9.8986 | 0.0510 | 0.1010 | 56.554 | 5.7133 | 14 |
| 15 | 2.0789 | 0.4810 | 21.5786 | 10.3797 | 0.0463 | 0.0963 | 63.288 | 6.0973 | 15 |
| 16 | 2.1829 | 0.4581 | 23.6575 | 10.8378 | 0.0423 | 0.0923 | 70.160 | 6.4736 | 16 |
| 17 | 2.2920 | 0.4363 | 25.8404 | 11.2741 | 0.0387 | 0.0887 | 77.141 | 6.8423 | 17 |
| 18 | 2.4066 | 0.4155 | 28.1324 | 11.6896 | 0.0355 | 0.0855 | 84.204 | 7.2034 | 18 |
| 19 | 2.5270 | 0.3957 | 30.5390 | 12.0853 | 0.0327 | 0.0827 | 91.328 | 7.5569 | 19 |
| 20 | 2.6533 | 0.3769 | 33.0660 | 12.4622 | 0.0302 | 0.0802 | 98.488 | 7.9030 | 20 |
| 21 | 2.7860 | 0.3589 | 35.7193 | 12.8212 | 0.0280 | 0.0780 | 105.667 | 8.2416 | 21 |
| 22 | 2.9253 | 0.3418 | 38.5052 | 13.1630 | 0.0260 | 0.0760 | 112.846 | 8.5730 | 22 |
| 23 | 3.0715 | 0.3256 | 41.4305 | 13.4886 | 0.0241 | 0.0741 | 120.009 | 8.8971 | 23 |
| 24 | 3.2251 | 0.3101 | 44.5020 | 13.7986 | 0.0225 | 0.0725 | 127.140 | 9.2140 | 24 |
| 25 | 3.3864 | 0.2953 | 47.7271 | 14.0939 | 0.0210 | 0.0710 | 134.228 | 9.5238 | 25 |
| 30 | 4.3219 | 0.2314 | 66.4388 | 15.3725 | 0.0151 | 0.0651 | 168.623 | 10.9691 | 30 |
| 35 | 5.5160 | 0.1813 | 90.3203 | 16.3742 | 0.0111 | 0.0611 | 200.581 | 12.2498 | 35 |
| 40 | 7.0400 | 0.1420 | 120.7998 | 17.1591 | 0.0083 | 0.0583 | 229.545 | 13.3775 | 40 |
| 45 | 8.9850 | 0.1113 | 159.7002 | 17.7741 | 0.0063 | 0.0563 | 255.315 | 14.3644 | 45 |
| 50 | 11.4674 | 0.0872 | 209.3480 | 18.2559 | 0.0048 | 0.0548 | 277.915 | 15.2233 | 50 |
| 60 | 18.6792 | 0.0535 | 353.5837 | 18.9293 | 0.0028 | 0.0528 | 314.343 | 16.6062 | 60 |
| 80 | 49.5614 | 0.0202 | 971.2288 | 19.5965 | 0.0010 | 0.0510 | 359.646 | 18.3526 | 80 |
| 100 | 131.5013 | 0.0076 | 2610.0252 | 19.8479 | 0.0004 | 0.0504 | 381.749 | 19.2337 | 100 |
| ∞ | | | | 20.0000 | | 0.0500 | | | ∞ |

## TABLE C-9 Discrete Compounding; $i = 6\%$

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| *N* | To Find *F* Given *P* *F/P* | To Find *P* Given *F* *P/F* | To Find *F* Given *A* *F/A* | To Find *P* Given *A* *P/A* | To Find *A* Given *F* *A/F* | To Find *A* Given *P* *A/P* | To Find *P* Given *G* *P/G* | To Find *A* Given *G* *A/G* | *N* |
| 1 | 1.0600 | 0.9434 | 1.0000 | 0.9434 | 1.0000 | 1.0600 | 0.000 | 0.0000 | 1 |
| 2 | 1.1236 | 0.8900 | 2.0600 | 1.8334 | 0.4854 | 0.5454 | 0.890 | 0.4854 | 2 |
| 3 | 1.1910 | 0.8396 | 3.1836 | 2.6730 | 0.3141 | 0.3741 | 2.569 | 0.9612 | 3 |
| 4 | 1.2625 | 0.7921 | 4.3746 | 3.4651 | 0.2286 | 0.2886 | 4.946 | 1.4272 | 4 |
| 5 | 1.3382 | 0.7473 | 5.6371 | 4.2124 | 0.1774 | 0.2374 | 7.935 | 1.8836 | 5 |
| 6 | 1.4185 | 0.7050 | 6.9753 | 4.9173 | 0.1434 | 0.2034 | 11.459 | 2.3304 | 6 |
| 7 | 1.5036 | 0.6651 | 8.3938 | 5.5824 | 0.1191 | 0.1791 | 15.450 | 2.7676 | 7 |
| 8 | 1.5938 | 0.6274 | 9.8975 | 6.2098 | 0.1010 | 0.1610 | 19.842 | 3.1952 | 8 |
| 9 | 1.6895 | 0.5919 | 11.4913 | 6.8017 | 0.0870 | 0.1470 | 24.577 | 3.6133 | 9 |
| 10 | 1.7908 | 0.5584 | 13.1808 | 7.3601 | 0.0759 | 0.1359 | 29.602 | 4.0220 | 10 |
| 11 | 1.8983 | 0.5268 | 14.9716 | 7.8869 | 0.0668 | 0.1268 | 34.870 | 4.4213 | 11 |
| 12 | 2.0122 | 0.4970 | 16.8699 | 8.3838 | 0.0593 | 0.1193 | 40.337 | 4.8113 | 12 |
| 13 | 2.1329 | 0.4688 | 18.8821 | 8.8527 | 0.0530 | 0.1130 | 45.963 | 5.1920 | 13 |
| 14 | 2.2609 | 0.4423 | 21.0151 | 9.2950 | 0.0476 | 0.1076 | 51.713 | 5.5635 | 14 |
| 15 | 2.3966 | 0.4173 | 23.2760 | 9.7122 | 0.0430 | 0.1030 | 57.555 | 5.9260 | 15 |
| 16 | 2.5404 | 0.3936 | 25.6725 | 10.1059 | 0.0390 | 0.0990 | 63.459 | 6.2794 | 16 |
| 17 | 2.6928 | 0.3714 | 28.2129 | 10.4773 | 0.0354 | 0.0954 | 69.401 | 6.6240 | 17 |
| 18 | 2.8543 | 0.3503 | 30.9057 | 10.8276 | 0.0324 | 0.0924 | 75.357 | 6.9597 | 18 |
| 19 | 3.0256 | 0.3305 | 33.7600 | 11.1581 | 0.0296 | 0.0896 | 81.306 | 7.2867 | 19 |
| 20 | 3.2071 | 0.3118 | 36.7856 | 11.4699 | 0.0272 | 0.0872 | 87.230 | 7.6051 | 20 |
| 21 | 3.3996 | 0.2942 | 39.9927 | 11.7641 | 0.0250 | 0.0850 | 93.114 | 7.9151 | 21 |
| 22 | 3.6035 | 0.2775 | 43.3923 | 12.0416 | 0.0230 | 0.0830 | 98.941 | 8.2166 | 22 |
| 23 | 3.8197 | 0.2618 | 46.9958 | 12.3034 | 0.0213 | 0.0813 | 104.701 | 8.5099 | 23 |
| 24 | 4.0489 | 0.2470 | 50.8156 | 12.5504 | 0.0197 | 0.0797 | 110.381 | 8.7951 | 24 |
| 25 | 4.2919 | 0.2330 | 54.8645 | 12.7834 | 0.0182 | 0.0782 | 115.973 | 9.0722 | 25 |
| 30 | 5.7435 | 0.1741 | 79.0582 | 13.7648 | 0.0126 | 0.0726 | 142.359 | 10.3422 | 30 |
| 35 | 7.6861 | 0.1301 | 111.4348 | 14.4982 | 0.0090 | 0.0690 | 165.743 | 11.4319 | 35 |
| 40 | 10.2857 | 0.0972 | 154.7620 | 15.0463 | 0.0065 | 0.0665 | 185.957 | 12.3590 | 40 |
| 45 | 13.7646 | 0.0727 | 212.7435 | 15.4558 | 0.0047 | 0.0647 | 203.110 | 13.1413 | 45 |
| 50 | 18.4202 | 0.0543 | 290.3359 | 15.7619 | 0.0034 | 0.0634 | 217.457 | 13.7964 | 50 |
| 60 | 32.9877 | 0.0303 | 533.1282 | 16.1614 | 0.0019 | 0.0619 | 239.043 | 14.7909 | 60 |
| 80 | 105.7960 | 0.0095 | 1746.5999 | 16.5091 | 0.0006 | 0.0606 | 262.549 | 15.9033 | 80 |
| 100 | 339.3021 | 0.0029 | 5638.3681 | 16.6175 | 0.0002 | 0.0602 | 272.047 | 16.3711 | 100 |
| ∞ | | | | 16.6667 | | 0.0600 | | | ∞ |

## TABLE C-10 Discrete Compounding; $i = 7\%$

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| $N$ | To Find $F$ Given $P$ $F/P$ | To Find $P$ Given $F$ $P/F$ | To Find $F$ Given $A$ $F/A$ | To Find $P$ Given $A$ $P/A$ | To Find $A$ Given $F$ $A/F$ | To Find $A$ Given $P$ $A/P$ | To Find $P$ Given $G$ $P/G$ | To Find $A$ Given $G$ $A/G$ | $N$ |
| 1 | 1.0700 | 0.9346 | 1.0000 | 0.9346 | 1.0000 | 1.0700 | 0.000 | 0.0000 | 1 |
| 2 | 1.1449 | 0.8734 | 2.0700 | 1.8080 | 0.4831 | 0.5531 | 0.873 | 0.4831 | 2 |
| 3 | 1.2250 | 0.8163 | 3.2149 | 2.6243 | 0.3111 | 0.3811 | 2.506 | 0.9549 | 3 |
| 4 | 1.3108 | 0.7629 | 4.4399 | 3.3872 | 0.2252 | 0.2952 | 4.795 | 1.4155 | 4 |
| 5 | 1.4026 | 0.7130 | 5.7507 | 4.1002 | 0.1739 | 0.2439 | 7.647 | 1.8650 | 5 |
| 6 | 1.5007 | 0.6663 | 7.1533 | 4.7665 | 0.1398 | 0.2098 | 10.978 | 2.3032 | 6 |
| 7 | 1.6058 | 0.6227 | 8.6540 | 5.3893 | 0.1156 | 0.1856 | 14.715 | 2.7304 | 7 |
| 8 | 1.7182 | 0.5820 | 10.2598 | 5.9713 | 0.0975 | 0.1675 | 18.789 | 3.1465 | 8 |
| 9 | 1.8385 | 0.5439 | 11.9780 | 6.5152 | 0.0835 | 0.1535 | 23.140 | 3.5517 | 9 |
| 10 | 1.9672 | 0.5083 | 13.8164 | 7.0236 | 0.0724 | 0.1424 | 27.716 | 3.9461 | 10 |
| 11 | 2.1049 | 0.4751 | 15.7836 | 7.4987 | 0.0634 | 0.1334 | 32.467 | 4.3296 | 11 |
| 12 | 2.2522 | 0.4440 | 17.8885 | 7.9427 | 0.0559 | 0.1259 | 37.351 | 4.7025 | 12 |
| 13 | 2.4098 | 0.4150 | 20.1406 | 8.3577 | 0.0497 | 0.1197 | 42.330 | 5.0648 | 13 |
| 14 | 2.5785 | 0.3878 | 22.5505 | 8.7455 | 0.0443 | 0.1143 | 47.372 | 5.4167 | 14 |
| 15 | 2.7590 | 0.3624 | 25.1290 | 9.1079 | 0.0398 | 0.1098 | 52.446 | 5.7583 | 15 |
| 16 | 2.9522 | 0.3387 | 27.8881 | 9.4466 | 0.0359 | 0.1059 | 57.527 | 6.0897 | 16 |
| 17 | 3.1588 | 0.3166 | 30.8402 | 9.7632 | 0.0324 | 0.1024 | 62.592 | 6.4110 | 17 |
| 18 | 3.3799 | 0.2959 | 33.9990 | 10.0591 | 0.0294 | 0.0994 | 67.622 | 6.7225 | 18 |
| 19 | 3.6165 | 0.2765 | 37.3790 | 10.3356 | 0.0268 | 0.0968 | 72.599 | 7.0242 | 19 |
| 20 | 3.8697 | 0.2584 | 40.9955 | 10.5940 | 0.0244 | 0.0944 | 77.509 | 7.3163 | 20 |
| 21 | 4.1406 | 0.2415 | 44.8652 | 10.8355 | 0.0223 | 0.0923 | 82.339 | 7.5990 | 21 |
| 22 | 4.4304 | 0.2257 | 49.0057 | 11.0612 | 0.0204 | 0.0904 | 87.079 | 7.8725 | 22 |
| 23 | 4.7405 | 0.2109 | 53.4361 | 11.2722 | 0.0187 | 0.0887 | 91.720 | 8.1369 | 23 |
| 24 | 5.0724 | 0.1971 | 58.1767 | 11.4693 | 0.0172 | 0.0872 | 96.255 | 8.3923 | 24 |
| 25 | 5.4274 | 0.1842 | 63.2490 | 11.6536 | 0.0158 | 0.0858 | 100.677 | 8.6391 | 25 |
| 30 | 7.6123 | 0.1314 | 94.4608 | 12.4090 | 0.0106 | 0.0806 | 120.972 | 9.7487 | 30 |
| 35 | 10.6766 | 0.0937 | 138.2369 | 12.9477 | 0.0072 | 0.0772 | 138.135 | 10.6687 | 35 |
| 40 | 14.9745 | 0.0668 | 199.6351 | 13.3317 | 0.0050 | 0.0750 | 152.293 | 11.4233 | 40 |
| 45 | 21.0023 | 0.0476 | 285.7495 | 13.6055 | 0.0035 | 0.0735 | 163.756 | 12.0360 | 45 |
| 50 | 29.4570 | 0.0339 | 406.5289 | 13.8007 | 0.0025 | 0.0725 | 172.905 | 12.5287 | 50 |
| 60 | 57.9464 | 0.0173 | 813.5204 | 14.0392 | 0.0012 | 0.0712 | 185.768 | 13.2321 | 60 |
| 80 | 224.2344 | 0.0045 | 3189.0627 | 14.2220 | 0.0003 | 0.0703 | 198.075 | 13.9273 | 80 |
| 100 | 867.7163 | 0.0012 | 12381.6618 | 14.2693 | 0.0001 | 0.0701 | 202.200 | 14.1703 | 100 |
| ∞ | | | | 14.2857 | | 0.0700 | | | ∞ |

## TABLE C-11 Discrete Compounding; $i = 8\%$

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| $N$ | To Find $F$ Given $P$ $F/P$ | To Find $P$ Given $F$ $P/F$ | To Find $F$ Given $A$ $F/A$ | To Find $P$ Given $A$ $P/A$ | To Find $A$ Given $F$ $A/F$ | To Find $A$ Given $P$ $A/P$ | To Find $P$ Given $G$ $P/G$ | To Find $A$ Given $G$ $A/G$ | $N$ |
| 1 | 1.0800 | 0.9259 | 1.0000 | 0.9259 | 1.0000 | 1.0800 | 0.000 | 0.0000 | 1 |
| 2 | 1.1664 | 0.8573 | 2.0800 | 1.7833 | 0.4808 | 0.5608 | 0.857 | 0.4808 | 2 |
| 3 | 1.2597 | 0.7938 | 3.2464 | 2.5771 | 0.3080 | 0.3880 | 2.445 | 0.9487 | 3 |
| 4 | 1.3605 | 0.7350 | 4.5061 | 3.3121 | 0.2219 | 0.3019 | 4.650 | 1.4040 | 4 |
| 5 | 1.4693 | 0.6806 | 5.8666 | 3.9927 | 0.1705 | 0.2505 | 7.372 | 1.8465 | 5 |
| 6 | 1.5869 | 0.6302 | 7.3359 | 4.6229 | 0.1363 | 0.2163 | 10.523 | 2.2763 | 6 |
| 7 | 1.7138 | 0.5835 | 8.9228 | 5.2064 | 0.1121 | 0.1921 | 14.024 | 2.6937 | 7 |
| 8 | 1.8509 | 0.5403 | 10.6366 | 5.7466 | 0.0940 | 0.1740 | 17.806 | 3.0985 | 8 |
| 9 | 1.9990 | 0.5002 | 12.4876 | 6.2469 | 0.0801 | 0.1601 | 21.808 | 3.4910 | 9 |
| 10 | 2.1589 | 0.4632 | 14.4866 | 6.7101 | 0.0690 | 0.1490 | 25.977 | 3.8713 | 10 |
| 11 | 2.3316 | 0.4289 | 16.6455 | 7.1390 | 0.0601 | 0.1401 | 30.266 | 4.2395 | 11 |
| 12 | 2.5182 | 0.3971 | 18.9771 | 7.5361 | 0.0527 | 0.1327 | 34.634 | 4.5957 | 12 |
| 13 | 2.7196 | 0.3677 | 21.4953 | 7.9038 | 0.0465 | 0.1265 | 39.046 | 4.9402 | 13 |
| 14 | 2.9372 | 0.3405 | 24.2149 | 8.2442 | 0.0413 | 0.1213 | 43.472 | 5.2731 | 14 |
| 15 | 3.1722 | 0.3152 | 27.1521 | 8.5595 | 0.0368 | 0.1168 | 47.886 | 5.5945 | 15 |
| 16 | 3.4259 | 0.2919 | 30.3243 | 8.8514 | 0.0330 | 0.1130 | 52.264 | 5.9046 | 16 |
| 17 | 3.7000 | 0.2703 | 33.7502 | 9.1216 | 0.0296 | 0.1096 | 56.588 | 6.2037 | 17 |
| 18 | 3.9960 | 0.2502 | 37.4502 | 9.3719 | 0.0267 | 0.1067 | 60.843 | 6.4920 | 18 |
| 19 | 4.3157 | 0.2317 | 41.4463 | 9.6036 | 0.0241 | 0.1041 | 65.013 | 6.7697 | 19 |
| 20 | 4.6610 | 0.2145 | 45.7620 | 9.8181 | 0.0219 | 0.1019 | 69.090 | 7.0369 | 20 |
| 21 | 5.0338 | 0.1987 | 50.4229 | 10.0168 | 0.0198 | 0.0998 | 73.063 | 7.2940 | 21 |
| 22 | 5.4365 | 0.1839 | 55.4568 | 10.2007 | 0.0180 | 0.0980 | 76.926 | 7.5412 | 22 |
| 23 | 5.8715 | 0.1703 | 60.8933 | 10.3711 | 0.0164 | 0.0964 | 80.673 | 7.7786 | 23 |
| 24 | 6.3412 | 0.1577 | 66.7648 | 10.5288 | 0.0150 | 0.0950 | 84.300 | 8.0066 | 24 |
| 25 | 6.8485 | 0.1460 | 73.1059 | 10.6748 | 0.0137 | 0.0937 | 87.804 | 8.2254 | 25 |
| 30 | 10.0627 | 0.0994 | 113.2832 | 11.2578 | 0.0088 | 0.0888 | 103.456 | 9.1897 | 30 |
| 35 | 14.7853 | 0.0676 | 172.3168 | 11.6546 | 0.0058 | 0.0858 | 116.092 | 9.9611 | 35 |
| 40 | 21.7245 | 0.0460 | 259.0565 | 11.9246 | 0.0039 | 0.0839 | 126.042 | 10.5699 | 40 |
| 45 | 31.9204 | 0.0313 | 386.5056 | 12.1084 | 0.0026 | 0.0826 | 133.733 | 11.0447 | 45 |
| 50 | 46.9016 | 0.0213 | 573.7702 | 12.2335 | 0.0017 | 0.0817 | 139.593 | 11.4107 | 50 |
| 60 | 101.2571 | 0.0099 | 1253.2133 | 12.3766 | 0.0008 | 0.0808 | 147.300 | 11.9015 | 60 |
| 80 | 471.9548 | 0.0021 | 5886.9354 | 12.4735 | 0.0002 | 0.0802 | 153.800 | 12.3301 | 80 |
| 100 | 2199.7613 | 0.0005 | 27484.5157 | 12.4943 | [a] | 0.0800 | 155.611 | 12.4545 | 100 |
| ∞ | | | | 12.5000 | | 0.0800 | | | ∞ |

[a] Less than 0.0001.

## TABLE C-12 Discrete Compounding; $i = 9\%$

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| N | To Find F Given P F/P | To Find P Given F P/F | To Find F Given A F/A | To Find P Given A P/A | To Find A Given F A/F | To Find A Given P A/P | To Find P Given G P/G | To Find A Given G A/G | N |
| 1 | 1.0900 | 0.9174 | 1.0000 | 0.9174 | 1.0000 | 1.0900 | 0.000 | 0.0000 | 1 |
| 2 | 1.1881 | 0.8417 | 2.0900 | 1.7591 | 0.4785 | 0.5685 | 0.842 | 0.4785 | 2 |
| 3 | 1.2950 | 0.7722 | 3.2781 | 2.5313 | 0.3051 | 0.3951 | 2.386 | 0.9426 | 3 |
| 4 | 1.4116 | 0.7084 | 4.5731 | 3.2397 | 0.2187 | 0.3087 | 4.511 | 1.3925 | 4 |
| 5 | 1.5386 | 0.6499 | 5.9847 | 3.8897 | 0.1671 | 0.2571 | 7.111 | 1.8282 | 5 |
| 6 | 1.6771 | 0.5963 | 7.5233 | 4.4859 | 0.1329 | 0.2229 | 10.092 | 2.2498 | 6 |
| 7 | 1.8280 | 0.5470 | 9.2004 | 5.0330 | 0.1087 | 0.1987 | 13.375 | 2.6574 | 7 |
| 8 | 1.9926 | 0.5019 | 11.0285 | 5.5348 | 0.0907 | 0.1807 | 16.888 | 3.0512 | 8 |
| 9 | 2.1719 | 0.4604 | 13.0210 | 5.9952 | 0.0768 | 0.1668 | 20.571 | 3.4312 | 9 |
| 10 | 2.3674 | 0.4224 | 15.1929 | 6.4177 | 0.0658 | 0.1558 | 24.373 | 3.7978 | 10 |
| 11 | 2.5804 | 0.3875 | 17.5603 | 6.8052 | 0.0569 | 0.1469 | 28.248 | 4.1510 | 11 |
| 12 | 2.8127 | 0.3555 | 20.1407 | 7.1607 | 0.0497 | 0.1397 | 32.159 | 4.4910 | 12 |
| 13 | 3.0658 | 0.3262 | 22.9534 | 7.4869 | 0.0436 | 0.1336 | 36.073 | 4.8182 | 13 |
| 14 | 3.3417 | 0.2992 | 26.0192 | 7.7862 | 0.0384 | 0.1284 | 39.963 | 5.1326 | 14 |
| 15 | 3.6425 | 0.2745 | 29.3609 | 8.0607 | 0.0341 | 0.1241 | 43.807 | 5.4346 | 15 |
| 16 | 3.9703 | 0.2519 | 33.0034 | 8.3126 | 0.0303 | 0.1203 | 47.585 | 5.7245 | 16 |
| 17 | 4.3276 | 0.2311 | 36.9737 | 8.5436 | 0.0270 | 0.1170 | 51.282 | 6.0024 | 17 |
| 18 | 4.7171 | 0.2120 | 41.3013 | 8.7556 | 0.0242 | 0.1142 | 54.886 | 6.2687 | 18 |
| 19 | 5.1417 | 0.1945 | 46.0185 | 8.9501 | 0.0217 | 0.1117 | 58.387 | 6.5236 | 19 |
| 20 | 5.6044 | 0.1784 | 51.1601 | 9.1285 | 0.0195 | 0.1095 | 61.777 | 6.7674 | 20 |
| 21 | 6.1088 | 0.1637 | 56.7645 | 9.2922 | 0.0176 | 0.1076 | 65.051 | 7.0006 | 21 |
| 22 | 6.6586 | 0.1502 | 62.8733 | 9.4424 | 0.0159 | 0.1059 | 68.205 | 7.2232 | 22 |
| 23 | 7.2579 | 0.1378 | 69.5319 | 9.5802 | 0.0144 | 0.1044 | 71.236 | 7.4357 | 23 |
| 24 | 7.9111 | 0.1264 | 76.7898 | 9.7066 | 0.0130 | 0.1030 | 74.143 | 7.6384 | 24 |
| 25 | 8.6231 | 0.1160 | 84.7009 | 9.8226 | 0.0118 | 0.1018 | 76.927 | 7.8316 | 25 |
| 30 | 13.2677 | 0.0754 | 136.3075 | 10.2737 | 0.0073 | 0.0973 | 89.028 | 8.6657 | 30 |
| 35 | 20.4140 | 0.0490 | 215.7108 | 10.5668 | 0.0046 | 0.0946 | 98.359 | 9.3083 | 35 |
| 40 | 31.4094 | 0.0318 | 337.8824 | 10.7574 | 0.0030 | 0.0930 | 105.376 | 9.7957 | 40 |
| 45 | 48.3273 | 0.0207 | 525.8587 | 10.8812 | 0.0019 | 0.0919 | 110.556 | 10.1603 | 45 |
| 50 | 74.3575 | 0.0134 | 815.0836 | 10.9617 | 0.0012 | 0.0912 | 114.325 | 10.4295 | 50 |
| 60 | 176.0313 | 0.0057 | 1944.7921 | 11.0480 | 0.0005 | 0.0905 | 118.968 | 10.7683 | 60 |
| 80 | 986.5517 | 0.0010 | 10950.5741 | 11.0998 | 0.0001 | 0.0901 | 122.431 | 11.0299 | 80 |
| 100 | 5529.0408 | 0.0002 | 61422.6755 | 11.1091 | [a] | 0.0900 | 123.234 | 11.0930 | 100 |
| ∞ | | | | 11.1111 | | 0.0900 | | | ∞ |

[a] Less than 0.0001.

## TABLE C-13 Discrete Compounding; $i = 10\%$

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| N | To Find F Given P F/P | To Find P Given F P/F | To Find F Given A F/A | To Find P Given A P/A | To Find A Given F A/F | To Find A Given P A/P | To Find P Given G P/G | To Find A Given G A/G | N |
| 1 | 1.1000 | 0.9091 | 1.0000 | 0.9091 | 1.0000 | 1.1000 | 0.000 | 0.0000 | 1 |
| 2 | 1.2100 | 0.8264 | 2.1000 | 1.7355 | 0.4762 | 0.5762 | 0.826 | 0.4762 | 2 |
| 3 | 1.3310 | 0.7513 | 3.3100 | 2.4869 | 0.3021 | 0.4021 | 2.329 | 0.9366 | 3 |
| 4 | 1.4641 | 0.6830 | 4.6410 | 3.1699 | 0.2155 | 0.3155 | 4.378 | 1.3812 | 4 |
| 5 | 1.6105 | 0.6209 | 6.1051 | 3.7908 | 0.1638 | 0.2638 | 6.862 | 1.8101 | 5 |
| 6 | 1.7716 | 0.5645 | 7.7156 | 4.3553 | 0.1296 | 0.2296 | 9.684 | 2.2236 | 6 |
| 7 | 1.9487 | 0.5132 | 9.4872 | 4.8684 | 0.1054 | 0.2054 | 12.763 | 2.6216 | 7 |
| 8 | 2.1436 | 0.4665 | 11.4359 | 5.3349 | 0.0874 | 0.1874 | 16.029 | 3.0045 | 8 |
| 9 | 2.3579 | 0.4241 | 13.5795 | 5.7590 | 0.0736 | 0.1736 | 19.422 | 3.3724 | 9 |
| 10 | 2.5937 | 0.3855 | 15.9374 | 6.1446 | 0.0627 | 0.1627 | 22.891 | 3.7255 | 10 |
| 11 | 2.8531 | 0.3505 | 18.5312 | 6.4951 | 0.0540 | 0.1540 | 26.396 | 4.0641 | 11 |
| 12 | 3.1384 | 0.3186 | 21.3843 | 6.8137 | 0.0468 | 0.1468 | 29.901 | 4.3884 | 12 |
| 13 | 3.4523 | 0.2897 | 24.5227 | 7.1034 | 0.0408 | 0.1408 | 33.377 | 4.6988 | 13 |
| 14 | 3.7975 | 0.2633 | 27.9750 | 7.3667 | 0.0357 | 0.1357 | 36.801 | 4.9955 | 14 |
| 15 | 4.1772 | 0.2394 | 31.7725 | 7.6061 | 0.0315 | 0.1315 | 40.152 | 5.2789 | 15 |
| 16 | 4.5950 | 0.2176 | 35.9497 | 7.8237 | 0.0278 | 0.1278 | 43.416 | 5.5493 | 16 |
| 17 | 5.0545 | 0.1978 | 40.5447 | 8.0216 | 0.0247 | 0.1247 | 46.582 | 5.8071 | 17 |
| 18 | 5.5599 | 0.1799 | 45.5992 | 8.2014 | 0.0219 | 0.1219 | 49.640 | 6.0526 | 18 |
| 19 | 6.1159 | 0.1635 | 51.1591 | 8.3649 | 0.0195 | 0.1195 | 52.583 | 6.2861 | 19 |
| 20 | 6.7275 | 0.1486 | 57.2750 | 8.5136 | 0.0175 | 0.1175 | 55.407 | 6.5081 | 20 |
| 21 | 7.4002 | 0.1351 | 64.0025 | 8.6487 | 0.0156 | 0.1156 | 58.110 | 6.7189 | 21 |
| 22 | 8.1403 | 0.1228 | 71.4027 | 8.7715 | 0.0140 | 0.1140 | 60.689 | 6.9189 | 22 |
| 23 | 8.9543 | 0.1117 | 79.5430 | 8.8832 | 0.0126 | 0.1126 | 63.146 | 7.1085 | 23 |
| 24 | 9.8497 | 0.1015 | 88.4973 | 8.9847 | 0.0113 | 0.1113 | 65.481 | 7.2881 | 24 |
| 25 | 10.8347 | 0.0923 | 98.3471 | 9.0770 | 0.0102 | 0.1102 | 67.696 | 7.4580 | 25 |
| 30 | 17.4494 | 0.0573 | 164.4940 | 9.4269 | 0.0061 | 0.1061 | 77.077 | 8.1762 | 30 |
| 35 | 28.1024 | 0.0356 | 271.0244 | 9.6442 | 0.0037 | 0.1037 | 83.987 | 8.7086 | 35 |
| 40 | 45.2593 | 0.0221 | 442.5926 | 9.7791 | 0.0023 | 0.1023 | 88.953 | 9.0962 | 40 |
| 45 | 72.8905 | 0.0137 | 718.9048 | 9.8628 | 0.0014 | 0.1014 | 92.454 | 9.3740 | 45 |
| 50 | 117.3909 | 0.0085 | 1163.9085 | 9.9148 | 0.0009 | 0.1009 | 94.889 | 9.5704 | 50 |
| 60 | 304.4816 | 0.0033 | 3034.8164 | 9.9672 | 0.0003 | 0.1003 | 97.701 | 9.8023 | 60 |
| 80 | 2048.4002 | 0.0005 | 20474.0021 | 9.9951 | [a] | 0.1000 | 99.561 | 9.9609 | 80 |
| 100 | 13780.6123 | 0.0001 | 137796.1234 | 9.9993 | [a] | 0.1000 | 99.920 | 9.9927 | 100 |
| ∞ | | | | 10.0000 | | 0.1000 | | | ∞ |

[a] Less than 0.0001.

**TABLE C-14 Discrete Compounding; $i = 12\%$**

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| N | To Find F Given P F/P | To Find P Given F P/F | To Find F Given A F/A | To Find P Given A P/A | To Find A Given F A/F | To Find A Given P A/P | To Find P Given G P/G | To Find A Given G A/G | N |
| 1 | 1.1200 | 0.8929 | 1.0000 | 0.8929 | 1.0000 | 1.1200 | 0.000 | 0.0000 | 1 |
| 2 | 1.2544 | 0.7972 | 2.1200 | 1.6901 | 0.4717 | 0.5917 | 0.797 | 0.4717 | 2 |
| 3 | 1.4049 | 0.7118 | 3.3744 | 2.4018 | 0.2963 | 0.4163 | 2.221 | 0.9246 | 3 |
| 4 | 1.5735 | 0.6355 | 4.7793 | 3.0373 | 0.2092 | 0.3292 | 4.127 | 1.3589 | 4 |
| 5 | 1.7623 | 0.5674 | 6.3528 | 3.6048 | 0.1574 | 0.2774 | 6.397 | 1.7746 | 5 |
| 6 | 1.9738 | 0.5066 | 8.1152 | 4.1114 | 0.1232 | 0.2432 | 8.930 | 2.1720 | 6 |
| 7 | 2.2107 | 0.4523 | 10.0890 | 4.5638 | 0.0991 | 0.2191 | 11.644 | 2.5515 | 7 |
| 8 | 2.4760 | 0.4039 | 12.2997 | 4.9676 | 0.0813 | 0.2013 | 14.471 | 2.9131 | 8 |
| 9 | 2.7731 | 0.3606 | 14.7757 | 5.3282 | 0.0677 | 0.1877 | 17.356 | 3.2574 | 9 |
| 10 | 3.1058 | 0.3220 | 17.5487 | 5.6502 | 0.0570 | 0.1770 | 20.254 | 3.5847 | 10 |
| 11 | 3.4785 | 0.2875 | 20.6546 | 5.9377 | 0.0484 | 0.1684 | 23.129 | 3.8953 | 11 |
| 12 | 3.8960 | 0.2567 | 24.1331 | 6.1944 | 0.0414 | 0.1614 | 25.952 | 4.1897 | 12 |
| 13 | 4.3635 | 0.2292 | 28.0291 | 6.4235 | 0.0357 | 0.1557 | 28.702 | 4.4683 | 13 |
| 14 | 4.8871 | 0.2046 | 32.3926 | 6.6282 | 0.0309 | 0.1509 | 31.362 | 4.7317 | 14 |
| 15 | 5.4736 | 0.1827 | 37.2797 | 6.8109 | 0.0268 | 0.1468 | 33.920 | 4.9803 | 15 |
| 16 | 6.1304 | 0.1631 | 42.7533 | 6.9740 | 0.0234 | 0.1434 | 36.367 | 5.2147 | 16 |
| 17 | 6.8660 | 0.1456 | 48.8837 | 7.1196 | 0.0205 | 0.1405 | 38.697 | 5.4353 | 17 |
| 18 | 7.6900 | 0.1300 | 55.7497 | 7.2497 | 0.0179 | 0.1379 | 40.908 | 5.6427 | 18 |
| 19 | 8.6128 | 0.1161 | 63.4397 | 7.3658 | 0.0158 | 0.1358 | 42.998 | 5.8375 | 19 |
| 20 | 9.6463 | 0.1037 | 72.0524 | 7.4694 | 0.0139 | 0.1339 | 44.968 | 6.0202 | 20 |
| 21 | 10.8038 | 0.0926 | 81.6987 | 7.5620 | 0.0122 | 0.1322 | 46.819 | 6.1913 | 21 |
| 22 | 12.1003 | 0.0826 | 92.5026 | 7.6446 | 0.0108 | 0.1308 | 48.554 | 6.3514 | 22 |
| 23 | 13.5523 | 0.0738 | 104.6029 | 7.7184 | 0.0096 | 0.1296 | 50.178 | 6.5010 | 23 |
| 24 | 15.1786 | 0.0659 | 118.1552 | 7.7843 | 0.0085 | 0.1285 | 51.693 | 6.6406 | 24 |
| 25 | 17.0001 | 0.0588 | 133.3339 | 7.8431 | 0.0075 | 0.1275 | 53.105 | 6.7708 | 25 |
| 30 | 29.9599 | 0.0334 | 241.3327 | 8.0552 | 0.0041 | 0.1241 | 58.782 | 7.2974 | 30 |
| 35 | 52.7996 | 0.0189 | 431.6635 | 8.1755 | 0.0023 | 0.1223 | 62.605 | 7.6577 | 35 |
| 40 | 93.0510 | 0.0107 | 767.0914 | 8.2438 | 0.0013 | 0.1213 | 65.116 | 7.8988 | 40 |
| 45 | 163.9876 | 0.0061 | 1358.2300 | 8.2825 | 0.0007 | 0.1207 | 66.734 | 8.0572 | 45 |
| 50 | 289.0022 | 0.0035 | 2400.0182 | 8.3045 | 0.0004 | 0.1204 | 67.762 | 8.1597 | 50 |
| 60 | 897.5969 | 0.0011 | 7471.6411 | 8.3240 | 0.0001 | 0.1201 | 68.810 | 8.2664 | 60 |
| 80 | 8658.4831 | 0.0001 | 72145.6925 | 8.3324 | [a] | 0.1200 | 69.359 | 8.3241 | 80 |
| 100 | 83522.2657 | [a] | 696010.5477 | 8.3332 | [a] | 0.1200 | 69.434 | 8.3321 | 100 |
| ∞ | | | | 8.3333 | | 0.1200 | | | ∞ |

[a] Less than 0.0001.

**TABLE C-15 Discrete Compounding; $i = 15\%$**

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| *N* | To Find *F* Given *P* *F/P* | To Find *P* Given *F* *P/F* | To Find *F* Given *A* *F/A* | To Find *P* Given *A* *P/A* | To Find *A* Given *F* *A/F* | To Find *A* Given *P* *A/P* | To Find *P* Given *G* *P/G* | To Find *A* Given *G* *A/G* | *N* |
| 1 | 1.1500 | 0.8696 | 1.0000 | 0.8696 | 1.0000 | 1.1500 | 0.000 | 0.0000 | 1 |
| 2 | 1.3225 | 0.7561 | 2.1500 | 1.6257 | 0.4651 | 0.6151 | 0.756 | 0.4651 | 2 |
| 3 | 1.5209 | 0.6575 | 3.4725 | 2.2832 | 0.2880 | 0.4380 | 2.071 | 0.9071 | 3 |
| 4 | 1.7490 | 0.5718 | 4.9934 | 2.8550 | 0.2003 | 0.3503 | 3.786 | 1.3263 | 4 |
| 5 | 2.0114 | 0.4972 | 6.7424 | 3.3522 | 0.1483 | 0.2983 | 5.775 | 1.7228 | 5 |
| 6 | 2.3131 | 0.4323 | 8.7537 | 3.7845 | 0.1142 | 0.2642 | 7.937 | 2.0972 | 6 |
| 7 | 2.6600 | 0.3759 | 11.0668 | 4.1604 | 0.0904 | 0.2404 | 10.192 | 2.4498 | 7 |
| 8 | 3.0590 | 0.3269 | 13.7268 | 4.4873 | 0.0729 | 0.2229 | 12.481 | 2.7813 | 8 |
| 9 | 3.5179 | 0.2843 | 16.7858 | 4.7716 | 0.0596 | 0.2096 | 14.755 | 3.0922 | 9 |
| 10 | 4.0456 | 0.2472 | 20.3037 | 5.0188 | 0.0493 | 0.1993 | 16.980 | 3.3832 | 10 |
| 11 | 4.6524 | 0.2149 | 24.3493 | 5.2337 | 0.0411 | 0.1911 | 19.129 | 3.6549 | 11 |
| 12 | 5.3503 | 0.1869 | 29.0017 | 5.4206 | 0.0345 | 0.1845 | 21.185 | 3.9082 | 12 |
| 13 | 6.1528 | 0.1625 | 34.3519 | 5.5831 | 0.0291 | 0.1791 | 23.135 | 4.1438 | 13 |
| 14 | 7.0757 | 0.1413 | 40.5047 | 5.7245 | 0.0247 | 0.1747 | 24.973 | 4.3624 | 14 |
| 15 | 8.1371 | 0.1229 | 47.5804 | 5.8474 | 0.0210 | 0.1710 | 26.693 | 4.5650 | 15 |
| 16 | 9.3576 | 0.1069 | 55.7175 | 5.9542 | 0.0179 | 0.1679 | 28.296 | 4.7522 | 16 |
| 17 | 10.7613 | 0.0929 | 65.0751 | 6.0472 | 0.0154 | 0.1654 | 29.783 | 4.9251 | 17 |
| 18 | 12.3755 | 0.0808 | 75.8364 | 6.1280 | 0.0132 | 0.1632 | 31.157 | 5.0843 | 18 |
| 19 | 14.2318 | 0.0703 | 88.2118 | 6.1982 | 0.0113 | 0.1613 | 32.421 | 5.2307 | 19 |
| 20 | 16.3665 | 0.0611 | 102.4436 | 6.2593 | 0.0098 | 0.1598 | 33.582 | 5.3651 | 20 |
| 21 | 18.8215 | 0.0531 | 118.8101 | 6.3125 | 0.0084 | 0.1584 | 34.645 | 5.4883 | 21 |
| 22 | 21.6447 | 0.0462 | 137.6316 | 6.3587 | 0.0073 | 0.1573 | 35.615 | 5.6010 | 22 |
| 23 | 24.8915 | 0.0402 | 159.2764 | 6.3988 | 0.0063 | 0.1563 | 36.499 | 5.7040 | 23 |
| 24 | 28.6252 | 0.0349 | 184.1678 | 6.4338 | 0.0054 | 0.1554 | 37.302 | 5.7979 | 24 |
| 25 | 32.9190 | 0.0304 | 212.7930 | 6.4641 | 0.0047 | 0.1547 | 38.031 | 5.8834 | 25 |
| 30 | 66.2118 | 0.0151 | 434.7451 | 6.5660 | 0.0023 | 0.1523 | 40.753 | 6.2066 | 30 |
| 35 | 133.1755 | 0.0075 | 881.1702 | 6.6166 | 0.0011 | 0.1511 | 42.359 | 6.4019 | 35 |
| 40 | 267.8635 | 0.0037 | 1779.0903 | 6.6418 | 0.0006 | 0.1506 | 43.283 | 6.5168 | 40 |
| 45 | 538.7693 | 0.0019 | 3585.1285 | 6.6543 | 0.0003 | 0.1503 | 43.805 | 6.5830 | 45 |
| 50 | 1083.6574 | 0.0009 | 7217.7163 | 6.6605 | 0.0001 | 0.1501 | 44.096 | 6.6205 | 50 |
| 60 | 4383.9987 | 0.0002 | 29219.9916 | 6.6651 | [a] | 0.1500 | 44.343 | 6.6530 | 60 |
| 80 | 71750.8794 | [a] | 478332.5293 | 6.6666 | [a] | 0.1500 | 44.436 | 6.6656 | 80 |
| 100 | 1174313.4507 | [a] | 7828749.6713 | 6.6667 | [a] | 0.1500 | 44.444 | 6.6666 | 100 |
| ∞ | | | | 6.6667 | | 0.1500 | | | ∞ |

[a] Less than 0.0001.

**TABLE C-16 Discrete Compounding; $i = 18\%$**

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| *N* | To Find *F* Given *P* *F/P* | To Find *P* Given *F* *P/F* | To Find *F* Given *A* *F/A* | To Find *P* Given *A* *P/A* | To Find *A* Given *F* *A/F* | To Find *A* Given *P* *A/P* | To Find *P* Given *G* *P/G* | To Find *A* Given *G* *A/G* | *N* |
| 1 | 1.1800 | 0.8475 | 1.0000 | 0.8475 | 1.0000 | 1.1800 | 0.000 | 0.0000 | 1 |
| 2 | 1.3924 | 0.7182 | 2.1800 | 1.5656 | 0.4587 | 0.6387 | 0.718 | 0.4587 | 2 |
| 3 | 1.6430 | 0.6086 | 3.5724 | 2.1743 | 0.2799 | 0.4599 | 1.935 | 0.8902 | 3 |
| 4 | 1.9388 | 0.5158 | 5.2154 | 2.6901 | 0.1917 | 0.3717 | 3.483 | 1.2947 | 4 |
| 5 | 2.2878 | 0.4371 | 7.1542 | 3.1272 | 0.1398 | 0.3198 | 5.231 | 1.6728 | 5 |
| 6 | 2.6996 | 0.3704 | 9.4420 | 3.4976 | 0.1059 | 0.2859 | 7.083 | 2.0252 | 6 |
| 7 | 3.1855 | 0.3139 | 12.1415 | 3.8115 | 0.0824 | 0.2624 | 8.967 | 2.3526 | 7 |
| 8 | 3.7589 | 0.2660 | 15.3270 | 4.0776 | 0.0652 | 0.2452 | 10.829 | 2.6558 | 8 |
| 9 | 4.4355 | 0.2255 | 19.0859 | 4.3030 | 0.0524 | 0.2324 | 12.633 | 2.9358 | 9 |
| 10 | 5.2338 | 0.1911 | 23.5213 | 4.4941 | 0.0425 | 0.2225 | 14.353 | 3.1936 | 10 |
| 11 | 6.1759 | 0.1619 | 28.7551 | 4.6560 | 0.0348 | 0.2148 | 15.972 | 3.4303 | 11 |
| 12 | 7.2876 | 0.1372 | 34.9311 | 4.7932 | 0.0286 | 0.2086 | 17.481 | 3.6470 | 12 |
| 13 | 8.5994 | 0.1163 | 42.2187 | 4.9095 | 0.0237 | 0.2037 | 18.877 | 3.8449 | 13 |
| 14 | 10.1472 | 0.0985 | 50.8180 | 5.0081 | 0.0197 | 0.1997 | 20.158 | 4.0250 | 14 |
| 15 | 11.9737 | 0.0835 | 60.9653 | 5.0916 | 0.0164 | 0.1964 | 21.327 | 4.1887 | 15 |
| 16 | 14.1290 | 0.0708 | 72.9390 | 5.1624 | 0.0137 | 0.1937 | 22.389 | 4.3369 | 16 |
| 17 | 16.6722 | 0.0600 | 87.0680 | 5.2223 | 0.0115 | 0.1915 | 23.348 | 4.4708 | 17 |
| 18 | 19.6733 | 0.0508 | 103.7403 | 5.2732 | 0.0096 | 0.1896 | 24.212 | 4.5916 | 18 |
| 19 | 23.2144 | 0.0431 | 123.4135 | 5.3162 | 0.0081 | 0.1881 | 24.988 | 4.7003 | 19 |
| 20 | 27.3930 | 0.0365 | 146.6280 | 5.3527 | 0.0068 | 0.1868 | 25.681 | 4.7978 | 20 |
| 21 | 32.3238 | 0.0309 | 174.0210 | 5.3837 | 0.0057 | 0.1857 | 26.300 | 4.8851 | 21 |
| 22 | 38.1421 | 0.0262 | 206.3448 | 5.4099 | 0.0048 | 0.1848 | 26.851 | 4.9632 | 22 |
| 23 | 45.0076 | 0.0222 | 244.4868 | 5.4321 | 0.0041 | 0.1841 | 27.339 | 5.0329 | 23 |
| 24 | 53.1090 | 0.0188 | 289.4945 | 5.4509 | 0.0035 | 0.1835 | 27.773 | 5.0950 | 24 |
| 25 | 62.6686 | 0.0160 | 342.6035 | 5.4669 | 0.0029 | 0.1829 | 28.156 | 5.1502 | 25 |
| 30 | 143.3706 | 0.0070 | 790.9480 | 5.5168 | 0.0013 | 0.1813 | 29.486 | 5.3448 | 30 |
| 35 | 327.9973 | 0.0030 | 1816.6516 | 5.5386 | 0.0006 | 0.1806 | 30.177 | 5.4485 | 35 |
| 40 | 750.3783 | 0.0013 | 4163.2130 | 5.5482 | 0.0002 | 0.1802 | 30.527 | 5.5022 | 40 |
| 45 | 1716.6839 | 0.0006 | 9531.5771 | 5.5523 | 0.0001 | 0.1801 | 30.701 | 5.5293 | 45 |
| 50 | 3927.3569 | 0.0003 | 21813.0937 | 5.5541 | [a] | 0.1800 | 30.786 | 5.5428 | 50 |
| 60 | 20555.1400 | [a] | 114189.6665 | 5.5553 | [a] | 0.1800 | 30.847 | 5.5526 | 60 |
| 80 | 563067.6604 | [a] | 3128148.1133 | 5.5555 | [a] | 0.1800 | 30.863 | 5.5554 | 80 |
| ∞ | | | | 5.5556 | | 0.1800 | | | ∞ |

[a] Less than 0.0001.

**TABLE C-17 Discrete Compounding; $i = 20\%$**

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| *N* | To Find *F* Given *P* *F*/*P* | To Find *P* Given *F* *P*/*F* | To Find *F* Given *A* *F*/*A* | To Find *P* Given *A* *P*/*A* | To Find *A* Given *F* *A*/*F* | To Find *A* Given *P* *A*/*P* | To Find *P* Given *G* *P*/*G* | To Find *A* Given *G* *A*/*G* | *N* |
| 1 | 1.2000 | 0.8333 | 1.0000 | 0.8333 | 1.0000 | 1.2000 | 0.000 | 0.0000 | 1 |
| 2 | 1.4400 | 0.6944 | 2.2000 | 1.5278 | 0.4545 | 0.6545 | 0.694 | 0.4545 | 2 |
| 3 | 1.7280 | 0.5787 | 3.6400 | 2.1065 | 0.2747 | 0.4747 | 1.852 | 0.8791 | 3 |
| 4 | 2.0736 | 0.4823 | 5.3680 | 2.5887 | 0.1863 | 0.3863 | 3.299 | 1.2742 | 4 |
| 5 | 2.4883 | 0.4019 | 7.4416 | 2.9906 | 0.1344 | 0.3344 | 4.906 | 1.6405 | 5 |
| 6 | 2.9860 | 0.3349 | 9.9299 | 3.3255 | 0.1007 | 0.3007 | 6.581 | 1.9788 | 6 |
| 7 | 3.5832 | 0.2791 | 12.9159 | 3.6046 | 0.0774 | 0.2774 | 8.255 | 2.2902 | 7 |
| 8 | 4.2998 | 0.2326 | 16.4991 | 3.8372 | 0.0606 | 0.2606 | 9.883 | 2.5756 | 8 |
| 9 | 5.1598 | 0.1938 | 20.7989 | 4.0310 | 0.0481 | 0.2481 | 11.434 | 2.8364 | 9 |
| 10 | 6.1917 | 0.1615 | 25.9587 | 4.1925 | 0.0385 | 0.2385 | 12.887 | 3.0739 | 10 |
| 11 | 7.4301 | 0.1346 | 32.1504 | 4.3271 | 0.0311 | 0.2311 | 14.233 | 3.2893 | 11 |
| 12 | 8.9161 | 0.1122 | 39.5805 | 4.4392 | 0.0253 | 0.2253 | 15.467 | 3.4841 | 12 |
| 13 | 10.6993 | 0.0935 | 48.4966 | 4.5327 | 0.0206 | 0.2206 | 16.588 | 3.6597 | 13 |
| 14 | 12.8392 | 0.0779 | 59.1959 | 4.6106 | 0.0169 | 0.2169 | 17.601 | 3.8175 | 14 |
| 15 | 15.4070 | 0.0649 | 72.0351 | 4.6755 | 0.0139 | 0.2139 | 18.510 | 3.9588 | 15 |
| 16 | 18.4884 | 0.0541 | 87.4421 | 4.7296 | 0.0114 | 0.2114 | 19.321 | 4.0851 | 16 |
| 17 | 22.1861 | 0.0451 | 105.9306 | 4.7746 | 0.0094 | 0.2094 | 20.042 | 4.1976 | 17 |
| 18 | 26.6233 | 0.0376 | 128.1167 | 4.8122 | 0.0078 | 0.2078 | 20.681 | 4.2975 | 18 |
| 19 | 31.9480 | 0.0313 | 154.7400 | 4.8435 | 0.0065 | 0.2065 | 21.244 | 4.3861 | 19 |
| 20 | 38.3376 | 0.0261 | 186.6880 | 4.8696 | 0.0054 | 0.2054 | 21.740 | 4.4643 | 20 |
| 21 | 46.0051 | 0.0217 | 225.0256 | 4.8913 | 0.0044 | 0.2044 | 22.174 | 4.5334 | 21 |
| 22 | 55.2061 | 0.0181 | 271.0307 | 4.9094 | 0.0037 | 0.2037 | 22.555 | 4.5941 | 22 |
| 23 | 66.2474 | 0.0151 | 326.2369 | 4.9245 | 0.0031 | 0.2031 | 22.887 | 4.6475 | 23 |
| 24 | 79.4968 | 0.0126 | 392.4842 | 4.9371 | 0.0025 | 0.2025 | 23.176 | 4.6943 | 24 |
| 25 | 95.3962 | 0.0105 | 471.9811 | 4.9476 | 0.0021 | 0.2021 | 23.428 | 4.7352 | 25 |
| 30 | 237.3763 | 0.0042 | 1181.8816 | 4.9789 | 0.0008 | 0.2008 | 24.263 | 4.8731 | 30 |
| 35 | 590.6682 | 0.0017 | 2948.3411 | 4.9915 | 0.0003 | 0.2003 | 24.661 | 4.9406 | 35 |
| 40 | 1469.7716 | 0.0007 | 7343.8578 | 4.9966 | 0.0001 | 0.2001 | 24.847 | 4.9728 | 40 |
| 45 | 3657.2620 | 0.0003 | 18281.3099 | 4.9986 | 0.0001 | 0.2001 | 24.932 | 4.9877 | 45 |
| 50 | 9100.4382 | 0.0001 | 45497.1908 | 4.9995 | [a] | 0.2000 | 24.970 | 4.9945 | 50 |
| 60 | 56347.5144 | [a] | 281732.5718 | 4.9999 | [a] | 0.2000 | 24.994 | 4.9989 | 60 |
| 80 | 2160228.4620 | [a] | 10801137.3101 | 5.0000 | [a] | 0.2000 | 25.000 | 5.0000 | 80 |
| ∞ | | | | 5.0000 | | 0.2000 | | | ∞ |

[a]Less than 0.0001.

## TABLE C-18 Discrete Compounding; $i = 25\%$

| | Single Payment | | Uniform Series | | | | Uniform Gradient | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Capital Recovery Factor | Gradient Present Worth Factor | Gradient Uniform Series Factor | |
| *N* | To Find *F* Given *P* *F/P* | To Find *P* Given *F* *P/F* | To Find *F* Given *A* *F/A* | To Find *P* Given *A* *P/A* | To Find *A* Given *F* *A/F* | To Find *A* Given *P* *A/P* | To Find *P* Given *G* *P/G* | To Find *A* Given *G* *A/G* | *N* |
| 1 | 1.2500 | 0.8000 | 1.0000 | 0.8000 | 1.0000 | 1.2500 | 0.000 | 0.0000 | 1 |
| 2 | 1.5625 | 0.6400 | 2.2500 | 1.4400 | 0.4444 | 0.6944 | 0.640 | 0.4444 | 2 |
| 3 | 1.9531 | 0.5120 | 3.8125 | 1.9520 | 0.2623 | 0.5123 | 1.664 | 0.8525 | 3 |
| 4 | 2.4414 | 0.4096 | 5.7656 | 2.3616 | 0.1734 | 0.4234 | 2.893 | 1.2249 | 4 |
| 5 | 3.0518 | 0.3277 | 8.2070 | 2.6893 | 0.1218 | 0.3718 | 4.204 | 1.5631 | 5 |
| 6 | 3.8147 | 0.2621 | 11.2588 | 2.9514 | 0.0888 | 0.3388 | 5.514 | 1.8683 | 6 |
| 7 | 4.7684 | 0.2097 | 15.0735 | 3.1611 | 0.0663 | 0.3163 | 6.773 | 2.1424 | 7 |
| 8 | 5.9605 | 0.1678 | 19.8419 | 3.3289 | 0.0504 | 0.3004 | 7.947 | 2.3872 | 8 |
| 9 | 7.4506 | 0.1342 | 25.8023 | 3.4631 | 0.0388 | 0.2888 | 9.021 | 2.6048 | 9 |
| 10 | 9.3132 | 0.1074 | 33.2529 | 3.5705 | 0.0301 | 0.2801 | 9.987 | 2.7971 | 10 |
| 11 | 11.6415 | 0.0859 | 42.5661 | 3.6564 | 0.0235 | 0.2735 | 10.846 | 2.9663 | 11 |
| 12 | 14.5519 | 0.0687 | 54.2077 | 3.7251 | 0.0184 | 0.2684 | 11.602 | 3.1145 | 12 |
| 13 | 18.1899 | 0.0550 | 68.7596 | 3.7801 | 0.0145 | 0.2645 | 12.262 | 3.2437 | 13 |
| 14 | 22.7374 | 0.0440 | 86.9495 | 3.8241 | 0.0115 | 0.2615 | 12.833 | 3.3559 | 14 |
| 15 | 28.4217 | 0.0352 | 109.6868 | 3.8593 | 0.0091 | 0.2591 | 13.326 | 3.4530 | 15 |
| 16 | 35.5271 | 0.0281 | 138.1085 | 3.8874 | 0.0072 | 0.2572 | 13.748 | 3.5366 | 16 |
| 17 | 44.4089 | 0.0225 | 173.6357 | 3.9099 | 0.0058 | 0.2558 | 14.109 | 3.6084 | 17 |
| 18 | 55.5112 | 0.0180 | 218.0446 | 3.9279 | 0.0046 | 0.2546 | 14.415 | 3.6698 | 18 |
| 19 | 69.3889 | 0.0144 | 273.5558 | 3.9424 | 0.0037 | 0.2537 | 14.674 | 3.7222 | 19 |
| 20 | 86.7362 | 0.0115 | 342.9447 | 3.9539 | 0.0029 | 0.2529 | 14.893 | 3.7667 | 20 |
| 21 | 108.4202 | 0.0092 | 429.6809 | 3.9631 | 0.0023 | 0.2523 | 15.078 | 3.8045 | 21 |
| 22 | 135.5253 | 0.0074 | 538.1011 | 3.9705 | 0.0019 | 0.2519 | 15.233 | 3.8365 | 22 |
| 23 | 169.4066 | 0.0059 | 673.6264 | 3.9764 | 0.0015 | 0.2515 | 15.363 | 3.8634 | 23 |
| 24 | 211.7582 | 0.0047 | 843.0329 | 3.9811 | 0.0012 | 0.2512 | 15.471 | 3.8861 | 24 |
| 25 | 264.6978 | 0.0038 | 1054.7912 | 3.9849 | 0.0009 | 0.2509 | 15.562 | 3.9052 | 25 |
| 30 | 807.7936 | 0.0012 | 3227.1743 | 3.9950 | 0.0003 | 0.2503 | 15.832 | 3.9628 | 30 |
| 35 | 2465.1903 | 0.0004 | 9856.7613 | 3.9984 | 0.0001 | 0.2501 | 15.937 | 3.9858 | 35 |
| 40 | 7523.1638 | 0.0001 | 30088.6554 | 3.9995 | [a] | 0.2500 | 15.977 | 3.9947 | 40 |
| 45 | 22958.8740 | [a] | 91831.4962 | 3.9998 | [a] | 0.2500 | 15.992 | 3.9980 | 45 |
| 50 | 70064.9232 | [a] | 280255.6929 | 3.9999 | [a] | 0.2500 | 15.997 | 3.9993 | 50 |
| 60 | 652530.4468 | [a] | 2610117.7872 | 4.0000 | [a] | 0.2500 | 16.000 | 3.9999 | 60 |
| ∞ | | | | 4.0000 | | 0.2500 | | | ∞ |

[a] Less than 0.0001.

# APPENDIX D

# Interest and Annuity Tables for Continuous Compounding

For various values of $\underline{r}$ from 8% to 20%,

$\underline{r}$ = nominal interest rate per period, compounded continuously;
$N$ = number of compounding periods;

$$(F/P, \underline{r}\%, N) = e^{rN};$$

$$(P/F, \underline{r}\%, N) = e^{-rN} = \frac{1}{e^{rN}};$$

$$(F/A, \underline{r}\%, N) = \frac{e^{rN} - 1}{e^{r} - 1};$$

$$(P/A, \underline{r}\%, N) = \frac{e^{rN} - 1}{e^{rN}\left(e^{r} - 1\right)}.$$